Computer algebra independent integration tests

4-Trig-functions/4.3-Tangent/4.3.4.2-a+b-tan-^m-c+d-tan-^n-A+B-tan+C-tan^2-

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [171]. This is test number [105].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (171)	% 0. (0)
Mathematica	% 98.83 (169)	% 1.17 (2)
Maple	% 71.35 (122)	% 28.65 (49)
Maxima	% 49.12 (84)	% 50.88 (87)
Fricas	% 49.12 (84)	% 50.88 (87)
Sympy	% 25.73 (44)	% 74.27 (127)
Giac	% 45.61 (78)	% 54.39 (93)

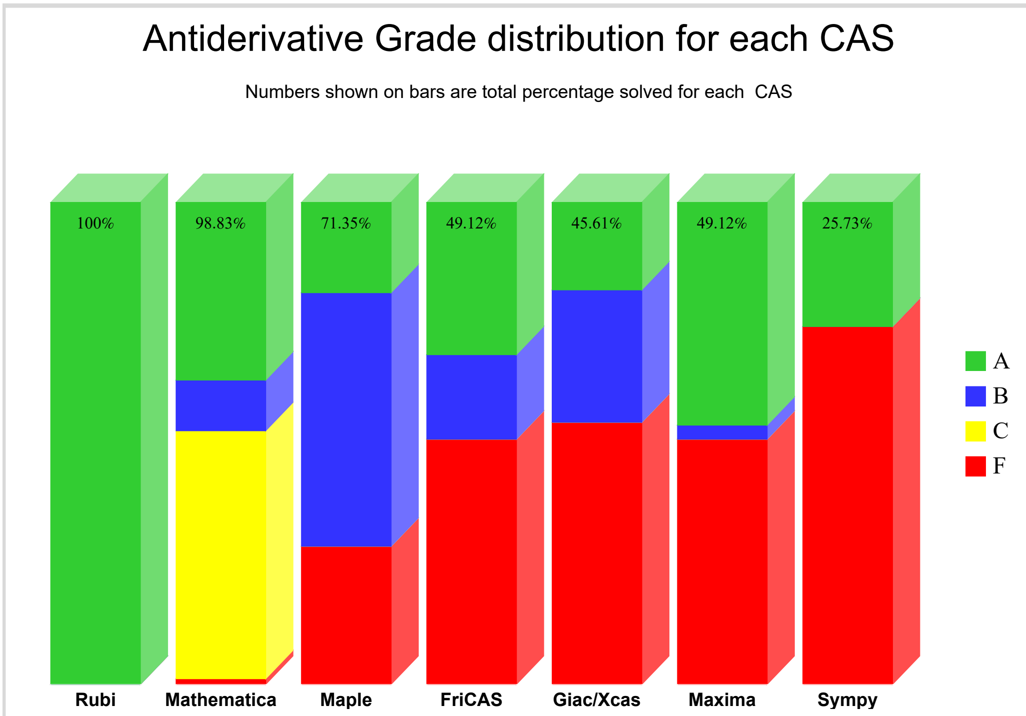
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

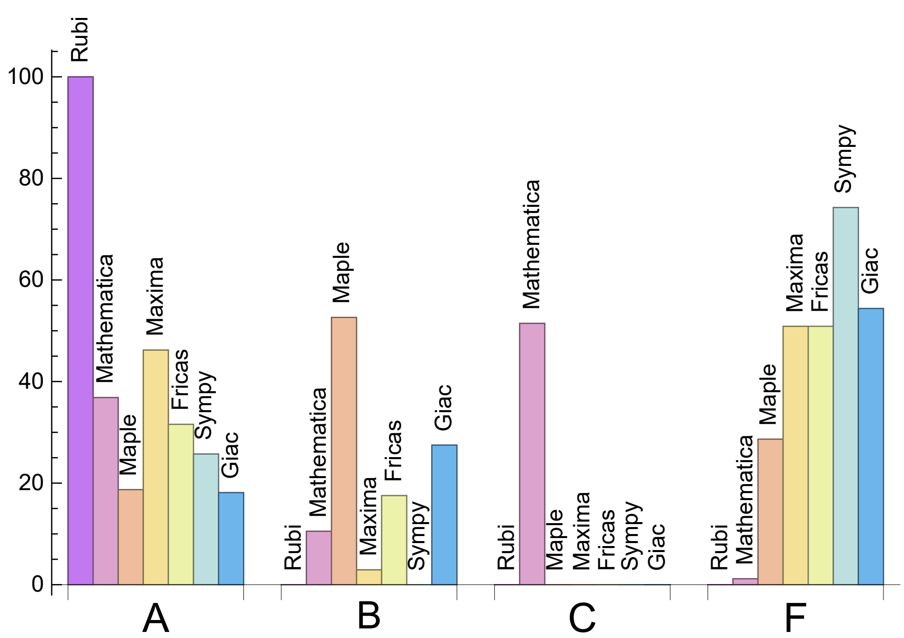
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	36.84	10.53	51.46	1.17
Maple	18.71	52.63	0.	28.65
Maxima	46.2	2.92	0.	50.88
Fricas	31.58	17.54	0.	50.88
Sympy	25.73	0.	0.	74.27
Giac	18.13	27.49	0.	54.39

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	1.99	323.47	1.	287.	1.
Mathematica	7.03	110174.	220.18	322.	1.39
Maple	0.11	6746.27	18.94	994.	3.4
Maxima	1.67	507.25	1.82	293.5	1.62
Fricas	5.76	1693.17	4.81	614.	3.53
Sympy	24.1	1041.09	5.12	312.	2.19
Giac	2.36	1273.74	5.72	570.5	2.99

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {69, 83, 84, 89, 128, 129, 132, 135, 138, 139, 141, 143, 144, 145, 146, 153, 154, 155, 159, 160}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

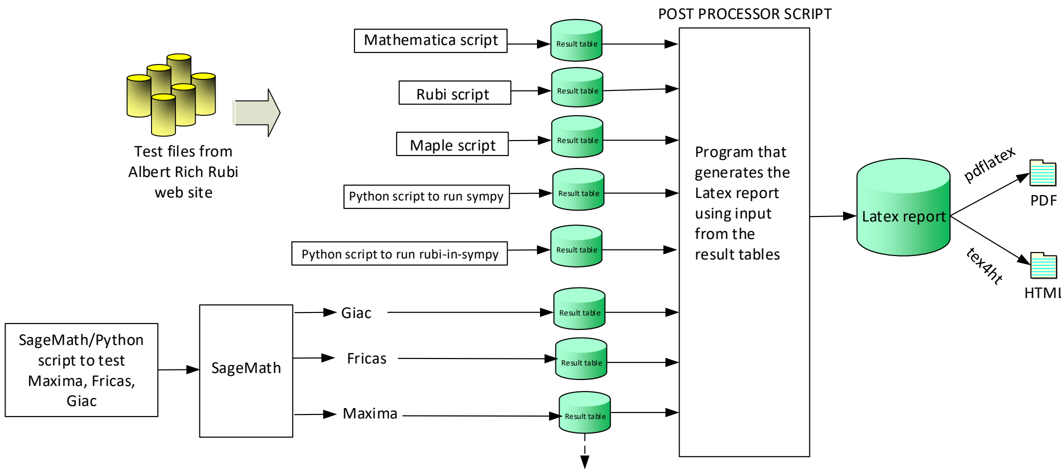
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 28, 45, 46, 47, 48, 53, 69, 74, 75, 76, 82, 84, 88, 91, 92, 93, 94, 98, 99, 100, 101, 104, 105, 106, 107, 111, 112, 113, 114, 115, 120, 128, 129, 130, 131, 133, 134, 135, 136, 137, 141, 142, 147, 148, 149, 150, 151, 152, 156, 157, 158, 161, 162, 163, 166, 167, 168, 169, 170 }

B grade: { 81, 83, 89, 90, 95, 96, 97, 102, 103, 108, 109, 110, 121, 126, 127, 140, 165, 171 }

C grade: { 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71, 72, 73, 77, 78, 79, 80, 85, 86, 87, 116, 117, 118, 119, 122, 123, 124, 125, 132, 138, 139, 143, 144, 145, 146, 153, 154, 155, 159, 160 }

F grade: { 49, 164}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 31, 32, 33, 38, 53 }

B grade: { 28, 29, 30, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127 }

C grade: { }

F grade: { 45, 46, 47, 48, 49, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 84, 85, 86, 87

B grade: { 76, 82, 83, 88, 89 }

C grade: { }

F grade: { 45, 46, 47, 48, 49, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 34, 35, 50, 51, 52, 53, 54, 57, 58, 59, 60, 61, 64, 65, 66, 67, 70, 71, 72, 73, 74, 79, 80 }

B grade: { 32, 33, 36, 37, 38, 39, 40, 41, 42, 43, 44, 55, 56, 62, 63, 68, 69, 75, 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 88, 89 }

C grade: { }

F grade: { 45, 46, 47, 48, 49, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 28, 50, 51, 52, 53, 54, 57, 58, 59, 60, 61, 64, 65, 66, 67, 70, 71, 72, 73 }

B grade: { }

C grade: { }

F grade: { 23, 24, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 55, 56, 62, 63, 68, 69, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171 }

2.1.7 Giac

A grade: { 3, 4, 11, 12, 13, 18, 19, 20, 21, 25, 26, 27, 28, 29, 30, 31, 32, 33, 37, 38, 39, 44, 54, 61, 67, 70, 71, 72, 73, 74, 79 }

B grade: { 1, 2, 5, 6, 7, 8, 9, 10, 14, 15, 16, 17, 22, 23, 24, 34, 35, 36, 40, 41, 42, 43, 51, 52, 53, 55, 56, 59, 60, 62, 63, 66, 68, 69, 75, 76, 77, 78, 80, 81, 83, 84, 85, 86, 87, 88, 89 }

C grade: { }

F grade: { 45, 46, 47, 48, 49, 50, 57, 58, 64, 65, 82, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	86	135	116	208	139	1373
normalized size	1	1.	0.99	1.55	1.33	2.39	1.6	15.78
time (sec)	N/A	0.134	0.55	0.013	1.61	1.372	1.525	2.509

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	67	105	89	161	105	832
normalized size	1	1.	1.02	1.59	1.35	2.44	1.59	12.61
time (sec)	N/A	0.046	0.293	0.013	1.765	1.439	0.588	1.923

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	59	66	68	122	82	68
normalized size	1	1.	1.4	1.57	1.62	2.9	1.95	1.62
time (sec)	N/A	0.06	0.054	0.061	1.752	1.351	2.891	1.421

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	44	51	70	146	85	72
normalized size	1	1.	1.19	1.38	1.89	3.95	2.3	1.95
time (sec)	N/A	0.11	0.069	0.068	1.668	1.415	4.868	1.494

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	78	65	92	178	116	161
normalized size	1	1.	1.81	1.51	2.14	4.14	2.7	3.74
time (sec)	N/A	0.124	0.162	0.059	1.734	1.338	15.03	1.538

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	77	96	116	234	143	242
normalized size	1	1.	1.17	1.45	1.76	3.55	2.17	3.67
time (sec)	N/A	0.159	0.435	0.076	1.708	1.392	14.001	1.559

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	101	124	140	292	180	320
normalized size	1	1.	1.16	1.43	1.61	3.36	2.07	3.68
time (sec)	N/A	0.193	0.996	0.076	1.639	1.267	28.298	1.621

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	100	150	165	340	211	404
normalized size	1	1.	0.93	1.39	1.53	3.15	1.95	3.74
time (sec)	N/A	0.226	1.141	0.073	1.679	1.386	88.916	1.587

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	221	249	198	340	250	3008
normalized size	1	1.	1.49	1.68	1.34	2.3	1.69	20.32
time (sec)	N/A	0.302	6.223	0.013	1.713	1.398	1.939	4.907

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	172	199	162	275	194	2037
normalized size	1	1.	1.54	1.78	1.45	2.46	1.73	18.19
time (sec)	N/A	0.112	1.789	0.012	1.728	1.32	1.584	3.218

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	96	140	123	209	151	128
normalized size	1	1.	1.1	1.61	1.41	2.4	1.74	1.47
time (sec)	N/A	0.135	0.45	0.079	1.749	1.368	5.89	1.744

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	91	109	115	217	136	116
normalized size	1	1.	1.3	1.56	1.64	3.1	1.94	1.66
time (sec)	N/A	0.185	0.268	0.076	1.748	1.421	11.254	1.876

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	100	110	126	274	158	159
normalized size	1	1.	1.39	1.53	1.75	3.81	2.19	2.21
time (sec)	N/A	0.207	0.247	0.076	1.756	1.404	18.927	1.836

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	123	141	162	285	206	320
normalized size	1	1.	1.4	1.6	1.84	3.24	2.34	3.64
time (sec)	N/A	0.263	0.342	0.095	1.694	1.433	30.604	2.05

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	152	188	201	367	258	451
normalized size	1	1.	1.29	1.59	1.7	3.11	2.19	3.82
time (sec)	N/A	0.311	1.146	0.093	1.688	1.506	47.642	2.085

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	180	238	236	446	311	587
normalized size	1	1.	1.19	1.58	1.56	2.95	2.06	3.89
time (sec)	N/A	0.369	3.021	0.098	1.76	1.428	83.336	2.138

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	209	314	242	408	313	3875
normalized size	1	1.	1.27	1.9	1.47	2.47	1.9	23.48
time (sec)	N/A	0.177	1.567	0.014	1.754	1.605	2.784	6.493

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	130	234	193	324	248	213
normalized size	1	1.	0.93	1.67	1.38	2.31	1.77	1.52
time (sec)	N/A	0.208	1.058	0.089	1.769	1.641	10.421	2.371

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	113	183	167	305	211	174
normalized size	1	1.	0.97	1.56	1.43	2.61	1.8	1.49
time (sec)	N/A	0.336	0.455	0.094	1.792	1.838	26.694	2.518

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	113	168	169	347	214	205
normalized size	1	1.	0.95	1.41	1.42	2.92	1.8	1.72
time (sec)	N/A	0.331	0.469	0.082	1.763	1.752	35.377	2.515

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	126	186	192	383	253	261
normalized size	1	1.	0.99	1.46	1.51	3.02	1.99	2.06
time (sec)	N/A	0.355	0.448	0.137	1.784	1.763	78.706	2.513

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	164	233	243	419	330	527
normalized size	1	1.	1.06	1.51	1.58	2.72	2.14	3.42
time (sec)	N/A	0.427	1.273	0.087	1.697	1.097	142.243	2.659

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	199	302	290	518	0	713
normalized size	1	1.	1.04	1.58	1.52	2.71	0.	3.73
time (sec)	N/A	0.514	0.746	0.101	1.72	1.145	0.	2.838

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	237	376	338	620	0	905
normalized size	1	1.	1.02	1.61	1.45	2.66	0.	3.88
time (sec)	N/A	0.558	1.168	0.115	1.634	1.126	0.	3.013

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	138	211	176	412	1306	182
normalized size	1	1.	1.09	1.66	1.39	3.24	10.28	1.43
time (sec)	N/A	0.468	1.366	0.037	1.759	1.264	21.148	1.808

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	118	179	147	333	1020	149
normalized size	1	1.	1.17	1.77	1.46	3.3	10.1	1.48
time (sec)	N/A	0.243	0.571	0.034	1.775	1.206	16.672	1.551

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	98	159	127	251	711	128
normalized size	1	1.	1.15	1.87	1.49	2.95	8.36	1.51
time (sec)	N/A	0.163	0.17	0.033	2.412	1.177	7.054	1.616

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	67	153	119	174	541	127
normalized size	1	1.	1.16	2.64	2.05	3.	9.33	2.19
time (sec)	N/A	0.144	0.115	0.109	1.738	1.086	89.498	1.553

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	113	174	144	267	0	153
normalized size	1	1.	1.41	2.17	1.8	3.34	0.	1.91
time (sec)	N/A	0.201	0.312	0.13	1.756	1.264	0.	1.663

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	138	214	177	404	0	212
normalized size	1	1.	1.34	2.08	1.72	3.92	0.	2.06
time (sec)	N/A	0.342	0.833	0.119	1.787	1.237	0.	1.693

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	163	266	213	518	0	289
normalized size	1	1.	1.19	1.94	1.55	3.78	0.	2.11
time (sec)	N/A	0.682	1.375	0.13	1.592	1.33	0.	1.77

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	444	364	297	936	0	392
normalized size	1	1.	2.13	1.75	1.43	4.5	0.	1.88
time (sec)	N/A	0.532	4.039	0.043	1.717	1.462	0.	1.889

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	324	313	266	682	0	329
normalized size	1	1.	2.06	1.99	1.69	4.34	0.	2.1
time (sec)	N/A	0.311	2.055	0.047	1.678	1.306	0.	1.634

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	140	305	250	490	0	325
normalized size	1	1.	1.22	2.65	2.17	4.26	0.	2.83
time (sec)	N/A	0.147	2.027	0.042	1.726	1.131	0.	1.659

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	190	301	239	489	0	316
normalized size	1	1.	1.71	2.71	2.15	4.41	0.	2.85
time (sec)	N/A	0.208	1.963	0.135	1.813	1.141	0.	1.631

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	159	325	281	701	0	377
normalized size	1	1.	1.16	2.37	2.05	5.12	0.	2.75
time (sec)	N/A	0.403	2.361	0.148	1.636	1.352	0.	1.867

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	193	399	354	1017	0	489
normalized size	1	1.	1.01	2.08	1.84	5.3	0.	2.55
time (sec)	N/A	0.608	3.434	0.137	1.716	1.531	0.	1.813

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	1146	619	525	1895	0	682
normalized size	1	1.	3.46	1.87	1.59	5.73	0.	2.06
time (sec)	N/A	0.861	6.678	0.048	1.754	1.853	0.	2.075

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	462	566	494	1432	0	618
normalized size	1	1.	1.85	2.26	1.98	5.73	0.	2.47
time (sec)	N/A	0.581	4.533	0.054	1.892	1.606	0.	1.932

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	288	495	450	1038	0	554
normalized size	1	1.	1.52	2.62	2.38	5.49	0.	2.93
time (sec)	N/A	0.427	4.647	0.051	1.817	1.176	0.	1.625

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	188	488	446	1058	0	554
normalized size	1	1.	1.05	2.73	2.49	5.91	0.	3.09
time (sec)	N/A	0.255	3.746	0.047	1.875	1.154	0.	1.533

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	243	483	433	1038	0	552
normalized size	1	1.	1.39	2.76	2.47	5.93	0.	3.15
time (sec)	N/A	0.316	3.807	0.154	1.621	1.191	0.	1.503

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	223	540	502	1451	0	647
normalized size	1	1.	1.04	2.51	2.33	6.75	0.	3.01
time (sec)	N/A	0.68	2.878	0.179	1.839	1.614	0.	1.583

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	288	651	613	1982	0	756
normalized size	1	1.	1.	2.27	2.14	6.91	0.	2.63
time (sec)	N/A	0.941	6.402	0.174	1.804	1.823	0.	1.637

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	110	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.155	0.402	0.326	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	115	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.138	0.376	180.	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	133	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.144	0.513	0.435	0.	0.	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	133	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.137	0.504	0.411	0.	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	328	328	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.561	27.173	0.6	0.	0.	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	300	994	562	915	1001	0
normalized size	1	1.	0.85	2.82	1.59	2.59	2.84	0.
time (sec)	N/A	0.785	6.365	0.018	1.507	1.223	5.487	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	243	631	370	608	617	8778
normalized size	1	1.	0.98	2.54	1.49	2.45	2.49	35.4
time (sec)	N/A	0.451	3.175	0.017	1.491	1.21	1.889	8.418

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	161	334	204	348	326	3939
normalized size	1	1.	1.	2.07	1.27	2.16	2.02	24.47
time (sec)	N/A	0.241	1.539	0.014	1.456	1.155	0.834	3.872

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	76	136	100	177	131	1239
normalized size	1	1.	1.04	1.86	1.37	2.42	1.79	16.97
time (sec)	N/A	0.061	0.446	0.014	1.478	1.068	0.705	1.818

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	155	148	506	247	483	2387	251
normalized size	1	0.99	0.95	3.24	1.58	3.1	15.3	1.61
time (sec)	N/A	0.349	1.111	0.041	1.466	1.966	23.099	1.401

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	589	948	456	1160	0	717
normalized size	1	1.	2.22	3.58	1.72	4.38	0.	2.71
time (sec)	N/A	0.474	6.484	0.056	1.489	2.324	0.	1.397

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	331	1513	775	2072	0	1400
normalized size	1	1.	1.03	4.73	2.42	6.48	0.	4.38
time (sec)	N/A	0.703	6.228	0.068	1.523	1.327	0.	1.496

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	661	661	573	1807	933	1490	1819	0
normalized size	1	1.	0.87	2.73	1.41	2.25	2.75	0.
time (sec)	N/A	2.384	6.648	0.024	1.488	1.282	6.121	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	443	443	383	1165	625	1007	1134	0
normalized size	1	1.	0.86	2.63	1.41	2.27	2.56	0.
time (sec)	N/A	1.278	6.501	0.017	1.481	1.22	4.66	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	264	241	631	351	583	617	8778
normalized size	1	0.99	0.91	2.37	1.32	2.19	2.32	33.
time (sec)	N/A	0.472	2.647	0.016	1.474	1.128	3.395	9.247

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	176	262	182	308	241	2873
normalized size	1	1.	1.34	2.	1.39	2.35	1.84	21.93
time (sec)	N/A	0.155	1.126	0.015	1.443	1.052	1.332	3.585

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	252	190	861	392	822	4444	456
normalized size	1	0.99	0.75	3.39	1.54	3.24	17.5	1.8
time (sec)	N/A	0.827	3.035	0.048	1.521	2.917	36.782	1.739

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	415	415	2640	1554	670	1987	0	1231
normalized size	1	1.	6.36	3.74	1.61	4.79	0.	2.97
time (sec)	N/A	1.053	7.783	0.061	1.529	3.562	0.	1.871

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	597	597	2499	2465	1133	3513	0	2314
normalized size	1	1.	4.19	4.13	1.9	5.88	0.	3.88
time (sec)	N/A	1.29	7.905	0.077	1.596	4.656	0.	1.87

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	603	603	419	1807	918	1461	1819	0
normalized size	1	1.	0.69	3.	1.52	2.42	3.02	0.
time (sec)	N/A	1.533	6.583	0.021	1.498	1.312	7.86	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	389	387	297	994	522	861	1001	0
normalized size	1	0.99	0.76	2.56	1.34	2.21	2.57	0.
time (sec)	N/A	0.705	6.342	0.017	1.56	1.236	5.768	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	212	420	273	456	410	5805
normalized size	1	1.	1.11	2.2	1.43	2.39	2.15	30.39
time (sec)	N/A	0.244	2.41	0.014	1.494	1.139	2.493	7.635

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	363	255	1304	589	1269	7096	774
normalized size	1	1.	0.7	3.59	1.62	3.5	19.55	2.13
time (sec)	N/A	1.512	4.738	0.051	1.534	5.578	46.978	2.252

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	574	574	2467	2250	925	3131	0	1832
normalized size	1	1.	4.3	3.92	1.61	5.45	0.	3.19
time (sec)	N/A	2.321	8.361	0.07	1.643	8.359	0.	2.423

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	798	798	1451	3522	1511	5261	0	3382
normalized size	1	1.	1.82	4.41	1.89	6.59	0.	4.24
time (sec)	N/A	2.839	15.051	0.079	1.771	10.924	0.	2.488

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	258	1304	601	1315	7096	774
normalized size	1	1.	0.77	3.87	1.78	3.9	21.06	2.3
time (sec)	N/A	1.588	4.285	0.054	1.546	5.581	46.64	2.428

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	190	861	397	830	4444	456
normalized size	1	1.	0.81	3.65	1.68	3.52	18.83	1.93
time (sec)	N/A	0.804	2.898	0.046	1.463	2.733	40.737	2.065

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	148	506	240	470	2387	251
normalized size	1	1.	0.95	3.24	1.54	3.01	15.3	1.61
time (sec)	N/A	0.342	1.055	0.05	1.485	1.591	23.743	1.683

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	117	234	143	269	966	147
normalized size	1	1.	1.18	2.36	1.44	2.72	9.76	1.48
time (sec)	N/A	0.098	0.213	0.037	1.456	1.222	14.003	1.621

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	164	313	647	328	633	0	367
normalized size	1	0.99	1.9	3.92	1.99	3.84	0.	2.22
time (sec)	N/A	0.256	1.525	0.076	1.491	2.491	0.	1.721

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	543	1262	702	2738	0	1142
normalized size	1	1.	1.93	4.49	2.5	9.74	0.	4.06
time (sec)	N/A	0.795	6.913	0.094	1.575	7.993	0.	1.742

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	477	477	898	2298	1480	7380	0	2871
normalized size	1	1.	1.88	4.82	3.1	15.47	0.	6.02
time (sec)	N/A	1.786	8.879	0.111	1.804	26.386	0.	1.92

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	579	579	2463	2250	923	3141	0	1829
normalized size	1	1.	4.25	3.89	1.59	5.42	0.	3.16
time (sec)	N/A	2.134	8.394	0.07	1.594	8.215	0.	2.439

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	417	417	2636	1554	666	1983	0	1231
normalized size	1	1.	6.32	3.73	1.6	4.76	0.	2.95
time (sec)	N/A	1.113	7.763	0.072	1.574	3.755	0.	1.909

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	288	606	948	431	1095	0	713
normalized size	1	0.99	2.08	3.25	1.48	3.75	0.	2.44
time (sec)	N/A	0.554	6.336	0.058	1.487	1.833	0.	1.628

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	207	438	277	566	0	404
normalized size	1	1.	1.48	3.13	1.98	4.04	0.	2.89
time (sec)	N/A	0.209	2.241	0.042	1.462	1.132	0.	1.585

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	592	1263	693	2620	0	1142
normalized size	1	1.	2.02	4.31	2.37	8.94	0.	3.9
time (sec)	N/A	0.812	7.452	0.1	1.575	8.541	0.	1.81

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	509	508	984	2012	1600	8519	0	0
normalized size	1	1.	1.93	3.95	3.14	16.74	0.	0.
time (sec)	N/A	2.151	8.905	0.144	1.812	35.291	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	841	841	1758	3364	3401	19950	0	4288
normalized size	1	1.	2.09	4.	4.04	23.72	0.	5.1
time (sec)	N/A	4.076	8.346	0.143	2.211	102.058	0.	3.14

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	804	804	1445	3522	1499	5218	0	3382
normalized size	1	1.	1.8	4.38	1.86	6.49	0.	4.21
time (sec)	N/A	2.747	14.923	0.085	1.742	13.21	0.	2.431

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	597	597	2499	2465	1116	3366	0	2307
normalized size	1	1.	4.19	4.13	1.87	5.64	0.	3.86
time (sec)	N/A	1.385	7.863	0.085	1.636	5.451	0.	1.969

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	352	349	331	1513	733	1905	0	1400
normalized size	1	0.99	0.94	4.3	2.08	5.41	0.	3.98
time (sec)	N/A	0.711	6.026	0.07	1.533	1.639	0.	1.822

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	261	713	495	1215	0	740
normalized size	1	1.	1.25	3.41	2.37	5.81	0.	3.54
time (sec)	N/A	0.376	4.632	0.055	1.515	1.356	0.	1.966

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	487	487	912	2298	1455	7109	0	2869
normalized size	1	1.	1.87	4.72	2.99	14.6	0.	5.89
time (sec)	N/A	1.83	8.882	0.117	1.84	38.508	0.	3.617

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	861	860	1732	3364	3425	20006	0	4288
normalized size	1	1.	2.01	3.91	3.98	23.24	0.	4.98
time (sec)	N/A	4.276	8.2	0.14	2.273	98.95	0.	2.953

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	464	464	1232	6661	0	0	0	0
normalized size	1	1.	2.66	14.36	0.	0.	0.	0.
time (sec)	N/A	2.089	6.426	0.227	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	314	4775	0	0	0	0
normalized size	1	1.	0.97	14.69	0.	0.	0.	0.
time (sec)	N/A	1.306	4.771	0.172	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	220	3028	0	0	0	0
normalized size	1	1.	0.98	13.52	0.	0.	0.	0.
time (sec)	N/A	0.628	1.96	0.148	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	150	1472	0	0	0	0
normalized size	1	1.	0.97	9.5	0.	0.	0.	0.
time (sec)	N/A	0.306	0.552	0.128	0.	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	233	3576	0	0	0	0
normalized size	1	1.	1.	15.28	0.	0.	0.	0.
time (sec)	N/A	1.087	0.673	0.191	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	764	5778	0	0	0	0
normalized size	1	1.	2.41	18.23	0.	0.	0.	0.
time (sec)	N/A	1.439	6.396	0.214	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	543	543	2819	9797	0	0	0	0
normalized size	1	1.	5.19	18.04	0.	0.	0.	0.
time (sec)	N/A	4.037	6.413	0.241	0.	0.	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	550	550	1290	11056	0	0	0	0
normalized size	1	1.	2.35	20.1	0.	0.	0.	0.
time (sec)	N/A	2.734	6.418	0.198	0.	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	396	396	350	8031	0	0	0	0
normalized size	1	1.	0.88	20.28	0.	0.	0.	0.
time (sec)	N/A	1.727	6.16	0.18	0.	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	260	5149	0	0	0	0
normalized size	1	1.	0.95	18.86	0.	0.	0.	0.
time (sec)	N/A	0.879	4.518	0.15	0.	0.	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	202	2517	0	0	0	0
normalized size	1	1.	1.08	13.46	0.	0.	0.	0.
time (sec)	N/A	0.46	1.234	0.113	0.	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	266	6055	0	0	0	0
normalized size	1	1.	0.98	22.34	0.	0.	0.	0.
time (sec)	N/A	1.814	2.421	0.192	0.	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	372	372	1732	9865	0	0	0	0
normalized size	1	1.	4.66	26.52	0.	0.	0.	0.
time (sec)	N/A	2.547	6.225	0.235	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	532	532	7678	14441	0	0	0	0
normalized size	1	1.	14.43	27.14	0.	0.	0.	0.
time (sec)	N/A	4.087	6.55	0.251	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	503	503	564	11478	0	0	0	0
normalized size	1	1.	1.12	22.82	0.	0.	0.	0.
time (sec)	N/A	2.312	6.436	0.192	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	351	324	7402	0	0	0	0
normalized size	1	0.99	0.92	20.97	0.	0.	0.	0.
time (sec)	N/A	1.212	4.999	0.164	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	262	3614	0	0	0	0
normalized size	1	1.	1.14	15.78	0.	0.	0.	0.
time (sec)	N/A	0.629	2.031	0.123	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	322	8698	0	0	0	0
normalized size	1	1.	0.96	25.89	0.	0.	0.	0.
time (sec)	N/A	2.812	5.328	0.214	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	473	473	6112	14119	0	0	0	0
normalized size	1	1.	12.92	29.85	0.	0.	0.	0.
time (sec)	N/A	3.896	6.55	0.258	0.	0.	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	643	643	18214	20663	0	0	0	0
normalized size	1	1.	28.33	32.14	0.	0.	0.	0.
time (sec)	N/A	6.065	6.898	0.282	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	407	1200	25426	0	0	0	0
normalized size	1	1.	2.95	62.47	0.	0.	0.	0.
time (sec)	N/A	1.699	6.456	0.211	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	275	18289	0	0	0	0
normalized size	1	1.	0.96	63.72	0.	0.	0.	0.
time (sec)	N/A	1.001	5.997	0.183	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	192	4132	0	0	0	0
normalized size	1	1.	0.99	21.3	0.	0.	0.	0.
time (sec)	N/A	0.498	1.476	0.154	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	129	5570	0	0	0	0
normalized size	1	1.	0.97	41.88	0.	0.	0.	0.
time (sec)	N/A	0.216	0.213	0.14	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	194	13474	0	0	0	0
normalized size	1	1.	0.92	64.16	0.	0.	0.	0.
time (sec)	N/A	0.615	0.373	0.193	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	327	521	20870	0	0	0	0
normalized size	1	1.	1.59	63.82	0.	0.	0.	0.
time (sec)	N/A	1.379	6.215	0.222	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	511	511	920	49725	0	0	0	0
normalized size	1	1.	1.8	97.31	0.	0.	0.	0.
time (sec)	N/A	2.465	6.774	0.244	0.	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	476	36710	0	0	0	0
normalized size	1	1.	1.39	107.03	0.	0.	0.	0.
time (sec)	N/A	1.354	6.577	0.198	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	290	23472	0	0	0	0
normalized size	1	1.	1.44	116.78	0.	0.	0.	0.
time (sec)	N/A	0.554	2.392	0.164	0.	0.	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	218	11427	0	0	0	0
normalized size	1	1.	1.39	72.78	0.	0.	0.	0.
time (sec)	N/A	0.294	0.925	0.125	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	296	26343	0	0	0	0
normalized size	1	1.	1.13	100.55	0.	0.	0.	0.
time (sec)	N/A	1.277	4.734	0.216	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	447	446	2078	40619	0	0	0	0
normalized size	1	1.	4.65	90.87	0.	0.	0.	0.
time (sec)	N/A	2.881	6.252	0.263	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	585	585	670	85156	0	0	0	0
normalized size	1	1.	1.15	145.57	0.	0.	0.	0.
time (sec)	N/A	2.968	6.832	0.284	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	358	358	502	61833	0	0	0	0
normalized size	1	1.	1.4	172.72	0.	0.	0.	0.
time (sec)	N/A	1.551	6.477	0.247	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	271	300	40201	0	0	0	0
normalized size	1	0.99	1.1	147.26	0.	0.	0.	0.
time (sec)	N/A	0.798	2.742	0.21	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	223	20647	0	0	0	0
normalized size	1	1.	1.07	98.79	0.	0.	0.	0.
time (sec)	N/A	0.486	0.853	0.149	0.	0.	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	365	1948	45119	0	0	0	0
normalized size	1	1.	5.34	123.61	0.	0.	0.	0.
time (sec)	N/A	2.466	6.264	0.25	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	679	678	6052	67570	0	0	0	0
normalized size	1	1.	8.91	99.51	0.	0.	0.	0.
time (sec)	N/A	5.062	6.418	0.327	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	679	679	1202	0	0	0	0	0
normalized size	1	1.	1.77	0.	0.	0.	0.	0.
time (sec)	N/A	9.926	9.73	180.	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	505	505	835	0	0	0	0	0
normalized size	1	1.	1.65	0.	0.	0.	0.	0.
time (sec)	N/A	7.338	8.815	180.	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	383	619	0	0	0	0	0
normalized size	1	1.01	1.62	0.	0.	0.	0.	0.
time (sec)	N/A	4.973	7.708	180.	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	441	0	0	0	0	0
normalized size	1	1.	1.54	0.	0.	0.	0.	0.
time (sec)	N/A	2.633	4.055	180.	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	F(-1)	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	300	300	621058	0	0	0	0	0
normalized size	1	1.	2070.19	0.	0.	0.	0.	0.
time (sec)	N/A	3.796	35.725	180.	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	370	370	603	0	0	0	0	0
normalized size	1	1.	1.63	0.	0.	0.	0.	0.
time (sec)	N/A	2.052	6.959	180.	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	597	597	1108	0	0	0	0	0
normalized size	1	1.	1.86	0.	0.	0.	0.	0.
time (sec)	N/A	3.589	7.154	180.	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	682	682	1316	0	0	0	0	0
normalized size	1	1.	1.93	0.	0.	0.	0.	0.
time (sec)	N/A	11.896	8.172	180.	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	508	508	867	0	0	0	0	0
normalized size	1	1.	1.71	0.	0.	0.	0.	0.
time (sec)	N/A	7.488	8.795	180.	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	384	384	613	0	0	0	0	0
normalized size	1	1.	1.6	0.	0.	0.	0.	0.
time (sec)	N/A	4.311	7.57	180.	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	F(-1)	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	382	382	1073629	0	0	0	0	0
normalized size	1	1.	2810.55	0.	0.	0.	0.	0.
time (sec)	N/A	5.739	39.48	180.	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	402	402	1347065	0	0	0	0	0
normalized size	1	1.	3350.91	0.	0.	0.	0.	0.
time (sec)	N/A	7.127	40.594	180.	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	586	586	3134	0	0	0	0	0
normalized size	1	1.	5.35	0.	0.	0.	0.	0.
time (sec)	N/A	3.668	9.006	180.	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	697	697	1261	0	0	0	0	0
normalized size	1	1.	1.81	0.	0.	0.	0.	0.
time (sec)	N/A	10.416	9.266	180.	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	505	505	780	0	0	0	0	0
normalized size	1	1.	1.54	0.	0.	0.	0.	0.
time (sec)	N/A	6.23	8.556	180.	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	535	535	1654245	0	0	0	0	0
normalized size	1	1.	3092.05	0.	0.	0.	0.	0.
time (sec)	N/A	8.314	44.009	180.	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	545	545	2018669	0	0	0	0	0
normalized size	1	1.	3703.98	0.	0.	0.	0.	0.
time (sec)	N/A	11.066	46.464	180.	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	590	590	2345519	0	0	0	0	0
normalized size	1	1.	3975.46	0.	0.	0.	0.	0.
time (sec)	N/A	14.02	48.323	180.	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	946	946	2719441	0	0	0	0	0
normalized size	1	1.	2874.67	0.	0.	0.	0.	0.
time (sec)	N/A	6.464	52.887	180.	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	505	505	785	0	0	0	0	0
normalized size	1	1.	1.55	0.	0.	0.	0.	0.
time (sec)	N/A	5.953	8.342	180.	0.	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	383	383	607	0	0	0	0	0
normalized size	1	1.	1.58	0.	0.	0.	0.	0.
time (sec)	N/A	4.077	7.453	180.	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	456	0	0	0	0	0
normalized size	1	1.	1.57	0.	0.	0.	0.	0.
time (sec)	N/A	2.555	6.857	180.	0.	0.	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	362	0	0	0	0	0
normalized size	1	1.	1.51	0.	0.	0.	0.	0.
time (sec)	N/A	1.457	2.315	180.	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	264	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.969	2.482	180.	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	375	388	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	1.765	6.006	180.	0.	0.	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	528	528	1653959	0	0	0	0	0
normalized size	1	1.	3132.5	0.	0.	0.	0.	0.
time (sec)	N/A	8.188	44.227	180.	0.	0.	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	F(-1)	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	380	380	1073499	0	0	0	0	0
normalized size	1	1.	2825.	0.	0.	0.	0.	0.
time (sec)	N/A	5.627	39.369	180.	0.	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	F(-1)	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	299	299	621084	0	0	0	0	0
normalized size	1	1.	2077.2	0.	0.	0.	0.	0.
time (sec)	N/A	3.333	35.438	180.	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	275	0	0	0	0	0
normalized size	1	1.	1.1	0.	0.	0.	0.	0.
time (sec)	N/A	1.001	3.116	180.	0.	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	383	382	484	0	0	0	0	0
normalized size	1	1.	1.26	0.	0.	0.	0.	0.
time (sec)	N/A	1.878	6.671	180.	0.	0.	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	598	598	902	0	0	0	0	0
normalized size	1	1.	1.51	0.	0.	0.	0.	0.
time (sec)	N/A	3.435	6.849	180.	0.	0.	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	549	549	2018643	0	0	0	0	0
normalized size	1	1.	3676.95	0.	0.	0.	0.	0.
time (sec)	N/A	10.5	46.635	180.	0.	0.	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	407	407	1347117	0	0	0	0	0
normalized size	1	1.	3309.87	0.	0.	0.	0.	0.
time (sec)	N/A	7.163	40.789	180.	0.	0.	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	373	373	609	0	0	0	0	0
normalized size	1	1.	1.63	0.	0.	0.	0.	0.
time (sec)	N/A	1.922	6.91	180.	0.	0.	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	379	379	403	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	1.814	5.365	180.	0.	0.	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	651	650	903	0	0	0	0	0
normalized size	1	1.	1.39	0.	0.	0.	0.	0.
time (sec)	N/A	3.431	6.875	180.	0.	0.	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	376	376	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.9	22.737	0.649	0.	0.	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	560	551	1390	0	0	0	0	0
normalized size	1	0.98	2.48	0.	0.	0.	0.	0.
time (sec)	N/A	2.377	6.387	0.839	0.	0.	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	360	505	0	0	0	0	0
normalized size	1	0.99	1.39	0.	0.	0.	0.	0.
time (sec)	N/A	1.152	6.332	0.591	0.	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	202	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.528	2.711	0.492	0.	0.	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	135	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.184	0.209	0.39	0.	0.	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	204	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.482	0.998	0.543	0.	0.	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	402	563	0	0	0	0	0
normalized size	1	1.	1.4	0.	0.	0.	0.	0.
time (sec)	N/A	1.215	6.184	0.641	0.	0.	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	702	702	2238	0	0	0	0	0
normalized size	1	1.	3.19	0.	0.	0.	0.	0.
time (sec)	N/A	2.938	6.235	0.805	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [38] had the largest ratio of [0.2]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	5	1.	36	0.139
2	A	3	3	1.	30	0.1
3	A	3	3	1.	36	0.083
4	A	5	4	1.	38	0.105
5	A	4	4	1.	38	0.105
6	A	5	5	1.	38	0.132
7	A	6	5	1.	38	0.132
8	A	7	5	1.	38	0.132
9	A	6	6	1.	38	0.158
10	A	4	4	1.	32	0.125
11	A	4	4	1.	38	0.105
12	A	5	4	1.	40	0.1
13	A	5	4	1.	40	0.1
14	A	5	5	1.	40	0.125
15	A	6	6	1.	40	0.15
16	A	7	6	1.	40	0.15
17	A	5	4	1.	32	0.125
18	A	5	4	1.	38	0.105
19	A	6	5	1.	40	0.125
20	A	6	5	1.	40	0.125
21	A	6	5	1.	40	0.125
22	A	6	6	1.	40	0.15
23	A	7	7	1.	40	0.175
24	A	8	7	1.	40	0.175
25	A	7	7	1.	40	0.175
26	A	6	6	1.	38	0.158
27	A	6	4	1.	32	0.125
28	A	3	3	1.	38	0.079
29	A	4	4	1.	40	0.1
30	A	5	5	1.	40	0.125
31	A	6	6	1.	40	0.15
32	A	7	7	1.	40	0.175
33	A	6	6	1.	38	0.158
34	A	3	3	1.	32	0.094
35	A	4	4	1.	38	0.105
36	A	5	5	1.	40	0.125
37	A	6	6	1.	40	0.15
38	A	8	8	1.	40	0.2
39	A	7	7	1.	40	0.175
40	A	5	5	1.	38	0.132
41	A	4	4	1.	32	0.125
42	A	5	4	1.	38	0.105
43	A	6	6	1.	40	0.15

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
44	A	7	6	1.	40	0.15
45	A	7	5	1.	39	0.128
46	A	7	5	1.	39	0.128
47	A	7	5	1.	41	0.122
48	A	7	5	1.	41	0.122
49	A	13	7	1.	43	0.163
50	A	6	5	1.	43	0.116
51	A	5	5	1.	43	0.116
52	A	4	4	1.	41	0.098
53	A	3	3	1.	31	0.097
54	A	5	5	0.99	43	0.116
55	A	5	5	1.	43	0.116
56	A	4	4	1.	43	0.093
57	A	7	6	1.	45	0.133
58	A	6	6	1.	45	0.133
59	A	5	5	0.99	43	0.116
60	A	4	4	1.	33	0.121
61	A	6	6	0.99	45	0.133
62	A	6	6	1.	45	0.133
63	A	6	6	1.	45	0.133
64	A	7	6	1.	45	0.133
65	A	6	5	0.99	43	0.116
66	A	5	4	1.	33	0.121
67	A	7	6	1.	45	0.133
68	A	7	7	1.	45	0.156
69	A	7	6	1.	45	0.133
70	A	7	6	1.	45	0.133
71	A	6	6	1.	45	0.133
72	A	5	5	1.	43	0.116
73	A	4	4	1.	33	0.121
74	A	3	2	0.99	45	0.044
75	A	4	3	1.	45	0.067
76	A	5	3	1.	45	0.067
77	A	7	7	1.	45	0.156
78	A	6	6	1.	45	0.133
79	A	5	5	0.99	43	0.116
80	A	3	3	1.	33	0.091
81	A	4	3	1.	45	0.067
82	A	5	3	1.	45	0.067
83	A	6	3	1.	45	0.067
84	A	7	6	1.	45	0.133
85	A	6	6	1.	45	0.133
86	A	4	4	0.99	43	0.093
87	A	4	4	1.	33	0.121

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	5	3	1.	45	0.067
89	A	6	3	1.	45	0.067
90	A	12	8	1.	47	0.17
91	A	11	8	1.	47	0.17
92	A	10	7	1.	45	0.156
93	A	9	6	1.	35	0.171
94	A	12	7	1.	47	0.149
95	A	12	7	1.	47	0.149
96	A	13	8	1.	47	0.17
97	A	13	8	1.	47	0.17
98	A	12	8	1.	47	0.17
99	A	11	7	1.	45	0.156
100	A	10	6	1.	35	0.171
101	A	13	7	1.	47	0.149
102	A	13	8	1.	47	0.17
103	A	13	7	1.	47	0.149
104	A	13	8	1.	47	0.17
105	A	12	7	0.99	45	0.156
106	A	11	6	1.	35	0.171
107	A	14	7	1.	47	0.149
108	A	14	8	1.	47	0.17
109	A	14	8	1.	47	0.17
110	A	11	7	1.	47	0.149
111	A	10	7	1.	47	0.149
112	A	9	6	1.	45	0.133
113	A	8	5	1.	35	0.143
114	A	11	6	1.	47	0.128
115	A	12	7	1.	47	0.149
116	A	11	8	1.	47	0.17
117	A	10	7	1.	47	0.149
118	A	9	6	1.	45	0.133
119	A	8	5	1.	35	0.143
120	A	12	7	1.	47	0.149
121	A	13	7	1.	47	0.149
122	A	11	7	1.	47	0.149
123	A	10	7	1.	47	0.149
124	A	9	6	0.99	45	0.133
125	A	9	6	1.	35	0.171
126	A	13	7	1.	47	0.149
127	A	14	7	1.	47	0.149
128	A	16	8	1.	49	0.163
129	A	15	8	1.	49	0.163
130	A	14	8	1.01	49	0.163
131	A	13	8	1.	49	0.163

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
132	A	13	8	1.	49	0.163
133	A	9	6	1.	49	0.122
134	A	10	6	1.	49	0.122
135	A	16	8	1.	49	0.163
136	A	15	8	1.	49	0.163
137	A	14	8	1.	49	0.163
138	A	14	9	1.	49	0.184
139	A	14	8	1.	49	0.163
140	A	10	6	1.	49	0.122
141	A	16	8	1.	49	0.163
142	A	15	8	1.	49	0.163
143	A	15	9	1.	49	0.184
144	A	15	9	1.	49	0.184
145	A	15	8	1.	49	0.163
146	A	11	6	1.	49	0.122
147	A	15	8	1.	49	0.163
148	A	14	8	1.	49	0.163
149	A	13	8	1.	49	0.163
150	A	12	7	1.	49	0.143
151	A	8	5	1.	49	0.102
152	A	9	5	1.	49	0.102
153	A	15	9	1.	49	0.184
154	A	14	9	1.	49	0.184
155	A	13	8	1.	49	0.163
156	A	8	5	1.	49	0.102
157	A	9	5	1.	49	0.102
158	A	10	5	1.	49	0.102
159	A	15	9	1.	49	0.184
160	A	14	8	1.	49	0.163
161	A	9	6	1.	49	0.122
162	A	9	5	1.	49	0.102
163	A	10	5	1.	49	0.102
164	A	9	6	1.	45	0.133
165	A	9	6	0.98	45	0.133
166	A	8	6	0.99	45	0.133
167	A	7	5	1.	43	0.116
168	A	6	4	1.	33	0.121
169	A	8	5	1.	45	0.111
170	A	9	6	1.	45	0.133
171	A	10	6	1.	45	0.133

Chapter 3

Listing of integrals

3.1 $\int \tan(c+dx)(a+b\tan(c+dx)) \left(B\tan(c+dx)+C\tan^2(c+d\right.$

Optimal. Leaf size=87

$$\frac{(aC+bB)\tan^2(c+dx)}{2d}+\frac{(aB-bC)\tan(c+dx)}{d}+\frac{(aC+bB)\log(\cos(c+dx))}{d}-x(aB-bC)+\frac{bC\tan^3(c+dx)}{3d}$$

```
[Out] -((a*B - b*C)*x) + ((b*B + a*C)*Log[Cos[c + d*x]])/d + ((a*B - b*C)*Tan[c +
 d*x])/d + ((b*B + a*C)*Tan[c + d*x]^2)/(2*d) + (b*C*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.133698, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {3632, 3592, 3528, 3525, 3475}

$$\frac{(aC+bB)\tan^2(c+dx)}{2d}+\frac{(aB-bC)\tan(c+dx)}{d}+\frac{(aC+bB)\log(\cos(c+dx))}{d}-x(aB-bC)+\frac{bC\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d*x]^2),x
]
```

```
[Out] -((a*B - b*C)*x) + ((b*B + a*C)*Log[Cos[c + d*x]])/d + ((a*B - b*C)*Tan[c +
 d*x])/d + ((b*B + a*C)*Tan[c + d*x]^2)/(2*d) + (b*C*Tan[c + d*x]^3)/(3*d)
```

Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3592

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(
B*d*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \tan(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx &= \int \tan^2(c+dx)(a+b\tan(c+dx))(B+C\tan(c\\ &= \frac{bC\tan^3(c+dx)}{3d}+\int \tan^2(c+dx)(aB-bC+(\\ &= \frac{(bB+aC)\tan^2(c+dx)}{2d}+\frac{bC\tan^3(c+dx)}{3d}+\int\\ &= -(aB-bC)x+\frac{(aB-bC)\tan(c+dx)}{d}+\frac{(bB+}{}\\ &= -(aB-bC)x+\frac{(bB+aC)\log(\cos(c+dx))}{d}+\frac{(a}{}\end{aligned}$$

Mathematica [A] time = 0.549577, size = 86, normalized size = 0.99

$$\frac{3(aC+bB)\tan^2(c+dx)+(6bC-6aB)\tan^{-1}(\tan(c+dx))+6(aB-bC)\tan(c+dx)+6(aC+bB)\log(\cos(c+dx))+2b}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d*x
]^2),x]
```

```
[Out] ((-6*a*B + 6*b*C)*ArcTan[Tan[c + d*x]] + 6*(b*B + a*C)*Log[Cos[c + d*x]] +
6*(a*B - b*C)*Tan[c + d*x] + 3*(b*B + a*C)*Tan[c + d*x]^2 + 2*b*C*Tan[c + d
*x]^3)/(6*d)
```

Maple [A] time = 0.013, size = 135, normalized size = 1.6

$$\frac{Cb(\tan(dx+c))^3}{3d}+\frac{B(\tan(dx+c))^2b}{2d}+\frac{C(\tan(dx+c))^2a}{2d}+\frac{aB\tan(dx+c)}{d}-\frac{Cb\tan(dx+c)}{d}-\frac{\ln\left(1+(\tan(dx+\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

```
[Out] 1/3*b*C*tan(d*x+c)^3/d+1/2/d*B*tan(d*x+c)^2*b+1/2/d*C*tan(d*x+c)^2*a+1/d*a*
B*tan(d*x+c)-b*C*tan(d*x+c)/d-1/2/d*ln(1+tan(d*x+c)^2)*B*b-1/2/d*ln(1+tan(d
*x+c)^2)*C*a-1/d*a*B*arctan(tan(d*x+c))+1/d*C*arctan(tan(d*x+c))*b
```

Maxima [A] time = 1.61027, size = 116, normalized size = 1.33

$$\frac{2\,Cb\tan\left(dx+c\right)^{3}+3\,(Ca+Bb)\tan\left(dx+c\right)^{2}-6\,(Ba-Cb)(dx+c)-3\,(Ca+Bb)\log\left(\tan\left(dx+c\right)^{2}+1\right)+6\,(Ba-C}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algo
rithm="maxima")
```

```
[Out] 1/6*(2*C*b*tan(d*x + c)^3 + 3*(C*a + B*b)*tan(d*x + c)^2 - 6*(B*a - C*b)*(d
*x + c) - 3*(C*a + B*b)*log(tan(d*x + c)^2 + 1) + 6*(B*a - C*b)*tan(d*x + c
))/d
```

Fricas [A] time = 1.37233, size = 208, normalized size = 2.39

$$\frac{2\,Cb\tan\left(dx+c\right)^{3}-6\,(Ba-Cb)dx+3\,(Ca+Bb)\tan\left(dx+c\right)^{2}+3\,(Ca+Bb)\log\left(\frac{1}{\tan(dx+c)^{2}+1}\right)+6\,(Ba-Cb)\tan\left(dx\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algo
rithm="fricas")
```

```
[Out] 1/6*(2*C*b*tan(d*x + c)^3 - 6*(B*a - C*b)*d*x + 3*(C*a + B*b)*tan(d*x + c)^
2 + 3*(C*a + B*b)*log(1/(tan(d*x + c)^2 + 1)) + 6*(B*a - C*b)*tan(d*x + c))
/d
```

Sympy [A] time = 1.52478, size = 139, normalized size = 1.6

$$\begin{cases}-Bax+\frac{Ba\tan\left(c+dx\right)}{d}-\frac{Bb\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d}+\frac{Bb\tan^{2}\left(c+dx\right)}{2d}-\frac{Ca\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d}+\frac{Ca\tan^{2}\left(c+dx\right)}{2d}+Cbx+\frac{Cb\tan^{3}\left(c+dx\right)}{3d}-C\\ x\left(a+b\tan\left(c\right)\right)\left(B\tan\left(c\right)+C\tan^{2}\left(c\right)\right)\tan\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

```
[Out] Piecewise((-B*a*x + B*a*tan(c + d*x)/d - B*b*log(tan(c + d*x)**2 + 1)/(2*d)
 + B*b*tan(c + d*x)**2/(2*d) - C*a*log(tan(c + d*x)**2 + 1)/(2*d) + C*a*tan
(c + d*x)**2/(2*d) + C*b*x + C*b*tan(c + d*x)**3/(3*d) - C*b*tan(c + d*x)/d
, Ne(d, 0)), (x*(a + b*tan(c))*(B*tan(c) + C*tan(c)**2)*tan(c), True))
```

Giac [B] time = 2.50916, size = 1373, normalized size = 15.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algo
rithm="giac")
```

```
[Out] -1/6*(6*B*a*d*x*tan(d*x)^3*tan(c)^3 - 6*C*b*d*x*tan(d*x)^3*tan(c)^3 - 3*C*a
*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)
^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^3*tan(c)^3 - 3*
B*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d
*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^3*tan(c)^3 -
 18*B*a*d*x*tan(d*x)^2*tan(c)^2 + 18*C*b*d*x*tan(d*x)^2*tan(c)^2 - 3*C*a*ta
n(d*x)^3*tan(c)^3 - 3*B*b*tan(d*x)^3*tan(c)^3 + 9*C*a*log(4*(tan(c)^2 + 1)/
(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)
^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^2 + 9*B*b*log(4*(tan(c)^2 +
1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d
*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^2 + 6*B*a*tan(d*x)^3*tan(
c)^2 - 6*C*b*tan(d*x)^3*tan(c)^2 + 6*B*a*tan(d*x)^2*tan(c)^3 - 6*C*b*tan(d*
x)^2*tan(c)^3 + 18*B*a*d*x*tan(d*x)*tan(c) - 18*C*b*d*x*tan(d*x)*tan(c) - 3
*C*a*tan(d*x)^3*tan(c) - 3*B*b*tan(d*x)^3*tan(c) + 3*C*a*tan(d*x)^2*tan(c)^
2 + 3*B*b*tan(d*x)^2*tan(c)^2 - 3*C*a*tan(d*x)*tan(c)^3 - 3*B*b*tan(d*x)*ta
n(c)^3 + 2*C*b*tan(d*x)^3 - 9*C*a*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2
 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(
c) + 1))*tan(d*x)*tan(c) - 9*B*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2
- 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c
) + 1))*tan(d*x)*tan(c) - 12*B*a*tan(d*x)^2*tan(c) + 18*C*b*tan(d*x)^2*tan(
c) - 12*B*a*tan(d*x)*tan(c)^2 + 18*C*b*tan(d*x)*tan(c)^2 + 2*C*b*tan(c)^3 -
 6*B*a*d*x + 6*C*b*d*x + 3*C*a*tan(d*x)^2 + 3*B*b*tan(d*x)^2 - 3*C*a*tan(d*
x)*tan(c) - 3*B*b*tan(d*x)*tan(c) + 3*C*a*tan(c)^2 + 3*B*b*tan(c)^2 + 3*C*a
*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)
^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)) + 3*B*b*log(4*(tan(c)^2
+ 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan
(d*x)^2 - 2*tan(d*x)*tan(c) + 1)) + 6*B*a*tan(d*x) - 6*C*b*tan(d*x) + 6*B*a
*tan(c) - 6*C*b*tan(c) + 3*C*a + 3*B*b)/(d*tan(d*x)^3*tan(c)^3 - 3*d*tan(d*
x)^2*tan(c)^2 + 3*d*tan(d*x)*tan(c) - d)
```

3.2 $\int (a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx$

Optimal. Leaf size=66

$$-\frac{(aB-bC)\log(\cos(c+dx))}{d}-x(aC+bB)+\frac{C(a+b\tan(c+dx))^2}{2bd}+\frac{bB\tan(c+dx)}{d}$$

```
[Out] -((b*B + a*C)*x) - ((a*B - b*C)*Log[Cos[c + d*x]])/d + (b*B*Tan[c + d*x])/d
 + (C*(a + b*Tan[c + d*x])^2)/(2*b*d)
```

Rubi [A] time = 0.0458897, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3630, 3525, 3475}

$$-\frac{(aB-bC)\log(\cos(c+dx))}{d}-x(aC+bB)+\frac{C(a+b\tan(c+dx))^2}{2bd}+\frac{bB\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d*x]^2),x]
```

```
[Out] -((b*B + a*C)*x) - ((a*B - b*C)*Log[Cos[c + d*x]])/d + (b*B*Tan[c + d*x])/d
 + (C*(a + b*Tan[c + d*x])^2)/(2*b*d)
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx=\frac{C(a+b\tan(c+dx))^2}{2bd}+\int (a+b\tan(c+dx))(-C+B\text{t}$$

$$=-(bB+aC)x+\frac{bB\tan(c+dx)}{d}+\frac{C(a+b\tan(c+dx))^2}{2bd}$$

$$=-(bB+aC)x-\frac{(aB-bC)\log(\cos(c+dx))}{d}+\frac{bB\tan(c+}{d}$$

Mathematica [A] time = 0.292894, size = 67, normalized size = 1.02

$$\frac{-2(aC+bB)\tan^{-1}(\tan(c+dx))+2(aC+bB)\tan(c+dx)+2(bC-aB)\log(\cos(c+dx))+bC\tan^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d*x]^2),x]
```

```
[Out] (-2*(b*B + a*C)*ArcTan[Tan[c + d*x]] + 2*(-(a*B) + b*C)*Log[Cos[c + d*x]] +
 2*(b*B + a*C)*Tan[c + d*x] + b*C*Tan[c + d*x]^2)/(2*d)
```

Maple [A] time = 0.013, size = 105, normalized size = 1.6

$$\frac{C(\tan(dx+c))^2 b}{2d}+\frac{B\tan(dx+c)b}{d}+\frac{C\tan(dx+c)a}{d}+\frac{a\ln\left(1+(\tan(dx+c))^2\right)B}{2d}-\frac{\ln\left(1+(\tan(dx+c))^2\right)Cb}{2d}-\frac{B}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

```
[Out] 1/2/d*C*b*tan(d*x+c)^2+b*B*tan(d*x+c)/d+1/d*C*tan(d*x+c)*a+1/2/d*a*ln(1+tan
(d*x+c)^2)*B-1/2/d*ln(1+tan(d*x+c)^2)*C*b-1/d*B*arctan(tan(d*x+c))*b-1/d*C*
arctan(tan(d*x+c))*a
```

Maxima [A] time = 1.76493, size = 89, normalized size = 1.35

$$\frac{Cb\tan(dx+c)^2-2(Ca+Bb)(dx+c)+(Ba-Cb)\log\left(\tan(dx+c)^2+1\right)+2(Ca+Bb)\tan(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algorithm="maxi
ma")
```

```
[Out] 1/2*(C*b*tan(d*x + c)^2 - 2*(C*a + B*b)*(d*x + c) + (B*a - C*b)*log(tan(d*x
 + c)^2 + 1) + 2*(C*a + B*b)*tan(d*x + c))/d
```

Fricas [A] time = 1.43942, size = 161, normalized size = 2.44

$$\frac{Cb\tan(dx+c)^2-2(Ca+Bb)dx-(Ba-Cb)\log\left(\frac{1}{\tan(dx+c)^2+1}\right)+2(Ca+Bb)\tan(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algorithm="fric
as")
```

```
[Out] 1/2*(C*b*tan(d*x + c)^2 - 2*(C*a + B*b)*d*x - (B*a - C*b)*log(1/(tan(d*x +
c)^2 + 1)) + 2*(C*a + B*b)*tan(d*x + c))/d
```

Sympy [A] time = 0.587673, size = 105, normalized size = 1.59

$$\begin{cases} \frac{Ba\log\left(\tan^2(c+dx)+1\right)}{2d} - Bbx + \frac{Bb\tan(c+dx)}{d} - Cax + \frac{Ca\tan(c+dx)}{d} - \frac{Cb\log\left(\tan^2(c+dx)+1\right)}{2d} + \frac{Cb\tan^2(c+dx)}{2d} & \text{for } d \neq 0 \\ x\left(a+b\tan(c)\right)\left(B\tan(c)+C\tan^2(c)\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

```
[Out] Piecewise((B*a*log(tan(c + d*x)**2 + 1)/(2*d) - B*b*x + B*b*tan(c + d*x)/d
- C*a*x + C*a*tan(c + d*x)/d - C*b*log(tan(c + d*x)**2 + 1)/(2*d) + C*b*tan
(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*tan(c))*(B*tan(c) + C*tan(c)**2),
True))
```

Giac [B] time = 1.92277, size = 832, normalized size = 12.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algorithm="giac
")
```

```
[Out] -1/2*(2*C*a*d*x*tan(d*x)^2*tan(c)^2 + 2*B*b*d*x*tan(d*x)^2*tan(c)^2 + B*a*l
og(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2
*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^2 - C*b*
log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^
2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^2 - 4*C
*a*d*x*tan(d*x)*tan(c) - 4*B*b*d*x*tan(d*x)*tan(c) - C*b*tan(d*x)^2*tan(c)^
2 - 2*B*a*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) +
 tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)*tan(c)
 + 2*C*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) +
tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)*tan(c)
+ 2*C*a*tan(d*x)^2*tan(c) + 2*B*b*tan(d*x)^2*tan(c) + 2*C*a*tan(d*x)*tan(c)
^2 + 2*B*b*tan(d*x)*tan(c)^2 + 2*C*a*d*x + 2*B*b*d*x - C*b*tan(d*x)^2 - C*b
*tan(c)^2 + B*a*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*ta
n(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)) - C*b*log
(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*t
an(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)) - 2*C*a*tan(d*x) - 2*B*b*tan
(d*x) - 2*C*a*tan(c) - 2*B*b*tan(c) - C*b)/(d*tan(d*x)^2*tan(c)^2 - 2*d*tan
(d*x)*tan(c) + d)
```

3.3 $\int \cot(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)$

Optimal. Leaf size=42

$$-\frac{(aC+bB)\log(\cos(c+dx))}{d}+x(aB-bC)+\frac{bC\tan(c+dx)}{d}$$

```
[Out] (a*B - b*C)*x - ((b*B + a*C)*Log[Cos[c + d*x]])/d + (b*C*Tan[c + d*x])/d
```

Rubi [A] time = 0.0604294, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3632, 3525, 3475}

$$-\frac{(aC+bB)\log(\cos(c+dx))}{d}+x(aB-bC)+\frac{bC\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d*x]^2),x
]
```

```
[Out] (a*B - b*C)*x - ((b*B + a*C)*Log[Cos[c + d*x]])/d + (b*C*Tan[c + d*x])/d
```

Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx &= \int (a+b\tan(c+dx))(B+C\tan(c+dx))\,dx\\ &= (aB-bC)x+\frac{bC\tan(c+dx)}{d}+(bB+aC)\int \text{ta}\\ &= (aB-bC)x-\frac{(bB+aC)\log(\cos(c+dx))}{d}+\frac{bC}{}\end{aligned}$$

Mathematica [A] time = 0.0538475, size = 59, normalized size = 1.4

$$aBx-\frac{aC\log(\cos(c+dx))}{d}-\frac{bB\log(\cos(c+dx))}{d}-\frac{bC\tan^{-1}(\tan(c+dx))}{d}+\frac{bC\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d*x
]^2),x]
```

```
[Out] a*B*x - (b*C*ArcTan[Tan[c + d*x]])/d - (b*B*Log[Cos[c + d*x]])/d - (a*C*Log
[Cos[c + d*x]])/d + (b*C*Tan[c + d*x])/d
```

Maple [A] time = 0.061, size = 66, normalized size = 1.6

$$aBx - Cbx - \frac{Bb\ln(\cos(dx+c))}{d} + \frac{Bac}{d} + \frac{Cb\tan(dx+c)}{d} - \frac{Ca\ln(\cos(dx+c))}{d} - \frac{Cbc}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

```
[Out] a*B*x-C*b*x-1/d*B*b*ln(cos(d*x+c))+1/d*B*a*c+b*C*tan(d*x+c)/d-1/d*C*a*ln(co
s(d*x+c))-1/d*C*b*c
```

Maxima [A] time = 1.75158, size = 68, normalized size = 1.62

$$\frac{2\,Cb\tan(dx+c) + 2\,(Ba-Cb)(dx+c) + (Ca+Bb)\log\left(\tan(dx+c)^2+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algo
rithm="maxima")
```

```
[Out] 1/2*(2*C*b*tan(d*x + c) + 2*(B*a - C*b)*(d*x + c) + (C*a + B*b)*log(tan(d*x
 + c)^2 + 1))/d
```

Fricas [A] time = 1.35061, size = 122, normalized size = 2.9

$$\frac{2\,(Ba-Cb)dx + 2\,Cb\tan(dx+c) - (Ca+Bb)\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algo
rithm="fricas")
```

```
[Out] 1/2*(2*(B*a - C*b)*d*x + 2*C*b*tan(d*x + c) - (C*a + B*b)*log(1/(tan(d*x +
c)^2 + 1)))/d
```

Sympy [A] time = 2.89056, size = 82, normalized size = 1.95

$$\begin{cases} Bax + \frac{Bb\log\left(\tan^2(c+dx)+1\right)}{2d} + \frac{Ca\log\left(\tan^2(c+dx)+1\right)}{2d} - Cbx + \frac{Cb\tan(c+dx)}{d} & \text{for } d \neq 0 \\ x\,(a+b\tan(c))\left(B\tan(c)+C\tan^2(c)\right)\cot(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

```
[Out] Piecewise((B*a*x + B*b*log(tan(c + d*x)**2 + 1)/(2*d) + C*a*log(tan(c + d*x
)**2 + 1)/(2*d) - C*b*x + C*b*tan(c + d*x)/d, Ne(d, 0)), (x*(a + b*tan(c))*
(B*tan(c) + C*tan(c)**2)*cot(c), True))
```

Giac [A] time = 1.42125, size = 68, normalized size = 1.62

$$\frac{2\,Cb\tan\left(dx+c\right)+2\,(Ba-Cb)(dx+c)+(Ca+Bb)\log\left(\tan\left(dx+c\right)^{2}+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algo
rithm="giac")
```

```
[Out] 1/2*(2*C*b*tan(d*x + c) + 2*(B*a - C*b)*(d*x + c) + (C*a + B*b)*log(tan(d*x
 + c)^2 + 1))/d
```

3.4 $\int \cot^2(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)$

Optimal. Leaf size=37

$$x(aC+bB)+\frac{aB\log(\sin(c+dx))}{d}-\frac{bC\log(\cos(c+dx))}{d}$$

```
[Out] (b*B + a*C)*x - (b*C*Log[Cos[c + d*x]])/d + (a*B*Log[Sin[c + d*x]])/d
```

Rubi [A] time = 0.109605, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3632, 3589, 3475, 3531}

$$x(aC+bB)+\frac{aB\log(\sin(c+dx))}{d}-\frac{bC\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d*x]^2)
,x]
```

```
[Out] (b*B + a*C)*x - (b*C*Log[Cos[c + d*x]])/d + (a*B*Log[Sin[c + d*x]])/d
```

Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3589

```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_
.)*(x_)]))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(B*d)
/b, Int[Tan[e + f*x], x], x] + Dist[1/b, Int[Simp[A*b*c + (A*b*d + B*(b*c -
 a*d))*Tan[e + f*x], x]/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d,
e, f, A, B}, x] && NeQ[b*c - a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \cot^2(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx = \int \cot(c+dx)(a+b\tan(c+dx))(B+C\tan(c$$
$$= (bC)\int \tan(c+dx)\,dx + \int \cot(c+dx)(aB+($$
$$= (bB+aC)x - \frac{bC\log(\cos(c+dx))}{d} + (aB)\int \text{co}$$
$$= (bB+aC)x - \frac{bC\log(\cos(c+dx))}{d} + \frac{aB\log(\text{si}}{}$$

Mathematica [A] time = 0.0689106, size = 44, normalized size = 1.19

$$\frac{aB(\log(\tan(c+dx))+\log(\cos(c+dx)))}{d} + aCx + bBx - \frac{bC\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d
*x]^2),x]
```

```
[Out] b*B*x + a*C*x - (b*C*Log[Cos[c + d*x]])/d + (a*B*(Log[Cos[c + d*x]] + Log[T
an[c + d*x]]))/d
```

Maple [A] time = 0.068, size = 51, normalized size = 1.4

$$Bxb + Cxa + \frac{aB\ln(\sin(dx+c))}{d} + \frac{Bbc}{d} - \frac{Cb\ln(\cos(dx+c))}{d} + \frac{Cac}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

```
[Out] B*x*b+C*x*a+1/d*a*B*ln(sin(d*x+c))+1/d*B*b*c-b*C*ln(cos(d*x+c))/d+1/d*C*a*c
```

Maxima [A] time = 1.66812, size = 70, normalized size = 1.89

$$\frac{2\,Ba\log(\tan(dx+c)) + 2\,(Ca+Bb)(dx+c) - (Ba-Cb)\log\left(\tan(dx+c)^2+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/2*(2*B*a*log(tan(d*x + c)) + 2*(C*a + B*b)*(d*x + c) - (B*a - C*b)*log(ta
n(d*x + c)^2 + 1))/d
```

Fricas [A] time = 1.41548, size = 146, normalized size = 3.95

$$\frac{2\,(Ca+Bb)dx + Ba\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right) - Cb\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/2*(2*(C*a + B*b)*d*x + B*a*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1)) - C*b
*log(1/(tan(d*x + c)^2 + 1)))/d
```

Sympy [A] time = 4.86821, size = 85, normalized size = 2.3

$$\begin{cases} -\frac{Ba\log\left(\tan^2(c+dx)+1\right)}{2d} + \frac{Ba\log(\tan(c+dx))}{d} + Bbx + Cax + \frac{Cb\log\left(\tan^2(c+dx)+1\right)}{2d} & \text{for } d \neq 0 \\ x(a+b\tan(c))\left(B\tan(c) + C\tan^2(c)\right)\cot^2(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

```
[Out] Piecewise((-B*a*log(tan(c + d*x)**2 + 1)/(2*d) + B*a*log(tan(c + d*x))/d +
B*b*x + C*a*x + C*b*log(tan(c + d*x)**2 + 1)/(2*d), Ne(d, 0)), (x*(a + b*ta
n(c))*(B*tan(c) + C*tan(c)**2)*cot(c)**2, True))
```

Giac [A] time = 1.49365, size = 72, normalized size = 1.95

$$\frac{2\,Ba\log\left(|\tan(dx+c)|\right) + 2\,(Ca+Bb)(dx+c) - (Ba-Cb)\log\left(\tan(dx+c)^2+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] 1/2*(2*B*a*log(abs(tan(d*x + c))) + 2*(C*a + B*b)*(d*x + c) - (B*a - C*b)*l
og(tan(d*x + c)^2 + 1))/d
```

3.5 $\int \cot^3(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx\right.$

Optimal. Leaf size=43

$$\frac{(aC+bB)\log(\sin(c+dx))}{d}+x(-(aB-bC))-\frac{aB\cot(c+dx)}{d}$$

```
[Out] -((a*B - b*C)*x) - (a*B*Cot[c + d*x])/d + ((b*B + a*C)*Log[Sin[c + d*x]])/d
```

Rubi [A] time = 0.123954, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3632, 3591, 3531, 3475}

$$\frac{(aC+bB)\log(\sin(c+dx))}{d}+x(-(aB-bC))-\frac{aB\cot(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d*x]^2)
,x]
```

```
[Out] -((a*B - b*C)*x) - (a*B*Cot[c + d*x])/d + ((b*B + a*C)*Log[Sin[c + d*x]])/d
```

Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3591

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((
b*c - a*d)*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 + b^
2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c +
 b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m,
 -1] && NeQ[a^2 + b^2, 0]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^3(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx = \int \cot^2(c+dx)(a+b\tan(c+dx))(B+C\tan$$
$$= -\frac{aB\cot(c+dx)}{d} + \int \cot(c+dx)(bB+aC$$
$$= -(aB-bC)x - \frac{aB\cot(c+dx)}{d} + (bB+aC)$$
$$= -(aB-bC)x - \frac{aB\cot(c+dx)}{d} + \frac{(bB+aC)}{}$$

Mathematica [C] time = 0.161645, size = 78, normalized size = 1.81

$$-\frac{aB\cot(c+dx)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(c+dx)\right)}{d} + \frac{aC(\log(\tan(c+dx))+\log(\cos(c+dx)))}{d} + \frac{bB(\log}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d
*x]^2),x]
```

```
[Out] b*C*x - (a*B*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d*x]^2])
/d + (b*B*(Log[Cos[c + d*x]] + Log[Tan[c + d*x]]))/d + (a*C*(Log[Cos[c + d*
x]] + Log[Tan[c + d*x]]))/d
```

Maple [A] time = 0.059, size = 65, normalized size = 1.5

$$-aBx + Cbx - \frac{B\cot(dx+c)a}{d} + \frac{Bb\ln(\sin(dx+c))}{d} - \frac{Bac}{d} + \frac{Ca\ln(\sin(dx+c))}{d} + \frac{Cbc}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

```
[Out] -a*B*x+C*b*x-1/d*B*cot(d*x+c)*a+1/d*B*b*ln(sin(d*x+c))-1/d*B*a*c+1/d*C*a*ln
(sin(d*x+c))+1/d*C*b*c
```

Maxima [A] time = 1.73373, size = 92, normalized size = 2.14

$$-\frac{2(Ba-Cb)(dx+c) + (Ca+Bb)\log\left(\tan(dx+c)^2+1\right) - 2(Ca+Bb)\log(\tan(dx+c)) + \frac{2Ba}{\tan(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] -1/2*(2*(B*a - C*b)*(d*x + c) + (C*a + B*b)*log(tan(d*x + c)^2 + 1) - 2*(C*
a + B*b)*log(tan(d*x + c)) + 2*B*a/tan(d*x + c))/d
```

Fricas [A] time = 1.33783, size = 178, normalized size = 4.14

$$-\frac{2\,(Ba-Cb)dx\tan\left(dx+c\right)-(Ca+Bb)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)+2\,Ba}{2\,d\tan\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] -1/2*(2*(B*a - C*b)*d*x*tan(d*x + c) - (C*a + B*b)*log(tan(d*x + c)^2/(tan(
d*x + c)^2 + 1))*tan(d*x + c) + 2*B*a)/(d*tan(d*x + c))
```

Sympy [A] time = 15.0302, size = 116, normalized size = 2.7

$$\begin{cases} \text{NaN} & \text{for}\: c = 0 \wedge d \\ x\left(a+b\tan\left(c\right)\right)\left(B\tan\left(c\right)+C\tan^{2}\left(c\right)\right)\cot^{3}\left(c\right) & \text{for}\: d = 0 \\ \text{NaN} & \text{for}\: c = -dx \\ -Bax-\frac{Ba}{d\tan\left(c+dx\right)}-\frac{Bb\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d}+\frac{Bb\log\left(\tan\left(c+dx\right)\right)}{d}-\frac{Ca\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d}+\frac{Ca\log\left(\tan\left(c+dx\right)\right)}{d}+Cbx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

```
[Out] Piecewise((nan, Eq(c, 0) & Eq(d, 0)), (x*(a + b*tan(c))*(B*tan(c) + C*tan(c
)**2)*cot(c)**3, Eq(d, 0)), (nan, Eq(c, -d*x)), (-B*a*x - B*a/(d*tan(c + d*
x)) - B*b*log(tan(c + d*x)**2 + 1)/(2*d) + B*b*log(tan(c + d*x))/d - C*a*lo
g(tan(c + d*x)**2 + 1)/(2*d) + C*a*log(tan(c + d*x))/d + C*b*x, True))
```

Giac [B] time = 1.53822, size = 161, normalized size = 3.74

$$\frac{Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,(Ba-Cb)(dx+c)-2\,(Ca+Bb)\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)+2\,(Ca+Bb)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] 1/2*(B*a*tan(1/2*d*x + 1/2*c) - 2*(B*a - C*b)*(d*x + c) - 2*(C*a + B*b)*log
(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(C*a + B*b)*log(abs(tan(1/2*d*x + 1/2*c)))
 - (2*C*a*tan(1/2*d*x + 1/2*c) + 2*B*b*tan(1/2*d*x + 1/2*c) + B*a)/tan(1/2*
d*x + 1/2*c))/d
```

3.6 $\int \cot^4(c+dx)(a+b\tan(c+dx)) \left(B\tan(c+dx) + C\tan^2(c+\right.$

Optimal. Leaf size=66

$$-\frac{(aC+bB)\cot(c+dx)}{d} - \frac{(aB-bC)\log(\sin(c+dx))}{d} - x(aC+bB) - \frac{aB\cot^2(c+dx)}{2d}$$

```
[Out] -((b*B + a*C)*x) - ((b*B + a*C)*Cot[c + d*x])/d - (a*B*Cot[c + d*x]^2)/(2*d
) - ((a*B - b*C)*Log[Sin[c + d*x]])/d
```

Rubi [A] time = 0.159387, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {3632, 3591, 3529, 3531, 3475}

$$-\frac{(aC+bB)\cot(c+dx)}{d} - \frac{(aB-bC)\log(\sin(c+dx))}{d} - x(aC+bB) - \frac{aB\cot^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d*x]^2)
,x]
```

```
[Out] -((b*B + a*C)*x) - ((b*B + a*C)*Cot[c + d*x])/d - (a*B*Cot[c + d*x]^2)/(2*d
) - ((a*B - b*C)*Log[Sin[c + d*x]])/d
```

Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3591

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((
b*c - a*d)*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 + b^
2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c +
 b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m,
 -1] && NeQ[a^2 + b^2, 0]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
```

```
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx = \int \cot^3(c+dx)(a+b\tan(c+dx))(B+C\tan($$
$$= -\frac{aB\cot^2(c+dx)}{2d} + \int \cot^2(c+dx)(bB+aC -$$
$$= -\frac{(bB+aC)\cot(c+dx)}{d} - \frac{aB\cot^2(c+dx)}{2d} +$$
$$= -(bB+aC)x - \frac{(bB+aC)\cot(c+dx)}{d} - \frac{aB\cot}{}$$
$$= -(bB+aC)x - \frac{(bB+aC)\cot(c+dx)}{d} - \frac{aB\cot}{}$$

Mathematica [C] time = 0.435101, size = 77, normalized size = 1.17

$$-\frac{2(aC+bB)\cot(c+dx)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(c+dx)\right) + 2(aB-bC)(\log(\tan(c+dx)) + \log(\cos(c+d}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d
*x]^2),x]
```

```
[Out] -(a*B*Cot[c + d*x]^2 + 2*(b*B + a*C)*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1
, 1/2, -Tan[c + d*x]^2] + 2*(a*B - b*C)*(Log[Cos[c + d*x]] + Log[Tan[c + d*
x]]))/(2*d)
```

Maple [A] time = 0.076, size = 96, normalized size = 1.5

$$-Bxb - \frac{B\cot(dx+c)\,b}{d} - \frac{Bbc}{d} + \frac{Cb\ln(\sin(dx+c))}{d} - \frac{aB(\cot(dx+c))^2}{2d} - \frac{aB\ln(\sin(dx+c))}{d} - Cxa - \frac{C\cot(dx+c)\,a}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

```
[Out] -B*x*b-1/d*B*cot(d*x+c)*b-1/d*B*b*c+1/d*C*b*ln(sin(d*x+c))-1/2/d*a*B*cot(d*
x+c)^2-1/d*a*B*ln(sin(d*x+c))-C*x*a-1/d*C*cot(d*x+c)*a-1/d*C*a*c
```

Maxima [A] time = 1.70778, size = 116, normalized size = 1.76

$$-\frac{2(Ca+Bb)(dx+c) - (Ba-Cb)\log\left(\tan(dx+c)^2+1\right) + 2(Ba-Cb)\log(\tan(dx+c)) + \frac{Ba+2(Ca+Bb)\tan(dx+c)}{\tan(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] -1/2*(2*(C*a + B*b)*(d*x + c) - (B*a - C*b)*log(tan(d*x + c)^2 + 1) + 2*(B*
a - C*b)*log(tan(d*x + c)) + (B*a + 2*(C*a + B*b)*tan(d*x + c))/tan(d*x + c
)^2)/d
```

Fricas [A] time = 1.39201, size = 234, normalized size = 3.55

$$-\frac{(Ba-Cb)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan(dx+c)^2+(2(Ca+Bb)dx+Ba)\tan(dx+c)^2+Ba+2(Ca+Bb)\tan(dx+c)}{2d\tan(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] -1/2*((B*a - C*b)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^2 +
 (2*(C*a + B*b)*d*x + B*a)*tan(d*x + c)^2 + B*a + 2*(C*a + B*b)*tan(d*x + c
))/(d*tan(d*x + c)^2)
```

Sympy [A] time = 14.0008, size = 143, normalized size = 2.17

$$\begin{cases} \text{NaN} \\ x(a+b\tan(c))\left(B\tan(c)+C\tan^2(c)\right)\cot^4(c) \\ \text{NaN} \\ \frac{Ba\log\left(\tan^2(c+dx)+1\right)}{2d}-\frac{Ba\log(\tan(c+dx))}{d}-\frac{Ba}{2d\tan^2(c+dx)}-Bbx-\frac{Bb}{d\tan(c+dx)}-Cax-\frac{Ca}{d\tan(c+dx)}-\frac{Cb\log\left(\tan^2(c+dx)+1\right)}{2d}+\frac{Cb\log\ldots}{} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

```
[Out] Piecewise((nan, Eq(c, 0) & Eq(d, 0)), (x*(a + b*tan(c))*(B*tan(c) + C*tan(c
)**2)*cot(c)**4, Eq(d, 0)), (nan, Eq(c, -d*x)), (B*a*log(tan(c + d*x)**2 +
1)/(2*d) - B*a*log(tan(c + d*x))/d - B*a/(2*d*tan(c + d*x)**2) - B*b*x - B*
b/(d*tan(c + d*x)) - C*a*x - C*a/(d*tan(c + d*x)) - C*b*log(tan(c + d*x)**2
 + 1)/(2*d) + C*b*log(tan(c + d*x))/d, True))
```

Giac [B] time = 1.55907, size = 242, normalized size = 3.67

$$-\frac{Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-4Ca\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-4Bb\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+8(Ca+Bb)(dx+c)-8(Ba-Cb)\log\left(\tan\left(\ldots\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] -1/8*(B*a*tan(1/2*d*x + 1/2*c)^2 - 4*C*a*tan(1/2*d*x + 1/2*c) - 4*B*b*tan(1
/2*d*x + 1/2*c) + 8*(C*a + B*b)*(d*x + c) - 8*(B*a - C*b)*log(tan(1/2*d*x +
 1/2*c)^2 + 1) + 8*(B*a - C*b)*log(abs(tan(1/2*d*x + 1/2*c))) - (12*B*a*tan
(1/2*d*x + 1/2*c)^2 - 12*C*b*tan(1/2*d*x + 1/2*c)^2 - 4*C*a*tan(1/2*d*x + 1
/2*c) - 4*B*b*tan(1/2*d*x + 1/2*c) - B*a)/tan(1/2*d*x + 1/2*c)^2)/d
```

3.7 $\int \cot^5(c+dx)(a+b\tan(c+dx)) \left(B\tan(c+dx)+C\tan^2(c+dx)\right)$

Optimal. Leaf size=87

$$-\frac{(aC+bB)\cot^2(c+dx)}{2d}+\frac{(aB-bC)\cot(c+dx)}{d}-\frac{(aC+bB)\log(\sin(c+dx))}{d}+x(aB-bC)-\frac{aB\cot^3(c+dx)}{3d}$$

```
[Out] (a*B - b*C)*x + ((a*B - b*C)*Cot[c + d*x])/d - ((b*B + a*C)*Cot[c + d*x]^2)
/(2*d) - (a*B*Cot[c + d*x]^3)/(3*d) - ((b*B + a*C)*Log[Sin[c + d*x]])/d
```

Rubi [A] time = 0.19293, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {3632, 3591, 3529, 3531, 3475}

$$-\frac{(aC+bB)\cot^2(c+dx)}{2d}+\frac{(aB-bC)\cot(c+dx)}{d}-\frac{(aC+bB)\log(\sin(c+dx))}{d}+x(aB-bC)-\frac{aB\cot^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d*x]^2)
,x]
```

```
[Out] (a*B - b*C)*x + ((a*B - b*C)*Cot[c + d*x])/d - ((b*B + a*C)*Cot[c + d*x]^2)
/(2*d) - (a*B*Cot[c + d*x]^3)/(3*d) - ((b*B + a*C)*Log[Sin[c + d*x]])/d
```

Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3591

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((
b*c - a*d)*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 + b^
2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c +
 b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m,
 -1] && NeQ[a^2 + b^2, 0]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
```

```
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^5(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx = \int \cot^4(c+dx)(a+b\tan(c+dx))(B+C\tan(\cdots$$
$$= -\frac{aB\cot^3(c+dx)}{3d} + \int \cot^3(c+dx)(bB+aC-\cdots$$
$$= -\frac{(bB+aC)\cot^2(c+dx)}{2d} - \frac{aB\cot^3(c+dx)}{3d} + \cdots$$
$$= \frac{(aB-bC)\cot(c+dx)}{d} - \frac{(bB+aC)\cot^2(c+d\cdots}{2d}\cdots$$
$$= (aB-bC)x + \frac{(aB-bC)\cot(c+dx)}{d} - \frac{(bB+a\cdots}{\cdots}$$
$$= (aB-bC)x + \frac{(aB-bC)\cot(c+dx)}{d} - \frac{(bB+a\cdots}{\cdots}$$

Mathematica [C] time = 0.995765, size = 101, normalized size = 1.16

$$-\frac{2aB\cot^3(c+dx)\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(c+dx)\right)+6bC\cot(c+dx)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\cdots\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d
*x]^2),x]
```

```
[Out] -(2*a*B*Cot[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2] +
6*b*C*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d*x]^2] + 3*(b*
B + a*C)*(Cot[c + d*x]^2 + 2*(Log[Cos[c + d*x]] + Log[Tan[c + d*x]])))/(6*d
)
```

Maple [A] time = 0.076, size = 124, normalized size = 1.4

$$-\frac{Bb(\cot(dx+c))^2}{2d} - \frac{Bb\ln(\sin(dx+c))}{d} - Cbx - \frac{C\cot(dx+c)b}{d} - \frac{Cbc}{d} - \frac{aB(\cot(dx+c))^3}{3d} + \frac{B\cot(dx+c)a}{d} + aBx\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

```
[Out] -1/2/d*B*b*cot(d*x+c)^2-1/d*B*b*ln(sin(d*x+c))-C*b*x-1/d*C*cot(d*x+c)*b-1/d
*C*b*c-1/3/d*a*B*cot(d*x+c)^3+1/d*B*cot(d*x+c)*a+a*B*x+1/d*B*a*c-1/2/d*C*a*
cot(d*x+c)^2-1/d*C*a*ln(sin(d*x+c))
```

Maxima [A] time = 1.63866, size = 140, normalized size = 1.61

$$\frac{6\,(Ba-Cb)(dx+c)+3\,(Ca+Bb)\log\left(\tan\left(dx+c\right)^{2}+1\right)-6\,(Ca+Bb)\log\left(\tan\left(dx+c\right)\right)+\frac{6\,(Ba-Cb)\tan(dx+c)^{2}-2\,Ba-3\,(C}{\tan(dx+c)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/6*(6*(B*a - C*b)*(d*x + c) + 3*(C*a + B*b)*log(tan(d*x + c)^2 + 1) - 6*(C
*a + B*b)*log(tan(d*x + c)) + (6*(B*a - C*b)*tan(d*x + c)^2 - 2*B*a - 3*(C*
a + B*b)*tan(d*x + c))/tan(d*x + c)^3)/d
```

Fricas [A] time = 1.2671, size = 292, normalized size = 3.36

$$-\frac{3\,(Ca+Bb)\log\left(\frac{\tan(dx+c)^{2}}{\tan(dx+c)^{2}+1}\right)\tan\left(dx+c\right)^{3}-3\,(2\,(Ba-Cb)dx-Ca-Bb)\tan\left(dx+c\right)^{3}-6\,(Ba-Cb)\tan\left(dx+c\right)^{2}}{6\,d\tan\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] -1/6*(3*(C*a + B*b)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^3
 - 3*(2*(B*a - C*b)*d*x - C*a - B*b)*tan(d*x + c)^3 - 6*(B*a - C*b)*tan(d*x
 + c)^2 + 2*B*a + 3*(C*a + B*b)*tan(d*x + c))/(d*tan(d*x + c)^3)
```

Sympy [A] time = 28.2979, size = 180, normalized size = 2.07

$$\begin{cases} \mathrm{NaN} \\ x\,(a+b\tan\left(c\right))\left(B\tan\left(c\right)+C\tan^{2}\left(c\right)\right)\cot^{5}\left(c\right) \\ Bax+\frac{Ba}{d\tan\left(c+dx\right)}-\frac{Ba}{3d\tan^{3}\left(c+dx\right)}+\frac{Bb\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d}-\frac{Bb\log\left(\tan\left(c+dx\right)\right)}{d}-\frac{Bb}{2d\tan^{2}\left(c+dx\right)}+\frac{Ca\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d}-\frac{Ca\log(\tan}{d} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

```
[Out] Piecewise((nan, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x))), (x*(a
 + b*tan(c))*(B*tan(c) + C*tan(c)**2)*cot(c)**5, Eq(d, 0)), (B*a*x + B*a/(d
*tan(c + d*x)) - B*a/(3*d*tan(c + d*x)**3) + B*b*log(tan(c + d*x)**2 + 1)/(
2*d) - B*b*log(tan(c + d*x))/d - B*b/(2*d*tan(c + d*x)**2) + C*a*log(tan(c
+ d*x)**2 + 1)/(2*d) - C*a*log(tan(c + d*x))/d - C*a/(2*d*tan(c + d*x)**2)
- C*b*x - C*b/(d*tan(c + d*x)), True))
```

Giac [B] time = 1.62126, size = 320, normalized size = 3.68

$$Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-3\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-3\,Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-15\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+12\,Cb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] 1/24*(B*a*tan(1/2*d*x + 1/2*c)^3 - 3*C*a*tan(1/2*d*x + 1/2*c)^2 - 3*B*b*tan
(1/2*d*x + 1/2*c)^2 - 15*B*a*tan(1/2*d*x + 1/2*c) + 12*C*b*tan(1/2*d*x + 1/
2*c) + 24*(B*a - C*b)*(d*x + c) + 24*(C*a + B*b)*log(tan(1/2*d*x + 1/2*c)^2
 + 1) - 24*(C*a + B*b)*log(abs(tan(1/2*d*x + 1/2*c))) + (44*C*a*tan(1/2*d*x
 + 1/2*c)^3 + 44*B*b*tan(1/2*d*x + 1/2*c)^3 + 15*B*a*tan(1/2*d*x + 1/2*c)^2
 - 12*C*b*tan(1/2*d*x + 1/2*c)^2 - 3*C*a*tan(1/2*d*x + 1/2*c) - 3*B*b*tan(1
/2*d*x + 1/2*c) - B*a)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.8 $\int \cot^6(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+\right.$

Optimal. Leaf size=108

$$-\frac{(aC+bB)\cot^3(c+dx)}{3d}+\frac{(aB-bC)\cot^2(c+dx)}{2d}+\frac{(aC+bB)\cot(c+dx)}{d}+\frac{(aB-bC)\log(\sin(c+dx))}{d}+x(aC+b$$

```
[Out] (b*B + a*C)*x + ((b*B + a*C)*Cot[c + d*x])/d + ((a*B - b*C)*Cot[c + d*x]^2)
/(2*d) - ((b*B + a*C)*Cot[c + d*x]^3)/(3*d) - (a*B*Cot[c + d*x]^4)/(4*d) +
((a*B - b*C)*Log[Sin[c + d*x]])/d
```

Rubi [A] time = 0.226194, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {3632, 3591, 3529, 3531, 3475}

$$-\frac{(aC+bB)\cot^3(c+dx)}{3d}+\frac{(aB-bC)\cot^2(c+dx)}{2d}+\frac{(aC+bB)\cot(c+dx)}{d}+\frac{(aB-bC)\log(\sin(c+dx))}{d}+x(aC+b$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d*x]^2)
,x]
```

```
[Out] (b*B + a*C)*x + ((b*B + a*C)*Cot[c + d*x])/d + ((a*B - b*C)*Cot[c + d*x]^2)
/(2*d) - ((b*B + a*C)*Cot[c + d*x]^3)/(3*d) - (a*B*Cot[c + d*x]^4)/(4*d) +
((a*B - b*C)*Log[Sin[c + d*x]])/d
```

Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3591

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((
b*c - a*d)*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 + b^
2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c +
 b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m,
 -1] && NeQ[a^2 + b^2, 0]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cot^6(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx &= \int \cot^5(c+dx)(a+b\tan(c+dx))(B+C\tan(\\
&= -\frac{aB\cot^4(c+dx)}{4d}+\int \cot^4(c+dx)(bB+aC- \\
&= -\frac{(bB+aC)\cot^3(c+dx)}{3d}-\frac{aB\cot^4(c+dx)}{4d}+ \\
&= \frac{(aB-bC)\cot^2(c+dx)}{2d}-\frac{(bB+aC)\cot^3(c+ \\
&= \frac{(bB+aC)\cot(c+dx)}{d}+\frac{(aB-bC)\cot^2(c+d \\
&= (bB+aC)x+\frac{(bB+aC)\cot(c+dx)}{d}+\frac{(aB-b \\
&= (bB+aC)x+\frac{(bB+aC)\cot(c+dx)}{d}+\frac{(aB-b
\end{aligned}
$$

Mathematica [C] time = 1.14117, size = 100, normalized size = 0.93

$$
-\frac{4(aC+bB)\cot^3(c+dx)\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(c+dx)\right)+3\left((2bC-2aB)\cot^2(c+dx)-4(aB-bC)(\right.}{12d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d
*x]^2),x]
```

```
[Out] -(4*(b*B + a*C)*Cot[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*
x]^2] + 3*((-2*a*B + 2*b*C)*Cot[c + d*x]^2 + a*B*Cot[c + d*x]^4 - 4*(a*B -
b*C)*(Log[Cos[c + d*x]] + Log[Tan[c + d*x]])))/(12*d)
```

Maple [A] time = 0.073, size = 150, normalized size = 1.4

$$
-\frac{Bb(\cot(dx+c))^3}{3d}+\frac{B\cot(dx+c)b}{d}+Bxb+\frac{Bbc}{d}-\frac{Cb(\cot(dx+c))^2}{2d}-\frac{Cb\ln(\sin(dx+c))}{d}-\frac{aB(\cot(dx+c))^4}{4d}+\frac{al
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

```
[Out] -1/3/d*B*b*cot(d*x+c)^3+1/d*B*cot(d*x+c)*b+B*x*b+1/d*B*b*c-1/2/d*C*b*cot(d*
x+c)^2-1/d*C*b*ln(sin(d*x+c))-1/4*a*B*cot(d*x+c)^4/d+1/2/d*a*B*cot(d*x+c)^2
```

```
+1/d*a*B*ln(sin(d*x+c))-1/3/d*C*a*cot(d*x+c)^3+1/d*C*cot(d*x+c)*a+C*x*a+1/d
*C*a*c
```

Maxima [A] time = 1.67937, size = 165, normalized size = 1.53

$$\frac{12\,(Ca+Bb)(dx+c)-6\,(Ba-Cb)\log\left(\tan\left(dx+c\right)^{2}+1\right)+12\,(Ba-Cb)\log\left(\tan\left(dx+c\right)\right)+\frac{12\,(Ca+Bb)\tan(dx+c)^{3}+6\,(Ba\cdots}{\cdots}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/12*(12*(C*a + B*b)*(d*x + c) - 6*(B*a - C*b)*log(tan(d*x + c)^2 + 1) + 12
*(B*a - C*b)*log(tan(d*x + c)) + (12*(C*a + B*b)*tan(d*x + c)^3 + 6*(B*a -
C*b)*tan(d*x + c)^2 - 3*B*a - 4*(C*a + B*b)*tan(d*x + c))/tan(d*x + c)^4)/d
```

Fricas [A] time = 1.38618, size = 340, normalized size = 3.15

$$\frac{6\,(Ba-Cb)\log\left(\frac{\tan(dx+c)^{2}}{\tan(dx+c)^{2}+1}\right)\tan\left(dx+c\right)^{4}+3\,(4\,(Ca+Bb)dx+3\,Ba-2\,Cb)\tan\left(dx+c\right)^{4}+12\,(Ca+Bb)\tan\left(dx+\cdots\right.}{12\,d\tan\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/12*(6*(B*a - C*b)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^4
 + 3*(4*(C*a + B*b)*d*x + 3*B*a - 2*C*b)*tan(d*x + c)^4 + 12*(C*a + B*b)*ta
n(d*x + c)^3 + 6*(B*a - C*b)*tan(d*x + c)^2 - 3*B*a - 4*(C*a + B*b)*tan(d*x
 + c))/(d*tan(d*x + c)^4)
```

Sympy [A] time = 88.9161, size = 211, normalized size = 1.95

$$\begin{cases} \text{NaN} \\ x\left(a+b\tan\left(c\right)\right)\left(B\tan\left(c\right)+C\tan^{2}\left(c\right)\right)\cot^{6}\left(c\right) \\ -\frac{Ba\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d}+\frac{Ba\log\left(\tan\left(c+dx\right)\right)}{d}+\frac{Ba}{2d\tan^{2}\left(c+dx\right)}-\frac{Ba}{4d\tan^{4}\left(c+dx\right)}+Bbx+\frac{Bb}{d\tan\left(c+dx\right)}-\frac{Bb}{3d\tan^{3}\left(c+dx\right)}+Cax+\frac{Ca}{d\tan\left(c\cdots\right.} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

```
[Out] Piecewise((nan, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x))), (x*(a
 + b*tan(c))*(B*tan(c) + C*tan(c)**2)*cot(c)**6, Eq(d, 0)), (-B*a*log(tan(c
 + d*x)**2 + 1)/(2*d) + B*a*log(tan(c + d*x))/d + B*a/(2*d*tan(c + d*x)**2)
 - B*a/(4*d*tan(c + d*x)**4) + B*b*x + B*b/(d*tan(c + d*x)) - B*b/(3*d*tan(
c + d*x)**3) + C*a*x + C*a/(d*tan(c + d*x)) - C*a/(3*d*tan(c + d*x)**3) + C
*b*log(tan(c + d*x)**2 + 1)/(2*d) - C*b*log(tan(c + d*x))/d - C*b/(2*d*tan(
c + d*x)**2), True))
```

Giac [B] time = 1.58677, size = 404, normalized size = 3.74

$$-\frac{3\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-8\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-8\,Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-36\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+24\,Cb\tan\left(\frac{1}{2}\,dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] -1/192*(3*B*a*tan(1/2*d*x + 1/2*c)^4 - 8*C*a*tan(1/2*d*x + 1/2*c)^3 - 8*B*b
*tan(1/2*d*x + 1/2*c)^3 - 36*B*a*tan(1/2*d*x + 1/2*c)^2 + 24*C*b*tan(1/2*d*
x + 1/2*c)^2 + 120*C*a*tan(1/2*d*x + 1/2*c) + 120*B*b*tan(1/2*d*x + 1/2*c)
- 192*(C*a + B*b)*(d*x + c) + 192*(B*a - C*b)*log(tan(1/2*d*x + 1/2*c)^2 +
1) - 192*(B*a - C*b)*log(abs(tan(1/2*d*x + 1/2*c))) + (400*B*a*tan(1/2*d*x
+ 1/2*c)^4 - 400*C*b*tan(1/2*d*x + 1/2*c)^4 - 120*C*a*tan(1/2*d*x + 1/2*c)^
3 - 120*B*b*tan(1/2*d*x + 1/2*c)^3 - 36*B*a*tan(1/2*d*x + 1/2*c)^2 + 24*C*b
*tan(1/2*d*x + 1/2*c)^2 + 8*C*a*tan(1/2*d*x + 1/2*c) + 8*B*b*tan(1/2*d*x +
1/2*c) + 3*B*a)/tan(1/2*d*x + 1/2*c)^4)/d
```

3.9 $\int \tan(c+dx)(a+b\tan(c+dx))^2 \left(B\tan(c+dx)+C\tan^2(c+$

Optimal. Leaf size=148

$$\frac{\left(a^2C+2abB-b^2C\right)\log(\cos(c+dx))}{d}-x\left(a^2B-2abC-b^2B\right)+\frac{(4bB-aC)(a+b\tan(c+dx))^3}{12b^2d}-\frac{b(aC+bB)\tan(c}{d}$$

```
[Out] -((a^2*B - b^2*B - 2*a*b*C)*x) + ((2*a*b*B + a^2*C - b^2*C)*Log[Cos[c + d*x
]])/d - (b*(b*B + a*C)*Tan[c + d*x])/d - (C*(a + b*Tan[c + d*x])^2)/(2*d) +
 ((4*b*B - a*C)*(a + b*Tan[c + d*x])^3)/(12*b^2*d) + (C*Tan[c + d*x]*(a + b
*Tan[c + d*x])^3)/(4*b*d)
```

Rubi [A] time = 0.301735, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3632, 3607, 3630, 3528, 3525, 3475}

$$\frac{\left(a^2C+2abB-b^2C\right)\log(\cos(c+dx))}{d}-x\left(a^2B-2abC-b^2B\right)+\frac{(4bB-aC)(a+b\tan(c+dx))^3}{12b^2d}-\frac{b(aC+bB)\tan(c}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2)
,x]
```



```
[Out] -((a^2*B - b^2*B - 2*a*b*C)*x) + ((2*a*b*B + a^2*C - b^2*C)*Log[Cos[c + d*x
]])/d - (b*(b*B + a*C)*Tan[c + d*x])/d - (C*(a + b*Tan[c + d*x])^2)/(2*d) +
 ((4*b*B - a*C)*(a + b*Tan[c + d*x])^3)/(12*b^2*d) + (C*Tan[c + d*x]*(a + b
*Tan[c + d*x])^3)/(4*b*d)
```


Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3607

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*B*(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(m
+ n)), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan
[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
+ n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b*
(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1] &
& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
```

```
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \tan(c+dx)(a+b\tan(c+dx))^2 \left(B\tan(c+dx)+C\tan^2(c+dx)\right) dx = \int \tan^2(c+dx)(a+b\tan(c+dx))^2(B+C\tan$$
$$= \frac{C\tan(c+dx)(a+b\tan(c+dx))^3}{4bd} + \frac{\int (a+b\,\mathrm{t}}{}$$
$$= \frac{(4bB-aC)(a+b\tan(c+dx))^3}{12b^2d} + \frac{C\tan(c+d}{}$$
$$= -\frac{C(a+b\tan(c+dx))^2}{2d} + \frac{(4bB-aC)(a+b\,\mathrm{ta}}{12b^2d}$$
$$= -\left(a^2B-b^2B-2abC\right)x - \frac{b(bB+aC)\tan(c+}{d}$$
$$= -\left(a^2B-b^2B-2abC\right)x + \frac{\left(2abB+a^2C-b^2C\right.}{d}$$

Mathematica [C] time = 6.22293, size = 221, normalized size = 1.49

$$\frac{C\tan(c+dx)(a+b\tan(c+dx))^3}{4bd} + \frac{\frac{(4bB-aC)(a+b\tan(c+dx))^3}{3bd} + \frac{2\left((bB-aC)\left(-i(a-ib)^2\log(\tan(c+dx)+i)+i(a+ib)^2\log(-\tan(c+dx)+i)-2b^2\tan(\right.\right.}{}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c + d
*x]^2),x]
```

```
[Out] (C*Tan[c + d*x]*(a + b*Tan[c + d*x])^3)/(4*b*d) + (((4*b*B - a*C)*(a + b*Ta
n[c + d*x])^3)/(3*b*d) + (2*((b*B - a*C)*(I*(a + I*b)^2*Log[I - Tan[c + d*x
]] - I*(a - I*b)^2*Log[I + Tan[c + d*x]] - 2*b^2*Tan[c + d*x]) - C*((I*a -
b)^3*Log[I - Tan[c + d*x]] - (I*a + b)^3*Log[I + Tan[c + d*x]] + 6*a*b^2*Ta
n[c + d*x] + b^3*Tan[c + d*x]^2)))/d)/(4*b)
```

Maple [A] time = 0.013, size = 249, normalized size = 1.7

$$\frac{b^2C(\tan(dx+c))^4}{4d}+\frac{B(\tan(dx+c))^3b^2}{3d}+\frac{2C(\tan(dx+c))^3ab}{3d}+\frac{Bab(\tan(dx+c))^2}{d}+\frac{C(\tan(dx+c))^2a^2}{2d}-\frac{b^2\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

```
[Out] 1/4/d*b^2*C*tan(d*x+c)^4+1/3/d*B*tan(d*x+c)^3*b^2+2/3/d*C*tan(d*x+c)^3*a*b+
1/d*B*a*b*tan(d*x+c)^2+1/2/d*C*tan(d*x+c)^2*a^2-1/2/d*b^2*C*tan(d*x+c)^2+1/
d*a^2*B*tan(d*x+c)-1/d*b^2*B*tan(d*x+c)-2/d*C*a*b*tan(d*x+c)-1/d*ln(1+tan(d
*x+c)^2)*B*a*b-1/2/d*ln(1+tan(d*x+c)^2)*C*a^2+1/2/d*ln(1+tan(d*x+c)^2)*b^2*
C-1/d*a^2*B*arctan(tan(d*x+c))+1/d*B*arctan(tan(d*x+c))*b^2+2/d*C*arctan(ta
n(d*x+c))*a*b
```

Maxima [A] time = 1.71334, size = 198, normalized size = 1.34

$$\frac{3Cb^2\tan(dx+c)^4+4\left(2Cab+Bb^2\right)\tan(dx+c)^3+6\left(Ca^2+2Bab-Cb^2\right)\tan(dx+c)^2-12\left(Ba^2-2Cab-Bb^2\right)(\cdots}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/12*(3*C*b^2*tan(d*x + c)^4 + 4*(2*C*a*b + B*b^2)*tan(d*x + c)^3 + 6*(C*a^
2 + 2*B*a*b - C*b^2)*tan(d*x + c)^2 - 12*(B*a^2 - 2*C*a*b - B*b^2)*(d*x + c
) - 6*(C*a^2 + 2*B*a*b - C*b^2)*log(tan(d*x + c)^2 + 1) + 12*(B*a^2 - 2*C*a
*b - B*b^2)*tan(d*x + c))/d
```

Fricas [A] time = 1.39793, size = 340, normalized size = 2.3

$$\frac{3Cb^2\tan(dx+c)^4+4\left(2Cab+Bb^2\right)\tan(dx+c)^3-12\left(Ba^2-2Cab-Bb^2\right)dx+6\left(Ca^2+2Bab-Cb^2\right)\tan(dx+c)^{\cdots}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/12*(3*C*b^2*tan(d*x + c)^4 + 4*(2*C*a*b + B*b^2)*tan(d*x + c)^3 - 12*(B*a
^2 - 2*C*a*b - B*b^2)*d*x + 6*(C*a^2 + 2*B*a*b - C*b^2)*tan(d*x + c)^2 + 6*
(C*a^2 + 2*B*a*b - C*b^2)*log(1/(tan(d*x + c)^2 + 1)) + 12*(B*a^2 - 2*C*a*b
 - B*b^2)*tan(d*x + c))/d
```

Sympy [A] time = 1.93907, size = 250, normalized size = 1.69

$$\begin{cases}-Ba^2x+\frac{Ba^2\tan(c+dx)}{d}-\frac{Bab\log\left(\tan^2(c+dx)+1\right)}{d}+\frac{Bab\tan^2(c+dx)}{d}+Bb^2x+\frac{Bb^2\tan^3(c+dx)}{3d}-\frac{Bb^2\tan(c+dx)}{d}-\frac{Ca^2\log\left(\tan^2(c+dx)+1\right)}{2d}\cdots \\ x\left(a+b\tan(c)\right)^2\left(B\tan(c)+C\tan^2(c)\right)\tan(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))**2*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

```
[Out] Piecewise((-B*a**2*x + B*a**2*tan(c + d*x)/d - B*a*b*log(tan(c + d*x)**2 +
1)/d + B*a*b*tan(c + d*x)**2/d + B*b**2*x + B*b**2*tan(c + d*x)**3/(3*d) -
B*b**2*tan(c + d*x)/d - C*a**2*log(tan(c + d*x)**2 + 1)/(2*d) + C*a**2*tan(
c + d*x)**2/(2*d) + 2*C*a*b*x + 2*C*a*b*tan(c + d*x)**3/(3*d) - 2*C*a*b*tan
(c + d*x)/d + C*b**2*log(tan(c + d*x)**2 + 1)/(2*d) + C*b**2*tan(c + d*x)**
4/(4*d) - C*b**2*tan(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*tan(c))**2*(B*
tan(c) + C*tan(c)**2)*tan(c), True))
```

Giac [B] time = 4.9075, size = 3008, normalized size = 20.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] -1/12*(12*B*a^2*d*x*tan(d*x)^4*tan(c)^4 - 24*C*a*b*d*x*tan(d*x)^4*tan(c)^4
- 12*B*b^2*d*x*tan(d*x)^4*tan(c)^4 - 6*C*a^2*log(4*(tan(c)^2 + 1)/(tan(d*x)
^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*ta
n(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^4 - 12*B*a*b*log(4*(tan(c)^2 + 1)/(ta
n(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2
- 2*tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^4 + 6*C*b^2*log(4*(tan(c)^2 + 1
)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*
x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^4 - 48*B*a^2*d*x*tan(d*x)^
3*tan(c)^3 + 96*C*a*b*d*x*tan(d*x)^3*tan(c)^3 + 48*B*b^2*d*x*tan(d*x)^3*tan
(c)^3 - 6*C*a^2*tan(d*x)^4*tan(c)^4 - 12*B*a*b*tan(d*x)^4*tan(c)^4 + 9*C*b^
2*tan(d*x)^4*tan(c)^4 + 24*C*a^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2
- 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c
) + 1))*tan(d*x)^3*tan(c)^3 + 48*B*a*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan
(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)
*tan(c) + 1))*tan(d*x)^3*tan(c)^3 - 24*C*b^2*log(4*(tan(c)^2 + 1)/(tan(d*x)
^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*ta
n(d*x)*tan(c) + 1))*tan(d*x)^3*tan(c)^3 + 12*B*a^2*tan(d*x)^4*tan(c)^3 - 24
*C*a*b*tan(d*x)^4*tan(c)^3 - 12*B*b^2*tan(d*x)^4*tan(c)^3 + 12*B*a^2*tan(d*
x)^3*tan(c)^4 - 24*C*a*b*tan(d*x)^3*tan(c)^4 - 12*B*b^2*tan(d*x)^3*tan(c)^4
 + 72*B*a^2*d*x*tan(d*x)^2*tan(c)^2 - 144*C*a*b*d*x*tan(d*x)^2*tan(c)^2 - 7
2*B*b^2*d*x*tan(d*x)^2*tan(c)^2 - 6*C*a^2*tan(d*x)^4*tan(c)^2 - 12*B*a*b*ta
n(d*x)^4*tan(c)^2 + 6*C*b^2*tan(d*x)^4*tan(c)^2 + 12*C*a^2*tan(d*x)^3*tan(c
)^3 + 24*B*a*b*tan(d*x)^3*tan(c)^3 - 24*C*b^2*tan(d*x)^3*tan(c)^3 - 6*C*a^2
*tan(d*x)^2*tan(c)^4 - 12*B*a*b*tan(d*x)^2*tan(c)^4 + 6*C*b^2*tan(d*x)^2*ta
n(c)^4 + 8*C*a*b*tan(d*x)^4*tan(c) + 4*B*b^2*tan(d*x)^4*tan(c) - 36*C*a^2*l
og(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2
*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^2 - 72*B
*a*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(
d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^2
+ 36*C*b^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c)
+ tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan
(c)^2 - 36*B*a^2*tan(d*x)^3*tan(c)^2 + 96*C*a*b*tan(d*x)^3*tan(c)^2 + 48*B*
b^2*tan(d*x)^3*tan(c)^2 - 36*B*a^2*tan(d*x)^2*tan(c)^3 + 96*C*a*b*tan(d*x)^
2*tan(c)^3 + 48*B*b^2*tan(d*x)^2*tan(c)^3 + 8*C*a*b*tan(d*x)*tan(c)^4 + 4*B
*b^2*tan(d*x)*tan(c)^4 - 3*C*b^2*tan(d*x)^4 - 48*B*a^2*d*x*tan(d*x)*tan(c)
+ 96*C*a*b*d*x*tan(d*x)*tan(c) + 48*B*b^2*d*x*tan(d*x)*tan(c) + 12*C*a^2*ta
```

```
n(d*x)^3*tan(c) + 24*B*a*b*tan(d*x)^3*tan(c) - 24*C*b^2*tan(d*x)^3*tan(c) -
 12*C*a^2*tan(d*x)^2*tan(c)^2 - 24*B*a*b*tan(d*x)^2*tan(c)^2 + 12*C*b^2*tan
(d*x)^2*tan(c)^2 + 12*C*a^2*tan(d*x)*tan(c)^3 + 24*B*a*b*tan(d*x)*tan(c)^3
- 24*C*b^2*tan(d*x)*tan(c)^3 - 3*C*b^2*tan(c)^4 - 8*C*a*b*tan(d*x)^3 - 4*B*
b^2*tan(d*x)^3 + 24*C*a^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan
(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))
*tan(d*x)*tan(c) + 48*B*a*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*t
an(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1
))*tan(d*x)*tan(c) - 24*C*b^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2
*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) +
 1))*tan(d*x)*tan(c) + 36*B*a^2*tan(d*x)^2*tan(c) - 96*C*a*b*tan(d*x)^2*tan
(c) - 48*B*b^2*tan(d*x)^2*tan(c) + 36*B*a^2*tan(d*x)*tan(c)^2 - 96*C*a*b*ta
n(d*x)*tan(c)^2 - 48*B*b^2*tan(d*x)*tan(c)^2 - 8*C*a*b*tan(c)^3 - 4*B*b^2*t
an(c)^3 + 12*B*a^2*d*x - 24*C*a*b*d*x - 12*B*b^2*d*x - 6*C*a^2*tan(d*x)^2 -
 12*B*a*b*tan(d*x)^2 + 6*C*b^2*tan(d*x)^2 + 12*C*a^2*tan(d*x)*tan(c) + 24*B
*a*b*tan(d*x)*tan(c) - 24*C*b^2*tan(d*x)*tan(c) - 6*C*a^2*tan(c)^2 - 12*B*a
*b*tan(c)^2 + 6*C*b^2*tan(c)^2 - 6*C*a^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*t
an(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*
x)*tan(c) + 1)) - 12*B*a*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*ta
n(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)
) + 6*C*b^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c)
 + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)) - 12*B*a^2*ta
n(d*x) + 24*C*a*b*tan(d*x) + 12*B*b^2*tan(d*x) - 12*B*a^2*tan(c) + 24*C*a*b
*tan(c) + 12*B*b^2*tan(c) - 6*C*a^2 - 12*B*a*b + 9*C*b^2)/(d*tan(d*x)^4*tan
(c)^4 - 4*d*tan(d*x)^3*tan(c)^3 + 6*d*tan(d*x)^2*tan(c)^2 - 4*d*tan(d*x)*ta
n(c) + d)
```

3.10 $\int (a+b\tan(c+dx))^2 \left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx$

Optimal. Leaf size=112

$$-\frac{\left(a^2B-2abC-b^2B\right)\log(\cos(c+dx))}{d}-x\left(a^2C+2abB-b^2C\right)+\frac{b(aB-bC)\tan(c+dx)}{d}+\frac{B(a+b\tan(c+dx))^2}{2d}+\frac{C(a}{}$$

```
[Out] -((2*a*b*B + a^2*C - b^2*C)*x) - ((a^2*B - b^2*B - 2*a*b*C)*Log[Cos[c + d*x
]])/d + (b*(a*B - b*C)*Tan[c + d*x])/d + (B*(a + b*Tan[c + d*x])^2)/(2*d) +
 (C*(a + b*Tan[c + d*x])^3)/(3*b*d)
```

Rubi [A] time = 0.111664, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3630, 3528, 3525, 3475}

$$-\frac{\left(a^2B-2abC-b^2B\right)\log(\cos(c+dx))}{d}-x\left(a^2C+2abB-b^2C\right)+\frac{b(aB-bC)\tan(c+dx)}{d}+\frac{B(a+b\tan(c+dx))^2}{2d}+\frac{C(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2),x]
```

```
[Out] -((2*a*b*B + a^2*C - b^2*C)*x) - ((a^2*B - b^2*B - 2*a*b*C)*Log[Cos[c + d*x
]])/d + (b*(a*B - b*C)*Tan[c + d*x])/d + (B*(a + b*Tan[c + d*x])^2)/(2*d) +
 (C*(a + b*Tan[c + d*x])^3)/(3*b*d)
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx = \frac{C(a+b\tan(c+dx))^3}{3bd} + \int (a+b\tan(c+dx))^2(-C+B$$
$$= \frac{B(a+b\tan(c+dx))^2}{2d} + \frac{C(a+b\tan(c+dx))^3}{3bd} + \int (a$$
$$= -\left(2abB+a^2C-b^2C\right)x + \frac{b(aB-bC)\tan(c+dx)}{d} + B$$
$$= -\left(2abB+a^2C-b^2C\right)x - \frac{\left(a^2B-b^2B-2abC\right)\log(\cos}{d}$$

Mathematica [C] time = 1.78949, size = 172, normalized size = 1.54

$$\frac{3(aB+bC)\left(-2b^2\tan(c+dx)+i\left((a+ib)^2\log(-\tan(c+dx)+i)-(a-ib)^2\log(\tan(c+dx)+i)\right)\right)+3B\left(6ab^2\tan(c\right.}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2),x]
```

```
[Out] (2*C*(a + b*Tan[c + d*x])^3 + 3*(a*B + b*C)*(I*((a + I*b)^2*Log[I - Tan[c +
 d*x]] - (a - I*b)^2*Log[I + Tan[c + d*x]]) - 2*b^2*Tan[c + d*x]) + 3*B*((I
*a - b)^3*Log[I - Tan[c + d*x]] - (I*a + b)^3*Log[I + Tan[c + d*x]] + 6*a*b
^2*Tan[c + d*x] + b^3*Tan[c + d*x]^2))/(6*b*d)
```

Maple [A] time = 0.012, size = 199, normalized size = 1.8

$$\frac{b^2C(\tan(dx+c))^3}{3d} + \frac{B(\tan(dx+c))^2b^2}{2d} + \frac{C(\tan(dx+c))^2ab}{d} + 2\frac{Bab\tan(dx+c)}{d} + \frac{C\tan(dx+c)a^2}{d} - \frac{b^2C\tan}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

```
[Out] 1/3/d*b^2*C*tan(d*x+c)^3+1/2/d*B*tan(d*x+c)^2*b^2+1/d*C*tan(d*x+c)^2*a*b+2/
d*B*a*b*tan(d*x+c)+1/d*C*tan(d*x+c)*a^2-b^2*C*tan(d*x+c)/d+1/2/d*a^2*B*ln(1
+tan(d*x+c)^2)-1/2/d*ln(1+tan(d*x+c)^2)*b^2*B-1/d*ln(1+tan(d*x+c)^2)*C*a*b-
2/d*B*arctan(tan(d*x+c))*a*b-1/d*C*arctan(tan(d*x+c))*a^2+1/d*C*arctan(tan(
d*x+c))*b^2
```

Maxima [A] time = 1.7285, size = 162, normalized size = 1.45

$$\frac{2Cb^2\tan(dx+c)^3+3\left(2Cab+Bb^2\right)\tan(dx+c)^2-6\left(Ca^2+2Bab-Cb^2\right)(dx+c)+3\left(Ba^2-2Cab-Bb^2\right)\log\left(\tan\right.}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] 1/6*(2*C*b^2*tan(d*x + c)^3 + 3*(2*C*a*b + B*b^2)*tan(d*x + c)^2 - 6*(C*a^2
 + 2*B*a*b - C*b^2)*(d*x + c) + 3*(B*a^2 - 2*C*a*b - B*b^2)*log(tan(d*x + c
```

```
)^2 + 1) + 6*(C*a^2 + 2*B*a*b - C*b^2)*tan(d*x + c))/d
```

Fricas [A] time = 1.32028, size = 275, normalized size = 2.46

$$\frac{2\,Cb^2\tan\left(dx+c\right)^3-6\left(Ca^2+2\,Bab-Cb^2\right)dx+3\left(2\,Cab+Bb^2\right)\tan\left(dx+c\right)^2-3\left(Ba^2-2\,Cab-Bb^2\right)\log\left(\frac{1}{\tan(dx+c)^2+1}\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/6*(2*C*b^2*tan(d*x + c)^3 - 6*(C*a^2 + 2*B*a*b - C*b^2)*d*x + 3*(2*C*a*b
+ B*b^2)*tan(d*x + c)^2 - 3*(B*a^2 - 2*C*a*b - B*b^2)*log(1/(tan(d*x + c)^2
 + 1)) + 6*(C*a^2 + 2*B*a*b - C*b^2)*tan(d*x + c))/d
```

Sympy [A] time = 1.58424, size = 194, normalized size = 1.73

$$\begin{cases} \frac{Ba^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}-2Babx+\frac{2Bab\tan\left(c+dx\right)}{d}-\frac{Bb^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{Bb^2\tan^2\left(c+dx\right)}{2d}-Ca^2x+\frac{Ca^2\tan\left(c+dx\right)}{d}-\frac{Cab\log\left(\tan^2\left(c+\right.\right.}{d} \\ x\left(a+b\tan\left(c\right)\right)^2\left(B\tan\left(c\right)+C\tan^2\left(c\right)\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**2*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

```
[Out] Piecewise((B*a**2*log(tan(c + d*x)**2 + 1)/(2*d) - 2*B*a*b*x + 2*B*a*b*tan(
c + d*x)/d - B*b**2*log(tan(c + d*x)**2 + 1)/(2*d) + B*b**2*tan(c + d*x)**2
/(2*d) - C*a**2*x + C*a**2*tan(c + d*x)/d - C*a*b*log(tan(c + d*x)**2 + 1)/
d + C*a*b*tan(c + d*x)**2/d + C*b**2*x + C*b**2*tan(c + d*x)**3/(3*d) - C*b
**2*tan(c + d*x)/d, Ne(d, 0)), (x*(a + b*tan(c))**2*(B*tan(c) + C*tan(c)**2
), True))
```

Giac [B] time = 3.21836, size = 2037, normalized size = 18.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] -1/6*(6*C*a^2*d*x*tan(d*x)^3*tan(c)^3 + 12*B*a*b*d*x*tan(d*x)^3*tan(c)^3 -
6*C*b^2*d*x*tan(d*x)^3*tan(c)^3 + 3*B*a^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*
tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d
*x)*tan(c) + 1))*tan(d*x)^3*tan(c)^3 - 6*C*a*b*log(4*(tan(c)^2 + 1)/(tan(d*
x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*
tan(d*x)*tan(c) + 1))*tan(d*x)^3*tan(c)^3 - 3*B*b^2*log(4*(tan(c)^2 + 1)/(t
an(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2
 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^3*tan(c)^3 - 18*C*a^2*d*x*tan(d*x)^2*ta
n(c)^2 - 36*B*a*b*d*x*tan(d*x)^2*tan(c)^2 + 18*C*b^2*d*x*tan(d*x)^2*tan(c)^
2 - 6*C*a*b*tan(d*x)^3*tan(c)^3 - 3*B*b^2*tan(d*x)^3*tan(c)^3 - 9*B*a^2*log
```

```
(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*t
an(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^2 + 18*C*a
*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*
x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^2 +
9*B*b^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + t
an(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)
^2 + 6*C*a^2*tan(d*x)^3*tan(c)^2 + 12*B*a*b*tan(d*x)^3*tan(c)^2 - 6*C*b^2*t
an(d*x)^3*tan(c)^2 + 6*C*a^2*tan(d*x)^2*tan(c)^3 + 12*B*a*b*tan(d*x)^2*tan(
c)^3 - 6*C*b^2*tan(d*x)^2*tan(c)^3 + 18*C*a^2*d*x*tan(d*x)*tan(c) + 36*B*a*
b*d*x*tan(d*x)*tan(c) - 18*C*b^2*d*x*tan(d*x)*tan(c) - 6*C*a*b*tan(d*x)^3*t
an(c) - 3*B*b^2*tan(d*x)^3*tan(c) + 6*C*a*b*tan(d*x)^2*tan(c)^2 + 3*B*b^2*t
an(d*x)^2*tan(c)^2 - 6*C*a*b*tan(d*x)*tan(c)^3 - 3*B*b^2*tan(d*x)*tan(c)^3
+ 2*C*b^2*tan(d*x)^3 + 9*B*a^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 -
2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c)
+ 1))*tan(d*x)*tan(c) - 18*C*a*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2
- 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c
) + 1))*tan(d*x)*tan(c) - 9*B*b^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2
 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(
c) + 1))*tan(d*x)*tan(c) - 12*C*a^2*tan(d*x)^2*tan(c) - 24*B*a*b*tan(d*x)^2
*tan(c) + 18*C*b^2*tan(d*x)^2*tan(c) - 12*C*a^2*tan(d*x)*tan(c)^2 - 24*B*a*
b*tan(d*x)*tan(c)^2 + 18*C*b^2*tan(d*x)*tan(c)^2 + 2*C*b^2*tan(c)^3 - 6*C*a
^2*d*x - 12*B*a*b*d*x + 6*C*b^2*d*x + 6*C*a*b*tan(d*x)^2 + 3*B*b^2*tan(d*x)
^2 - 6*C*a*b*tan(d*x)*tan(c) - 3*B*b^2*tan(d*x)*tan(c) + 6*C*a*b*tan(c)^2 +
 3*B*b^2*tan(c)^2 - 3*B*a^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*t
an(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1
)) + 6*C*a*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c
) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)) + 3*B*b^2*lo
g(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*
tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)) + 6*C*a^2*tan(d*x) + 12*B*a
*b*tan(d*x) - 6*C*b^2*tan(d*x) + 6*C*a^2*tan(c) + 12*B*a*b*tan(c) - 6*C*b^2
*tan(c) + 6*C*a*b + 3*B*b^2)/(d*tan(d*x)^3*tan(c)^3 - 3*d*tan(d*x)^2*tan(c)
^2 + 3*d*tan(d*x)*tan(c) - d)
```

3.11 $\int \cot(c+dx)(a+b\tan(c+dx))^2 \left(B\tan(c+dx)+C\tan^2(c+d$

Optimal. Leaf size=87

$$-\frac{\left(a^2C+2abB-b^2C\right)\log(\cos(c+dx))}{d}+x\left(a^2B-2abC-b^2B\right)+\frac{b(aC+bB)\tan(c+dx)}{d}+\frac{C(a+b\tan(c+dx))^2}{2d}$$

```
[Out] (a^2*B - b^2*B - 2*a*b*C)*x - ((2*a*b*B + a^2*C - b^2*C)*Log[Cos[c + d*x]])
/d + (b*(b*B + a*C)*Tan[c + d*x])/d + (C*(a + b*Tan[c + d*x])^2)/(2*d)
```

Rubi [A] time = 0.135311, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3632, 3528, 3525, 3475}

$$-\frac{\left(a^2C+2abB-b^2C\right)\log(\cos(c+dx))}{d}+x\left(a^2B-2abC-b^2B\right)+\frac{b(aC+bB)\tan(c+dx)}{d}+\frac{C(a+b\tan(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2)
,x]
```

```
[Out] (a^2*B - b^2*B - 2*a*b*C)*x - ((2*a*b*B + a^2*C - b^2*C)*Log[Cos[c + d*x]])
/d + (b*(b*B + a*C)*Tan[c + d*x])/d + (C*(a + b*Tan[c + d*x])^2)/(2*d)
```

Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx = \int (a+b\tan(c+dx))^2(B+C\tan(c+dx))\,d$$
$$= \frac{C(a+b\tan(c+dx))^2}{2d} + \int (a+b\tan(c+d$$
$$= \left(a^2B-b^2B-2abC\right)x + \frac{b(bB+aC)\tan(c+}{d}$$
$$= \left(a^2B-b^2B-2abC\right)x - \frac{\left(2abB+a^2C-b^2C\right.}{d}$$

Mathematica [C] time = 0.450116, size = 96, normalized size = 1.1

$$\frac{2b(2aC+bB)\tan(c+dx)+(a-ib)^2(C+iB)\log(\tan(c+dx)+i)+(a+ib)^2(C-iB)\log(-\tan(c+dx)+i)+b^2C\,\mathrm{ta}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c + d
*x]^2),x]
```

```
[Out] ((a + I*b)^2*((-I)*B + C)*Log[I - Tan[c + d*x]] + (a - I*b)^2*(I*B + C)*Log
[I + Tan[c + d*x]] + 2*b*(b*B + 2*a*C)*Tan[c + d*x] + b^2*C*Tan[c + d*x]^2)
/(2*d)
```

Maple [A] time = 0.079, size = 140, normalized size = 1.6

$$-b^2Bx+\frac{b^2B\tan(dx+c)}{d}-\frac{Bb^2c}{d}+\frac{b^2C(\tan(dx+c))^2}{2d}+\frac{b^2C\ln(\cos(dx+c))}{d}-2\frac{Bab\ln(\cos(dx+c))}{d}-2Cabx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

```
[Out] -b^2*B*x+1/d*b^2*B*tan(d*x+c)-1/d*B*b^2*c+1/2/d*b^2*C*tan(d*x+c)^2+b^2*C*ln
(cos(d*x+c))/d-2/d*B*a*b*ln(cos(d*x+c))-2*C*a*b*x+2/d*C*a*b*tan(d*x+c)-2/d*
C*a*b*c+a^2*B*x+1/d*B*a^2*c-1/d*C*a^2*ln(cos(d*x+c))
```

Maxima [A] time = 1.74923, size = 123, normalized size = 1.41

$$\frac{Cb^2\tan(dx+c)^2+2\left(Ba^2-2Cab-Bb^2\right)(dx+c)+\left(Ca^2+2Bab-Cb^2\right)\log\left(\tan(dx+c)^2+1\right)+2\left(2Cab+Bb^2\right)\mathrm{ta}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/2*(C*b^2*tan(d*x + c)^2 + 2*(B*a^2 - 2*C*a*b - B*b^2)*(d*x + c) + (C*a^2
+ 2*B*a*b - C*b^2)*log(tan(d*x + c)^2 + 1) + 2*(2*C*a*b + B*b^2)*tan(d*x +
c))/d
```

Fricas [A] time = 1.36793, size = 209, normalized size = 2.4

$$\frac{Cb^2 \tan(dx+c)^2 + 2\left(Ba^2 - 2Cab - Bb^2\right)dx - \left(Ca^2 + 2Bab - Cb^2\right)\log\left(\frac{1}{\tan(dx+c)^2+1}\right) + 2\left(2Cab + Bb^2\right)\tan(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/2*(C*b^2*tan(d*x + c)^2 + 2*(B*a^2 - 2*C*a*b - B*b^2)*d*x - (C*a^2 + 2*B*
a*b - C*b^2)*log(1/(tan(d*x + c)^2 + 1)) + 2*(2*C*a*b + B*b^2)*tan(d*x + c)
)/d
```

Sympy [A] time = 5.89005, size = 151, normalized size = 1.74

$$\begin{cases} Ba^2x + \frac{Bab\log\left(\tan^2(c+dx)+1\right)}{d} - Bb^2x + \frac{Bb^2\tan(c+dx)}{d} + \frac{Ca^2\log\left(\tan^2(c+dx)+1\right)}{2d} - 2Cabx + \frac{2Cab\tan(c+dx)}{d} - \frac{Cb^2\log\left(\tan^2(c+dx)+1\right)}{2d} + \\ x\left(a+b\tan(c)\right)^2\left(B\tan(c) + C\tan^2(c)\right)\cot(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))**2*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

```
[Out] Piecewise((B*a**2*x + B*a*b*log(tan(c + d*x)**2 + 1)/d - B*b**2*x + B*b**2*
tan(c + d*x)/d + C*a**2*log(tan(c + d*x)**2 + 1)/(2*d) - 2*C*a*b*x + 2*C*a*
b*tan(c + d*x)/d - C*b**2*log(tan(c + d*x)**2 + 1)/(2*d) + C*b**2*tan(c + d
*x)**2/(2*d), Ne(d, 0)), (x*(a + b*tan(c))**2*(B*tan(c) + C*tan(c)**2)*cot(
c), True))
```

Giac [A] time = 1.74418, size = 128, normalized size = 1.47

$$\frac{Cb^2\tan(dx+c)^2 + 4Cab\tan(dx+c) + 2Bb^2\tan(dx+c) + 2\left(Ba^2 - 2Cab - Bb^2\right)(dx+c) + \left(Ca^2 + 2Bab - Cb^2\right)\log\left(\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] 1/2*(C*b^2*tan(d*x + c)^2 + 4*C*a*b*tan(d*x + c) + 2*B*b^2*tan(d*x + c) + 2
*(B*a^2 - 2*C*a*b - B*b^2)*(d*x + c) + (C*a^2 + 2*B*a*b - C*b^2)*log(tan(d*
x + c)^2 + 1))/d
```

3.12 $\int \cot^2(c+dx)(a+b\tan(c+dx))^2 \left(B\tan(c+dx)+C\tan^2(c\right.$

Optimal. Leaf size=70

$$x\left(a^2C+2abB-b^2C\right)+\frac{a^2B\log(\sin(c+dx))}{d}-\frac{b(2aC+bB)\log(\cos(c+dx))}{d}+\frac{b^2C\tan(c+dx)}{d}$$

```
[Out] (2*a*b*B + a^2*C - b^2*C)*x - (b*(b*B + 2*a*C)*Log[Cos[c + d*x]])/d + (a^2*
B*Log[Sin[c + d*x]])/d + (b^2*C*Tan[c + d*x])/d
```

Rubi [A] time = 0.184683, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3632, 3606, 3624, 3475}

$$x\left(a^2C+2abB-b^2C\right)+\frac{a^2B\log(\sin(c+dx))}{d}-\frac{b(2aC+bB)\log(\cos(c+dx))}{d}+\frac{b^2C\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^
2),x]
```

```
[Out] (2*a*b*B + a^2*C - b^2*C)*x - (b*(b*B + 2*a*C)*Log[Cos[c + d*x]])/d + (a^2*
B*Log[Sin[c + d*x]])/d + (b^2*C*Tan[c + d*x])/d
```

Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3606

```
Int[(((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (
f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b^
2*B*Tan[e + f*x])/(d*f), x] + Dist[1/d, Int[(a^2*A*d - b^2*B*c + (2*a*A*b +
 B*(a^2 - b^2))*d*Tan[e + f*x] + (A*b^2*d - b*B*(b*c - 2*a*d))*Tan[e + f*x]
^2)/(c + d*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && N
eQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3624

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[B*x, x] + (Dist[A, Int[1/Tan[e
 + f*x], x], x] + Dist[C, Int[Tan[e + f*x], x], x]) /; FreeQ[{e, f, A, B, C
}, x] && NeQ[A, C]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx = \int \cot(c+dx)(a+b\tan(c+dx))^2(B+C\tan$$
$$= \frac{b^2C\tan(c+dx)}{d} + \int \cot(c+dx)\left(a^2B+\left(2ab\right.\right.$$
$$= \left(2abB+a^2C-b^2C\right)x + \frac{b^2C\tan(c+dx)}{d} + ($$
$$= \left(2abB+a^2C-b^2C\right)x - \frac{b(bB+2aC)\log(\cos}{d}$$

Mathematica [C] time = 0.26808, size = 91, normalized size = 1.3

$$-\frac{-2a^2B\log(\tan(c+dx))+(a+ib)^2(B+iC)\log(-\tan(c+dx)+i)+(a-ib)^2(B-iC)\log(\tan(c+dx)+i)-2b^2C\tan(c}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c +
 d*x]^2),x]
```

```
[Out] -((a + I*b)^2*(B + I*C)*Log[I - Tan[c + d*x]] - 2*a^2*B*Log[Tan[c + d*x]] +
 (a - I*b)^2*(B - I*C)*Log[I + Tan[c + d*x]] - 2*b^2*C*Tan[c + d*x])/(2*d)
```

Maple [A] time = 0.076, size = 109, normalized size = 1.6

$$2\,Babx+Cxa^2-b^2Cx+\frac{a^2B\ln(\sin(dx+c))}{d}-\frac{b^2B\ln(\cos(dx+c))}{d}+2\,\frac{Babc}{d}+\frac{b^2C\tan(dx+c)}{d}-2\,\frac{Cab\ln(\cos(dx+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

```
[Out] 2*B*a*b*x+C*x*a^2-b^2*C*x+1/d*a^2*B*ln(sin(d*x+c))-1/d*b^2*B*ln(cos(d*x+c))
+2/d*B*a*b*c+b^2*C*tan(d*x+c)/d-2/d*C*a*b*ln(cos(d*x+c))+1/d*C*a^2*c-1/d*C*
b^2*c
```

Maxima [A] time = 1.74779, size = 115, normalized size = 1.64

$$\frac{2\,Ba^2\log(\tan(dx+c))+2\,Cb^2\tan(dx+c)+2\left(Ca^2+2\,Bab-Cb^2\right)(dx+c)-\left(Ba^2-2\,Cab-Bb^2\right)\log\left(\tan(dx+c)^2+\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/2*(2*B*a^2*log(tan(d*x + c)) + 2*C*b^2*tan(d*x + c) + 2*(C*a^2 + 2*B*a*b
- C*b^2)*(d*x + c) - (B*a^2 - 2*C*a*b - B*b^2)*log(tan(d*x + c)^2 + 1))/d
```

Fricas [A] time = 1.42113, size = 217, normalized size = 3.1

$$\frac{Ba^2\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)+2\,Cb^2\tan(dx+c)+2\left(Ca^2+2\,Bab-Cb^2\right)dx-\left(2\,Cab+Bb^2\right)\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/2*(B*a^2*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1)) + 2*C*b^2*tan(d*x + c)
+ 2*(C*a^2 + 2*B*a*b - C*b^2)*d*x - (2*C*a*b + B*b^2)*log(1/(tan(d*x + c)^2
 + 1)))/d
```

Sympy [A] time = 11.2536, size = 136, normalized size = 1.94

$$\begin{cases}-\frac{Ba^2\log\left(\tan^2(c+dx)+1\right)}{2d}+\frac{Ba^2\log(\tan(c+dx))}{d}+2Babx+\frac{Bb^2\log\left(\tan^2(c+dx)+1\right)}{2d}+Ca^2x+\frac{Cab\log\left(\tan^2(c+dx)+1\right)}{d}-Cb^2x+\frac{Cb^2\tan\ldots}{} \\ x\,(a+b\tan(c))^2\left(B\tan(c)+C\tan^2(c)\right)\cot^2(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*tan(d*x+c))**2*(B*tan(d*x+c)+C*tan(d*x+c)**2),
x)
```

```
[Out] Piecewise((-B*a**2*log(tan(c + d*x)**2 + 1)/(2*d) + B*a**2*log(tan(c + d*x)
)/d + 2*B*a*b*x + B*b**2*log(tan(c + d*x)**2 + 1)/(2*d) + C*a**2*x + C*a*b*
log(tan(c + d*x)**2 + 1)/d - C*b**2*x + C*b**2*tan(c + d*x)/d, Ne(d, 0)), (
x*(a + b*tan(c))**2*(B*tan(c) + C*tan(c)**2)*cot(c)**2, True))
```

Giac [A] time = 1.87633, size = 116, normalized size = 1.66

$$\frac{2\,Ba^2\log(|\tan(dx+c)|)+2\,Cb^2\tan(dx+c)+2\left(Ca^2+2\,Bab-Cb^2\right)(dx+c)-\left(Ba^2-2\,Cab-Bb^2\right)\log\left(\tan(dx+c\ldots\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/2*(2*B*a^2*log(abs(tan(d*x + c))) + 2*C*b^2*tan(d*x + c) + 2*(C*a^2 + 2*B
*a*b - C*b^2)*(d*x + c) - (B*a^2 - 2*C*a*b - B*b^2)*log(tan(d*x + c)^2 + 1)
)/d
```

3.13 $\int \cot^3(c+dx)(a+b\tan(c+dx))^2 \left(B\tan(c+dx)+C\tan^2(c+\right.$

Optimal. Leaf size=72

$$-x\left(a^2B-2abC-b^2B\right)-\frac{a^2B\cot(c+dx)}{d}+\frac{a(aC+2bB)\log(\sin(c+dx))}{d}-\frac{b^2C\log(\cos(c+dx))}{d}$$

```
[Out] -((a^2*B - b^2*B - 2*a*b*C)*x) - (a^2*B*Cot[c + d*x])/d - (b^2*C*Log[Cos[c
+ d*x]])/d + (a*(2*b*B + a*C)*Log[Sin[c + d*x]])/d
```

Rubi [A] time = 0.206626, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3632, 3604, 3624, 3475}

$$-x\left(a^2B-2abC-b^2B\right)-\frac{a^2B\cot(c+dx)}{d}+\frac{a(aC+2bB)\log(\sin(c+dx))}{d}-\frac{b^2C\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^
2),x]
```

```
[Out] -((a^2*B - b^2*B - 2*a*b*C)*x) - (a^2*B*Cot[c + d*x])/d - (b^2*C*Log[Cos[c
+ d*x]])/d + (a*(2*b*B + a*C)*Log[Sin[c + d*x]])/d
```

Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3604

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp
[((B*c - A*d)*(b*c - a*d)^2*(c + d*Tan[e + f*x])^(n + 1))/(f*d^2*(n + 1)*(c
^2 + d^2)), x] + Dist[1/(d*(c^2 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*S
imp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*c +
 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2)*T
an[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

Rule 3624

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[B*x, x] + (Dist[A, Int[1/Tan[e
 + f*x], x], x] + Dist[C, Int[Tan[e + f*x], x], x]) /; FreeQ[{e, f, A, B, C
}, x] && NeQ[A, C]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx = \int \cot^2(c+dx)(a+b\tan(c+dx))^2(B+C$$
$$= -\frac{a^2B\cot(c+dx)}{d} + \int \cot(c+dx)\left(a(2bB\right.$$
$$= -\left(a^2B-b^2B-2abC\right)x - \frac{a^2B\cot(c+dx)}{d}$$
$$= -\left(a^2B-b^2B-2abC\right)x - \frac{a^2B\cot(c+dx)}{d}$$

Mathematica [C] time = 0.247485, size = 100, normalized size = 1.39

$$\frac{-2a^2B\cot(c+dx)+2a(aC+2bB)\log(\tan(c+dx))+i(a+ib)^2(B+iC)\log(-\tan(c+dx)+i)-(a-ib)^2(C+iB)\log}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c +
 d*x]^2),x]
```

```
[Out] (-2*a^2*B*Cot[c + d*x] + I*(a + I*b)^2*(B + I*C)*Log[I - Tan[c + d*x]] + 2*
a*(2*b*B + a*C)*Log[Tan[c + d*x]] - (a - I*b)^2*(I*B + C)*Log[I + Tan[c + d
*x]])/(2*d)
```

Maple [A] time = 0.076, size = 110, normalized size = 1.5

$$-a^2Bx+b^2Bx+2\,Cabx-\frac{B\cot(dx+c)\,a^2}{d}+2\,\frac{Bab\ln(\sin(dx+c))}{d}-\frac{Ba^2c}{d}+\frac{Bb^2c}{d}+\frac{Ca^2\ln(\sin(dx+c))}{d}-\frac{b^2C\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

```
[Out] -a^2*B*x+b^2*B*x+2*C*a*b*x-1/d*B*cot(d*x+c)*a^2+2/d*B*a*b*ln(sin(d*x+c))-1/
d*B*a^2*c+1/d*B*b^2*c+1/d*C*a^2*ln(sin(d*x+c))-b^2*C*ln(cos(d*x+c))/d+2/d*C
*a*b*c
```

Maxima [A] time = 1.75614, size = 126, normalized size = 1.75

$$-\frac{2\left(Ba^2-2\,Cab-Bb^2\right)(dx+c)+\left(Ca^2+2\,Bab-Cb^2\right)\log\left(\tan(dx+c)^2+1\right)-2\left(Ca^2+2\,Bab\right)\log(\tan(dx+c))+}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] -1/2*(2*(B*a^2 - 2*C*a*b - B*b^2)*(d*x + c) + (C*a^2 + 2*B*a*b - C*b^2)*log
(tan(d*x + c)^2 + 1) - 2*(C*a^2 + 2*B*a*b)*log(tan(d*x + c)) + 2*B*a^2/tan(
d*x + c))/d
```

Fricas [A] time = 1.4045, size = 274, normalized size = 3.81

$$-\frac{Cb^2\log\left(\frac{1}{\tan(dx+c)^2+1}\right)\tan(dx+c)+2\left(Ba^2-2\,Cab-Bb^2\right)dx\tan(dx+c)+2\,Ba^2-\left(Ca^2+2\,Bab\right)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan(dx+c)}{2\,d\tan(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] -1/2*(C*b^2*log(1/(tan(d*x + c)^2 + 1))*tan(d*x + c) + 2*(B*a^2 - 2*C*a*b -
 B*b^2)*d*x*tan(d*x + c) + 2*B*a^2 - (C*a^2 + 2*B*a*b)*log(tan(d*x + c)^2/(
tan(d*x + c)^2 + 1))*tan(d*x + c))/(d*tan(d*x + c))
```

Sympy [A] time = 18.9267, size = 158, normalized size = 2.19

$$\begin{cases} \text{NaN} \\ x\left(a+b\tan(c)\right)^2\left(B\tan(c)+C\tan^2(c)\right)\cot^3(c) \\ \text{NaN} \\ -Ba^2x-\frac{Ba^2}{d\tan(c+dx)}-\frac{Bab\log\left(\tan^2(c+dx)+1\right)}{d}+\frac{2Bab\log(\tan(c+dx))}{d}+Bb^2x-\frac{Ca^2\log\left(\tan^2(c+dx)+1\right)}{2d}+\frac{Ca^2\log(\tan(c+dx))}{d}+2Cabx \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*tan(d*x+c))**2*(B*tan(d*x+c)+C*tan(d*x+c)**2),
x)
```

```
[Out] Piecewise((nan, Eq(c, 0) & Eq(d, 0)), (x*(a + b*tan(c))**2*(B*tan(c) + C*ta
n(c)**2)*cot(c)**3, Eq(d, 0)), (nan, Eq(c, -d*x)), (-B*a**2*x - B*a**2/(d*t
an(c + d*x)) - B*a*b*log(tan(c + d*x)**2 + 1)/d + 2*B*a*b*log(tan(c + d*x))
/d + B*b**2*x - C*a**2*log(tan(c + d*x)**2 + 1)/(2*d) + C*a**2*log(tan(c +
d*x))/d + 2*C*a*b*x + C*b**2*log(tan(c + d*x)**2 + 1)/(2*d), True))
```

Giac [A] time = 1.83617, size = 159, normalized size = 2.21

$$-\frac{2\left(Ba^2-2\,Cab-Bb^2\right)(dx+c)+\left(Ca^2+2\,Bab-Cb^2\right)\log\left(\tan(dx+c)^2+1\right)-2\left(Ca^2+2\,Bab\right)\log\left(|\tan(dx+c)|\right)+\frac{2(}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] -1/2*(2*(B*a^2 - 2*C*a*b - B*b^2)*(d*x + c) + (C*a^2 + 2*B*a*b - C*b^2)*log
(tan(d*x + c)^2 + 1) - 2*(C*a^2 + 2*B*a*b)*log(abs(tan(d*x + c))) + 2*(C*a^
2*tan(d*x + c) + 2*B*a*b*tan(d*x + c) + B*a^2)/tan(d*x + c))/d
```

3.14 $\int \cot^4(c+dx)(a+b\tan(c+dx))^2 \left(B\tan(c+dx)+C\tan^2(c\right.$

Optimal. Leaf size=88

$$-\frac{\left(a^2B-2abC-b^2B\right)\log(\sin(c+dx))}{d}-\frac{a^2B\cot^2(c+dx)}{2d}+x\left(b^2C-a(aC+2bB)\right)-\frac{a(aC+2bB)\cot(c+dx)}{d}$$

```
[Out] (b^2*C - a*(2*b*B + a*C))*x - (a*(2*b*B + a*C)*Cot[c + d*x])/d - (a^2*B*Cot
[c + d*x]^2)/(2*d) - ((a^2*B - b^2*B - 2*a*b*C)*Log[Sin[c + d*x]])/d
```

Rubi [A] time = 0.263472, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3632, 3604, 3628, 3531, 3475}

$$-\frac{\left(a^2B-2abC-b^2B\right)\log(\sin(c+dx))}{d}-\frac{a^2B\cot^2(c+dx)}{2d}+x\left(b^2C-a(aC+2bB)\right)-\frac{a(aC+2bB)\cot(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^
2),x]
```



```
[Out] (b^2*C - a*(2*b*B + a*C))*x - (a*(2*b*B + a*C)*Cot[c + d*x])/d - (a^2*B*Cot
[c + d*x]^2)/(2*d) - ((a^2*B - b^2*B - 2*a*b*C)*Log[Sin[c + d*x]])/d
```


Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3604

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp
[((B*c - A*d)*(b*c - a*d)^2*(c + d*Tan[e + f*x])^(n + 1))/(f*d^2*(n + 1)*(c
^2 + d^2)), x] + Dist[1/(d*(c^2 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*S
imp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*c +
 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2)*T
an[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

Rule 3628

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2
- a*b*B + a^2*C)*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 + b^2)), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^4(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx &= \int \cot^3(c+dx)(a+b\tan(c+dx))^2(B+C\tan\\ &= -\frac{a^2B\cot^2(c+dx)}{2d}+\int \cot^2(c+dx)\left(a(2bB+\right.\\ &= -\frac{a(2bB+aC)\cot(c+dx)}{d}-\frac{a^2B\cot^2(c+dx)}{2d}\\ &= \left(b^2C-a(2bB+aC)\right)x-\frac{a(2bB+aC)\cot(c+}{d}\\ &= \left(b^2C-a(2bB+aC)\right)x-\frac{a(2bB+aC)\cot(c+}{d}\end{aligned}$$

Mathematica [C] time = 0.342002, size = 123, normalized size = 1.4

$$\frac{-2\left(a^2B-2abC-b^2B\right)\log(\tan(c+dx))-a^2B\cot^2(c+dx)-2a(aC+2bB)\cot(c+dx)+(a-ib)^2(B-iC)\log(\tan(c+}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c +
 d*x]^2),x]
```

```
[Out] (-2*a*(2*b*B + a*C)*Cot[c + d*x] - a^2*B*Cot[c + d*x]^2 + (a + I*b)^2*(B +
I*C)*Log[I - Tan[c + d*x]] - 2*(a^2*B - b^2*B - 2*a*b*C)*Log[Tan[c + d*x]]
+ (a - I*b)^2*(B - I*C)*Log[I + Tan[c + d*x]])/(2*d)
```

Maple [A] time = 0.095, size = 141, normalized size = 1.6

$$\frac{b^2B\ln(\sin(dx+c))}{d}+b^2Cx+\frac{Cb^2c}{d}-2Babx-2\frac{B\cot(dx+c)ab}{d}-2\frac{Babc}{d}+2\frac{Cab\ln(\sin(dx+c))}{d}-\frac{a^2B(\cot(dx+c}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

```
[Out] 1/d*b^2*B*ln(sin(d*x+c))+b^2*C*x+1/d*C*b^2*c-2*B*a*b*x-2/d*B*cot(d*x+c)*a*b
-2/d*B*a*b*c+2/d*C*a*b*ln(sin(d*x+c))-1/2/d*a^2*B*cot(d*x+c)^2-1/d*a^2*B*ln
(sin(d*x+c))-C*x*a^2-1/d*C*cot(d*x+c)*a^2-1/d*C*a^2*c
```

Maxima [A] time = 1.69442, size = 162, normalized size = 1.84

$$-\frac{2\left(Ca^2+2\,Bab-Cb^2\right)(dx+c)-\left(Ba^2-2\,Cab-Bb^2\right)\log\left(\tan\left(dx+c\right)^2+1\right)+2\left(Ba^2-2\,Cab-Bb^2\right)\log\left(\tan\left(dx+\right.\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] -1/2*(2*(C*a^2 + 2*B*a*b - C*b^2)*(d*x + c) - (B*a^2 - 2*C*a*b - B*b^2)*log
(tan(d*x + c)^2 + 1) + 2*(B*a^2 - 2*C*a*b - B*b^2)*log(tan(d*x + c)) + (B*a
^2 + 2*(C*a^2 + 2*B*a*b)*tan(d*x + c))/tan(d*x + c)^2)/d
```

Fricas [A] time = 1.4326, size = 285, normalized size = 3.24

$$-\frac{\left(Ba^2-2\,Cab-Bb^2\right)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^2+Ba^2+\left(Ba^2+2\left(Ca^2+2\,Bab-Cb^2\right)dx\right)\tan\left(dx+c\right)^2+2\left(Ca\right.}{2\,d\tan\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] -1/2*((B*a^2 - 2*C*a*b - B*b^2)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*ta
n(d*x + c)^2 + B*a^2 + (B*a^2 + 2*(C*a^2 + 2*B*a*b - C*b^2)*d*x)*tan(d*x +
c)^2 + 2*(C*a^2 + 2*B*a*b)*tan(d*x + c))/(d*tan(d*x + c)^2)
```

Sympy [A] time = 30.6038, size = 206, normalized size = 2.34

$$\begin{cases} \text{NaN} \\ x\left(a+b\tan\left(c\right)\right)^2\left(B\tan\left(c\right)+C\tan^2\left(c\right)\right)\cot^4\left(c\right) \\ \text{NaN} \\ \frac{Ba^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}-\frac{Ba^2\log\left(\tan\left(c+dx\right)\right)}{d}-\frac{Ba^2}{2d\tan^2\left(c+dx\right)}-2Babx-\frac{2Bab}{d\tan\left(c+dx\right)}-\frac{Bb^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{Bb^2\log\left(\tan\left(c+dx\right)\right)}{d}- \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*tan(d*x+c))**2*(B*tan(d*x+c)+C*tan(d*x+c)**2),
x)
```

```
[Out] Piecewise((nan, Eq(c, 0) & Eq(d, 0)), (x*(a + b*tan(c))**2*(B*tan(c) + C*ta
n(c)**2)*cot(c)**4, Eq(d, 0)), (nan, Eq(c, -d*x)), (B*a**2*log(tan(c + d*x)
**2 + 1)/(2*d) - B*a**2*log(tan(c + d*x))/d - B*a**2/(2*d*tan(c + d*x)**2)
- 2*B*a*b*x - 2*B*a*b/(d*tan(c + d*x)) - B*b**2*log(tan(c + d*x)**2 + 1)/(2
*d) + B*b**2*log(tan(c + d*x))/d - C*a**2*x - C*a**2/(d*tan(c + d*x)) - C*a
*b*log(tan(c + d*x)**2 + 1)/d + 2*C*a*b*log(tan(c + d*x))/d + C*b**2*x, Tru
e))
```

<u>Giac</u> [B] time = 2.04962, size = 320, normalized size = 3.64

$$-\frac{Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4\,Ca^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-8\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\left(Ca^2+2\,Bab-Cb^2\right)(dx+c)-8\left(Ba^2-2\,C\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] -1/8*(B*a^2*tan(1/2*d*x + 1/2*c)^2 - 4*C*a^2*tan(1/2*d*x + 1/2*c) - 8*B*a*b
*tan(1/2*d*x + 1/2*c) + 8*(C*a^2 + 2*B*a*b - C*b^2)*(d*x + c) - 8*(B*a^2 -
2*C*a*b - B*b^2)*log(tan(1/2*d*x + 1/2*c)^2 + 1) + 8*(B*a^2 - 2*C*a*b - B*b
^2)*log(abs(tan(1/2*d*x + 1/2*c))) - (12*B*a^2*tan(1/2*d*x + 1/2*c)^2 - 24*
C*a*b*tan(1/2*d*x + 1/2*c)^2 - 12*B*b^2*tan(1/2*d*x + 1/2*c)^2 - 4*C*a^2*ta
n(1/2*d*x + 1/2*c) - 8*B*a*b*tan(1/2*d*x + 1/2*c) - B*a^2)/tan(1/2*d*x + 1/
2*c)^2)/d
```

3.15 $\int \cot^5(c+dx)(a+b\tan(c+dx))^2 \left(B\tan(c+dx)+C\tan^2(c\right.$

Optimal. Leaf size=118

$$\frac{\left(a^2B-2abC-b^2B\right)\cot(c+dx)}{d}+x\left(a^2B-2abC-b^2B\right)-\frac{a^2B\cot^3(c+dx)}{3d}+\frac{\left(b^2C-a(aC+2bB)\right)\log(\sin(c+dx))}{d}$$

```
[Out] (a^2*B - b^2*B - 2*a*b*C)*x + ((a^2*B - b^2*B - 2*a*b*C)*Cot[c + d*x])/d -
(a*(2*b*B + a*C)*Cot[c + d*x]^2)/(2*d) - (a^2*B*Cot[c + d*x]^3)/(3*d) + ((b
^2*C - a*(2*b*B + a*C))*Log[Sin[c + d*x]])/d
```

Rubi [A] time = 0.31102, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {3632, 3604, 3628, 3529, 3531, 3475}

$$\frac{\left(a^2B-2abC-b^2B\right)\cot(c+dx)}{d}+x\left(a^2B-2abC-b^2B\right)-\frac{a^2B\cot^3(c+dx)}{3d}+\frac{\left(b^2C-a(aC+2bB)\right)\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^
2),x]
```


```
[Out] (a^2*B - b^2*B - 2*a*b*C)*x + ((a^2*B - b^2*B - 2*a*b*C)*Cot[c + d*x])/d -
(a*(2*b*B + a*C)*Cot[c + d*x]^2)/(2*d) - (a^2*B*Cot[c + d*x]^3)/(3*d) + ((b
^2*C - a*(2*b*B + a*C))*Log[Sin[c + d*x]])/d
```


Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3604

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp
[((B*c - A*d)*(b*c - a*d)^2*(c + d*Tan[e + f*x])^(n + 1))/(f*d^2*(n + 1)*(c
^2 + d^2)), x] + Dist[1/(d*(c^2 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*S
imp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*c +
 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2)*T
an[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

Rule 3628

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2
- a*b*B + a^2*C)*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 + b^2)), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx = \int \cot^4(c+dx)(a+b\tan(c+dx))^2(B+C\tan$$

$$= -\frac{a^2B\cot^3(c+dx)}{3d} + \int \cot^3(c+dx)\left(a(2bB+\right.$$

$$= -\frac{a(2bB+aC)\cot^2(c+dx)}{2d} - \frac{a^2B\cot^3(c+d}{3d}$$

$$= \frac{\left(a^2B-b^2B-2abC\right)\cot(c+dx)}{d} - \frac{a(2bB+a}{}$$

$$= \left(a^2B-b^2B-2abC\right)x + \frac{\left(a^2B-b^2B-2abC\right)}{d}$$

$$= \left(a^2B-b^2B-2abC\right)x + \frac{\left(a^2B-b^2B-2abC\right)}{d}$$

Mathematica [C] time = 1.1463, size = 152, normalized size = 1.29

$$\frac{6\left(a^2B-2abC-b^2B\right)\cot(c+dx)-6\left(a^2C+2abB-b^2C\right)\log(\tan(c+dx))-2a^2B\cot^3(c+dx)-3a(aC+2bB)\cot^2(c+}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c +
 d*x]^2),x]
```

```
[Out] (6*(a^2*B - b^2*B - 2*a*b*C)*Cot[c + d*x] - 3*a*(2*b*B + a*C)*Cot[c + d*x]^
2 - 2*a^2*B*Cot[c + d*x]^3 + 3*(a + I*b)^2*((-I)*B + C)*Log[I - Tan[c + d*x
]] - 6*(2*a*b*B + a^2*C - b^2*C)*Log[Tan[c + d*x]] + 3*(a - I*b)^2*(I*B + C
)*Log[I + Tan[c + d*x]])/(6*d)
```

Maple [A] time = 0.093, size = 188, normalized size = 1.6

$$-b^2Bx - \frac{B\cot(dx+c)\,b^2}{d} - \frac{Bb^2c}{d} + \frac{b^2C\ln(\sin(dx+c))}{d} - \frac{Bab(\cot(dx+c))^2}{d} - 2\frac{Bab\ln(\sin(dx+c))}{d} - 2Cabx - 2\frac{C}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

```
[Out] -b^2*B*x-1/d*B*cot(d*x+c)*b^2-1/d*B*b^2*c+1/d*b^2*C*ln(sin(d*x+c))-1/d*B*a*
b*cot(d*x+c)^2-2/d*B*a*b*ln(sin(d*x+c))-2*C*a*b*x-2/d*C*cot(d*x+c)*a*b-2/d*
C*a*b*c-1/3/d*a^2*B*cot(d*x+c)^3+1/d*B*cot(d*x+c)*a^2+a^2*B*x+1/d*B*a^2*c-1
/2/d*C*a^2*cot(d*x+c)^2-1/d*C*a^2*ln(sin(d*x+c))
```

Maxima [A] time = 1.68782, size = 201, normalized size = 1.7

$$\frac{6\left(Ba^2-2\,Cab-Bb^2\right)(dx+c)+3\left(Ca^2+2\,Bab-Cb^2\right)\log\left(\tan\left(dx+c\right)^2+1\right)-6\left(Ca^2+2\,Bab-Cb^2\right)\log\left(\tan\left(dx+\right.\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/6*(6*(B*a^2 - 2*C*a*b - B*b^2)*(d*x + c) + 3*(C*a^2 + 2*B*a*b - C*b^2)*lo
g(tan(d*x + c)^2 + 1) - 6*(C*a^2 + 2*B*a*b - C*b^2)*log(tan(d*x + c)) - (2*
B*a^2 - 6*(B*a^2 - 2*C*a*b - B*b^2)*tan(d*x + c)^2 + 3*(C*a^2 + 2*B*a*b)*ta
n(d*x + c))/tan(d*x + c)^3)/d
```

Fricas [A] time = 1.50593, size = 367, normalized size = 3.11

$$-\frac{3\left(Ca^2+2\,Bab-Cb^2\right)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^3+3\left(Ca^2+2\,Bab-2\left(Ba^2-2\,Cab-Bb^2\right)dx\right)\tan\left(dx+c\right)^3+}{6\,d\tan\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] -1/6*(3*(C*a^2 + 2*B*a*b - C*b^2)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*
tan(d*x + c)^3 + 3*(C*a^2 + 2*B*a*b - 2*(B*a^2 - 2*C*a*b - B*b^2)*d*x)*tan(
d*x + c)^3 + 2*B*a^2 - 6*(B*a^2 - 2*C*a*b - B*b^2)*tan(d*x + c)^2 + 3*(C*a^
2 + 2*B*a*b)*tan(d*x + c))/(d*tan(d*x + c)^3)
```

Sympy [A] time = 47.6416, size = 258, normalized size = 2.19

$$\begin{cases} \text{NaN} \\ x\left(a+b\tan\left(c\right)\right)^2\left(B\tan\left(c\right)+C\tan^2\left(c\right)\right)\cot^5\left(c\right) \\ Ba^2x+\frac{Ba^2}{d\tan\left(c+dx\right)}-\frac{Ba^2}{3d\tan^3\left(c+dx\right)}+\frac{Bab\log\left(\tan^2\left(c+dx\right)+1\right)}{d}-\frac{2Bab\log\left(\tan\left(c+dx\right)\right)}{d}-\frac{Bab}{d\tan^2\left(c+dx\right)}-Bb^2x-\frac{Bb^2}{d\tan\left(c+dx\right)}+\frac{Ca^2\log\left(\right.}{} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+b*tan(d*x+c))**2*(B*tan(d*x+c)+C*tan(d*x+c)**2),
x)
```

```
[Out] Piecewise((nan, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x))), (x*(a
 + b*tan(c))**2*(B*tan(c) + C*tan(c)**2)*cot(c)**5, Eq(d, 0)), (B*a**2*x +
B*a**2/(d*tan(c + d*x)) - B*a**2/(3*d*tan(c + d*x)**3) + B*a*b*log(tan(c +
d*x)**2 + 1)/d - 2*B*a*b*log(tan(c + d*x))/d - B*a*b/(d*tan(c + d*x)**2) -
B*b**2*x - B*b**2/(d*tan(c + d*x)) + C*a**2*log(tan(c + d*x)**2 + 1)/(2*d)
- C*a**2*log(tan(c + d*x))/d - C*a**2/(2*d*tan(c + d*x)**2) - 2*C*a*b*x - 2
*C*a*b/(d*tan(c + d*x)) - C*b**2*log(tan(c + d*x)**2 + 1)/(2*d) + C*b**2*lo
g(tan(c + d*x))/d, True))
```

<u>Giac</u> [B] time = 2.0847, size = 451, normalized size = 3.82

$$\frac{Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,Ca^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-6\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+24\,Cab\tan\left(\frac{1}{2}\,dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/24*(B*a^2*tan(1/2*d*x + 1/2*c)^3 - 3*C*a^2*tan(1/2*d*x + 1/2*c)^2 - 6*B*a
*b*tan(1/2*d*x + 1/2*c)^2 - 15*B*a^2*tan(1/2*d*x + 1/2*c) + 24*C*a*b*tan(1/
2*d*x + 1/2*c) + 12*B*b^2*tan(1/2*d*x + 1/2*c) + 24*(B*a^2 - 2*C*a*b - B*b^
2)*(d*x + c) + 24*(C*a^2 + 2*B*a*b - C*b^2)*log(tan(1/2*d*x + 1/2*c)^2 + 1)
 - 24*(C*a^2 + 2*B*a*b - C*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + (44*C*a^2*
tan(1/2*d*x + 1/2*c)^3 + 88*B*a*b*tan(1/2*d*x + 1/2*c)^3 - 44*C*b^2*tan(1/2
*d*x + 1/2*c)^3 + 15*B*a^2*tan(1/2*d*x + 1/2*c)^2 - 24*C*a*b*tan(1/2*d*x +
1/2*c)^2 - 12*B*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*C*a^2*tan(1/2*d*x + 1/2*c) -
 6*B*a*b*tan(1/2*d*x + 1/2*c) - B*a^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.16 $\int \cot^6(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c\right.$

Optimal. Leaf size=151

$$\frac{\left(a^2B-2abC-b^2B\right)\cot^2(c+dx)}{2d}+\frac{\left(a^2B-2abC-b^2B\right)\log(\sin(c+dx))}{d}+x\left(a^2C+2abB-b^2C\right)-\frac{a^2B\cot^4(c+dx)}{4d}$$

```
[Out] (2*a*b*B + a^2*C - b^2*C)*x - ((b^2*C - a*(2*b*B + a*C))*Cot[c + d*x])/d +
((a^2*B - b^2*B - 2*a*b*C)*Cot[c + d*x]^2)/(2*d) - (a*(2*b*B + a*C)*Cot[c +
 d*x]^3)/(3*d) - (a^2*B*Cot[c + d*x]^4)/(4*d) + ((a^2*B - b^2*B - 2*a*b*C)*
Log[Sin[c + d*x]])/d
```

Rubi [A] time = 0.369397, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {3632, 3604, 3628, 3529, 3531, 3475}

$$\frac{\left(a^2B-2abC-b^2B\right)\cot^2(c+dx)}{2d}+\frac{\left(a^2B-2abC-b^2B\right)\log(\sin(c+dx))}{d}+x\left(a^2C+2abB-b^2C\right)-\frac{a^2B\cot^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^
2),x]
```

```
[Out] (2*a*b*B + a^2*C - b^2*C)*x - ((b^2*C - a*(2*b*B + a*C))*Cot[c + d*x])/d +
((a^2*B - b^2*B - 2*a*b*C)*Cot[c + d*x]^2)/(2*d) - (a*(2*b*B + a*C)*Cot[c +
 d*x]^3)/(3*d) - (a^2*B*Cot[c + d*x]^4)/(4*d) + ((a^2*B - b^2*B - 2*a*b*C)*
Log[Sin[c + d*x]])/d
```

Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3604

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp
[((B*c - A*d)*(b*c - a*d)^2*(c + d*Tan[e + f*x])^(n + 1))/(f*d^2*(n + 1)*(c
^2 + d^2)), x] + Dist[1/(d*(c^2 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*S
imp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*c +
 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2)*T
an[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

Rule 3628

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2
- a*b*B + a^2*C)*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 + b^2)), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
```

```
C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cot^6(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx &= \int \cot^5(c+dx)(a+b\tan(c+dx))^2(B+C\tan \\
&= -\frac{a^2B\cot^4(c+dx)}{4d}+\int \cot^4(c+dx)\left(a(2bB+\right. \\
&= -\frac{a(2bB+aC)\cot^3(c+dx)}{3d}-\frac{a^2B\cot^4(c+dx}{4d} \\
&= \frac{\left(a^2B-b^2B-2abC\right)\cot^2(c+dx)}{2d}-\frac{a(2bB+a}{} \\
&= -\frac{\left(b^2C-a(2bB+aC)\right)\cot(c+dx)}{d}+\frac{\left(a^2B-\right.}{} \\
&= \left(2abB+a^2C-b^2C\right)x-\frac{\left(b^2C-a(2bB+aC)\right)}{d} \\
&= \left(2abB+a^2C-b^2C\right)x-\frac{\left(b^2C-a(2bB+aC)\right)}{d}
\end{aligned}$$

Mathematica [C] time = 3.02095, size = 180, normalized size = 1.19

$$\frac{6\left(a^2B-2abC-b^2B\right)\cot^2(c+dx)+12\left(a^2C+2abB-b^2C\right)\cot(c+dx)-6\left(\left(-2a^2B+4abC+2b^2B\right)\log(\tan(c+dx))+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c +
 d*x]^2),x]
```

```
[Out] (12*(2*a*b*B + a^2*C - b^2*C)*Cot[c + d*x] + 6*(a^2*B - b^2*B - 2*a*b*C)*Co
t[c + d*x]^2 - 4*a*(2*b*B + a*C)*Cot[c + d*x]^3 - 3*a^2*B*Cot[c + d*x]^4 -
6*((a + I*b)^2*(B + I*C)*Log[I - Tan[c + d*x]] + (-2*a^2*B + 2*b^2*B + 4*a*
b*C)*Log[Tan[c + d*x]] + (a - I*b)^2*(B - I*C)*Log[I + Tan[c + d*x]]))/(12*
d)
```

Maple [A] time = 0.098, size = 238, normalized size = 1.6

$$-\frac{b^2B(\cot(dx+c))^2}{2d}-\frac{b^2B\ln(\sin(dx+c))}{d}-b^2Cx-\frac{C\cot(dx+c)\,b^2}{d}-\frac{Cb^2c}{d}-\frac{2\,Bab(\cot(dx+c))^3}{3d}+2\,\frac{B\cot(dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

```
[Out] -1/2/d*b^2*B*cot(d*x+c)^2-1/d*b^2*B*ln(sin(d*x+c))-b^2*C*x-1/d*C*cot(d*x+c)
*b^2-1/d*C*b^2*c-2/3/d*B*a*b*cot(d*x+c)^3+2/d*B*cot(d*x+c)*a*b+2*B*a*b*x+2/
d*B*a*b*c-1/d*C*a*b*cot(d*x+c)^2-2/d*C*a*b*ln(sin(d*x+c))-1/4*a^2*B*cot(d*x
+c)^4/d+1/2/d*a^2*B*cot(d*x+c)^2+1/d*a^2*B*ln(sin(d*x+c))-1/3/d*C*a^2*cot(d
*x+c)^3+1/d*C*cot(d*x+c)*a^2+C*x*a^2+1/d*C*a^2*c
```

Maxima [A] time = 1.76037, size = 236, normalized size = 1.56

$$\frac{12\left(Ca^2+2\,Bab-Cb^2\right)(dx+c)-6\left(Ba^2-2\,Cab-Bb^2\right)\log\left(\tan(dx+c)^2+1\right)+12\left(Ba^2-2\,Cab-Bb^2\right)\log(\tan(dx}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/12*(12*(C*a^2 + 2*B*a*b - C*b^2)*(d*x + c) - 6*(B*a^2 - 2*C*a*b - B*b^2)*
log(tan(d*x + c)^2 + 1) + 12*(B*a^2 - 2*C*a*b - B*b^2)*log(tan(d*x + c)) +
(12*(C*a^2 + 2*B*a*b - C*b^2)*tan(d*x + c)^3 - 3*B*a^2 + 6*(B*a^2 - 2*C*a*b
 - B*b^2)*tan(d*x + c)^2 - 4*(C*a^2 + 2*B*a*b)*tan(d*x + c))/tan(d*x + c)^4
)/d
```

Fricas [A] time = 1.42823, size = 446, normalized size = 2.95

$$\frac{6\left(Ba^2-2\,Cab-Bb^2\right)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan(dx+c)^4+3\left(3\,Ba^2-4\,Cab-2\,Bb^2+4\left(Ca^2+2\,Bab-Cb^2\right)dx\right)\tan(dx}{12\,d\,\mathrm{ta}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/12*(6*(B*a^2 - 2*C*a*b - B*b^2)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*
tan(d*x + c)^4 + 3*(3*B*a^2 - 4*C*a*b - 2*B*b^2 + 4*(C*a^2 + 2*B*a*b - C*b^
2)*d*x)*tan(d*x + c)^4 + 12*(C*a^2 + 2*B*a*b - C*b^2)*tan(d*x + c)^3 - 3*B*
a^2 + 6*(B*a^2 - 2*C*a*b - B*b^2)*tan(d*x + c)^2 - 4*(C*a^2 + 2*B*a*b)*tan(
d*x + c))/(d*tan(d*x + c)^4)
```

Sympy [A] time = 83.3357, size = 311, normalized size = 2.06

$$\begin{cases} \text{NaN} \\ x(a+b\tan(c))^2\left(B\tan(c)+C\tan^2(c)\right)\cot^6(c) \\ -\frac{Ba^2\log\left(\tan^2(c+dx)+1\right)}{2d}+\frac{Ba^2\log(\tan(c+dx))}{d}+\frac{Ba^2}{2d\tan^2(c+dx)}-\frac{Ba^2}{4d\tan^4(c+dx)}+2Babx+\frac{2Bab}{d\tan(c+dx)}-\frac{2Bab}{3d\tan^3(c+dx)}+\frac{Bb^2\log\left(\tan^2(c\right.}{2d} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+b*tan(d*x+c))**2*(B*tan(d*x+c)+C*tan(d*x+c)**2),
x)
```

```
[Out] Piecewise((nan, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x))), (x*(a
 + b*tan(c))**2*(B*tan(c) + C*tan(c)**2)*cot(c)**6, Eq(d, 0)), (-B*a**2*log
(tan(c + d*x)**2 + 1)/(2*d) + B*a**2*log(tan(c + d*x))/d + B*a**2/(2*d*tan(
c + d*x)**2) - B*a**2/(4*d*tan(c + d*x)**4) + 2*B*a*b*x + 2*B*a*b/(d*tan(c
+ d*x)) - 2*B*a*b/(3*d*tan(c + d*x)**3) + B*b**2*log(tan(c + d*x)**2 + 1)/(
2*d) - B*b**2*log(tan(c + d*x))/d - B*b**2/(2*d*tan(c + d*x)**2) + C*a**2*x
 + C*a**2/(d*tan(c + d*x)) - C*a**2/(3*d*tan(c + d*x)**3) + C*a*b*log(tan(c
 + d*x)**2 + 1)/d - 2*C*a*b*log(tan(c + d*x))/d - C*a*b/(d*tan(c + d*x)**2)
 - C*b**2*x - C*b**2/(d*tan(c + d*x)), True))
```

Giac [B] time = 2.13815, size = 587, normalized size = 3.89

$$-\frac{3\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-8\,Ca^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-16\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-36\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+48\,Cab\tan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] -1/192*(3*B*a^2*tan(1/2*d*x + 1/2*c)^4 - 8*C*a^2*tan(1/2*d*x + 1/2*c)^3 - 1
6*B*a*b*tan(1/2*d*x + 1/2*c)^3 - 36*B*a^2*tan(1/2*d*x + 1/2*c)^2 + 48*C*a*b
*tan(1/2*d*x + 1/2*c)^2 + 24*B*b^2*tan(1/2*d*x + 1/2*c)^2 + 120*C*a^2*tan(1
/2*d*x + 1/2*c) + 240*B*a*b*tan(1/2*d*x + 1/2*c) - 96*C*b^2*tan(1/2*d*x + 1
/2*c) - 192*(C*a^2 + 2*B*a*b - C*b^2)*(d*x + c) + 192*(B*a^2 - 2*C*a*b - B*
b^2)*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 192*(B*a^2 - 2*C*a*b - B*b^2)*log(ab
s(tan(1/2*d*x + 1/2*c))) + (400*B*a^2*tan(1/2*d*x + 1/2*c)^4 - 800*C*a*b*ta
n(1/2*d*x + 1/2*c)^4 - 400*B*b^2*tan(1/2*d*x + 1/2*c)^4 - 120*C*a^2*tan(1/2
*d*x + 1/2*c)^3 - 240*B*a*b*tan(1/2*d*x + 1/2*c)^3 + 96*C*b^2*tan(1/2*d*x +
 1/2*c)^3 - 36*B*a^2*tan(1/2*d*x + 1/2*c)^2 + 48*C*a*b*tan(1/2*d*x + 1/2*c)
^2 + 24*B*b^2*tan(1/2*d*x + 1/2*c)^2 + 8*C*a^2*tan(1/2*d*x + 1/2*c) + 16*B*
a*b*tan(1/2*d*x + 1/2*c) + 3*B*a^2)/tan(1/2*d*x + 1/2*c)^4)/d
```

3.17 $\int (a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+dx)\right) dx$

Optimal. Leaf size=165

$$\frac{b\left(a^2B-2abC-b^2B\right)\tan(c+dx)}{d}-\frac{\left(-3a^2bC+a^3B-3ab^2B+b^3C\right)\log(\cos(c+dx))}{d}-x\left(3a^2bB+a^3C-3ab^2C-b^3\right.$$

```
[Out] -((3*a^2*b*B - b^3*B + a^3*C - 3*a*b^2*C)*x) - ((a^3*B - 3*a*b^2*B - 3*a^2*
b*C + b^3*C)*Log[Cos[c + d*x]])/d + (b*(a^2*B - b^2*B - 2*a*b*C)*Tan[c + d*
x])/d + ((a*B - b*C)*(a + b*Tan[c + d*x])^2)/(2*d) + (B*(a + b*Tan[c + d*x]
)^3)/(3*d) + (C*(a + b*Tan[c + d*x])^4)/(4*b*d)
```

Rubi [A] time = 0.176974, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3630, 3528, 3525, 3475}

$$\frac{b\left(a^2B-2abC-b^2B\right)\tan(c+dx)}{d}-\frac{\left(-3a^2bC+a^3B-3ab^2B+b^3C\right)\log(\cos(c+dx))}{d}-x\left(3a^2bB+a^3C-3ab^2C-b^3\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^2),x]
```

```
[Out] -((3*a^2*b*B - b^3*B + a^3*C - 3*a*b^2*C)*x) - ((a^3*B - 3*a*b^2*B - 3*a^2*
b*C + b^3*C)*Log[Cos[c + d*x]])/d + (b*(a^2*B - b^2*B - 2*a*b*C)*Tan[c + d*
x])/d + ((a*B - b*C)*(a + b*Tan[c + d*x])^2)/(2*d) + (B*(a + b*Tan[c + d*x]
)^3)/(3*d) + (C*(a + b*Tan[c + d*x])^4)/(4*b*d)
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx = \frac{C(a+b\tan(c+dx))^4}{4bd}+\int (a+b\tan(c+dx))^3(-C+B\tan$$
$$= \frac{B(a+b\tan(c+dx))^3}{3d}+\frac{C(a+b\tan(c+dx))^4}{4bd}+\int (a+b$$
$$= \frac{(aB-bC)(a+b\tan(c+dx))^2}{2d}+\frac{B(a+b\tan(c+dx))^3}{3d}+$$
$$= -\left(3a^2bB-b^3B+a^3C-3ab^2C\right)x+\frac{b\left(a^2B-b^2B-2abC\right)}{d}$$
$$= -\left(3a^2bB-b^3B+a^3C-3ab^2C\right)x-\frac{\left(a^3B-3ab^2B-3a^2bC\right.}{}$$

Mathematica [C] time = 1.56657, size = 209, normalized size = 1.27

$$\frac{-12b^2B\left(b^2-6a^2\right)\tan(c+dx)-6(aB+bC)\left(6ab^2\tan(c+dx)+(-b+ia)^3\log(-\tan(c+dx)+i)-(b+ia)^3\log(\tan(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^2),x]
```

```
[Out] ((-6*I)*(a + I*b)^4*B*Log[I - Tan[c + d*x]] + (6*I)*(a - I*b)^4*B*Log[I + T
an[c + d*x]] - 12*b^2*(-6*a^2 + b^2)*B*Tan[c + d*x] + 24*a*b^3*B*Tan[c + d*
x]^2 + 4*b^4*B*Tan[c + d*x]^3 + 3*C*(a + b*Tan[c + d*x])^4 - 6*(a*B + b*C)*
((I*a - b)^3*Log[I - Tan[c + d*x]] - (I*a + b)^3*Log[I + Tan[c + d*x]] + 6*
a*b^2*Tan[c + d*x] + b^3*Tan[c + d*x]^2))/(12*b*d)
```

Maple [A] time = 0.014, size = 314, normalized size = 1.9

$$\frac{Cb^3(\tan(dx+c))^4}{4d}+\frac{B(\tan(dx+c))^3b^3}{3d}+\frac{C(\tan(dx+c))^3ab^2}{d}+\frac{3B(\tan(dx+c))^2ab^2}{2d}+\frac{3C(\tan(dx+c))^2a^2b}{2d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

```
[Out] 1/4/d*C*b^3*tan(d*x+c)^4+1/3/d*B*tan(d*x+c)^3*b^3+1/d*C*tan(d*x+c)^3*a*b^2+
3/2/d*B*tan(d*x+c)^2*a*b^2+3/2/d*C*tan(d*x+c)^2*a^2*b-1/2/d*C*b^3*tan(d*x+c
)^2+3/d*B*a^2*b*tan(d*x+c)-1/d*B*tan(d*x+c)*b^3+1/d*C*tan(d*x+c)*a^3-3/d*C*
a*b^2*tan(d*x+c)+1/2/d*a^3*B*ln(1+tan(d*x+c)^2)-3/2/d*ln(1+tan(d*x+c)^2)*B*
a*b^2-3/2/d*ln(1+tan(d*x+c)^2)*C*a^2*b+1/2/d*ln(1+tan(d*x+c)^2)*C*b^3-3/d*B
*arctan(tan(d*x+c))*a^2*b+1/d*B*arctan(tan(d*x+c))*b^3-1/d*C*arctan(tan(d*x
+c))*a^3+3/d*C*arctan(tan(d*x+c))*a*b^2
```

Maxima [A] time = 1.75413, size = 242, normalized size = 1.47

$$\frac{3Cb^3\tan(dx+c)^4+4\left(3Cab^2+Bb^3\right)\tan(dx+c)^3+6\left(3Ca^2b+3Bab^2-Cb^3\right)\tan(dx+c)^2-12\left(Ca^3+3Ba^2b-3C\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] 1/12*(3*C*b^3*tan(d*x + c)^4 + 4*(3*C*a*b^2 + B*b^3)*tan(d*x + c)^3 + 6*(3*
C*a^2*b + 3*B*a*b^2 - C*b^3)*tan(d*x + c)^2 - 12*(C*a^3 + 3*B*a^2*b - 3*C*a
*b^2 - B*b^3)*(d*x + c) + 6*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*log(tan
(d*x + c)^2 + 1) + 12*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*tan(d*x + c))
/d
```

Fricas [A] time = 1.60532, size = 408, normalized size = 2.47

$$\frac{3\,Cb^3\tan\left(dx+c\right)^4+4\left(3\,Cab^2+Bb^3\right)\tan\left(dx+c\right)^3-12\left(Ca^3+3\,Ba^2b-3\,Cab^2-Bb^3\right)dx+6\left(3\,Ca^2b+3\,Bab^2-C\cdots\right.}{12\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/12*(3*C*b^3*tan(d*x + c)^4 + 4*(3*C*a*b^2 + B*b^3)*tan(d*x + c)^3 - 12*(C
*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*d*x + 6*(3*C*a^2*b + 3*B*a*b^2 - C*b^
3)*tan(d*x + c)^2 - 6*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*log(1/(tan(d*
x + c)^2 + 1)) + 12*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*tan(d*x + c))/d
```

Sympy [A] time = 2.78369, size = 313, normalized size = 1.9

$$\begin{cases} \frac{Ba^3\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}-3Ba^2bx+\frac{3Ba^2b\tan\left(c+dx\right)}{d}-\frac{3Bab^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{3Bab^2\tan^2\left(c+dx\right)}{2d}+Bb^3x+\frac{Bb^3\tan^3\left(c+dx\right)}{3d}-\frac{Bb^3\cdots}{\cdots} \\ x\left(a+b\tan\left(c\right)\right)^3\left(B\tan\left(c\right)+C\tan^2\left(c\right)\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**3*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

```
[Out] Piecewise((B*a**3*log(tan(c + d*x)**2 + 1)/(2*d) - 3*B*a**2*b*x + 3*B*a**2*
b*tan(c + d*x)/d - 3*B*a*b**2*log(tan(c + d*x)**2 + 1)/(2*d) + 3*B*a*b**2*t
an(c + d*x)**2/(2*d) + B*b**3*x + B*b**3*tan(c + d*x)**3/(3*d) - B*b**3*tan
(c + d*x)/d - C*a**3*x + C*a**3*tan(c + d*x)/d - 3*C*a**2*b*log(tan(c + d*x
)**2 + 1)/(2*d) + 3*C*a**2*b*tan(c + d*x)**2/(2*d) + 3*C*a*b**2*x + C*a*b**
2*tan(c + d*x)**3/d - 3*C*a*b**2*tan(c + d*x)/d + C*b**3*log(tan(c + d*x)**
2 + 1)/(2*d) + C*b**3*tan(c + d*x)**4/(4*d) - C*b**3*tan(c + d*x)**2/(2*d),
 Ne(d, 0)), (x*(a + b*tan(c))**3*(B*tan(c) + C*tan(c)**2), True))
```

Giac [B] time = 6.49328, size = 3875, normalized size = 23.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] -1/12*(12*C*a^3*d*x*tan(d*x)^4*tan(c)^4 + 36*B*a^2*b*d*x*tan(d*x)^4*tan(c)^
4 - 36*C*a*b^2*d*x*tan(d*x)^4*tan(c)^4 - 12*B*b^3*d*x*tan(d*x)^4*tan(c)^4 +
 6*B*a^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) +
tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c
)^4 - 18*C*a^2*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*t
an(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)
^4*tan(c)^4 - 18*B*a*b^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(
d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*
tan(d*x)^4*tan(c)^4 + 6*C*b^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2
*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) +
 1))*tan(d*x)^4*tan(c)^4 - 48*C*a^3*d*x*tan(d*x)^3*tan(c)^3 - 144*B*a^2*b*d
*x*tan(d*x)^3*tan(c)^3 + 144*C*a*b^2*d*x*tan(d*x)^3*tan(c)^3 + 48*B*b^3*d*x
*tan(d*x)^3*tan(c)^3 - 18*C*a^2*b*tan(d*x)^4*tan(c)^4 - 18*B*a*b^2*tan(d*x)
^4*tan(c)^4 + 9*C*b^3*tan(d*x)^4*tan(c)^4 - 24*B*a^3*log(4*(tan(c)^2 + 1)/(
tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^
2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^3*tan(c)^3 + 72*C*a^2*b*log(4*(tan(c)^
2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + t
an(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^3*tan(c)^3 + 72*B*a*b^2*log(4*
(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(
c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^3*tan(c)^3 - 24*C*b^3*
log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^
2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^3*tan(c)^3 + 12*
C*a^3*tan(d*x)^4*tan(c)^3 + 36*B*a^2*b*tan(d*x)^4*tan(c)^3 - 36*C*a*b^2*tan
(d*x)^4*tan(c)^3 - 12*B*b^3*tan(d*x)^4*tan(c)^3 + 12*C*a^3*tan(d*x)^3*tan(c
)^4 + 36*B*a^2*b*tan(d*x)^3*tan(c)^4 - 36*C*a*b^2*tan(d*x)^3*tan(c)^4 - 12*
B*b^3*tan(d*x)^3*tan(c)^4 + 72*C*a^3*d*x*tan(d*x)^2*tan(c)^2 + 216*B*a^2*b*
d*x*tan(d*x)^2*tan(c)^2 - 216*C*a*b^2*d*x*tan(d*x)^2*tan(c)^2 - 72*B*b^3*d*
x*tan(d*x)^2*tan(c)^2 - 18*C*a^2*b*tan(d*x)^4*tan(c)^2 - 18*B*a*b^2*tan(d*x
)^4*tan(c)^2 + 6*C*b^3*tan(d*x)^4*tan(c)^2 + 36*C*a^2*b*tan(d*x)^3*tan(c)^3
 + 36*B*a*b^2*tan(d*x)^3*tan(c)^3 - 24*C*b^3*tan(d*x)^3*tan(c)^3 - 18*C*a^2
*b*tan(d*x)^2*tan(c)^4 - 18*B*a*b^2*tan(d*x)^2*tan(c)^4 + 6*C*b^3*tan(d*x)^
2*tan(c)^4 + 12*C*a*b^2*tan(d*x)^4*tan(c) + 4*B*b^3*tan(d*x)^4*tan(c) + 36*
B*a^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan
(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^2
 - 108*C*a^2*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan
(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2
*tan(c)^2 - 108*B*a*b^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d
*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*t
an(d*x)^2*tan(c)^2 + 36*C*b^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2
*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) +
 1))*tan(d*x)^2*tan(c)^2 - 36*C*a^3*tan(d*x)^3*tan(c)^2 - 108*B*a^2*b*tan(d
*x)^3*tan(c)^2 + 144*C*a*b^2*tan(d*x)^3*tan(c)^2 + 48*B*b^3*tan(d*x)^3*tan(
c)^2 - 36*C*a^3*tan(d*x)^2*tan(c)^3 - 108*B*a^2*b*tan(d*x)^2*tan(c)^3 + 144
*C*a*b^2*tan(d*x)^2*tan(c)^3 + 48*B*b^3*tan(d*x)^2*tan(c)^3 + 12*C*a*b^2*ta
n(d*x)*tan(c)^4 + 4*B*b^3*tan(d*x)*tan(c)^4 - 3*C*b^3*tan(d*x)^4 - 48*C*a^3
*d*x*tan(d*x)*tan(c) - 144*B*a^2*b*d*x*tan(d*x)*tan(c) + 144*C*a*b^2*d*x*ta
n(d*x)*tan(c) + 48*B*b^3*d*x*tan(d*x)*tan(c) + 36*C*a^2*b*tan(d*x)^3*tan(c)
 + 36*B*a*b^2*tan(d*x)^3*tan(c) - 24*C*b^3*tan(d*x)^3*tan(c) - 36*C*a^2*b*t
an(d*x)^2*tan(c)^2 - 36*B*a*b^2*tan(d*x)^2*tan(c)^2 + 12*C*b^3*tan(d*x)^2*t
an(c)^2 + 36*C*a^2*b*tan(d*x)*tan(c)^3 + 36*B*a*b^2*tan(d*x)*tan(c)^3 - 24*
C*b^3*tan(d*x)*tan(c)^3 - 3*C*b^3*tan(c)^4 - 12*C*a*b^2*tan(d*x)^3 - 4*B*b^
3*tan(d*x)^3 - 24*B*a^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d
*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*t
an(d*x)*tan(c) + 72*C*a^2*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*t
an(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1
))*tan(d*x)*tan(c) + 72*B*a*b^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 -
 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c)
 + 1))*tan(d*x)*tan(c) - 24*C*b^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2
 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(
```

```
c) + 1))*tan(d*x)*tan(c) + 36*C*a^3*tan(d*x)^2*tan(c) + 108*B*a^2*b*tan(d*x
)^2*tan(c) - 144*C*a*b^2*tan(d*x)^2*tan(c) - 48*B*b^3*tan(d*x)^2*tan(c) + 3
6*C*a^3*tan(d*x)*tan(c)^2 + 108*B*a^2*b*tan(d*x)*tan(c)^2 - 144*C*a*b^2*tan
(d*x)*tan(c)^2 - 48*B*b^3*tan(d*x)*tan(c)^2 - 12*C*a*b^2*tan(c)^3 - 4*B*b^3
*tan(c)^3 + 12*C*a^3*d*x + 36*B*a^2*b*d*x - 36*C*a*b^2*d*x - 12*B*b^3*d*x -
 18*C*a^2*b*tan(d*x)^2 - 18*B*a*b^2*tan(d*x)^2 + 6*C*b^3*tan(d*x)^2 + 36*C*
a^2*b*tan(d*x)*tan(c) + 36*B*a*b^2*tan(d*x)*tan(c) - 24*C*b^3*tan(d*x)*tan(
c) - 18*C*a^2*b*tan(c)^2 - 18*B*a*b^2*tan(c)^2 + 6*C*b^3*tan(c)^2 + 6*B*a^3
*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)
^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)) - 18*C*a^2*b*log(4*(tan(
c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2
+ tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)) - 18*B*a*b^2*log(4*(tan(c)^2 + 1)/(t
an(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2
 - 2*tan(d*x)*tan(c) + 1)) + 6*C*b^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c
)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*t
an(c) + 1)) - 12*C*a^3*tan(d*x) - 36*B*a^2*b*tan(d*x) + 36*C*a*b^2*tan(d*x)
 + 12*B*b^3*tan(d*x) - 12*C*a^3*tan(c) - 36*B*a^2*b*tan(c) + 36*C*a*b^2*tan
(c) + 12*B*b^3*tan(c) - 18*C*a^2*b - 18*B*a*b^2 + 9*C*b^3)/(d*tan(d*x)^4*ta
n(c)^4 - 4*d*tan(d*x)^3*tan(c)^3 + 6*d*tan(d*x)^2*tan(c)^2 - 4*d*tan(d*x)*t
an(c) + d)
```

3.18 $\int \cot(c+dx)(a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+d$

Optimal. Leaf size=140

$$\frac{b\left(a^2C+2abB-b^2C\right)\tan(c+dx)}{d}-\frac{\left(3a^2bB+a^3C-3ab^2C-b^3B\right)\log(\cos(c+dx))}{d}+x\left(-3a^2bC+a^3B-3ab^2B+b^3C\right)$$

```
[Out] (a^3*B - 3*a*b^2*B - 3*a^2*b*C + b^3*C)*x - ((3*a^2*b*B - b^3*B + a^3*C - 3
*a*b^2*C)*Log[Cos[c + d*x]])/d + (b*(2*a*b*B + a^2*C - b^2*C)*Tan[c + d*x])
/d + ((b*B + a*C)*(a + b*Tan[c + d*x])^2)/(2*d) + (C*(a + b*Tan[c + d*x])^3
)/(3*d)
```

Rubi [A] time = 0.207881, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3632, 3528, 3525, 3475}

$$\frac{b\left(a^2C+2abB-b^2C\right)\tan(c+dx)}{d}-\frac{\left(3a^2bB+a^3C-3ab^2C-b^3B\right)\log(\cos(c+dx))}{d}+x\left(-3a^2bC+a^3B-3ab^2B+b^3C\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^2)
,x]
```

```
[Out] (a^3*B - 3*a*b^2*B - 3*a^2*b*C + b^3*C)*x - ((3*a^2*b*B - b^3*B + a^3*C - 3
*a*b^2*C)*Log[Cos[c + d*x]])/d + (b*(2*a*b*B + a^2*C - b^2*C)*Tan[c + d*x])
/d + ((b*B + a*C)*(a + b*Tan[c + d*x])^2)/(2*d) + (C*(a + b*Tan[c + d*x])^3
)/(3*d)
```

Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx = \int (a+b\tan(c+dx))^3(B+C\tan(c+dx))\,d$$

$$= \frac{C(a+b\tan(c+dx))^3}{3d} + \int (a+b\tan(c+d$$

$$= \frac{(bB+aC)(a+b\tan(c+dx))^2}{2d} + \frac{C(a+b\tan}{}$$

$$= \left(a^3B-3ab^2B-3a^2bC+b^3C\right)x + \frac{b\left(2abB\right.}{}$$

$$= \left(a^3B-3ab^2B-3a^2bC+b^3C\right)x - \frac{\left(3a^2bB\right.}{}$$

Mathematica [C] time = 1.05835, size = 130, normalized size = 0.93

$$\frac{6b\left(3a^2C+3abB-b^2C\right)\tan(c+dx)+3b^2(3aC+bB)\tan^2(c+dx)+3(a-ib)^3(C+iB)\log(\tan(c+dx)+i)+3(a+}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c + d
*x]^2),x]
```

```
[Out] (3*(a + I*b)^3*((-I)*B + C)*Log[I - Tan[c + d*x]] + 3*(a - I*b)^3*(I*B + C)
*Log[I + Tan[c + d*x]] + 6*b*(3*a*b*B + 3*a^2*C - b^2*C)*Tan[c + d*x] + 3*b
^2*(b*B + 3*a*C)*Tan[c + d*x]^2 + 2*b^3*C*Tan[c + d*x]^3)/(6*d)
```

Maple [A] time = 0.089, size = 234, normalized size = 1.7

$$\frac{Bb^3(\tan(dx+c))^2}{2d} + \frac{Bb^3\ln(\cos(dx+c))}{d} + \frac{Cb^3(\tan(dx+c))^3}{3d} - \frac{Cb^3\tan(dx+c)}{d} + Cb^3x + \frac{Cb^3c}{d} - 3Bab^2x + 3\frac{B}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

```
[Out] 1/2/d*B*b^3*tan(d*x+c)^2+1/d*B*b^3*ln(cos(d*x+c))+1/3/d*C*b^3*tan(d*x+c)^3-
1/d*C*b^3*tan(d*x+c)+C*b^3*x+1/d*C*b^3*c-3*B*a*b^2*x+3/d*B*tan(d*x+c)*a*b^2
-3/d*B*a*b^2*c+3/2/d*C*a*b^2*tan(d*x+c)^2+3/d*C*a*b^2*ln(cos(d*x+c))-3/d*B*
a^2*b*ln(cos(d*x+c))-3*C*x*a^2*b+3/d*C*tan(d*x+c)*a^2*b-3/d*C*a^2*b*c+B*a^3
*x+1/d*B*a^3*c-1/d*C*a^3*ln(cos(d*x+c))
```

Maxima [A] time = 1.76893, size = 193, normalized size = 1.38

$$\frac{2Cb^3\tan(dx+c)^3+3\left(3Cab^2+Bb^3\right)\tan(dx+c)^2+6\left(Ba^3-3Ca^2b-3Bab^2+Cb^3\right)(dx+c)+3\left(Ca^3+3Ba^2b-\right.}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/6*(2*C*b^3*tan(d*x + c)^3 + 3*(3*C*a*b^2 + B*b^3)*tan(d*x + c)^2 + 6*(B*a
^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*(d*x + c) + 3*(C*a^3 + 3*B*a^2*b - 3*C*
a*b^2 - B*b^3)*log(tan(d*x + c)^2 + 1) + 6*(3*C*a^2*b + 3*B*a*b^2 - C*b^3)*
tan(d*x + c))/d
```

Fricas [A] time = 1.64078, size = 324, normalized size = 2.31

$$\frac{2\,Cb^3\tan\left(dx+c\right)^3+6\left(Ba^3-3\,Ca^2b-3\,Bab^2+Cb^3\right)dx+3\left(3\,Cab^2+Bb^3\right)\tan\left(dx+c\right)^2-3\left(Ca^3+3\,Ba^2b-3\,Cab^2-\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/6*(2*C*b^3*tan(d*x + c)^3 + 6*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*d*x
 + 3*(3*C*a*b^2 + B*b^3)*tan(d*x + c)^2 - 3*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2
- B*b^3)*log(1/(tan(d*x + c)^2 + 1)) + 6*(3*C*a^2*b + 3*B*a*b^2 - C*b^3)*ta
n(d*x + c))/d
```

Sympy [A] time = 10.4206, size = 248, normalized size = 1.77

$$\begin{cases} Ba^3x+\frac{3Ba^2b\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}-3Bab^2x+\frac{3Bab^2\tan\left(c+dx\right)}{d}-\frac{Bb^3\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{Bb^3\tan^2\left(c+dx\right)}{2d}+\frac{Ca^3\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}-3C \\ x\left(a+b\tan\left(c\right)\right)^3\left(B\tan\left(c\right)+C\tan^2\left(c\right)\right)\cot\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))**3*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

```
[Out] Piecewise((B*a**3*x + 3*B*a**2*b*log(tan(c + d*x)**2 + 1)/(2*d) - 3*B*a*b**
2*x + 3*B*a*b**2*tan(c + d*x)/d - B*b**3*log(tan(c + d*x)**2 + 1)/(2*d) + B
*b**3*tan(c + d*x)**2/(2*d) + C*a**3*log(tan(c + d*x)**2 + 1)/(2*d) - 3*C*a
**2*b*x + 3*C*a**2*b*tan(c + d*x)/d - 3*C*a*b**2*log(tan(c + d*x)**2 + 1)/(
2*d) + 3*C*a*b**2*tan(c + d*x)**2/(2*d) + C*b**3*x + C*b**3*tan(c + d*x)**3
/(3*d) - C*b**3*tan(c + d*x)/d, Ne(d, 0)), (x*(a + b*tan(c))**3*(B*tan(c) +
 C*tan(c)**2)*cot(c), True))
```

Giac [A] time = 2.37083, size = 213, normalized size = 1.52

$$\frac{2\,Cb^3\tan\left(dx+c\right)^3+9\,Cab^2\tan\left(dx+c\right)^2+3\,Bb^3\tan\left(dx+c\right)^2+18\,Ca^2b\tan\left(dx+c\right)+18\,Bab^2\tan\left(dx+c\right)-6\,Cb^3\,\mathrm{ta}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] 1/6*(2*C*b^3*tan(d*x + c)^3 + 9*C*a*b^2*tan(d*x + c)^2 + 3*B*b^3*tan(d*x +
c)^2 + 18*C*a^2*b*tan(d*x + c) + 18*B*a*b^2*tan(d*x + c) - 6*C*b^3*tan(d*x
+ c) + 6*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*(d*x + c) + 3*(C*a^3 + 3*B
*a^2*b - 3*C*a*b^2 - B*b^3)*log(tan(d*x + c)^2 + 1))/d
```

3.19 $\int \cot^2(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+\right.$

Optimal. Leaf size=117

$$-\frac{b\left(3a^2C+3abB-b^2C\right)\log(\cos(c+dx))}{d}+x\left(3a^2bB+a^3C-3ab^2C-b^3B\right)+\frac{a^3B\log(\sin(c+dx))}{d}+\frac{b^2(2aC+bB)\tan}{d}$$

```
[Out] (3*a^2*b*B - b^3*B + a^3*C - 3*a*b^2*C)*x - (b*(3*a*b*B + 3*a^2*C - b^2*C)*
Log[Cos[c + d*x]])/d + (a^3*B*Log[Sin[c + d*x]])/d + (b^2*(b*B + 2*a*C)*Tan
[c + d*x])/d + (b*C*(a + b*Tan[c + d*x])^2)/(2*d)
```

Rubi [A] time = 0.335582, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3632, 3607, 3637, 3624, 3475}

$$-\frac{b\left(3a^2C+3abB-b^2C\right)\log(\cos(c+dx))}{d}+x\left(3a^2bB+a^3C-3ab^2C-b^3B\right)+\frac{a^3B\log(\sin(c+dx))}{d}+\frac{b^2(2aC+bB)\tan}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^
2),x]
```

```
[Out] (3*a^2*b*B - b^3*B + a^3*C - 3*a*b^2*C)*x - (b*(3*a*b*B + 3*a^2*C - b^2*C)*
Log[Cos[c + d*x]])/d + (a^3*B*Log[Sin[c + d*x]])/d + (b^2*(b*B + 2*a*C)*Tan
[c + d*x])/d + (b*C*(a + b*Tan[c + d*x])^2)/(2*d)
```

Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3607

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*B*(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(m
+ n)), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan
[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
+ n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b*
(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1] &
& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
```

```
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3624

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[B*x, x] + (Dist[A, Int[1/Tan[e
 + f*x], x], x] + Dist[C, Int[Tan[e + f*x], x], x]) /; FreeQ[{e, f, A, B, C
}, x] && NeQ[A, C]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx = \int \cot(c+dx)(a+b\tan(c+dx))^3(B+C\tan$$
$$= \frac{bC(a+b\tan(c+dx))^2}{2d}+\frac{1}{2}\int \cot(c+dx)$$
$$= \frac{b^2(bB+2aC)\tan(c+dx)}{d}+\frac{bC(a+b\tan}{2d}$$
$$= \left(3a^2bB-b^3B+a^3C-3ab^2C\right)x+\frac{b^2(bB+}{}$$
$$= \left(3a^2bB-b^3B+a^3C-3ab^2C\right)x-\frac{b\left(3abB\right.}{}$$

Mathematica [C] time = 0.455175, size = 113, normalized size = 0.97

$$\frac{2a^3B\log(\tan(c+dx))+2b^2(3aC+bB)\tan(c+dx)-(a+ib)^3(B+iC)\log(-\tan(c+dx)+i)-(a-ib)^3(B-iC)\log(}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c +
 d*x]^2),x]
```

```
[Out] (-((a + I*b)^3*(B + I*C)*Log[I - Tan[c + d*x]]) + 2*a^3*B*Log[Tan[c + d*x]]
 - (a - I*b)^3*(B - I*C)*Log[I + Tan[c + d*x]] + 2*b^2*(b*B + 3*a*C)*Tan[c
+ d*x] + b^3*C*Tan[c + d*x]^2)/(2*d)
```

Maple [A] time = 0.094, size = 183, normalized size = 1.6

$$-Bxb^3+\frac{B\tan(dx+c)\,b^3}{d}-\frac{Bb^3c}{d}+\frac{Cb^3(\tan(dx+c))^2}{2d}+\frac{Cb^3\ln(\cos(dx+c))}{d}-3\frac{Bab^2\ln(\cos(dx+c))}{d}-3Cab^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

```
[Out] -B*x*b^3+1/d*B*tan(d*x+c)*b^3-1/d*B*b^3*c+1/2/d*C*b^3*tan(d*x+c)^2+b^3*C*ln
(cos(d*x+c))/d-3/d*B*a*b^2*ln(cos(d*x+c))-3*C*a*b^2*x+3/d*C*a*b^2*tan(d*x+c
```

```
)-3/d*C*a*b^2*c+3*B*a^2*b*x+3/d*B*a^2*b*c-3/d*C*a^2*b*ln(cos(d*x+c))+1/d*B*
a^3*ln(sin(d*x+c))+C*x*a^3+1/d*C*a^3*c
```

Maxima [A] time = 1.79214, size = 167, normalized size = 1.43

$$\frac{Cb^3\tan(dx+c)^2+2\,Ba^3\log(\tan(dx+c))+2\left(Ca^3+3\,Ba^2b-3\,Cab^2-Bb^3\right)(dx+c)-\left(Ba^3-3\,Ca^2b-3\,Bab^2+Cb^3\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/2*(C*b^3*tan(d*x + c)^2 + 2*B*a^3*log(tan(d*x + c)) + 2*(C*a^3 + 3*B*a^2*
b - 3*C*a*b^2 - B*b^3)*(d*x + c) - (B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*
log(tan(d*x + c)^2 + 1) + 2*(3*C*a*b^2 + B*b^3)*tan(d*x + c))/d
```

Fricas [A] time = 1.83794, size = 305, normalized size = 2.61

$$\frac{Cb^3\tan(dx+c)^2+Ba^3\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)+2\left(Ca^3+3\,Ba^2b-3\,Cab^2-Bb^3\right)dx-\left(3\,Ca^2b+3\,Bab^2-Cb^3\right)\log\left(\frac{1}{\tan(dx+c)^2}\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/2*(C*b^3*tan(d*x + c)^2 + B*a^3*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))
+ 2*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*d*x - (3*C*a^2*b + 3*B*a*b^2 -
C*b^3)*log(1/(tan(d*x + c)^2 + 1)) + 2*(3*C*a*b^2 + B*b^3)*tan(d*x + c))/d
```

Sympy [A] time = 26.6943, size = 211, normalized size = 1.8

$$\begin{cases} -\frac{Ba^3\log\left(\tan^2(c+dx)+1\right)}{2d}+\frac{Ba^3\log(\tan(c+dx))}{d}+3Ba^2bx+\frac{3Bab^2\log\left(\tan^2(c+dx)+1\right)}{2d}-Bb^3x+\frac{Bb^3\tan(c+dx)}{d}+Ca^3x+\frac{3Ca^2b\log\left(\tan^2(\right.}{2d} \\ x\,(a+b\tan(c))^3\left(B\tan(c)+C\tan^2(c)\right)\cot^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*tan(d*x+c))**3*(B*tan(d*x+c)+C*tan(d*x+c)**2),
x)
```

```
[Out] Piecewise((-B*a**3*log(tan(c + d*x)**2 + 1)/(2*d) + B*a**3*log(tan(c + d*x)
)/d + 3*B*a**2*b*x + 3*B*a*b**2*log(tan(c + d*x)**2 + 1)/(2*d) - B*b**3*x +
 B*b**3*tan(c + d*x)/d + C*a**3*x + 3*C*a**2*b*log(tan(c + d*x)**2 + 1)/(2*
d) - 3*C*a*b**2*x + 3*C*a*b**2*tan(c + d*x)/d - C*b**3*log(tan(c + d*x)**2
+ 1)/(2*d) + C*b**3*tan(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*tan(c))**3*
(B*tan(c) + C*tan(c)**2)*cot(c)**2, True))
```

Giac [A] time = 2.51779, size = 174, normalized size = 1.49

$$\frac{Cb^3 \tan(dx+c)^2 + 2\,Ba^3 \log(|\tan(dx+c)|) + 6\,Cab^2 \tan(dx+c) + 2\,Bb^3 \tan(dx+c) + 2\left(Ca^3 + 3\,Ba^2b - 3\,Cab^2 - \right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/2*(C*b^3*tan(d*x + c)^2 + 2*B*a^3*log(abs(tan(d*x + c))) + 6*C*a*b^2*tan(
d*x + c) + 2*B*b^3*tan(d*x + c) + 2*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)
*(d*x + c) - (B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*log(tan(d*x + c)^2 + 1
))/d
```

3.20 $\int \cot^3(c+dx)(a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+\right.$

Optimal. Leaf size=119

$$-x\left(-3a^2bC+a^3B-3ab^2B+b^3C\right)+\frac{a^2(aC+3bB)\log(\sin(c+dx))}{d}+\frac{b^2(aB+bC)\tan(c+dx)}{d}-\frac{b^2(3aC+bB)\log(\cos(}{d}$$

```
[Out] -((a^3*B - 3*a*b^2*B - 3*a^2*b*C + b^3*C)*x) - (b^2*(b*B + 3*a*C)*Log[Cos[c
 + d*x]])/d + (a^2*(3*b*B + a*C)*Log[Sin[c + d*x]])/d + (b^2*(a*B + b*C)*Ta
n[c + d*x])/d - (a*B*Cot[c + d*x]*(a + b*Tan[c + d*x])^2)/d
```

Rubi [A] time = 0.331394, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3632, 3605, 3637, 3624, 3475}

$$-x\left(-3a^2bC+a^3B-3ab^2B+b^3C\right)+\frac{a^2(aC+3bB)\log(\sin(c+dx))}{d}+\frac{b^2(aB+bC)\tan(c+dx)}{d}-\frac{b^2(3aC+bB)\log(\cos(}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^
2),x]
```

```
[Out] -((a^3*B - 3*a*b^2*B - 3*a^2*b*C + b^3*C)*x) - (b^2*(b*B + 3*a*C)*Log[Cos[c
 + d*x]])/d + (a^2*(3*b*B + a*C)*Log[Sin[c + d*x]])/d + (b^2*(a*B + b*C)*Ta
n[c + d*x])/d - (a*B*Cot[c + d*x]*(a + b*Tan[c + d*x])^2)/d
```

Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3605

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x
])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)),
 Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(
b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
+ 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e
 + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&
 LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
```

```
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3624

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[B*x, x] + (Dist[A, Int[1/Tan[e
 + f*x], x], x] + Dist[C, Int[Tan[e + f*x], x], x]) /; FreeQ[{e, f, A, B, C
}, x] && NeQ[A, C]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx=\int \cot^2(c+dx)(a+b\tan(c+dx))^3(B+C$$

$$=-\frac{aB\cot(c+dx)(a+b\tan(c+dx))^2}{d}+\int$$

$$=\frac{b^2(aB+bC)\tan(c+dx)}{d}-\frac{aB\cot(c+dx)}{}$$

$$=-\left(a^3B-3ab^2B-3a^2bC+b^3C\right)x+\frac{b^2(aB}{}$$

$$=-\left(a^3B-3ab^2B-3a^2bC+b^3C\right)x-\frac{b^2(bB}{}$$

Mathematica [C] time = 0.469283, size = 113, normalized size = 0.95

$$\frac{2a^2(aC+3bB)\log(\tan(c+dx))-2a^3B\cot(c+dx)+i(a+ib)^3(B+iC)\log(-\tan(c+dx)+i)+(b+ia)^3(B-iC)\log}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c +
 d*x]^2),x]
```

```
[Out] (-2*a^3*B*Cot[c + d*x] + I*(a + I*b)^3*(B + I*C)*Log[I - Tan[c + d*x]] + 2*
a^2*(3*b*B + a*C)*Log[Tan[c + d*x]] + (I*a + b)^3*(B - I*C)*Log[I + Tan[c +
 d*x]] + 2*b^3*C*Tan[c + d*x])/(2*d)
```

Maple [A] time = 0.082, size = 168, normalized size = 1.4

$$-Ba^3x+3\,Bab^2x+3\,Cxa^2b-Cb^3x-\frac{B\cot(dx+c)\,a^3}{d}+3\,\frac{Ba^2b\ln(\sin(dx+c))}{d}-\frac{Bb^3\ln(\cos(dx+c))}{d}-\frac{Ba^3c}{d}+3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

```
[Out] -B*a^3*x+3*B*a*b^2*x+3*C*x*a^2*b-C*b^3*x-1/d*B*cot(d*x+c)*a^3+3/d*B*a^2*b*l
n(sin(d*x+c))-1/d*B*b^3*ln(cos(d*x+c))-1/d*B*a^3*c+3/d*B*a*b^2*c+1/d*C*b^3*
```

```
tan(d*x+c)+1/d*C*a^3*ln(sin(d*x+c))-3/d*C*a*b^2*ln(cos(d*x+c))+3/d*C*a^2*b*
c-1/d*C*b^3*c
```

Maxima [A] time = 1.76325, size = 169, normalized size = 1.42

$$\frac{2\,Cb^3\tan\left(dx+c\right)-\frac{2\,Ba^3}{\tan(dx+c)}-2\left(Ba^3-3\,Ca^2b-3\,Bab^2+Cb^3\right)(dx+c)-\left(Ca^3+3\,Ba^2b-3\,Cab^2-Bb^3\right)\log\left(\tan\left(dx+\right.\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/2*(2*C*b^3*tan(d*x + c) - 2*B*a^3/tan(d*x + c) - 2*(B*a^3 - 3*C*a^2*b - 3
*B*a*b^2 + C*b^3)*(d*x + c) - (C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*log(t
an(d*x + c)^2 + 1) + 2*(C*a^3 + 3*B*a^2*b)*log(tan(d*x + c)))/d
```

Fricas [A] time = 1.75226, size = 347, normalized size = 2.92

$$\frac{2\,Cb^3\tan\left(dx+c\right)^2-2\,Ba^3-2\left(Ba^3-3\,Ca^2b-3\,Bab^2+Cb^3\right)dx\tan\left(dx+c\right)+\left(Ca^3+3\,Ba^2b\right)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx\right.}{2\,d\tan\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/2*(2*C*b^3*tan(d*x + c)^2 - 2*B*a^3 - 2*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2 +
C*b^3)*d*x*tan(d*x + c) + (C*a^3 + 3*B*a^2*b)*log(tan(d*x + c)^2/(tan(d*x +
 c)^2 + 1))*tan(d*x + c) - (3*C*a*b^2 + B*b^3)*log(1/(tan(d*x + c)^2 + 1))*
tan(d*x + c))/(d*tan(d*x + c))
```

Sympy [A] time = 35.377, size = 214, normalized size = 1.8

$$\begin{cases}\text{NaN} \\ x\left(a+b\tan\left(c\right)\right)^3\left(B\tan\left(c\right)+C\tan^2\left(c\right)\right)\cot^3\left(c\right) \\ \text{NaN} \\ -Ba^3x-\frac{Ba^3}{d\tan\left(c+dx\right)}-\frac{3Ba^2b\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{3Ba^2b\log\left(\tan\left(c+dx\right)\right)}{d}+3Bab^2x+\frac{Bb^3\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}-\frac{Ca^3\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*tan(d*x+c))**3*(B*tan(d*x+c)+C*tan(d*x+c)**2),
x)
```

```
[Out] Piecewise((nan, Eq(c, 0) & Eq(d, 0)), (x*(a + b*tan(c))**3*(B*tan(c) + C*ta
n(c)**2)*cot(c)**3, Eq(d, 0)), (nan, Eq(c, -d*x)), (-B*a**3*x - B*a**3/(d*t
an(c + d*x)) - 3*B*a**2*b*log(tan(c + d*x)**2 + 1)/(2*d) + 3*B*a**2*b*log(t
an(c + d*x))/d + 3*B*a*b**2*x + B*b**3*log(tan(c + d*x)**2 + 1)/(2*d) - C*a
**3*log(tan(c + d*x)**2 + 1)/(2*d) + C*a**3*log(tan(c + d*x))/d + 3*C*a**2*
b*x + 3*C*a*b**2*log(tan(c + d*x)**2 + 1)/(2*d) - C*b**3*x + C*b**3*tan(c +
```

```
d*x)/d, True))
```

Giac [A] time = 2.51525, size = 205, normalized size = 1.72

$$\frac{2\,Cb^3\tan\left(dx+c\right)-2\left(Ba^3-3\,Ca^2b-3\,Bab^2+Cb^3\right)(dx+c)-\left(Ca^3+3\,Ba^2b-3\,Cab^2-Bb^3\right)\log\left(\tan\left(dx+c\right)^2+1\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/2*(2*C*b^3*tan(d*x + c) - 2*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*(d*x
+ c) - (C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*log(tan(d*x + c)^2 + 1) + 2*
(C*a^3 + 3*B*a^2*b)*log(abs(tan(d*x + c))) - 2*(C*a^3*tan(d*x + c) + 3*B*a^
2*b*tan(d*x + c) + B*a^3)/tan(d*x + c))/d
```

3.21 $\int \cot^4(c+dx)(a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+\right.$

Optimal. Leaf size=127

$$-\frac{a\left(a^2B-3abC-3b^2B\right)\log(\sin(c+dx))}{d}-x\left(3a^2bB+a^3C-3ab^2C-b^3B\right)-\frac{a^2(aC+2bB)\cot(c+dx)}{d}-\frac{aB\cot^2(c+d}{}$$

```
[Out] -((3*a^2*b*B - b^3*B + a^3*C - 3*a*b^2*C)*x) - (a^2*(2*b*B + a*C)*Cot[c + d
*x])/d - (b^3*C*Log[Cos[c + d*x]])/d - (a*(a^2*B - 3*b^2*B - 3*a*b*C)*Log[S
in[c + d*x]])/d - (a*B*Cot[c + d*x]^2*(a + b*Tan[c + d*x])^2)/(2*d)
```

Rubi [A] time = 0.354905, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3632, 3605, 3635, 3624, 3475}

$$-\frac{a\left(a^2B-3abC-3b^2B\right)\log(\sin(c+dx))}{d}-x\left(3a^2bB+a^3C-3ab^2C-b^3B\right)-\frac{a^2(aC+2bB)\cot(c+dx)}{d}-\frac{aB\cot^2(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^
2),x]
```


```
[Out] -((3*a^2*b*B - b^3*B + a^3*C - 3*a*b^2*C)*x) - (a^2*(2*b*B + a*C)*Cot[c + d
*x])/d - (b^3*C*Log[Cos[c + d*x]])/d - (a*(a^2*B - 3*b^2*B - 3*a*b*C)*Log[S
in[c + d*x]])/d - (a*B*Cot[c + d*x]^2*(a + b*Tan[c + d*x])^2)/(2*d)
```


Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3605

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x
])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)),
 Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(
b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
+ 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e
 + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&
 LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3635

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(c^2*C - B*c*d + A*d^2)*(c +
d*Tan[e + f*x])^(n + 1))/(d^2*f*(n + 1)*(c^2 + d^2)), x] + Dist[1/(d*(c^2 +
```

```
 d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(c^
2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*Ta
n[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -
1]
```

Rule 3624

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[B*x, x] + (Dist[A, Int[1/Tan[e
 + f*x], x], x] + Dist[C, Int[Tan[e + f*x], x], x]) /; FreeQ[{e, f, A, B, C
}, x] && NeQ[A, C]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx = \int \cot^3(c+dx)(a+b\tan(c+dx))^3(B+C$$

$$= -\frac{aB\cot^2(c+dx)(a+b\tan(c+dx))^2}{2d} + \frac{1}{2}$$

$$= -\frac{a^2(2bB+aC)\cot(c+dx)}{d} - \frac{aB\cot^2(c+$$

$$= -\left(3a^2bB-b^3B+a^3C-3ab^2C\right)x - \frac{a^2(2b$$

$$= -\left(3a^2bB-b^3B+a^3C-3ab^2C\right)x - \frac{a^2(2b$$

Mathematica [C] time = 0.447553, size = 126, normalized size = 0.99

$$\frac{-2a\left(a^2B-3abC-3b^2B\right)\log(\tan(c+dx))-2a^2(aC+3bB)\cot(c+dx)+a^3(-B)\cot^2(c+dx)+(a+ib)^3(B+iC)\log}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c +
 d*x]^2),x]
```

```
[Out] (-2*a^2*(3*b*B + a*C)*Cot[c + d*x] - a^3*B*Cot[c + d*x]^2 + (a + I*b)^3*(B
+ I*C)*Log[I - Tan[c + d*x]] - 2*a*(a^2*B - 3*b^2*B - 3*a*b*C)*Log[Tan[c +
d*x]] + (a - I*b)^3*(B - I*C)*Log[I + Tan[c + d*x]])/(2*d)
```

Maple [A] time = 0.137, size = 186, normalized size = 1.5

$$Bxb^3+\frac{Bb^3c}{d}-\frac{Cb^3\ln(\cos(dx+c))}{d}+3\frac{Bab^2\ln(\sin(dx+c))}{d}+3Cab^2x+3\frac{Cab^2c}{d}-3Ba^2bx-3\frac{B\cot(dx+c)a^2b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

```
[Out] B*x*b^3+1/d*B*b^3*c-b^3*C*ln(cos(d*x+c))/d+3/d*B*a*b^2*ln(sin(d*x+c))+3*C*a
*b^2*x+3/d*C*a*b^2*c-3*B*a^2*b*x-3/d*B*cot(d*x+c)*a^2*b-3/d*B*a^2*b*c+3/d*C
*a^2*b*ln(sin(d*x+c))-1/2/d*B*a^3*cot(d*x+c)^2-1/d*B*a^3*ln(sin(d*x+c))-C*x
*a^3-1/d*C*cot(d*x+c)*a^3-1/d*C*a^3*c
```

Maxima [A] time = 1.78442, size = 192, normalized size = 1.51

$$-\frac{2\left(Ca^3+3\,Ba^2b-3\,Cab^2-Bb^3\right)(dx+c)-\left(Ba^3-3\,Ca^2b-3\,Bab^2+Cb^3\right)\log\left(\tan\left(dx+c\right)^2+1\right)+2\left(Ba^3-3\,Ca^2b-3\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] -1/2*(2*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*(d*x + c) - (B*a^3 - 3*C*a^
2*b - 3*B*a*b^2 + C*b^3)*log(tan(d*x + c)^2 + 1) + 2*(B*a^3 - 3*C*a^2*b - 3
*B*a*b^2)*log(tan(d*x + c)) + (B*a^3 + 2*(C*a^3 + 3*B*a^2*b)*tan(d*x + c))/
tan(d*x + c)^2)/d
```

Fricas [A] time = 1.76333, size = 383, normalized size = 3.02

$$-\frac{Cb^3\log\left(\frac{1}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^2+Ba^3+\left(Ba^3-3\,Ca^2b-3\,Bab^2\right)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^2+\left(Ba^3+2\left(Ca^3+\right.\right.}{2\,d\tan\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] -1/2*(C*b^3*log(1/(tan(d*x + c)^2 + 1))*tan(d*x + c)^2 + B*a^3 + (B*a^3 - 3
*C*a^2*b - 3*B*a*b^2)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)
^2 + (B*a^3 + 2*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*d*x)*tan(d*x + c)^2
 + 2*(C*a^3 + 3*B*a^2*b)*tan(d*x + c))/(d*tan(d*x + c)^2)
```

Sympy [A] time = 78.7063, size = 253, normalized size = 1.99

$$\begin{cases} \text{NaN} \\ x\left(a+b\tan\left(c\right)\right)^3\left(B\tan\left(c\right)+C\tan^2\left(c\right)\right)\cot^4\left(c\right) \\ \text{NaN} \\ \frac{Ba^3\log\left(\tan^2(c+dx)+1\right)}{2d}-\frac{Ba^3\log\left(\tan(c+dx)\right)}{d}-\frac{Ba^3}{2d\tan^2(c+dx)}-3Ba^2bx-\frac{3Ba^2b}{d\tan(c+dx)}-\frac{3Bab^2\log\left(\tan^2(c+dx)+1\right)}{2d}+\frac{3Bab^2\log\left(\tan(c+dx)\right)}{d} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*tan(d*x+c))**3*(B*tan(d*x+c)+C*tan(d*x+c)**2),
x)
```

```
[Out] Piecewise((nan, Eq(c, 0) & Eq(d, 0)), (x*(a + b*tan(c))**3*(B*tan(c) + C*ta
n(c)**2)*cot(c)**4, Eq(d, 0)), (nan, Eq(c, -d*x)), (B*a**3*log(tan(c + d*x)
```

```
**2 + 1)/(2*d) - B*a**3*log(tan(c + d*x))/d - B*a**3/(2*d*tan(c + d*x)**2)
- 3*B*a**2*b*x - 3*B*a**2*b/(d*tan(c + d*x)) - 3*B*a*b**2*log(tan(c + d*x)*
*2 + 1)/(2*d) + 3*B*a*b**2*log(tan(c + d*x))/d + B*b**3*x - C*a**3*x - C*a*
*3/(d*tan(c + d*x)) - 3*C*a**2*b*log(tan(c + d*x)**2 + 1)/(2*d) + 3*C*a**2*
b*log(tan(c + d*x))/d + 3*C*a*b**2*x + C*b**3*log(tan(c + d*x)**2 + 1)/(2*d
), True))
```

Giac [A] time = 2.51342, size = 261, normalized size = 2.06

$$-\frac{2\left(Ca^3 + 3\,Ba^2b - 3\,Cab^2 - Bb^3\right)(dx + c) - \left(Ba^3 - 3\,Ca^2b - 3\,Bab^2 + Cb^3\right)\log\left(\tan\left(dx + c\right)^2 + 1\right) + 2\left(Ba^3 - 3\,Ca^2b\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] -1/2*(2*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*(d*x + c) - (B*a^3 - 3*C*a^
2*b - 3*B*a*b^2 + C*b^3)*log(tan(d*x + c)^2 + 1) + 2*(B*a^3 - 3*C*a^2*b - 3
*B*a*b^2)*log(abs(tan(d*x + c))) - (3*B*a^3*tan(d*x + c)^2 - 9*C*a^2*b*tan(
d*x + c)^2 - 9*B*a*b^2*tan(d*x + c)^2 - 2*C*a^3*tan(d*x + c) - 6*B*a^2*b*ta
n(d*x + c) - B*a^3)/tan(d*x + c)^2)/d
```

3.22 $\int \cot^5(c+dx)(a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+\right.$

Optimal. Leaf size=154

$$\frac{a\left(3a^2B-9abC-8b^2B\right)\cot(c+dx)}{3d}-\frac{\left(3a^2bB+a^3C-3ab^2C-b^3B\right)\log(\sin(c+dx))}{d}+x\left(-3a^2bC+a^3B-3ab^2B+b^3C\right.$$

```
[Out] (a^3*B - 3*a*b^2*B - 3*a^2*b*C + b^3*C)*x + (a*(3*a^2*B - 8*b^2*B - 9*a*b*C
)*Cot[c + d*x])/(3*d) - (a^2*(5*b*B + 3*a*C)*Cot[c + d*x]^2)/(6*d) - ((3*a^
2*b*B - b^3*B + a^3*C - 3*a*b^2*C)*Log[Sin[c + d*x]])/d - (a*B*Cot[c + d*x]
^3*(a + b*Tan[c + d*x])^2)/(3*d)
```

Rubi [A] time = 0.426631, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {3632, 3605, 3635, 3628, 3531, 3475}

$$\frac{a\left(3a^2B-9abC-8b^2B\right)\cot(c+dx)}{3d}-\frac{\left(3a^2bB+a^3C-3ab^2C-b^3B\right)\log(\sin(c+dx))}{d}+x\left(-3a^2bC+a^3B-3ab^2B+b^3C\right.$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^
2),x]
```

```
[Out] (a^3*B - 3*a*b^2*B - 3*a^2*b*C + b^3*C)*x + (a*(3*a^2*B - 8*b^2*B - 9*a*b*C
)*Cot[c + d*x])/(3*d) - (a^2*(5*b*B + 3*a*C)*Cot[c + d*x]^2)/(6*d) - ((3*a^
2*b*B - b^3*B + a^3*C - 3*a*b^2*C)*Log[Sin[c + d*x]])/d - (a*B*Cot[c + d*x]
^3*(a + b*Tan[c + d*x])^2)/(3*d)
```

Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3605

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x
])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)),
 Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(
b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
+ 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e
 + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&
 LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3635

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
```

```
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(c^2*C - B*c*d + A*d^2)*(c +
d*Tan[e + f*x])^(n + 1))/(d^2*f*(n + 1)*(c^2 + d^2)), x] + Dist[1/(d*(c^2 +
 d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(c^
2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*Ta
n[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -
1]
```

Rule 3628

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2
- a*b*B + a^2*C)*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 + b^2)), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = \int \cot^4(c+dx)(a+b\tan(c+dx))^3(B+C$$
$$= -\frac{aB\cot^3(c+dx)(a+b\tan(c+dx))^2}{3d}+\frac{1}{3}$$
$$= -\frac{a^2(5bB+3aC)\cot^2(c+dx)}{6d}-\frac{aB\cot^3(c}{}$$
$$= \frac{a\left(3a^2B-8b^2B-9abC\right)\cot(c+dx)}{3d}-\frac{a^2(}{}$$
$$= \left(a^3B-3ab^2B-3a^2bC+b^3C\right)x+\frac{a\left(3a^2B\right.}{}$$
$$= \left(a^3B-3ab^2B-3a^2bC+b^3C\right)x+\frac{a\left(3a^2B\right.}{}$$

Mathematica [C] time = 1.27275, size = 164, normalized size = 1.06

$$\frac{6a\left(a^2B-3abC-3b^2B\right)\cot(c+dx)-6\left(3a^2bB+a^3C-3ab^2C-b^3B\right)\log(\tan(c+dx))-3a^2(aC+3bB)\cot^2(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c +
 d*x]^2),x]
```

```
[Out] (6*a*(a^2*B - 3*b^2*B - 3*a*b*C)*Cot[c + d*x] - 3*a^2*(3*b*B + a*C)*Cot[c +
 d*x]^2 - 2*a^3*B*Cot[c + d*x]^3 + 3*(a + I*b)^3*((-I)*B + C)*Log[I - Tan[c
 + d*x]] - 6*(3*a^2*b*B - b^3*B + a^3*C - 3*a*b^2*C)*Log[Tan[c + d*x]] + 3*
(a - I*b)^3*(I*B + C)*Log[I + Tan[c + d*x]])/(6*d)
```

Maple [A] time = 0.087, size = 233, normalized size = 1.5

$$\frac{Bb^3\ln(\sin(dx+c))}{d}+Cb^3x+\frac{Cb^3c}{d}-3\,Bab^2x-3\,\frac{B\cot(dx+c)\,ab^2}{d}-3\,\frac{Bab^2c}{d}+3\,\frac{Cab^2\ln(\sin(dx+c))}{d}-\frac{3\,Ba^2b\,(\cot}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

```
[Out] 1/d*B*b^3*ln(sin(d*x+c))+C*b^3*x+1/d*C*b^3*c-3*B*a*b^2*x-3/d*B*cot(d*x+c)*a
*b^2-3/d*B*a*b^2*c+3/d*C*a*b^2*ln(sin(d*x+c))-3/2/d*B*a^2*b*cot(d*x+c)^2-3/
d*B*a^2*b*ln(sin(d*x+c))-3*C*x*a^2*b-3/d*C*cot(d*x+c)*a^2*b-3/d*C*a^2*b*c-1
/3/d*B*a^3*cot(d*x+c)^3+1/d*B*cot(d*x+c)*a^3+B*a^3*x+1/d*B*a^3*c-1/2/d*C*a^
3*cot(d*x+c)^2-1/d*C*a^3*ln(sin(d*x+c))
```

Maxima [A] time = 1.69671, size = 243, normalized size = 1.58

$$\frac{6\left(Ba^3-3\,Ca^2b-3\,Bab^2+Cb^3\right)(dx+c)+3\left(Ca^3+3\,Ba^2b-3\,Cab^2-Bb^3\right)\log\left(\tan(dx+c)^2+1\right)-6\left(Ca^3+3\,Ba^2b-3\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/6*(6*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*(d*x + c) + 3*(C*a^3 + 3*B*a
^2*b - 3*C*a*b^2 - B*b^3)*log(tan(d*x + c)^2 + 1) - 6*(C*a^3 + 3*B*a^2*b -
3*C*a*b^2 - B*b^3)*log(tan(d*x + c)) - (2*B*a^3 - 6*(B*a^3 - 3*C*a^2*b - 3*
B*a*b^2)*tan(d*x + c)^2 + 3*(C*a^3 + 3*B*a^2*b)*tan(d*x + c))/tan(d*x + c)^
3)/d
```

Fricas [A] time = 1.09676, size = 419, normalized size = 2.72

$$-\frac{3\left(Ca^3+3\,Ba^2b-3\,Cab^2-Bb^3\right)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan(dx+c)^3+2\,Ba^3+3\left(Ca^3+3\,Ba^2b-2\left(Ba^3-3\,Ca^2b-3\,Bab^2\right.\right.}{6\,d\tan(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] -1/6*(3*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*log(tan(d*x + c)^2/(tan(d*x
 + c)^2 + 1))*tan(d*x + c)^3 + 2*B*a^3 + 3*(C*a^3 + 3*B*a^2*b - 2*(B*a^3 -
3*C*a^2*b - 3*B*a*b^2 + C*b^3)*d*x)*tan(d*x + c)^3 - 6*(B*a^3 - 3*C*a^2*b -
 3*B*a*b^2)*tan(d*x + c)^2 + 3*(C*a^3 + 3*B*a^2*b)*tan(d*x + c))/(d*tan(d*x
```

```
+ c)^3)
```

Sympy [A] time = 142.243, size = 330, normalized size = 2.14

$$\begin{cases} \text{NaN} \\ x(a+b\tan(c))^3\left(B\tan(c)+C\tan^2(c)\right)\cot^5(c) \\ Ba^3x+\frac{Ba^3}{d\tan(c+dx)}-\frac{Ba^3}{3d\tan^3(c+dx)}+\frac{3Ba^2b\log\left(\tan^2(c+dx)+1\right)}{2d}-\frac{3Ba^2b\log(\tan(c+dx))}{d}-\frac{3Ba^2b}{2d\tan^2(c+dx)}-3Bab^2x-\frac{3Bab^2}{d\tan(c+dx)}- \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+b*tan(d*x+c))**3*(B*tan(d*x+c)+C*tan(d*x+c)**2),
x)
```

```
[Out] Piecewise((nan, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x))), (x*(a
 + b*tan(c))**3*(B*tan(c) + C*tan(c)**2)*cot(c)**5, Eq(d, 0)), (B*a**3*x +
B*a**3/(d*tan(c + d*x)) - B*a**3/(3*d*tan(c + d*x)**3) + 3*B*a**2*b*log(tan
(c + d*x)**2 + 1)/(2*d) - 3*B*a**2*b*log(tan(c + d*x))/d - 3*B*a**2*b/(2*d*
tan(c + d*x)**2) - 3*B*a*b**2*x - 3*B*a*b**2/(d*tan(c + d*x)) - B*b**3*log(
tan(c + d*x)**2 + 1)/(2*d) + B*b**3*log(tan(c + d*x))/d + C*a**3*log(tan(c
+ d*x)**2 + 1)/(2*d) - C*a**3*log(tan(c + d*x))/d - C*a**3/(2*d*tan(c + d*x
)**2) - 3*C*a**2*b*x - 3*C*a**2*b/(d*tan(c + d*x)) - 3*C*a*b**2*log(tan(c +
 d*x)**2 + 1)/(2*d) + 3*C*a*b**2*log(tan(c + d*x))/d + C*b**3*x, True))
```

Giac [B] time = 2.65894, size = 527, normalized size = 3.42

$$\frac{Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3Ca^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-9Ba^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-15Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+36Ca^2b\tan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/24*(B*a^3*tan(1/2*d*x + 1/2*c)^3 - 3*C*a^3*tan(1/2*d*x + 1/2*c)^2 - 9*B*a
^2*b*tan(1/2*d*x + 1/2*c)^2 - 15*B*a^3*tan(1/2*d*x + 1/2*c) + 36*C*a^2*b*ta
n(1/2*d*x + 1/2*c) + 36*B*a*b^2*tan(1/2*d*x + 1/2*c) + 24*(B*a^3 - 3*C*a^2*
b - 3*B*a*b^2 + C*b^3)*(d*x + c) + 24*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^
3)*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 24*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*
b^3)*log(abs(tan(1/2*d*x + 1/2*c))) + (44*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 13
2*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 132*C*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 44*
B*b^3*tan(1/2*d*x + 1/2*c)^3 + 15*B*a^3*tan(1/2*d*x + 1/2*c)^2 - 36*C*a^2*b
*tan(1/2*d*x + 1/2*c)^2 - 36*B*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*C*a^3*tan(1
/2*d*x + 1/2*c) - 9*B*a^2*b*tan(1/2*d*x + 1/2*c) - B*a^3)/tan(1/2*d*x + 1/2
*c)^3)/d
```

3.23 $\int \cot^6(c+dx)(a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+\right.$

Optimal. Leaf size=191

$$\frac{a\left(2a^2B-6abC-5b^2B\right)\cot^2(c+dx)}{4d}+\frac{\left(3a^2bB+a^3C-3ab^2C-b^3B\right)\cot(c+dx)}{d}+\frac{\left(-3a^2bC+a^3B-3ab^2B+b^3C\right)\log}{d}$$

```
[Out] (3*a^2*b*B - b^3*B + a^3*C - 3*a*b^2*C)*x + ((3*a^2*b*B - b^3*B + a^3*C - 3
*a*b^2*C)*Cot[c + d*x])/d + (a*(2*a^2*B - 5*b^2*B - 6*a*b*C)*Cot[c + d*x]^2
)/(4*d) - (a^2*(3*b*B + 2*a*C)*Cot[c + d*x]^3)/(6*d) + ((a^3*B - 3*a*b^2*B
- 3*a^2*b*C + b^3*C)*Log[Sin[c + d*x]])/d - (a*B*Cot[c + d*x]^4*(a + b*Tan[
c + d*x])^2)/(4*d)
```

Rubi [A] time = 0.513793, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {3632, 3605, 3635, 3628, 3529, 3531, 3475}

$$\frac{a\left(2a^2B-6abC-5b^2B\right)\cot^2(c+dx)}{4d}+\frac{\left(3a^2bB+a^3C-3ab^2C-b^3B\right)\cot(c+dx)}{d}+\frac{\left(-3a^2bC+a^3B-3ab^2B+b^3C\right)\log}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^
2),x]
```

```
[Out] (3*a^2*b*B - b^3*B + a^3*C - 3*a*b^2*C)*x + ((3*a^2*b*B - b^3*B + a^3*C - 3
*a*b^2*C)*Cot[c + d*x])/d + (a*(2*a^2*B - 5*b^2*B - 6*a*b*C)*Cot[c + d*x]^2
)/(4*d) - (a^2*(3*b*B + 2*a*C)*Cot[c + d*x]^3)/(6*d) + ((a^3*B - 3*a*b^2*B
- 3*a^2*b*C + b^3*C)*Log[Sin[c + d*x]])/d - (a*B*Cot[c + d*x]^4*(a + b*Tan[
c + d*x])^2)/(4*d)
```

Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3605

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x
])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)),
 Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(
b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
+ 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e
 + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&
 LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3635

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(c^2*C - B*c*d + A*d^2)*(c +
d*Tan[e + f*x])^(n + 1))/(d^2*f*(n + 1)*(c^2 + d^2)), x] + Dist[1/(d*(c^2 +
 d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(c^
2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*Ta
n[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -
1]
```

Rule 3628

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2
- a*b*B + a^2*C)*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 + b^2)), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cot^6(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx &= \int \cot^5(c+dx)(a+b\tan(c+dx))^3(B+C \\
&= -\frac{aB\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d}+\frac{1}{4} \\
&= -\frac{a^2(3bB+2aC)\cot^3(c+dx)}{6d}-\frac{aB\cot^4(c \\
&= \frac{a\left(2a^2B-5b^2B-6abC\right)\cot^2(c+dx)}{4d}-\frac{a^2 \\
&= \frac{\left(3a^2bB-b^3B+a^3C-3ab^2C\right)\cot(c+dx)}{d} \\
&= \left(3a^2bB-b^3B+a^3C-3ab^2C\right)x+\frac{\left(3a^2bB\right. \\
&= \left(3a^2bB-b^3B+a^3C-3ab^2C\right)x+\frac{\left(3a^2bB\right.
\end{aligned}$$

Mathematica [C] time = 0.74581, size = 199, normalized size = 1.04

$$6a\left(a^2B-3abC-3b^2B\right)\cot^2(c+dx)+12\left(3a^2bB+a^3C-3ab^2C-b^3B\right)\cot(c+dx)+12\left(-3a^2bC+a^3B-3ab^2B+b^3C\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c +
 d*x]^2),x]
```

```
[Out] (12*(3*a^2*b*B - b^3*B + a^3*C - 3*a*b^2*C)*Cot[c + d*x] + 6*a*(a^2*B - 3*b
^2*B - 3*a*b*C)*Cot[c + d*x]^2 - 4*a^2*(3*b*B + a*C)*Cot[c + d*x]^3 - 3*a^3
*B*Cot[c + d*x]^4 - 6*(a + I*b)^3*(B + I*C)*Log[I - Tan[c + d*x]] + 12*(a^3
*B - 3*a*b^2*B - 3*a^2*b*C + b^3*C)*Log[Tan[c + d*x]] - 6*(a - I*b)^3*(B -
I*C)*Log[I + Tan[c + d*x]])/(12*d)
```

Maple [A] time = 0.101, size = 302, normalized size = 1.6

$$-Bxb^3-\frac{B\cot(dx+c)\,b^3}{d}-\frac{Bb^3c}{d}+\frac{Cb^3\ln(\sin(dx+c))}{d}-\frac{3\,Bab^2\,(\cot(dx+c))^2}{2\,d}-3\,\frac{Bab^2\ln(\sin(dx+c))}{d}-3\,Cab^2x-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

```
[Out] -B*x*b^3-1/d*B*cot(d*x+c)*b^3-1/d*B*b^3*c+1/d*C*b^3*ln(sin(d*x+c))-3/2/d*B*
a*b^2*cot(d*x+c)^2-3/d*B*a*b^2*ln(sin(d*x+c))-3*C*a*b^2*x-3/d*C*cot(d*x+c)*
a*b^2-3/d*C*a*b^2*c-1/d*B*a^2*b*cot(d*x+c)^3+3*B*a^2*b*x+3/d*B*cot(d*x+c)*a
^2*b+3/d*B*a^2*b*c-3/2/d*C*a^2*b*cot(d*x+c)^2-3/d*C*a^2*b*ln(sin(d*x+c))-1/
4/d*B*a^3*cot(d*x+c)^4+1/2/d*B*a^3*cot(d*x+c)^2+1/d*B*a^3*ln(sin(d*x+c))-1/
3/d*C*a^3*cot(d*x+c)^3+1/d*C*cot(d*x+c)*a^3+C*x*a^3+1/d*C*a^3*c
```

Maxima [A] time = 1.71969, size = 290, normalized size = 1.52

$$\frac{12\left(Ca^3+3\,Ba^2b-3\,Cab^2-Bb^3\right)(dx+c)-6\left(Ba^3-3\,Ca^2b-3\,Bab^2+Cb^3\right)\log\left(\tan(dx+c)^2+1\right)+12\left(Ba^3-3\,Ca^2b\cdots\right.}{1\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/12*(12*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*(d*x + c) - 6*(B*a^3 - 3*C
*a^2*b - 3*B*a*b^2 + C*b^3)*log(tan(d*x + c)^2 + 1) + 12*(B*a^3 - 3*C*a^2*b
 - 3*B*a*b^2 + C*b^3)*log(tan(d*x + c)) - (3*B*a^3 - 12*(C*a^3 + 3*B*a^2*b
- 3*C*a*b^2 - B*b^3)*tan(d*x + c)^3 - 6*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2)*tan
(d*x + c)^2 + 4*(C*a^3 + 3*B*a^2*b)*tan(d*x + c))/tan(d*x + c)^4)/d
```

Fricas [A] time = 1.14495, size = 518, normalized size = 2.71

$$6\left(Ba^3-3\,Ca^2b-3\,Bab^2+Cb^3\right)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan(dx+c)^4+3\left(3\,Ba^3-6\,Ca^2b-6\,Bab^2+4\left(Ca^3+3\,Ba^2b-3\,Cab^2\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/12*(6*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*log(tan(d*x + c)^2/(tan(d*x
 + c)^2 + 1))*tan(d*x + c)^4 + 3*(3*B*a^3 - 6*C*a^2*b - 6*B*a*b^2 + 4*(C*a^
3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*d*x)*tan(d*x + c)^4 - 3*B*a^3 + 12*(C*a^
3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*tan(d*x + c)^3 + 6*(B*a^3 - 3*C*a^2*b -
3*B*a*b^2)*tan(d*x + c)^2 - 4*(C*a^3 + 3*B*a^2*b)*tan(d*x + c))/(d*tan(d*x
+ c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+b*tan(d*x+c))**3*(B*tan(d*x+c)+C*tan(d*x+c)**2),
x)
```

```
[Out] Timed out
```

Giac [B] time = 2.83771, size = 713, normalized size = 3.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] -1/192*(3*B*a^3*tan(1/2*d*x + 1/2*c)^4 - 8*C*a^3*tan(1/2*d*x + 1/2*c)^3 - 2
4*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 36*B*a^3*tan(1/2*d*x + 1/2*c)^2 + 72*C*a
^2*b*tan(1/2*d*x + 1/2*c)^2 + 72*B*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 120*C*a^3
*tan(1/2*d*x + 1/2*c) + 360*B*a^2*b*tan(1/2*d*x + 1/2*c) - 288*C*a*b^2*tan(
1/2*d*x + 1/2*c) - 96*B*b^3*tan(1/2*d*x + 1/2*c) - 192*(C*a^3 + 3*B*a^2*b -
 3*C*a*b^2 - B*b^3)*(d*x + c) + 192*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)
*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 192*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b
^3)*log(abs(tan(1/2*d*x + 1/2*c))) + (400*B*a^3*tan(1/2*d*x + 1/2*c)^4 - 12
00*C*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 1200*B*a*b^2*tan(1/2*d*x + 1/2*c)^4 + 4
00*C*b^3*tan(1/2*d*x + 1/2*c)^4 - 120*C*a^3*tan(1/2*d*x + 1/2*c)^3 - 360*B*
a^2*b*tan(1/2*d*x + 1/2*c)^3 + 288*C*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 96*B*b^
3*tan(1/2*d*x + 1/2*c)^3 - 36*B*a^3*tan(1/2*d*x + 1/2*c)^2 + 72*C*a^2*b*tan
(1/2*d*x + 1/2*c)^2 + 72*B*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 8*C*a^3*tan(1/2*d
*x + 1/2*c) + 24*B*a^2*b*tan(1/2*d*x + 1/2*c) + 3*B*a^3)/tan(1/2*d*x + 1/2*
c)^4)/d
```

3.24 $\int \cot^7(c+dx)(a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+\right.$

Optimal. Leaf size=233

$$\frac{a\left(5a^2B-15abC-12b^2B\right)\cot^3(c+dx)}{15d}+\frac{\left(3a^2bB+a^3C-3ab^2C-b^3B\right)\cot^2(c+dx)}{2d}-\frac{\left(-3a^2bC+a^3B-3ab^2B+b^3C\right)}{d}$$

```
[Out] -((a^3*B - 3*a*b^2*B - 3*a^2*b*C + b^3*C)*x) - ((a^3*B - 3*a*b^2*B - 3*a^2*
b*C + b^3*C)*Cot[c + d*x])/d + ((3*a^2*b*B - b^3*B + a^3*C - 3*a*b^2*C)*Cot
[c + d*x]^2)/(2*d) + (a*(5*a^2*B - 12*b^2*B - 15*a*b*C)*Cot[c + d*x]^3)/(15
*d) - (a^2*(7*b*B + 5*a*C)*Cot[c + d*x]^4)/(20*d) + ((3*a^2*b*B - b^3*B + a
^3*C - 3*a*b^2*C)*Log[Sin[c + d*x]])/d - (a*B*Cot[c + d*x]^5*(a + b*Tan[c +
 d*x])^2)/(5*d)
```

Rubi [A] time = 0.558215, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {3632, 3605, 3635, 3628, 3529, 3531, 3475}

$$\frac{a\left(5a^2B-15abC-12b^2B\right)\cot^3(c+dx)}{15d}+\frac{\left(3a^2bB+a^3C-3ab^2C-b^3B\right)\cot^2(c+dx)}{2d}-\frac{\left(-3a^2bC+a^3B-3ab^2B+b^3C\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^
2),x]
```


```
[Out] -((a^3*B - 3*a*b^2*B - 3*a^2*b*C + b^3*C)*x) - ((a^3*B - 3*a*b^2*B - 3*a^2*
b*C + b^3*C)*Cot[c + d*x])/d + ((3*a^2*b*B - b^3*B + a^3*C - 3*a*b^2*C)*Cot
[c + d*x]^2)/(2*d) + (a*(5*a^2*B - 12*b^2*B - 15*a*b*C)*Cot[c + d*x]^3)/(15
*d) - (a^2*(7*b*B + 5*a*C)*Cot[c + d*x]^4)/(20*d) + ((3*a^2*b*B - b^3*B + a
^3*C - 3*a*b^2*C)*Log[Sin[c + d*x]])/d - (a*B*Cot[c + d*x]^5*(a + b*Tan[c +
 d*x])^2)/(5*d)
```


Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3605

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x
])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)),
 Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(
b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
+ 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e
 + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&
 LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3635

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(c^2*C - B*c*d + A*d^2)*(c +
d*Tan[e + f*x])^(n + 1))/(d^2*f*(n + 1)*(c^2 + d^2)), x] + Dist[1/(d*(c^2 +
 d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(c^
2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*Ta
n[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -
1]
```

Rule 3628

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2
- a*b*B + a^2*C)*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 + b^2)), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^7(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx = \int \cot^6(c+dx)(a+b\tan(c+dx))^3(B+C\tan$$

$$= -\frac{aB\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}+\frac{1}{5}\int$$

$$= -\frac{a^2(7bB+5aC)\cot^4(c+dx)}{20d}-\frac{aB\cot^5(c+}{}$$

$$= \frac{a\left(5a^2B-12b^2B-15abC\right)\cot^3(c+dx)}{15d}-\frac{a^2(}{}$$

$$= \frac{\left(3a^2bB-b^3B+a^3C-3ab^2C\right)\cot^2(c+dx)}{2d}+$$

$$= -\frac{\left(a^3B-3ab^2B-3a^2bC+b^3C\right)\cot(c+dx)}{d}+$$

$$= -\left(a^3B-3ab^2B-3a^2bC+b^3C\right)x-\frac{\left(a^3B-3\right.}{}$$

$$= -\left(a^3B-3ab^2B-3a^2bC+b^3C\right)x-\frac{\left(a^3B-3\right.}{}$$

Mathematica [C] time = 1.16752, size = 237, normalized size = 1.02

$$\frac{20a\left(a^2B-3abC-3b^2B\right)\cot^3(c+dx)+30\left(3a^2bB+a^3C-3ab^2C-b^3B\right)\cot^2(c+dx)-60\left(-3a^2bC+a^3B-3ab^2B+b^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c +
 d*x]^2),x]
```

```
[Out] (-60*(a^3*B - 3*a*b^2*B - 3*a^2*b*C + b^3*C)*Cot[c + d*x] + 30*(3*a^2*b*B -
 b^3*B + a^3*C - 3*a*b^2*C)*Cot[c + d*x]^2 + 20*a*(a^2*B - 3*b^2*B - 3*a*b*
C)*Cot[c + d*x]^3 - 15*a^2*(3*b*B + a*C)*Cot[c + d*x]^4 - 12*a^3*B*Cot[c +
d*x]^5 + (30*I)*(a + I*b)^3*(B + I*C)*Log[I - Tan[c + d*x]] + 60*(3*a^2*b*B
 - b^3*B + a^3*C - 3*a*b^2*C)*Log[Tan[c + d*x]] + 30*(I*a + b)^3*(B - I*C)*
Log[I + Tan[c + d*x]])/(60*d)
```

Maple [A] time = 0.115, size = 376, normalized size = 1.6

$$-\frac{Bab^2(\cot(dx+c))^3}{d}-\frac{B\cot(dx+c)a^3}{d}-Ba^3x-Cb^3x-\frac{Ba^3c}{d}-\frac{3Cab^2(\cot(dx+c))^2}{2d}-\frac{3Ba^2b(\cot(dx+c))^4}{4d}-\frac{Ca^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^7*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

```
[Out] -1/d*B*a*b^2*cot(d*x+c)^3-1/d*B*cot(d*x+c)*a^3-B*a^3*x-C*b^3*x-1/d*B*a^3*c-
3/2/d*C*a*b^2*cot(d*x+c)^2-3/4/d*B*a^2*b*cot(d*x+c)^4-1/d*C*a^2*b*cot(d*x+c
)^3-1/d*C*b^3*c+1/3/d*B*a^3*cot(d*x+c)^3+3*B*a*b^2*x+3*C*x*a^2*b+1/d*C*a^3*
ln(sin(d*x+c))-1/d*B*b^3*ln(sin(d*x+c))+1/2/d*C*a^3*cot(d*x+c)^2-1/d*C*cot(
d*x+c)*b^3-1/4/d*C*a^3*cot(d*x+c)^4-1/2/d*B*b^3*cot(d*x+c)^2-1/5/d*B*a^3*co
t(d*x+c)^5-3/d*C*a*b^2*ln(sin(d*x+c))+3/2/d*B*a^2*b*cot(d*x+c)^2+3/d*C*cot(
d*x+c)*a^2*b+3/d*B*a^2*b*ln(sin(d*x+c))+3/d*B*a*b^2*c+3/d*C*a^2*b*c+3/d*B*c
ot(d*x+c)*a*b^2
```

Maxima [A] time = 1.6342, size = 338, normalized size = 1.45

$$-\frac{60\left(Ba^3-3\,Ca^2b-3\,Bab^2+Cb^3\right)(dx+c)+30\left(Ca^3+3\,Ba^2b-3\,Cab^2-Bb^3\right)\log\left(\tan\left(dx+c\right)^2+1\right)-60\left(Ca^3+3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] -1/60*(60*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*(d*x + c) + 30*(C*a^3 + 3
*B*a^2*b - 3*C*a*b^2 - B*b^3)*log(tan(d*x + c)^2 + 1) - 60*(C*a^3 + 3*B*a^2
*b - 3*C*a*b^2 - B*b^3)*log(tan(d*x + c)) + (60*(B*a^3 - 3*C*a^2*b - 3*B*a*
b^2 + C*b^3)*tan(d*x + c)^4 + 12*B*a^3 - 30*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2
- B*b^3)*tan(d*x + c)^3 - 20*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2)*tan(d*x + c)^2
 + 15*(C*a^3 + 3*B*a^2*b)*tan(d*x + c))/tan(d*x + c)^5)/d
```

Fricas [A] time = 1.12562, size = 620, normalized size = 2.66

$$\frac{30\left(Ca^3+3\,Ba^2b-3\,Cab^2-Bb^3\right)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^5+15\left(3\,Ca^3+9\,Ba^2b-6\,Cab^2-2\,Bb^3-4\left(Ba^3-3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/60*(30*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*log(tan(d*x + c)^2/(tan(d*
x + c)^2 + 1))*tan(d*x + c)^5 + 15*(3*C*a^3 + 9*B*a^2*b - 6*C*a*b^2 - 2*B*b
^3 - 4*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*d*x)*tan(d*x + c)^5 - 60*(B*
a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*tan(d*x + c)^4 - 12*B*a^3 + 30*(C*a^3
+ 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*tan(d*x + c)^3 + 20*(B*a^3 - 3*C*a^2*b - 3
*B*a*b^2)*tan(d*x + c)^2 - 15*(C*a^3 + 3*B*a^2*b)*tan(d*x + c))/(d*tan(d*x
+ c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7*(a+b*tan(d*x+c))**3*(B*tan(d*x+c)+C*tan(d*x+c)**2),
x)
```

```
[Out] Timed out
```

Giac [B] time = 3.01282, size = 905, normalized size = 3.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/960*(6*B*a^3*tan(1/2*d*x + 1/2*c)^5 - 15*C*a^3*tan(1/2*d*x + 1/2*c)^4 - 4
5*B*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 70*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 120*C*
a^2*b*tan(1/2*d*x + 1/2*c)^3 + 120*B*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 180*C*a
^3*tan(1/2*d*x + 1/2*c)^2 + 540*B*a^2*b*tan(1/2*d*x + 1/2*c)^2 - 360*C*a*b^
2*tan(1/2*d*x + 1/2*c)^2 - 120*B*b^3*tan(1/2*d*x + 1/2*c)^2 + 660*B*a^3*tan
(1/2*d*x + 1/2*c) - 1800*C*a^2*b*tan(1/2*d*x + 1/2*c) - 1800*B*a*b^2*tan(1/
2*d*x + 1/2*c) + 480*C*b^3*tan(1/2*d*x + 1/2*c) - 960*(B*a^3 - 3*C*a^2*b -
3*B*a*b^2 + C*b^3)*(d*x + c) - 960*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*
log(tan(1/2*d*x + 1/2*c)^2 + 1) + 960*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^
3)*log(abs(tan(1/2*d*x + 1/2*c))) - (2192*C*a^3*tan(1/2*d*x + 1/2*c)^5 + 65
76*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 6576*C*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 2
192*B*b^3*tan(1/2*d*x + 1/2*c)^5 + 660*B*a^3*tan(1/2*d*x + 1/2*c)^4 - 1800*
C*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 1800*B*a*b^2*tan(1/2*d*x + 1/2*c)^4 + 480*
C*b^3*tan(1/2*d*x + 1/2*c)^4 - 180*C*a^3*tan(1/2*d*x + 1/2*c)^3 - 540*B*a^2
*b*tan(1/2*d*x + 1/2*c)^3 + 360*C*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 120*B*b^3*
tan(1/2*d*x + 1/2*c)^3 - 70*B*a^3*tan(1/2*d*x + 1/2*c)^2 + 120*C*a^2*b*tan(
1/2*d*x + 1/2*c)^2 + 120*B*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 15*C*a^3*tan(1/2*
d*x + 1/2*c) + 45*B*a^2*b*tan(1/2*d*x + 1/2*c) + 6*B*a^3)/tan(1/2*d*x + 1/2
*c)^5)/d
```

3.25 $\int \frac{\tan^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=127

$$-\frac{a^3(bB-aC)\log(a+b\tan(c+dx))}{b^3d\left(a^2+b^2\right)}+\frac{(aB+bC)\log(\cos(c+dx))}{d\left(a^2+b^2\right)}-\frac{x(bB-aC)}{a^2+b^2}+\frac{(bB-aC)\tan(c+dx)}{b^2d}+\frac{C\tan^2(\cdots}{2b\cdots}$$

```
[Out] -(((b*B - a*C)*x)/(a^2 + b^2)) + ((a*B + b*C)*Log[Cos[c + d*x]])/((a^2 + b^
2)*d) - (a^3*(b*B - a*C)*Log[a + b*Tan[c + d*x]])/(b^3*(a^2 + b^2)*d) + ((b
*B - a*C)*Tan[c + d*x])/(b^2*d) + (C*Tan[c + d*x]^2)/(2*b*d)
```

Rubi [A] time = 0.468378, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {3632, 3607, 3647, 3626, 3617, 31, 3475}

$$-\frac{a^3(bB-aC)\log(a+b\tan(c+dx))}{b^3d\left(a^2+b^2\right)}+\frac{(aB+bC)\log(\cos(c+dx))}{d\left(a^2+b^2\right)}-\frac{x(bB-aC)}{a^2+b^2}+\frac{(bB-aC)\tan(c+dx)}{b^2d}+\frac{C\tan^2(\cdots}{2b\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Tan[c + d*x]^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x
]),x]
```

```
[Out] -(((b*B - a*C)*x)/(a^2 + b^2)) + ((a*B + b*C)*Log[Cos[c + d*x]])/((a^2 + b^
2)*d) - (a^3*(b*B - a*C)*Log[a + b*Tan[c + d*x]])/(b^3*(a^2 + b^2)*d) + ((b
*B - a*C)*Tan[c + d*x])/(b^2*d) + (C*Tan[c + d*x]^2)/(2*b*d)
```

Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3607

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*B*(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(m
+ n)), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan
[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
+ n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b*
(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1] &
& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
```

```
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3626

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((a*A + b*B -
a*C)*x)/(a^2 + b^2), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3617

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx=\int \frac{\tan^3(c+dx)(B+C\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$

$$=\frac{C\tan^2(c+dx)}{2bd}+\frac{\int \frac{\tan(c+dx)\left(-2aC-2bC\tan(c+dx)+2(bB-aC)\tan^2(c+dx)\right)}{a+b\tan(c+dx)}$$

$$=\frac{(bB-aC)\tan(c+dx)}{b^2d}+\frac{C\tan^2(c+dx)}{2bd}+\frac{\int \frac{-2a(bB-aC)-2b^2B\tan(c}{a+}$$

$$=-\frac{(bB-aC)x}{a^2+b^2}+\frac{(bB-aC)\tan(c+dx)}{b^2d}+\frac{C\tan^2(c+dx)}{2bd}-\frac{\left(a^3(b\right.}{}$$

$$=-\frac{(bB-aC)x}{a^2+b^2}+\frac{(aB+bC)\log(\cos(c+dx))}{\left(a^2+b^2\right)d}+\frac{(bB-aC)\tan(c+}{b^2d}$$

$$=-\frac{(bB-aC)x}{a^2+b^2}+\frac{(aB+bC)\log(\cos(c+dx))}{\left(a^2+b^2\right)d}-\frac{a^3(bB-aC)\log(a}{b^3\left(a^2+\right.}$$

Mathematica [C] time = 1.36556, size = 138, normalized size = 1.09

$$\frac{\frac{2a^3(aC-bB)\log(a+b\tan(c+dx))}{b^2\left(a^2+b^2\right)}+\frac{2(bB-aC)\tan(c+dx)}{b}-\frac{b(B+iC)\log(-\tan(c+dx)+i)}{a+ib}-\frac{b(B-iC)\log(\tan(c+dx)+i)}{a-ib}+C\tan^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c
 + d*x]),x]
```

```
[Out] (-((b*(B + I*C)*Log[I - Tan[c + d*x]])/(a + I*b)) - (b*(B - I*C)*Log[I + Ta
n[c + d*x]])/(a - I*b) + (2*a^3*(-(b*B) + a*C)*Log[a + b*Tan[c + d*x]])/(b^
2*(a^2 + b^2)) + (2*(b*B - a*C)*Tan[c + d*x])/b + C*Tan[c + d*x]^2)/(2*b*d)
```

Maple [A] time = 0.037, size = 211, normalized size = 1.7

$$\frac{C\left(\tan\left(dx+c\right)\right)^{2}}{2\,bd}+\frac{B\tan\left(dx+c\right)}{bd}-\frac{C\tan\left(dx+c\right)a}{b^{2}d}-\frac{\ln\left(1+\left(\tan\left(dx+c\right)\right)^{2}\right)aB}{2\,d\left(a^{2}+b^{2}\right)}-\frac{\ln\left(1+\left(\tan\left(dx+c\right)\right)^{2}\right)Cb}{2\,d\left(a^{2}+b^{2}\right)}-\frac{B}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x)
```

```
[Out] 1/2*C*tan(d*x+c)^2/b/d+1/d/b*B*tan(d*x+c)-1/d/b^2*C*tan(d*x+c)*a-1/2/d/(a^2
+b^2)*ln(1+tan(d*x+c)^2)*a*B-1/2/d/(a^2+b^2)*ln(1+tan(d*x+c)^2)*C*b-1/d/(a^
2+b^2)*B*arctan(tan(d*x+c))*b+1/d/(a^2+b^2)*C*arctan(tan(d*x+c))*a-1/d/b^2*
a^3/(a^2+b^2)*ln(a+b*tan(d*x+c))*B+1/d/b^3*a^4/(a^2+b^2)*ln(a+b*tan(d*x+c))
*C
```

Maxima [A] time = 1.75865, size = 176, normalized size = 1.39

$$\frac{\frac{2\,(Ca-Bb)(dx+c)}{a^{2}+b^{2}}+\frac{2\left(Ca^{4}-Ba^{3}b\right)\log\left(b\tan\left(dx+c\right)+a\right)}{a^{2}b^{3}+b^{5}}-\frac{(Ba+Cb)\log\left(\tan\left(dx+c\right)^{2}+1\right)}{a^{2}+b^{2}}+\frac{Cb\tan\left(dx+c\right)^{2}-2\,(Ca-Bb)\tan\left(dx+c\right)}{b^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, al
gorithm="maxima")
```

```
[Out] 1/2*(2*(C*a - B*b)*(d*x + c)/(a^2 + b^2) + 2*(C*a^4 - B*a^3*b)*log(b*tan(d*
x + c) + a)/(a^2*b^3 + b^5) - (B*a + C*b)*log(tan(d*x + c)^2 + 1)/(a^2 + b^
2) + (C*b*tan(d*x + c)^2 - 2*(C*a - B*b)*tan(d*x + c))/b^2)/d
```

Fricas [A] time = 1.26386, size = 412, normalized size = 3.24

$$\frac{2\left(Cab^{3}-Bb^{4}\right)dx+\left(Ca^{2}b^{2}+Cb^{4}\right)\tan\left(dx+c\right)^{2}+\left(Ca^{4}-Ba^{3}b\right)\log\left(\frac{b^{2}\tan\left(dx+c\right)^{2}+2\,ab\tan\left(dx+c\right)+a^{2}}{\tan\left(dx+c\right)^{2}+1}\right)-\left(Ca^{4}-Ba^{3}b-Ba\right.}{2\left(a^{2}b^{3}+b^{5}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, al
gorithm="fricas")
```

```
[Out] 1/2*(2*(C*a*b^3 - B*b^4)*d*x + (C*a^2*b^2 + C*b^4)*tan(d*x + c)^2 + (C*a^4
- B*a^3*b)*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c
```

```
)^2 + 1)) - (C*a^4 - B*a^3*b - B*a*b^3 - C*b^4)*log(1/(tan(d*x + c)^2 + 1))
 - 2*(C*a^3*b - B*a^2*b^2 + C*a*b^3 - B*b^4)*tan(d*x + c))/((a^2*b^3 + b^5)
*d)
```

Sympy [A] time = 21.1479, size = 1306, normalized size = 10.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x*(B*tan(c) + C*tan(c)**2)*tan(c), Eq(a, 0) & Eq(b, 0) & Eq(
d, 0)), (3*B*d*x*tan(c + d*x)/(-2*b*d*tan(c + d*x) + 2*I*b*d) - 3*I*B*d*x/(
-2*b*d*tan(c + d*x) + 2*I*b*d) - I*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/
(-2*b*d*tan(c + d*x) + 2*I*b*d) - B*log(tan(c + d*x)**2 + 1)/(-2*b*d*tan(c
+ d*x) + 2*I*b*d) - 2*B*tan(c + d*x)**2/(-2*b*d*tan(c + d*x) + 2*I*b*d) - 3
*B/(-2*b*d*tan(c + d*x) + 2*I*b*d) + 3*I*C*d*x*tan(c + d*x)/(-2*b*d*tan(c +
 d*x) + 2*I*b*d) + 3*C*d*x/(-2*b*d*tan(c + d*x) + 2*I*b*d) + 2*C*log(tan(c
+ d*x)**2 + 1)*tan(c + d*x)/(-2*b*d*tan(c + d*x) + 2*I*b*d) - 2*I*C*log(tan
(c + d*x)**2 + 1)/(-2*b*d*tan(c + d*x) + 2*I*b*d) - C*tan(c + d*x)**3/(-2*b
*d*tan(c + d*x) + 2*I*b*d) - I*C*tan(c + d*x)**2/(-2*b*d*tan(c + d*x) + 2*I
*b*d) - 3*I*C/(-2*b*d*tan(c + d*x) + 2*I*b*d), Eq(a, -I*b)), (-3*B*d*x*tan(
c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) - 3*I*B*d*x/(2*b*d*tan(c + d*x) + 2
*I*b*d) - I*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2
*I*b*d) + B*log(tan(c + d*x)**2 + 1)/(2*b*d*tan(c + d*x) + 2*I*b*d) + 2*B*t
an(c + d*x)**2/(2*b*d*tan(c + d*x) + 2*I*b*d) + 3*B/(2*b*d*tan(c + d*x) + 2
*I*b*d) + 3*I*C*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) - 3*C*d*x/(
2*b*d*tan(c + d*x) + 2*I*b*d) - 2*C*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(
2*b*d*tan(c + d*x) + 2*I*b*d) - 2*I*C*log(tan(c + d*x)**2 + 1)/(2*b*d*tan(c
 + d*x) + 2*I*b*d) + C*tan(c + d*x)**3/(2*b*d*tan(c + d*x) + 2*I*b*d) - I*C
*tan(c + d*x)**2/(2*b*d*tan(c + d*x) + 2*I*b*d) - 3*I*C/(2*b*d*tan(c + d*x)
 + 2*I*b*d), Eq(a, I*b)), ((-B*log(tan(c + d*x)**2 + 1)/(2*d) + B*tan(c + d
*x)**2/(2*d) + C*x + C*tan(c + d*x)**3/(3*d) - C*tan(c + d*x)/d)/a, Eq(b, 0
)), (x*(B*tan(c) + C*tan(c)**2)*tan(c)**2/(a + b*tan(c)), Eq(d, 0)), (-2*B*
a**3*b*log(a/b + tan(c + d*x))/(2*a**2*b**3*d + 2*b**5*d) + 2*B*a**2*b**2*t
an(c + d*x)/(2*a**2*b**3*d + 2*b**5*d) - B*a*b**3*log(tan(c + d*x)**2 + 1)/
(2*a**2*b**3*d + 2*b**5*d) - 2*B*b**4*d*x/(2*a**2*b**3*d + 2*b**5*d) + 2*B*
b**4*tan(c + d*x)/(2*a**2*b**3*d + 2*b**5*d) + 2*C*a**4*log(a/b + tan(c + d
*x))/(2*a**2*b**3*d + 2*b**5*d) - 2*C*a**3*b*tan(c + d*x)/(2*a**2*b**3*d +
2*b**5*d) + C*a**2*b**2*tan(c + d*x)**2/(2*a**2*b**3*d + 2*b**5*d) + 2*C*a*
b**3*d*x/(2*a**2*b**3*d + 2*b**5*d) - 2*C*a*b**3*tan(c + d*x)/(2*a**2*b**3*
d + 2*b**5*d) - C*b**4*log(tan(c + d*x)**2 + 1)/(2*a**2*b**3*d + 2*b**5*d)
+ C*b**4*tan(c + d*x)**2/(2*a**2*b**3*d + 2*b**5*d), True))
```

Giac [A] time = 1.80761, size = 182, normalized size = 1.43

$$\frac{\frac{2\,(Ca-Bb)(dx+c)}{a^2+b^2}-\frac{(Ba+Cb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}+\frac{2\left(Ca^4-Ba^3b\right)\log(|b\tan(dx+c)+a|)}{a^2b^3+b^5}+\frac{Cb\tan(dx+c)^2-2\,Ca\tan(dx+c)+2\,Bb\tan(dx+c)}{b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, al
gorithm="giac")
```

```
[Out] 1/2*(2*(C*a - B*b)*(d*x + c)/(a^2 + b^2) - (B*a + C*b)*log(tan(d*x + c)^2 +
 1)/(a^2 + b^2) + 2*(C*a^4 - B*a^3*b)*log(abs(b*tan(d*x + c) + a))/(a^2*b^3
 + b^5) + (C*b*tan(d*x + c)^2 - 2*C*a*tan(d*x + c) + 2*B*b*tan(d*x + c))/b^
2)/d
```

3.26 $\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=101

$$\frac{a^2(bB-aC)\log(a+b\tan(c+dx))}{b^2d\left(a^2+b^2\right)}-\frac{(bB-aC)\log(\cos(c+dx))}{d\left(a^2+b^2\right)}-\frac{x(aB+bC)}{a^2+b^2}+\frac{C\tan(c+dx)}{bd}$$

```
[Out] -(((a*B + b*C)*x)/(a^2 + b^2)) - ((b*B - a*C)*Log[Cos[c + d*x]])/((a^2 + b^
2)*d) + (a^2*(b*B - a*C)*Log[a + b*Tan[c + d*x]])/(b^2*(a^2 + b^2)*d) + (C*
Tan[c + d*x])/(b*d)
```

Rubi [A] time = 0.243285, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3632, 3606, 3626, 3617, 31, 3475}

$$\frac{a^2(bB-aC)\log(a+b\tan(c+dx))}{b^2d\left(a^2+b^2\right)}-\frac{(bB-aC)\log(\cos(c+dx))}{d\left(a^2+b^2\right)}-\frac{x(aB+bC)}{a^2+b^2}+\frac{C\tan(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Tan[c + d*x]*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x])
,x]
```


```
[Out] -(((a*B + b*C)*x)/(a^2 + b^2)) - ((b*B - a*C)*Log[Cos[c + d*x]])/((a^2 + b^
2)*d) + (a^2*(b*B - a*C)*Log[a + b*Tan[c + d*x]])/(b^2*(a^2 + b^2)*d) + (C*
Tan[c + d*x])/(b*d)
```


Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3606

```
Int[(((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (
f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b^
2*B*Tan[e + f*x])/(d*f), x] + Dist[1/d, Int[(a^2*A*d - b^2*B*c + (2*a*A*b +
 B*(a^2 - b^2))*d*Tan[e + f*x] + (A*b^2*d - b*B*(b*c - 2*a*d))*Tan[e + f*x]
^2)/(c + d*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && N
eQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3626

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((a*A + b*B -
a*C)*x)/(a^2 + b^2), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3617

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx = \int \frac{\tan^2(c+dx)(B+C\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$

$$= \frac{C\tan(c+dx)}{bd} + \frac{\int \frac{-aC-bC\tan(c+dx)+(bB-aC)\tan^2(c+dx)}{a+b\tan(c+dx)}\,dx}{b}$$

$$= -\frac{(aB+bC)x}{a^2+b^2} + \frac{C\tan(c+dx)}{bd} + \frac{(bB-aC)\int \tan(c+dx)\,dx}{a^2+b^2} +$$

$$= -\frac{(aB+bC)x}{a^2+b^2} - \frac{(bB-aC)\log(\cos(c+dx))}{\left(a^2+b^2\right)d} + \frac{C\tan(c+dx)}{bd} +$$

$$= -\frac{(aB+bC)x}{a^2+b^2} - \frac{(bB-aC)\log(\cos(c+dx))}{\left(a^2+b^2\right)d} + \frac{a^2(bB-aC)\log(}{b^2\left(a^2\right.}$$

Mathematica [C] time = 0.571322, size = 118, normalized size = 1.17

$$\frac{\frac{2a^2(bB-aC)\log(a+b\tan(c+dx))}{b^2\left(a^2+b^2\right)} + \frac{i(B+iC)\log(-\tan(c+dx)+i)}{a+ib} - \frac{(C+iB)\log(\tan(c+dx)+i)}{a-ib} + \frac{2C\tan(c+dx)}{b}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c +
 d*x]),x]
```

```
[Out] ((I*(B + I*C)*Log[I - Tan[c + d*x]])/(a + I*b) - ((I*B + C)*Log[I + Tan[c +
 d*x]])/(a - I*b) + (2*a^2*(b*B - a*C)*Log[a + b*Tan[c + d*x]])/(b^2*(a^2 +
 b^2)) + (2*C*Tan[c + d*x])/b)/(2*d)
```

Maple [A] time = 0.034, size = 179, normalized size = 1.8

$$\frac{C\tan(dx+c)}{bd} + \frac{\ln\left(1+(\tan(dx+c))^2\right)Bb}{2d\left(a^2+b^2\right)} - \frac{\ln\left(1+(\tan(dx+c))^2\right)Ca}{2d\left(a^2+b^2\right)} - \frac{B\arctan(\tan(dx+c))a}{d\left(a^2+b^2\right)} - \frac{C\arctan(\tan}{d\left(a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x)
```

```
[Out] C*tan(d*x+c)/b/d+1/2/d/(a^2+b^2)*ln(1+tan(d*x+c)^2)*B*b-1/2/d/(a^2+b^2)*ln(
1+tan(d*x+c)^2)*C*a-1/d/(a^2+b^2)*B*arctan(tan(d*x+c))*a-1/d/(a^2+b^2)*C*ar
ctan(tan(d*x+c))*b+1/d/b*a^2/(a^2+b^2)*ln(a+b*tan(d*x+c))*B-1/d/b^2*a^3/(a^
2+b^2)*ln(a+b*tan(d*x+c))*C
```

Maxima [A] time = 1.77521, size = 147, normalized size = 1.46

$$-\frac{\frac{2\,(Ba+Cb)(dx+c)}{a^2+b^2}+\frac{2\left(Ca^3-Ba^2b\right)\log(b\tan(dx+c)+a)}{a^2b^2+b^4}+\frac{(Ca-Bb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}-\frac{2\,C\tan(dx+c)}{b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, algo
rithm="maxima")
```

```
[Out] -1/2*(2*(B*a + C*b)*(d*x + c)/(a^2 + b^2) + 2*(C*a^3 - B*a^2*b)*log(b*tan(d
*x + c) + a)/(a^2*b^2 + b^4) + (C*a - B*b)*log(tan(d*x + c)^2 + 1)/(a^2 + b
^2) - 2*C*tan(d*x + c)/b)/d
```

Fricas [A] time = 1.20611, size = 333, normalized size = 3.3

$$-\frac{2\left(Bab^2+Cb^3\right)dx+\left(Ca^3-Ba^2b\right)\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)-\left(Ca^3-Ba^2b+Cab^2-Bb^3\right)\log\left(\frac{1}{\tan(dx+c)^2+1}\right)-2\,(}{2\left(a^2b^2+b^4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, algo
rithm="fricas")
```

```
[Out] -1/2*(2*(B*a*b^2 + C*b^3)*d*x + (C*a^3 - B*a^2*b)*log((b^2*tan(d*x + c)^2 +
 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) - (C*a^3 - B*a^2*b + C*a*b
^2 - B*b^3)*log(1/(tan(d*x + c)^2 + 1)) - 2*(C*a^2*b + C*b^3)*tan(d*x + c))
/((a^2*b^2 + b^4)*d)
```

Sympy [A] time = 16.6724, size = 1020, normalized size = 10.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x*(B*tan(c) + C*tan(c)**2), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)),
 ((-B*x + B*tan(c + d*x)/d - C*log(tan(c + d*x)**2 + 1)/(2*d) + C*tan(c + d
*x)**2/(2*d))/a, Eq(b, 0)), (-I*B*d*x*tan(c + d*x)/(-2*b*d*tan(c + d*x) + 2
*I*b*d) - B*d*x/(-2*b*d*tan(c + d*x) + 2*I*b*d) - B*log(tan(c + d*x)**2 + 1
)*tan(c + d*x)/(-2*b*d*tan(c + d*x) + 2*I*b*d) + I*B*log(tan(c + d*x)**2 +
1)/(-2*b*d*tan(c + d*x) + 2*I*b*d) + I*B/(-2*b*d*tan(c + d*x) + 2*I*b*d) +
```

```
3*C*d*x*tan(c + d*x)/(-2*b*d*tan(c + d*x) + 2*I*b*d) - 3*I*C*d*x/(-2*b*d*ta
n(c + d*x) + 2*I*b*d) - I*C*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(-2*b*d*t
an(c + d*x) + 2*I*b*d) - C*log(tan(c + d*x)**2 + 1)/(-2*b*d*tan(c + d*x) +
2*I*b*d) - 2*C*tan(c + d*x)**2/(-2*b*d*tan(c + d*x) + 2*I*b*d) - 3*C/(-2*b*
d*tan(c + d*x) + 2*I*b*d), Eq(a, -I*b)), (-I*B*d*x*tan(c + d*x)/(2*b*d*tan(
c + d*x) + 2*I*b*d) + B*d*x/(2*b*d*tan(c + d*x) + 2*I*b*d) + B*log(tan(c +
d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) + I*B*log(tan(c +
d*x)**2 + 1)/(2*b*d*tan(c + d*x) + 2*I*b*d) + I*B/(2*b*d*tan(c + d*x) + 2*I
*b*d) - 3*C*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) - 3*I*C*d*x/(2*
b*d*tan(c + d*x) + 2*I*b*d) - I*C*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*
b*d*tan(c + d*x) + 2*I*b*d) + C*log(tan(c + d*x)**2 + 1)/(2*b*d*tan(c + d*x
) + 2*I*b*d) + 2*C*tan(c + d*x)**2/(2*b*d*tan(c + d*x) + 2*I*b*d) + 3*C/(2*
b*d*tan(c + d*x) + 2*I*b*d), Eq(a, I*b)), (x*(B*tan(c) + C*tan(c)**2)*tan(c
)/(a + b*tan(c)), Eq(d, 0)), (2*B*a**2*b*log(a/b + tan(c + d*x))/(2*a**2*b*
*2*d + 2*b**4*d) - 2*B*a*b**2*d*x/(2*a**2*b**2*d + 2*b**4*d) + B*b**3*log(t
an(c + d*x)**2 + 1)/(2*a**2*b**2*d + 2*b**4*d) - 2*C*a**3*log(a/b + tan(c +
 d*x))/(2*a**2*b**2*d + 2*b**4*d) + 2*C*a**2*b*tan(c + d*x)/(2*a**2*b**2*d
+ 2*b**4*d) - C*a*b**2*log(tan(c + d*x)**2 + 1)/(2*a**2*b**2*d + 2*b**4*d)
- 2*C*b**3*d*x/(2*a**2*b**2*d + 2*b**4*d) + 2*C*b**3*tan(c + d*x)/(2*a**2*b
**2*d + 2*b**4*d), True))
```

Giac [A] time = 1.5514, size = 149, normalized size = 1.48

$$-\frac{\frac{2\,(Ba+Cb)(dx+c)}{a^2+b^2}+\frac{(Ca-Bb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}+\frac{2\left(Ca^3-Ba^2b\right)\log(|b\tan(dx+c)+a|)}{a^2b^2+b^4}-\frac{2\,C\tan(dx+c)}{b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, algo
rithm="giac")
```

```
[Out] -1/2*(2*(B*a + C*b)*(d*x + c)/(a^2 + b^2) + (C*a - B*b)*log(tan(d*x + c)^2
+ 1)/(a^2 + b^2) + 2*(C*a^3 - B*a^2*b)*log(abs(b*tan(d*x + c) + a))/(a^2*b^
2 + b^4) - 2*C*tan(d*x + c)/b)/d
```

3.27 $\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=85

$$-\frac{a(bB-aC)\log(a+b\tan(c+dx))}{bd\left(a^2+b^2\right)}-\frac{(aB+bC)\log(\cos(c+dx))}{d\left(a^2+b^2\right)}+\frac{x(bB-aC)}{a^2+b^2}$$

```
[Out] ((b*B - a*C)*x)/(a^2 + b^2) - ((a*B + b*C)*Log[Cos[c + d*x]])/((a^2 + b^2)*
d) - (a*(b*B - a*C)*Log[a + b*Tan[c + d*x]])/(b*(a^2 + b^2)*d)
```

Rubi [A] time = 0.162711, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1629, 635, 203, 260}

$$-\frac{a(bB-aC)\log(a+b\tan(c+dx))}{bd\left(a^2+b^2\right)}-\frac{(aB+bC)\log(\cos(c+dx))}{d\left(a^2+b^2\right)}+\frac{x(bB-aC)}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Tan[c + d*x] + C*Tan[c + d*x]^2)/(a + b*Tan[c + d*x]),x]
```

```
[Out] ((b*B - a*C)*x)/(a^2 + b^2) - ((a*B + b*C)*Log[Cos[c + d*x]])/((a^2 + b^2)*
d) - (a*(b*B - a*C)*Log[a + b*Tan[c + d*x]])/(b*(a^2 + b^2)*d)
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x(B+Cx)}{(a+bx)(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a(-bB+aC)}{(a^2+b^2)(a+bx)} + \frac{bB-aC+(aB+bC)x}{(a^2+b^2)(1+x^2)}\right) dx, x, \tan(c+dx)\right)}{d}$$
$$= -\frac{a(bB-aC)\log(a+b\tan(c+dx))}{b(a^2+b^2)d} + \frac{\text{Subst}\left(\int \frac{bB-aC+(aB+bC)x}{1+x^2}\,dx, x, \tan(c+\right.}{(a^2+b^2)d}$$
$$= -\frac{a(bB-aC)\log(a+b\tan(c+dx))}{b(a^2+b^2)d} + \frac{(bB-aC)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+\right.}{(a^2+b^2)d}$$
$$= \frac{(bB-aC)x}{a^2+b^2} - \frac{(aB+bC)\log(\cos(c+dx))}{(a^2+b^2)d} - \frac{a(bB-aC)\log(a+b\tan(c+dx}{b(a^2+b^2)d}$$

Mathematica [C] time = 0.169879, size = 98, normalized size = 1.15

$$\frac{b(a-ib)(B+iC)\log(-\tan(c+dx)+i)+b(a+ib)(B-iC)\log(\tan(c+dx)+i)+2a(aC-bB)\log(a+b\tan(c+dx))}{2bd(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Tan[c + d*x] + C*Tan[c + d*x]^2)/(a + b*Tan[c + d*x]),x]
```

```
[Out] ((a - I*b)*b*(B + I*C)*Log[I - Tan[c + d*x]] + (a + I*b)*b*(B - I*C)*Log[I
+ Tan[c + d*x]] + 2*a*(-(b*B) + a*C)*Log[a + b*Tan[c + d*x]])/(2*b*(a^2 + b
^2)*d)
```

Maple [A] time = 0.033, size = 159, normalized size = 1.9

$$\frac{\ln\left(1+(\tan(dx+c))^2\right)aB}{2d(a^2+b^2)} + \frac{\ln\left(1+(\tan(dx+c))^2\right)Cb}{2d(a^2+b^2)} + \frac{B\arctan(\tan(dx+c))\,b}{d(a^2+b^2)} - \frac{C\arctan(\tan(dx+c))\,a}{d(a^2+b^2)} - a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x)
```

```
[Out] 1/2/d/(a^2+b^2)*ln(1+tan(d*x+c)^2)*a*B+1/2/d/(a^2+b^2)*ln(1+tan(d*x+c)^2)*C
*b+1/d/(a^2+b^2)*B*arctan(tan(d*x+c))*b-1/d/(a^2+b^2)*C*arctan(tan(d*x+c))*
a-1/d*a/(a^2+b^2)*ln(a+b*tan(d*x+c))*B+1/d*a^2/(a^2+b^2)/b*ln(a+b*tan(d*x+c
))*C
```

Maxima [A] time = 2.41237, size = 127, normalized size = 1.49

$$-\frac{\frac{2(Ca-Bb)(dx+c)}{a^2+b^2} - \frac{2(Ca^2-Bab)\log(b\tan(dx+c)+a)}{a^2b+b^3} - \frac{(Ba+Cb)\log(\tan(dx+c)^2+1)}{a^2+b^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/2*(2*(C*a - B*b)*(d*x + c)/(a^2 + b^2) - 2*(C*a^2 - B*a*b)*log(b*tan(d*x
 + c) + a)/(a^2*b + b^3) - (B*a + C*b)*log(tan(d*x + c)^2 + 1)/(a^2 + b^2))
/d
```

Fricas [A] time = 1.17741, size = 251, normalized size = 2.95

$$-\frac{2\left(Cab-Bb^{2}\right)dx-\left(Ca^{2}-Bab\right)\log\left(\frac{b^{2}\tan(dx+c)^{2}+2\,ab\tan(dx+c)+a^{2}}{\tan(dx+c)^{2}+1}\right)+\left(Ca^{2}+Cb^{2}\right)\log\left(\frac{1}{\tan(dx+c)^{2}+1}\right)}{2\left(a^{2}b+b^{3}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/2*(2*(C*a*b - B*b^2)*d*x - (C*a^2 - B*a*b)*log((b^2*tan(d*x + c)^2 + 2*a
*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) + (C*a^2 + C*b^2)*log(1/(tan(d
*x + c)^2 + 1)))/((a^2*b + b^3)*d)
```

Sympy [A] time = 7.0537, size = 711, normalized size = 8.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x*(B*tan(c) + C*tan(c)**2)/tan(c), Eq(a, 0) & Eq(b, 0) & Eq(
d, 0)), (-B*d*x*tan(c + d*x)/(-2*b*d*tan(c + d*x) + 2*I*b*d) + I*B*d*x/(-2*
b*d*tan(c + d*x) + 2*I*b*d) + B/(-2*b*d*tan(c + d*x) + 2*I*b*d) - I*C*d*x*t
an(c + d*x)/(-2*b*d*tan(c + d*x) + 2*I*b*d) - C*d*x/(-2*b*d*tan(c + d*x) +
2*I*b*d) - C*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(-2*b*d*tan(c + d*x) + 2
*I*b*d) + I*C*log(tan(c + d*x)**2 + 1)/(-2*b*d*tan(c + d*x) + 2*I*b*d) + I*
C/(-2*b*d*tan(c + d*x) + 2*I*b*d), Eq(a, -I*b)), (B*d*x*tan(c + d*x)/(2*b*d
*tan(c + d*x) + 2*I*b*d) + I*B*d*x/(2*b*d*tan(c + d*x) + 2*I*b*d) - B/(2*b*
d*tan(c + d*x) + 2*I*b*d) - I*C*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*
b*d) + C*d*x/(2*b*d*tan(c + d*x) + 2*I*b*d) + C*log(tan(c + d*x)**2 + 1)*ta
n(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) + I*C*log(tan(c + d*x)**2 + 1)/(2
*b*d*tan(c + d*x) + 2*I*b*d) + I*C/(2*b*d*tan(c + d*x) + 2*I*b*d), Eq(a, I*
b)), ((B*log(tan(c + d*x)**2 + 1)/(2*d) - C*x + C*tan(c + d*x)/d)/a, Eq(b,
0)), (x*(B*tan(c) + C*tan(c)**2)/(a + b*tan(c)), Eq(d, 0)), (-2*B*a*b*log(a
/b + tan(c + d*x))/(2*a**2*b*d + 2*b**3*d) + B*a*b*log(tan(c + d*x)**2 + 1)
/(2*a**2*b*d + 2*b**3*d) + 2*B*b**2*d*x/(2*a**2*b*d + 2*b**3*d) + 2*C*a**2*
log(a/b + tan(c + d*x))/(2*a**2*b*d + 2*b**3*d) - 2*C*a*b*d*x/(2*a**2*b*d +
 2*b**3*d) + C*b**2*log(tan(c + d*x)**2 + 1)/(2*a**2*b*d + 2*b**3*d), True)
)
```

Giac [A] time = 1.61591, size = 128, normalized size = 1.51

$$-\frac{\frac{2\,(Ca-Bb)(dx+c)}{a^2+b^2}-\frac{(Ba+Cb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}-\frac{2\,\left(Ca^2-Bab\right)\log(|b\tan(dx+c)+a|)}{a^2b+b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/2*(2*(C*a - B*b)*(d*x + c)/(a^2 + b^2) - (B*a + C*b)*log(tan(d*x + c)^2
+ 1)/(a^2 + b^2) - 2*(C*a^2 - B*a*b)*log(abs(b*tan(d*x + c) + a))/(a^2*b +
b^3))/d
```

3.28 $\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=58

$$\frac{(bB-aC)\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)}+\frac{x(aB+bC)}{a^2+b^2}$$

```
[Out] ((a*B + b*C)*x)/(a^2 + b^2) + ((b*B - a*C)*Log[a*Cos[c + d*x] + b*Sin[c + d
*x]])/((a^2 + b^2)*d)
```

Rubi [A] time = 0.143978, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {3632, 3531, 3530}

$$\frac{(bB-aC)\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)}+\frac{x(aB+bC)}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x])
,x]
```

```
[Out] ((a*B + b*C)*x)/(a^2 + b^2) + ((b*B - a*C)*Log[a*Cos[c + d*x] + b*Sin[c + d
*x]])/((a^2 + b^2)*d)
```

Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx = \int \frac{B+C\tan(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{(aB+bC)x}{a^2+b^2} + \frac{(bB-aC)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}\,dx}{a^2+b^2}$$
$$= \frac{(aB+bC)x}{a^2+b^2} + \frac{(bB-aC)\log(a\cos(c+dx)+b\sin(c+dx))}{\left(a^2+b^2\right)d}$$

Mathematica [A] time = 0.115209, size = 67, normalized size = 1.16

$$\frac{(bB-aC)\left(2\log(a\cot(c+dx)+b)-\log\left(\csc^2(c+dx)\right)\right)-2(aB+bC)\tan^{-1}(\cot(c+dx))}{2d\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c +
 d*x]),x]
```

```
[Out] (-2*(a*B + b*C)*ArcTan[Cot[c + d*x]] + (b*B - a*C)*(2*Log[b + a*Cot[c + d*x
]] - Log[Csc[c + d*x]^2]))/(2*(a^2 + b^2)*d)
```

Maple [B] time = 0.109, size = 153, normalized size = 2.6

$$-\frac{\ln\left(1+(\tan(dx+c))^2\right)Bb}{2d\left(a^2+b^2\right)} + \frac{\ln\left(1+(\tan(dx+c))^2\right)Ca}{2d\left(a^2+b^2\right)} + \frac{B\arctan(\tan(dx+c))\,a}{d\left(a^2+b^2\right)} + \frac{C\arctan(\tan(dx+c))\,b}{d\left(a^2+b^2\right)} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x)
```

```
[Out] -1/2/d/(a^2+b^2)*ln(1+tan(d*x+c)^2)*B*b+1/2/d/(a^2+b^2)*ln(1+tan(d*x+c)^2)*
C*a+1/d/(a^2+b^2)*B*arctan(tan(d*x+c))*a+1/d/(a^2+b^2)*C*arctan(tan(d*x+c))
*b+1/d/(a^2+b^2)*ln(a+b*tan(d*x+c))*B*b-1/d/(a^2+b^2)*ln(a+b*tan(d*x+c))*C*
a
```

Maxima [A] time = 1.7382, size = 119, normalized size = 2.05

$$\frac{\frac{2(Ba+Cb)(dx+c)}{a^2+b^2} - \frac{2(Ca-Bb)\log(b\tan(dx+c)+a)}{a^2+b^2} + \frac{(Ca-Bb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, algo
rithm="maxima")
```

```
[Out] 1/2*(2*(B*a + C*b)*(d*x + c)/(a^2 + b^2) - 2*(C*a - B*b)*log(b*tan(d*x + c)
 + a)/(a^2 + b^2) + (C*a - B*b)*log(tan(d*x + c)^2 + 1)/(a^2 + b^2))/d
```

Fricas [A] time = 1.08596, size = 174, normalized size = 3.

$$\frac{2\,(Ba+Cb)dx-(Ca-Bb)\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)}{2\left(a^2+b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, algo
rithm="fricas")
```

```
[Out] 1/2*(2*(B*a + C*b)*d*x - (C*a - B*b)*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*
x + c) + a^2)/(tan(d*x + c)^2 + 1)))/((a^2 + b^2)*d)
```

Sympy [A] time = 89.4981, size = 541, normalized size = 9.33

$$\begin{cases} \frac{\tilde{\infty}x\left(B\tan(c)+C\tan^2(c)\right)\cot(c)}{\tan(c)} \\ \frac{Bx+\frac{C\log\left(\tan^2(c+dx)+1\right)}{2d}}{a} \\ -\frac{iBdx\tan(c+dx)}{-2bd\tan(c+dx)+2ibd}-\frac{Bdx}{-2bd\tan(c+dx)+2ibd}-\frac{iB}{-2bd\tan(c+dx)+2ibd}-\frac{Cdx\tan(c+dx)}{-2bd\tan(c+dx)+2ibd}+\frac{iCdx}{-2bd\tan(c+dx)+2ibd}+\frac{C}{-2bd\tan(c+dx)+2ibd} \\ -\frac{iBdx\tan(c+dx)}{2bd\tan(c+dx)+2ibd}+\frac{Bdx}{2bd\tan(c+dx)+2ibd}-\frac{iB}{2bd\tan(c+dx)+2ibd}+\frac{Cdx\tan(c+dx)}{2bd\tan(c+dx)+2ibd}+\frac{iCdx}{2bd\tan(c+dx)+2ibd}-\frac{C}{2bd\tan(c+dx)+2ibd} \\ \frac{x\left(B\tan(c)+C\tan^2(c)\right)\cot(c)}{a+b\tan(c)} \\ \frac{2Badx}{2a^2d+2b^2d}+\frac{2Bb\log\left(\frac{a}{b}+\tan(c+dx)\right)}{2a^2d+2b^2d}-\frac{Bb\log\left(\tan^2(c+dx)+1\right)}{2a^2d+2b^2d}-\frac{2Ca\log\left(\frac{a}{b}+\tan(c+dx)\right)}{2a^2d+2b^2d}+\frac{Ca\log\left(\tan^2(c+dx)+1\right)}{2a^2d+2b^2d}+\frac{2Cbdx}{2a^2d+2b^2d} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x*(B*tan(c) + C*tan(c)**2)*cot(c)/tan(c), Eq(a, 0) & Eq(b, 0
) & Eq(d, 0)), ((B*x + C*log(tan(c + d*x)**2 + 1)/(2*d))/a, Eq(b, 0)), (-I*
B*d*x*tan(c + d*x)/(-2*b*d*tan(c + d*x) + 2*I*b*d) - B*d*x/(-2*b*d*tan(c +
d*x) + 2*I*b*d) - I*B/(-2*b*d*tan(c + d*x) + 2*I*b*d) - C*d*x*tan(c + d*x)/
(-2*b*d*tan(c + d*x) + 2*I*b*d) + I*C*d*x/(-2*b*d*tan(c + d*x) + 2*I*b*d) +
 C/(-2*b*d*tan(c + d*x) + 2*I*b*d), Eq(a, -I*b)), (-I*B*d*x*tan(c + d*x)/(2
*b*d*tan(c + d*x) + 2*I*b*d) + B*d*x/(2*b*d*tan(c + d*x) + 2*I*b*d) - I*B/(
2*b*d*tan(c + d*x) + 2*I*b*d) + C*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*
I*b*d) + I*C*d*x/(2*b*d*tan(c + d*x) + 2*I*b*d) - C/(2*b*d*tan(c + d*x) + 2
*I*b*d), Eq(a, I*b)), (x*(B*tan(c) + C*tan(c)**2)*cot(c)/(a + b*tan(c)), Eq
(d, 0)), (2*B*a*d*x/(2*a**2*d + 2*b**2*d) + 2*B*b*log(a/b + tan(c + d*x))/(
2*a**2*d + 2*b**2*d) - B*b*log(tan(c + d*x)**2 + 1)/(2*a**2*d + 2*b**2*d) -
 2*C*a*log(a/b + tan(c + d*x))/(2*a**2*d + 2*b**2*d) + C*a*log(tan(c + d*x)
**2 + 1)/(2*a**2*d + 2*b**2*d) + 2*C*b*d*x/(2*a**2*d + 2*b**2*d), True))
```

Giac [A] time = 1.55263, size = 127, normalized size = 2.19

$$\frac{\frac{2\,(Ba+Cb)(dx+c)}{a^2+b^2}+\frac{(Ca-Bb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}-\frac{2\left(Cab-Bb^2\right)\log(|b\tan(dx+c)+a|)}{a^2b+b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, algo
rithm="giac")
```

```
[Out] 1/2*(2*(B*a + C*b)*(d*x + c)/(a^2 + b^2) + (C*a - B*b)*log(tan(d*x + c)^2 +
 1)/(a^2 + b^2) - 2*(C*a*b - B*b^2)*log(abs(b*tan(d*x + c) + a))/(a^2*b + b
^3))/d
```

3.29 $\int \frac{\cot^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=80

$$-\frac{b(bB-aC)\log(a\cos(c+dx)+b\sin(c+dx))}{ad\left(a^2+b^2\right)}-\frac{x(bB-aC)}{a^2+b^2}+\frac{B\log(\sin(c+dx))}{ad}$$

```
[Out] -(((b*B - a*C)*x)/(a^2 + b^2)) + (B*Log[Sin[c + d*x]])/(a*d) - (b*(b*B - a*
C)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/(a*(a^2 + b^2)*d)
```

Rubi [A] time = 0.200835, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3632, 3611, 3530, 3475}

$$-\frac{b(bB-aC)\log(a\cos(c+dx)+b\sin(c+dx))}{ad\left(a^2+b^2\right)}-\frac{x(bB-aC)}{a^2+b^2}+\frac{B\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x
]),x]
```

```
[Out] -(((b*B - a*C)*x)/(a^2 + b^2)) + (B*Log[Sin[c + d*x]])/(a*d) - (b*(b*B - a*
C)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/(a*(a^2 + b^2)*d)
```

Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3611

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*tan[(e_.) + (f_.
)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[((B*(b
*c + a*d) + A*(a*c - b*d))*x)/((a^2 + b^2)*(c^2 + d^2)), x] + (Dist[(b*(A*b
 - a*B))/((b*c - a*d)*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e +
 f*x]), x], x] + Dist[(d*(B*c - A*d))/((b*c - a*d)*(c^2 + d^2)), Int[(d - c
*Tan[e + f*x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A,
 B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx = \int \frac{\cot(c+dx)(B+C\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$
$$= -\frac{(bB-aC)x}{a^2+b^2} + \frac{B\int\cot(c+dx)\,dx}{a} - \frac{(b(bB-aC))\int\frac{b-a\tan(c+}{a+b\tan(c+}}{a\left(a^2+b^2\right)}$$
$$= -\frac{(bB-aC)x}{a^2+b^2} + \frac{B\log(\sin(c+dx))}{ad} - \frac{b(bB-aC)\log(a\cos(c+}{a\left(a^2+b\right.}$$

Mathematica [C] time = 0.312373, size = 113, normalized size = 1.41

$$-\frac{\frac{2b(bB-aC)\log(a+b\tan(c+dx))}{a\left(a^2+b^2\right)}+\frac{(B+iC)\log(-\tan(c+dx)+i)}{a+ib}+\frac{(B-iC)\log(\tan(c+dx)+i)}{a-ib}-\frac{2B\log(\tan(c+dx))}{a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c
 + d*x]),x]
```

```
[Out] -(((B + I*C)*Log[I - Tan[c + d*x]])/(a + I*b) - (2*B*Log[Tan[c + d*x]])/a +
 ((B - I*C)*Log[I + Tan[c + d*x]])/(a - I*b) + (2*b*(b*B - a*C)*Log[a + b*T
an[c + d*x]])/(a*(a^2 + b^2)))/(2*d)
```

Maple [B] time = 0.13, size = 174, normalized size = 2.2

$$-\frac{\ln\left(1+(\tan(dx+c))^2\right)aB}{2d\left(a^2+b^2\right)}-\frac{\ln\left(1+(\tan(dx+c))^2\right)Cb}{2d\left(a^2+b^2\right)}-\frac{B\arctan(\tan(dx+c))b}{d\left(a^2+b^2\right)}+\frac{C\arctan(\tan(dx+c))a}{d\left(a^2+b^2\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x)
```

```
[Out] -1/2/d/(a^2+b^2)*ln(1+tan(d*x+c)^2)*a*B-1/2/d/(a^2+b^2)*ln(1+tan(d*x+c)^2)*
C*b-1/d/(a^2+b^2)*B*arctan(tan(d*x+c))*b+1/d/(a^2+b^2)*C*arctan(tan(d*x+c))
*a+1/d/a*B*ln(tan(d*x+c))-1/d*b^2/a/(a^2+b^2)*ln(a+b*tan(d*x+c))*B+1/d*b/(a
^2+b^2)*ln(a+b*tan(d*x+c))*C
```

Maxima [A] time = 1.75587, size = 144, normalized size = 1.8

$$\frac{\frac{2(Ca-Bb)(dx+c)}{a^2+b^2}+\frac{2\left(Cab-Bb^2\right)\log(b\tan(dx+c)+a)}{a^3+ab^2}-\frac{(Ba+Cb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}+\frac{2B\log(\tan(dx+c))}{a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, al
gorithm="maxima")
```

```
[Out] 1/2*(2*(C*a - B*b)*(d*x + c)/(a^2 + b^2) + 2*(C*a*b - B*b^2)*log(b*tan(d*x
+ c) + a)/(a^3 + a*b^2) - (B*a + C*b)*log(tan(d*x + c)^2 + 1)/(a^2 + b^2) +
```

```
2*B*log(tan(d*x + c))/a)/d
```

Fricas [A] time = 1.26378, size = 267, normalized size = 3.34

$$\frac{2\left(Ca^2-Bab\right)dx+\left(Ba^2+Bb^2\right)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)+\left(Cab-Bb^2\right)\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)}{2\left(a^3+ab^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, al
gorithm="fricas")
```

```
[Out] 1/2*(2*(C*a^2 - B*a*b)*d*x + (B*a^2 + B*b^2)*log(tan(d*x + c)^2/(tan(d*x +
c)^2 + 1)) + (C*a*b - B*b^2)*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) +
 a^2)/(tan(d*x + c)^2 + 1)))/((a^3 + a*b^2)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.66261, size = 153, normalized size = 1.91

$$\frac{\frac{2\,(Ca-Bb)(dx+c)}{a^2+b^2}-\frac{(Ba+Cb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}+\frac{2\left(Cab^2-Bb^3\right)\log(|b\tan(dx+c)+a|)}{a^3b+ab^3}+\frac{2\,B\log(|\tan(dx+c)|)}{a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, al
gorithm="giac")
```

```
[Out] 1/2*(2*(C*a - B*b)*(d*x + c)/(a^2 + b^2) - (B*a + C*b)*log(tan(d*x + c)^2 +
 1)/(a^2 + b^2) + 2*(C*a*b^2 - B*b^3)*log(abs(b*tan(d*x + c) + a))/(a^3*b +
 a*b^3) + 2*B*log(abs(tan(d*x + c)))/a)/d
```

3.30 $\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=103

$$\frac{b^2(bB-aC)\log(a\cos(c+dx)+b\sin(c+dx))}{a^2d\left(a^2+b^2\right)}-\frac{x(aB+bC)}{a^2+b^2}-\frac{(bB-aC)\log(\sin(c+dx))}{a^2d}-\frac{B\cot(c+dx)}{ad}$$

```
[Out] -(((a*B + b*C)*x)/(a^2 + b^2)) - (B*Cot[c + d*x])/(a*d) - ((b*B - a*C)*Log[
Sin[c + d*x]])/(a^2*d) + (b^2*(b*B - a*C)*Log[a*Cos[c + d*x] + b*Sin[c + d*
x]])/(a^2*(a^2 + b^2)*d)
```

Rubi [A] time = 0.342237, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3632, 3609, 3651, 3530, 3475}

$$\frac{b^2(bB-aC)\log(a\cos(c+dx)+b\sin(c+dx))}{a^2d\left(a^2+b^2\right)}-\frac{x(aB+bC)}{a^2+b^2}-\frac{(bB-aC)\log(\sin(c+dx))}{a^2d}-\frac{B\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x
]),x]
```

```
[Out] -(((a*B + b*C)*x)/(a^2 + b^2)) - (B*Cot[c + d*x])/(a*d) - ((b*B - a*C)*Log[
Sin[c + d*x]])/(a^2*d) + (b^2*(b*B - a*C)*Log[a*Cos[c + d*x] + b*Sin[c + d*
x]])/(a^2*(a^2 + b^2)*d)
```

Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1)
)/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B
*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)
) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n +
 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &
& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3651

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[((a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*x)
```

```
/((a^2 + b^2)*(c^2 + d^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx &= \int \frac{\cot^2(c+dx)(B+C\tan(c+dx))}{a+b\tan(c+dx)}\,dx\\ &= -\frac{B\cot(c+dx)}{ad}-\frac{\int \frac{\cot(c+dx)\left(bB-aC+aB\tan(c+dx)+bB\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx}{a}\\ &= -\frac{(aB+bC)x}{a^2+b^2}-\frac{B\cot(c+dx)}{ad}-\frac{(bB-aC)\int\cot(c+dx)\,dx}{a^2}+\frac{(b^{\cdots}}{\cdots}\\ &= -\frac{(aB+bC)x}{a^2+b^2}-\frac{B\cot(c+dx)}{ad}-\frac{(bB-aC)\log(\sin(c+dx))}{a^2d}+\frac{b^2(\cdots}{\cdots}\end{aligned}$$

Mathematica [C] time = 0.83286, size = 138, normalized size = 1.34

$$\frac{\frac{2b^2(bB-aC)\log(a+b\tan(c+dx))}{a^2\left(a^2+b^2\right)}+\frac{2(aC-bB)\log(\tan(c+dx))}{a^2}+\frac{i(B+iC)\log(-\tan(c+dx)+i)}{a+ib}-\frac{(C+iB)\log(\tan(c+dx)+i)}{a-ib}-\frac{2B\cot(c+dx)}{a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c
 + d*x]),x]
```

```
[Out] ((-2*B*Cot[c + d*x])/a + (I*(B + I*C)*Log[I - Tan[c + d*x]])/(a + I*b) + (2
*(-(b*B) + a*C)*Log[Tan[c + d*x]])/a^2 - ((I*B + C)*Log[I + Tan[c + d*x]])/
(a - I*b) + (2*b^2*(b*B - a*C)*Log[a + b*Tan[c + d*x]])/(a^2*(a^2 + b^2)))/
(2*d)
```

Maple [B] time = 0.119, size = 214, normalized size = 2.1

$$\frac{\ln\left(1+(\tan(dx+c))^2\right)Bb}{2d\left(a^2+b^2\right)}-\frac{\ln\left(1+(\tan(dx+c))^2\right)Ca}{2d\left(a^2+b^2\right)}-\frac{B\arctan(\tan(dx+c))\,a}{d\left(a^2+b^2\right)}-\frac{C\arctan(\tan(dx+c))\,b}{d\left(a^2+b^2\right)}-\frac{\cdots}{ad\tan\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x)
```

```
[Out] 1/2/d/(a^2+b^2)*ln(1+tan(d*x+c)^2)*B*b-1/2/d/(a^2+b^2)*ln(1+tan(d*x+c)^2)*C
*a-1/d/(a^2+b^2)*B*arctan(tan(d*x+c))*a-1/d/(a^2+b^2)*C*arctan(tan(d*x+c))*
b-1/d/a/tan(d*x+c)*B-1/d/a^2*ln(tan(d*x+c))*B*b+1/d/a*ln(tan(d*x+c))*C+1/d*
b^3/(a^2+b^2)/a^2*ln(a+b*tan(d*x+c))*B-1/d*b^2/(a^2+b^2)/a*ln(a+b*tan(d*x+c
))*C
```

Maxima [A] time = 1.78672, size = 177, normalized size = 1.72

$$-\frac{\frac{2\,(Ba+Cb)(dx+c)}{a^2+b^2}+\frac{2\left(Cab^2-Bb^3\right)\log(b\tan(dx+c)+a)}{a^4+a^2b^2}+\frac{(Ca-Bb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}-\frac{2\,(Ca-Bb)\log(\tan(dx+c))}{a^2}+\frac{2\,B}{a\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, al
gorithm="maxima")
```

```
[Out] -1/2*(2*(B*a + C*b)*(d*x + c)/(a^2 + b^2) + 2*(C*a*b^2 - B*b^3)*log(b*tan(d
*x + c) + a)/(a^4 + a^2*b^2) + (C*a - B*b)*log(tan(d*x + c)^2 + 1)/(a^2 + b
^2) - 2*(C*a - B*b)*log(tan(d*x + c))/a^2 + 2*B/(a*tan(d*x + c)))/d
```

Fricas [A] time = 1.23669, size = 404, normalized size = 3.92

$$-\frac{2\,Ba^3+2\,Bab^2+2\left(Ba^3+Ca^2b\right)dx\tan\left(dx+c\right)-\left(Ca^3-Ba^2b+Cab^2-Bb^3\right)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)+\left(Cab\right.}{2\left(a^4+a^2b^2\right)d\tan\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, al
gorithm="fricas")
```

```
[Out] -1/2*(2*B*a^3 + 2*B*a*b^2 + 2*(B*a^3 + C*a^2*b)*d*x*tan(d*x + c) - (C*a^3 -
 B*a^2*b + C*a*b^2 - B*b^3)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*
x + c) + (C*a*b^2 - B*b^3)*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a
^2)/(tan(d*x + c)^2 + 1))*tan(d*x + c))/((a^4 + a^2*b^2)*d*tan(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.69312, size = 212, normalized size = 2.06

$$-\frac{\frac{2\,(Ba+Cb)(dx+c)}{a^2+b^2}+\frac{(Ca-Bb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}+\frac{2\left(Cab^3-Bb^4\right)\log(|b\tan(dx+c)+a|)}{a^4b+a^2b^3}-\frac{2\,(Ca-Bb)\log(|\tan(dx+c)|)}{a^2}+\frac{2\,(Ca\tan(dx+c)-Bb\tan(dx+c)+}{a^2\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, al
gorithm="giac")
```

```
[Out] -1/2*(2*(B*a + C*b)*(d*x + c)/(a^2 + b^2) + (C*a - B*b)*log(tan(d*x + c)^2
+ 1)/(a^2 + b^2) + 2*(C*a*b^3 - B*b^4)*log(abs(b*tan(d*x + c) + a))/(a^4*b
+ a^2*b^3) - 2*(C*a - B*b)*log(abs(tan(d*x + c)))/a^2 + 2*(C*a*tan(d*x + c)
 - B*b*tan(d*x + c) + B*a)/(a^2*tan(d*x + c)))/d
```

3.31 $\int \frac{\cot^4(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=137

$$-\frac{\left(a^2B+abC-b^2B\right)\log(\sin(c+dx))}{a^3d}-\frac{b^3(bB-aC)\log(a\cos(c+dx)+b\sin(c+dx))}{a^3d\left(a^2+b^2\right)}+\frac{x(bB-aC)}{a^2+b^2}+\frac{(bB-aC)\cot}{a^2d}$$

```
[Out] ((b*B - a*C)*x)/(a^2 + b^2) + ((b*B - a*C)*Cot[c + d*x])/(a^2*d) - (B*Cot[c
 + d*x]^2)/(2*a*d) - ((a^2*B - b^2*B + a*b*C)*Log[Sin[c + d*x]])/(a^3*d) -
(b^3*(b*B - a*C)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/(a^3*(a^2 + b^2)*d)
```

Rubi [A] time = 0.68161, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {3632, 3609, 3649, 3651, 3530, 3475}

$$-\frac{\left(a^2B+abC-b^2B\right)\log(\sin(c+dx))}{a^3d}-\frac{b^3(bB-aC)\log(a\cos(c+dx)+b\sin(c+dx))}{a^3d\left(a^2+b^2\right)}+\frac{x(bB-aC)}{a^2+b^2}+\frac{(bB-aC)\cot}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x
]),x]
```


```
[Out] ((b*B - a*C)*x)/(a^2 + b^2) + ((b*B - a*C)*Cot[c + d*x])/(a^2*d) - (B*Cot[c
 + d*x]^2)/(2*a*d) - ((a^2*B - b^2*B + a*b*C)*Log[Sin[c + d*x]])/(a^3*d) -
(b^3*(b*B - a*C)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/(a^3*(a^2 + b^2)*d)
```


Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1)
)/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B
*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)
) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n +
 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &
& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
```

```
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
 b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3651

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[((a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*x)
/((a^2 + b^2)*(c^2 + d^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx = \int \frac{\cot^3(c+dx)(B+C\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$

$$= -\frac{B\cot^2(c+dx)}{2ad} - \frac{\int \frac{\cot^2(c+dx)\left(2(bB-aC)+2aB\tan(c+dx)+2bB\tan^2(c+dx)\right)}{a+b\tan(c+dx)}}{2a}$$

$$= \frac{(bB-aC)\cot(c+dx)}{a^2d} - \frac{B\cot^2(c+dx)}{2ad} + \frac{\int \frac{\cot(c+dx)\left(-2\left(a^2B-b^2B+a\right.\right.}{}}{}$$

$$= \frac{(bB-aC)x}{a^2+b^2} + \frac{(bB-aC)\cot(c+dx)}{a^2d} - \frac{B\cot^2(c+dx)}{2ad} - \frac{\left(b^3(bB-\right.}{}$$

$$= \frac{(bB-aC)x}{a^2+b^2} + \frac{(bB-aC)\cot(c+dx)}{a^2d} - \frac{B\cot^2(c+dx)}{2ad} - \frac{\left(a^2B-b\right.}{}$$

Mathematica [C] time = 1.37473, size = 163, normalized size = 1.19

$$\frac{\frac{2b^3(aC-bB)\log(a+b\tan(c+dx))}{a^3\left(a^2+b^2\right)} - \frac{2\left(a^2B+abC-b^2B\right)\log(\tan(c+dx))}{a^3} + \frac{2(bB-aC)\cot(c+dx)}{a^2} + \frac{(B+iC)\log(-\tan(c+dx)+i)}{a+ib} + \frac{(B-iC)\log(\tan(c+dx)+i)}{a-ib} - \cdots}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c
 + d*x]),x]
```

```
[Out] ((2*(b*B - a*C)*Cot[c + d*x])/a^2 - (B*Cot[c + d*x]^2)/a + ((B + I*C)*Log[I
 - Tan[c + d*x]])/(a + I*b) - (2*(a^2*B - b^2*B + a*b*C)*Log[Tan[c + d*x]])
/a^3 + ((B - I*C)*Log[I + Tan[c + d*x]])/(a - I*b) + (2*b^3*(-(b*B) + a*C)*
Log[a + b*Tan[c + d*x]])/(a^3*(a^2 + b^2)))/(2*d)
```

Maple [A] time = 0.13, size = 266, normalized size = 1.9

$$\frac{\ln\left(1+(\tan(dx+c))^2\right)aB}{2d\left(a^2+b^2\right)}+\frac{\ln\left(1+(\tan(dx+c))^2\right)Cb}{2d\left(a^2+b^2\right)}+\frac{B\arctan(\tan(dx+c))\,b}{d\left(a^2+b^2\right)}-\frac{C\arctan(\tan(dx+c))\,a}{d\left(a^2+b^2\right)}-\frac{}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x)
```

```
[Out] 1/2/d/(a^2+b^2)*ln(1+tan(d*x+c)^2)*a*B+1/2/d/(a^2+b^2)*ln(1+tan(d*x+c)^2)*C
*b+1/d/(a^2+b^2)*B*arctan(tan(d*x+c))*b-1/d/(a^2+b^2)*C*arctan(tan(d*x+c))*
a-1/2/d/a/tan(d*x+c)^2*B+1/d/a^2/tan(d*x+c)*B*b-1/d/a/tan(d*x+c)*C-1/d/a*B*
ln(tan(d*x+c))+1/d/a^3*ln(tan(d*x+c))*b^2*B-1/d/a^2*ln(tan(d*x+c))*C*b-1/d*
b^4/(a^2+b^2)/a^3*ln(a+b*tan(d*x+c))*B+1/d*b^3/(a^2+b^2)/a^2*ln(a+b*tan(d*x
+c))*C
```

Maxima [A] time = 1.59209, size = 213, normalized size = 1.55

$$-\frac{\frac{2(Ca-Bb)(dx+c)}{a^2+b^2}-\frac{2\left(Cab^3-Bb^4\right)\log(b\tan(dx+c)+a)}{a^5+a^3b^2}-\frac{(Ba+Cb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}+\frac{2\left(Ba^2+Cab-Bb^2\right)\log(\tan(dx+c))}{a^3}+\frac{Ba+2(Ca-Bb)\tan(dx+}{a^2\tan(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, al
gorithm="maxima")
```

```
[Out] -1/2*(2*(C*a - B*b)*(d*x + c)/(a^2 + b^2) - 2*(C*a*b^3 - B*b^4)*log(b*tan(d
*x + c) + a)/(a^5 + a^3*b^2) - (B*a + C*b)*log(tan(d*x + c)^2 + 1)/(a^2 + b
^2) + 2*(B*a^2 + C*a*b - B*b^2)*log(tan(d*x + c))/a^3 + (B*a + 2*(C*a - B*b
)*tan(d*x + c))/(a^2*tan(d*x + c)^2))/d
```

Fricas [A] time = 1.33, size = 518, normalized size = 3.78

$$-\frac{Ba^4+Ba^2b^2+\left(Ba^4+Ca^3b+Cab^3-Bb^4\right)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan(dx+c)^2-\left(Cab^3-Bb^4\right)\log\left(\frac{b^2\tan(dx+c)^2+2ab\tan(dx}{\tan(dx+c)^2+1}\right.}{2\left(a^5+a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, al
gorithm="fricas")
```

```
[Out] -1/2*(B*a^4 + B*a^2*b^2 + (B*a^4 + C*a^3*b + C*a*b^3 - B*b^4)*log(tan(d*x +
 c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^2 - (C*a*b^3 - B*b^4)*log((b^2*tan
(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1))*tan(d*x + c)^
2 + (B*a^4 + B*a^2*b^2 + 2*(C*a^4 - B*a^3*b)*d*x)*tan(d*x + c)^2 + 2*(C*a^4
 - B*a^3*b + C*a^2*b^2 - B*a*b^3)*tan(d*x + c))/((a^5 + a^3*b^2)*d*tan(d*x
+ c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.76978, size = 289, normalized size = 2.11

$$-\frac{\frac{2(Ca-Bb)(dx+c)}{a^2+b^2} - \frac{(Ba+Cb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2} - \frac{2\left(Cab^4-Bb^5\right)\log(|b\tan(dx+c)+a|)}{a^5b+a^3b^3} + \frac{2\left(Ba^2+Cab-Bb^2\right)\log(|\tan(dx+c)|)}{a^3} - \frac{3Ba^2\tan(dx+c)^2+3Cab\,\mathrm{t}\cdots}{\cdots}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, al
gorithm="giac")
```

```
[Out] -1/2*(2*(C*a - B*b)*(d*x + c)/(a^2 + b^2) - (B*a + C*b)*log(tan(d*x + c)^2
+ 1)/(a^2 + b^2) - 2*(C*a*b^4 - B*b^5)*log(abs(b*tan(d*x + c) + a))/(a^5*b
+ a^3*b^3) + 2*(B*a^2 + C*a*b - B*b^2)*log(abs(tan(d*x + c)))/a^3 - (3*B*a^
2*tan(d*x + c)^2 + 3*C*a*b*tan(d*x + c)^2 - 3*B*b^2*tan(d*x + c)^2 - 2*C*a^
2*tan(d*x + c) + 2*B*a*b*tan(d*x + c) - B*a^2)/(a^3*tan(d*x + c)^2))/d
```

3.32 $\int \frac{\tan^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=208

$$\frac{a(bB-aC)\tan^2(c+dx)}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{\left(-2a^2C+abB-b^2C\right)\tan(c+dx)}{b^2d\left(a^2+b^2\right)}+\frac{a^2\left(a^2bB-2a^3C-4ab^2C+3b^3B\right)\log(a+b\,\mathrm{ta}}{b^3d\left(a^2+b^2\right)^2}$$

```
[Out] -(((2*a*b*B - a^2*C + b^2*C)*x)/(a^2 + b^2)^2) + ((a^2*B - b^2*B + 2*a*b*C)
*Log[Cos[c + d*x]])/((a^2 + b^2)^2*d) + (a^2*(a^2*b*B + 3*b^3*B - 2*a^3*C -
 4*a*b^2*C)*Log[a + b*Tan[c + d*x]])/(b^3*(a^2 + b^2)^2*d) - ((a*b*B - 2*a^
2*C - b^2*C)*Tan[c + d*x])/(b^2*(a^2 + b^2)*d) + (a*(b*B - a*C)*Tan[c + d*x
]^2)/(b*(a^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

Rubi [A] time = 0.532155, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {3632, 3605, 3647, 3626, 3617, 31, 3475}

$$\frac{a(bB-aC)\tan^2(c+dx)}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{\left(-2a^2C+abB-b^2C\right)\tan(c+dx)}{b^2d\left(a^2+b^2\right)}+\frac{a^2\left(a^2bB-2a^3C-4ab^2C+3b^3B\right)\log(a+b\,\mathrm{ta}}{b^3d\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Tan[c + d*x]^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x
])^2,x]
```

```
[Out] -(((2*a*b*B - a^2*C + b^2*C)*x)/(a^2 + b^2)^2) + ((a^2*B - b^2*B + 2*a*b*C)
*Log[Cos[c + d*x]])/((a^2 + b^2)^2*d) + (a^2*(a^2*b*B + 3*b^3*B - 2*a^3*C -
 4*a*b^2*C)*Log[a + b*Tan[c + d*x]])/(b^3*(a^2 + b^2)^2*d) - ((a*b*B - 2*a^
2*C - b^2*C)*Tan[c + d*x])/(b^2*(a^2 + b^2)*d) + (a*(b*B - a*C)*Tan[c + d*x
]^2)/(b*(a^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3605

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x
])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)),
 Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(
b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
+ 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e
 + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&
 LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3626

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((a*A + b*B -
a*C)*x)/(a^2 + b^2), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3617

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}dx = \int \frac{\tan^3(c+dx)(B+C\tan(c+dx))}{(a+b\tan(c+dx))^2}dx$$

$$= \frac{a(bB-aC)\tan^2(c+dx)}{b\left(a^2+b^2\right)d(a+b\tan(c+dx))} + \frac{\int \frac{\tan(c+dx)\left(-2a(bB-aC)+b(bB-a\right.}{a}}{}$$

$$= -\frac{\left(abB-2a^2C-b^2C\right)\tan(c+dx)}{b^2\left(a^2+b^2\right)d} + \frac{a(bB-aC)\tan^2(c+}{b\left(a^2+b^2\right)d(a+b\tan(c}$$

$$= -\frac{\left(2abB-a^2C+b^2C\right)x}{\left(a^2+b^2\right)^2} - \frac{\left(abB-2a^2C-b^2C\right)\tan(c+dx)}{b^2\left(a^2+b^2\right)d} +$$

$$= -\frac{\left(2abB-a^2C+b^2C\right)x}{\left(a^2+b^2\right)^2} + \frac{\left(a^2B-b^2B+2abC\right)\log(\cos(c+dx}{\left(a^2+b^2\right)^2 d}$$

$$= -\frac{\left(2abB-a^2C+b^2C\right)x}{\left(a^2+b^2\right)^2} + \frac{\left(a^2B-b^2B+2abC\right)\log(\cos(c+dx}{\left(a^2+b^2\right)^2 d}$$

Mathematica [C] time = 4.03853, size = 444, normalized size = 2.13

$$\frac{2ia^2\left(-a^2bB+2a^3C+4ab^2C-3b^3B\right)\tan^{-1}(\tan(c+dx))(a+b\tan(c+dx))+a\left(2(a+ib)^2(c+dx)\left(ia^2b(B+4iC)-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c
 + d*x])^2,x]
```

```
[Out] (a*(2*(a + I*b)^2*(2*a*b^2*(B + I*C) + I*a^2*b*(B + (4*I)*C) - (2*I)*a^3*C
+ b^3*C)*(c + d*x) + 2*(a^2 + b^2)^2*(-(b*B) + 2*a*C)*Log[Cos[c + d*x]] + a
^2*(a^2*b*B + 3*b^3*B - 2*a^3*C - 4*a*b^2*C)*Log[(a*Cos[c + d*x] + b*Sin[c
+ d*x])^2]) + b*(2*(a^3*b^2*C*(3 - (4*I)*c - (4*I)*d*x) - b^5*C*(c + d*x) +
 I*a^4*b*B*(I + c + d*x) - (2*I)*a^5*C*(I + c + d*x) + a*b^4*(C - 2*B*(c +
d*x)) + a^2*b^3*(C*(c + d*x) + I*B*(I + 3*c + 3*d*x))) + 2*(a^2 + b^2)^2*(-
(b*B) + 2*a*C)*Log[Cos[c + d*x]] + a^2*(a^2*b*B + 3*b^3*B - 2*a^3*C - 4*a*b
^2*C)*Log[(a*Cos[c + d*x] + b*Sin[c + d*x])^2])*Tan[c + d*x] + 2*b^2*(a^2 +
 b^2)^2*C*Tan[c + d*x]^2 + (2*I)*a^2*(-(a^2*b*B) - 3*b^3*B + 2*a^3*C + 4*a*
b^2*C)*ArcTan[Tan[c + d*x]]*(a + b*Tan[c + d*x]))/(2*b^3*(a^2 + b^2)^2*d*(a
 + b*Tan[c + d*x]))
```

Maple [A] time = 0.043, size = 364, normalized size = 1.8

$$\frac{C\tan(dx+c)}{b^2d} - \frac{\ln\left(1+(\tan(dx+c))^2\right)a^2B}{2d\left(a^2+b^2\right)^2} + \frac{\ln\left(1+(\tan(dx+c))^2\right)b^2B}{2d\left(a^2+b^2\right)^2} - \frac{\ln\left(1+(\tan(dx+c))^2\right)Cab}{d\left(a^2+b^2\right)^2} - 2\frac{B\arct}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d*C/b^2*tan(d*x+c)-1/2/d/(a^2+b^2)^2*ln(1+tan(d*x+c)^2)*a^2*B+1/2/d/(a^2+
b^2)^2*ln(1+tan(d*x+c)^2)*b^2*B-1/d/(a^2+b^2)^2*ln(1+tan(d*x+c)^2)*C*a*b-2/
```

```
d/(a^2+b^2)^2*B*arctan(tan(d*x+c))*a*b+1/d/(a^2+b^2)^2*C*arctan(tan(d*x+c))
*a^2-1/d/(a^2+b^2)^2*C*arctan(tan(d*x+c))*b^2+1/d/b^2*a^4/(a^2+b^2)^2*ln(a+
b*tan(d*x+c))*B+3/d*a^2/(a^2+b^2)^2*ln(a+b*tan(d*x+c))*B-2/d/b^3*a^5/(a^2+b
^2)^2*ln(a+b*tan(d*x+c))*C-4/d/b*a^3/(a^2+b^2)^2*ln(a+b*tan(d*x+c))*C+1/d/b
^2*a^3/(a^2+b^2)/(a+b*tan(d*x+c))*B-1/d/b^3*a^4/(a^2+b^2)/(a+b*tan(d*x+c))*
C
```

Maxima [A] time = 1.7175, size = 297, normalized size = 1.43

$$\frac{\frac{2\left(Ca^2-2\,Bab-Cb^2\right)(dx+c)}{a^4+2\,a^2b^2+b^4}-\frac{2\left(2\,Ca^5-Ba^4b+4\,Ca^3b^2-3\,Ba^2b^3\right)\log(b\tan(dx+c)+a)}{a^4b^3+2\,a^2b^5+b^7}-\frac{\left(Ba^2+2\,Cab-Bb^2\right)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\left(Ca^4-Ba^3b\right)}{a^3b^3+ab^5+\left(a^2b^4+b^6\right)\tan(dx}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,
algorithm="maxima")
```

```
[Out] 1/2*(2*(C*a^2 - 2*B*a*b - C*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) - 2*(2*C
*a^5 - B*a^4*b + 4*C*a^3*b^2 - 3*B*a^2*b^3)*log(b*tan(d*x + c) + a)/(a^4*b^
3 + 2*a^2*b^5 + b^7) - (B*a^2 + 2*C*a*b - B*b^2)*log(tan(d*x + c)^2 + 1)/(a
^4 + 2*a^2*b^2 + b^4) - 2*(C*a^4 - B*a^3*b)/(a^3*b^3 + a*b^5 + (a^2*b^4 + b
^6)*tan(d*x + c)) + 2*C*tan(d*x + c)/b^2)/d
```

Fricas [B] time = 1.46194, size = 936, normalized size = 4.5

$$-\frac{2\,Ca^4b^2-2\,Ba^3b^3-2\left(Ca^3b^3-2\,Ba^2b^4-Cab^5\right)dx-2\left(Ca^4b^2+2\,Ca^2b^4+Cb^6\right)\tan\left(dx+c\right)^2+\left(2\,Ca^6-Ba^5b+4\,Ca^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,
algorithm="fricas")
```

```
[Out] -1/2*(2*C*a^4*b^2 - 2*B*a^3*b^3 - 2*(C*a^3*b^3 - 2*B*a^2*b^4 - C*a*b^5)*d*x
 - 2*(C*a^4*b^2 + 2*C*a^2*b^4 + C*b^6)*tan(d*x + c)^2 + (2*C*a^6 - B*a^5*b
+ 4*C*a^4*b^2 - 3*B*a^3*b^3 + (2*C*a^5*b - B*a^4*b^2 + 4*C*a^3*b^3 - 3*B*a^
2*b^4)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(t
an(d*x + c)^2 + 1)) - (2*C*a^6 - B*a^5*b + 4*C*a^4*b^2 - 2*B*a^3*b^3 + 2*C*
a^2*b^4 - B*a*b^5 + (2*C*a^5*b - B*a^4*b^2 + 4*C*a^3*b^3 - 2*B*a^2*b^4 + 2*
C*a*b^5 - B*b^6)*tan(d*x + c))*log(1/(tan(d*x + c)^2 + 1)) - 2*(2*C*a^5*b -
 B*a^4*b^2 + 2*C*a^3*b^3 + C*a*b^5 + (C*a^2*b^4 - 2*B*a*b^5 - C*b^6)*d*x)*t
an(d*x + c))/((a^4*b^4 + 2*a^2*b^6 + b^8)*d*tan(d*x + c) + (a^5*b^3 + 2*a^3
*b^5 + a*b^7)*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**2,
x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.88942, size = 392, normalized size = 1.88

$$\frac{\frac{2\left(Ca^2-2\,Bab-Cb^2\right)(dx+c)}{a^4+2\,a^2b^2+b^4}-\frac{\left(Ba^2+2\,Cab-Bb^2\right)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\left(2\,Ca^5-Ba^4b+4\,Ca^3b^2-3\,Ba^2b^3\right)\log(|b\tan(dx+c)+a|)}{a^4b^3+2\,a^2b^5+b^7}+\frac{2\,C\tan(dx+c)}{b^2}+\frac{2\,(2\,C\cdots}{\cdots}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,
algorithm="giac")
```

```
[Out] 1/2*(2*(C*a^2 - 2*B*a*b - C*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) - (B*a^2
 + 2*C*a*b - B*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2*(2*
C*a^5 - B*a^4*b + 4*C*a^3*b^2 - 3*B*a^2*b^3)*log(abs(b*tan(d*x + c) + a))/(
a^4*b^3 + 2*a^2*b^5 + b^7) + 2*C*tan(d*x + c)/b^2 + 2*(2*C*a^5*b*tan(d*x +
c) - B*a^4*b^2*tan(d*x + c) + 4*C*a^3*b^3*tan(d*x + c) - 3*B*a^2*b^4*tan(d*
x + c) + C*a^6 + 3*C*a^4*b^2 - 2*B*a^3*b^3)/((a^4*b^3 + 2*a^2*b^5 + b^7)*(b
*tan(d*x + c) + a)))/d
```

3.33 $\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=157

$$-\frac{a^2(bB-aC)}{b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{a\left(a^3(-C)-3ab^2C+2b^3B\right)\log(a+b\tan(c+dx))}{b^2d\left(a^2+b^2\right)^2}-\frac{\left(a^2(-C)+2abB+b^2C\right)\log(co$$

```
[Out] -(((a^2*B - b^2*B + 2*a*b*C)*x)/(a^2 + b^2)^2) - ((2*a*b*B - a^2*C + b^2*C)
*Log[Cos[c + d*x]])/((a^2 + b^2)^2*d) - (a*(2*b^3*B - a^3*C - 3*a*b^2*C)*Lo
g[a + b*Tan[c + d*x]])/(b^2*(a^2 + b^2)^2*d) - (a^2*(b*B - a*C))/(b^2*(a^2
+ b^2)*d*(a + b*Tan[c + d*x]))
```

Rubi [A] time = 0.311219, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3632, 3604, 3626, 3617, 31, 3475}

$$-\frac{a^2(bB-aC)}{b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{a\left(a^3(-C)-3ab^2C+2b^3B\right)\log(a+b\tan(c+dx))}{b^2d\left(a^2+b^2\right)^2}-\frac{\left(a^2(-C)+2abB+b^2C\right)\log(co$$

Antiderivative was successfully verified.

```
[In] Int[(Tan[c + d*x]*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x])
^2,x]
```



```
[Out] -(((a^2*B - b^2*B + 2*a*b*C)*x)/(a^2 + b^2)^2) - ((2*a*b*B - a^2*C + b^2*C)
*Log[Cos[c + d*x]])/((a^2 + b^2)^2*d) - (a*(2*b^3*B - a^3*C - 3*a*b^2*C)*Lo
g[a + b*Tan[c + d*x]])/(b^2*(a^2 + b^2)^2*d) - (a^2*(b*B - a*C))/(b^2*(a^2
+ b^2)*d*(a + b*Tan[c + d*x]))
```


Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3604

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp
[((B*c - A*d)*(b*c - a*d)^2*(c + d*Tan[e + f*x])^(n + 1))/(f*d^2*(n + 1)*(c
^2 + d^2)), x] + Dist[1/(d*(c^2 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*S
imp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*c +
 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2)*T
an[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

Rule 3626

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((a*A + b*B -
a*C)*x)/(a^2 + b^2), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
```

```
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3617

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\tan^2(c+dx)(B+C\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$

$$= -\frac{a^2(bB-aC)}{b^2\left(a^2+b^2\right)d(a+b\tan(c+dx))} + \frac{\int \frac{-a(bB-aC)+b(bB-aC)\tan(c+}{a+b\tan(}}{b\left(a^2+\right.}$$

$$= -\frac{\left(a^2B-b^2B+2abC\right)x}{\left(a^2+b^2\right)^2} - \frac{a^2(bB-aC)}{b^2\left(a^2+b^2\right)d(a+b\tan(c+dx))} + $$

$$= -\frac{\left(a^2B-b^2B+2abC\right)x}{\left(a^2+b^2\right)^2} - \frac{\left(2abB-a^2C+b^2C\right)\log(\cos(c+dx)}{\left(a^2+b^2\right)^2 d}$$

$$= -\frac{\left(a^2B-b^2B+2abC\right)x}{\left(a^2+b^2\right)^2} - \frac{\left(2abB-a^2C+b^2C\right)\log(\cos(c+dx)}{\left(a^2+b^2\right)^2 d}$$

Mathematica [C] time = 2.05516, size = 324, normalized size = 2.06

$$-2ia\left(a^3C+3ab^2C-2b^3B\right)\tan^{-1}(\tan(c+dx))(a+b\tan(c+dx))+a\left(2(a+ib)^2(c+dx)\left(ia^2C+2abC-b^2B\right)+a\left(a\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c +
 d*x])^2,x]
```

```
[Out] (a*(2*(a + I*b)^2*(-(b^2*B) + I*a^2*C + 2*a*b*C)*(c + d*x) - 2*(a^2 + b^2)^
2*C*Log[Cos[c + d*x]] + a*(-2*b^3*B + a^3*C + 3*a*b^2*C)*Log[(a*Cos[c + d*x
] + b*Sin[c + d*x])^2]) + b*(2*(a + I*b)*((-I)*b^3*B*(c + d*x) + I*a^3*C*(I
 + c + d*x) - a*b^2*((-2*I)*C*(c + d*x) + B*(I + c + d*x)) + a^2*b*(B + C*(
I + c + d*x))) - 2*(a^2 + b^2)^2*C*Log[Cos[c + d*x]] + a*(-2*b^3*B + a^3*C
+ 3*a*b^2*C)*Log[(a*Cos[c + d*x] + b*Sin[c + d*x])^2])*Tan[c + d*x] - (2*I)
```

```
*a*(-2*b^3*B + a^3*C + 3*a*b^2*C)*ArcTan[Tan[c + d*x]]*(a + b*Tan[c + d*x])
)/(2*b^2*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x]))
```

Maple [A] time = 0.047, size = 313, normalized size = 2.

$$\frac{\ln\left(1+(\tan(dx+c))^2\right)Bab}{d\left(a^2+b^2\right)^2}-\frac{\ln\left(1+(\tan(dx+c))^2\right)Ca^2}{2d\left(a^2+b^2\right)^2}+\frac{\ln\left(1+(\tan(dx+c))^2\right)b^2C}{2d\left(a^2+b^2\right)^2}-\frac{B\arctan(\tan(dx+c))\,a^2}{d\left(a^2+b^2\right)^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d/(a^2+b^2)^2*ln(1+tan(d*x+c)^2)*B*a*b-1/2/d/(a^2+b^2)^2*ln(1+tan(d*x+c)^
2)*C*a^2+1/2/d/(a^2+b^2)^2*ln(1+tan(d*x+c)^2)*b^2*C-1/d/(a^2+b^2)^2*B*arcta
n(tan(d*x+c))*a^2+1/d/(a^2+b^2)^2*B*arctan(tan(d*x+c))*b^2-2/d/(a^2+b^2)^2*
C*arctan(tan(d*x+c))*a*b-1/d*a^2/b/(a^2+b^2)/(a+b*tan(d*x+c))*B+1/d*a^3/b^2
/(a^2+b^2)/(a+b*tan(d*x+c))*C-2/d*a/(a^2+b^2)^2*b*ln(a+b*tan(d*x+c))*B+1/d*
a^4/(a^2+b^2)^2/b^2*ln(a+b*tan(d*x+c))*C+3/d*a^2/(a^2+b^2)^2*ln(a+b*tan(d*x
+c))*C
```

Maxima [A] time = 1.67767, size = 266, normalized size = 1.69

$$-\frac{\frac{2\left(Ba^2+2Cab-Bb^2\right)(dx+c)}{a^4+2a^2b^2+b^4}-\frac{2\left(Ca^4+3Ca^2b^2-2Bab^3\right)\log(b\tan(dx+c)+a)}{a^4b^2+2a^2b^4+b^6}+\frac{\left(Ca^2-2Bab-Cb^2\right)\log\left(\tan(dx+c)^2+1\right)}{a^4+2a^2b^2+b^4}-\frac{2\left(Ca^3-Ba^2b\right)}{a^3b^2+ab^4+\left(a^2b^3+b^5\right)\tan(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x, al
gorithm="maxima")
```

```
[Out] -1/2*(2*(B*a^2 + 2*C*a*b - B*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) - 2*(C*
a^4 + 3*C*a^2*b^2 - 2*B*a*b^3)*log(b*tan(d*x + c) + a)/(a^4*b^2 + 2*a^2*b^4
 + b^6) + (C*a^2 - 2*B*a*b - C*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^
2 + b^4) - 2*(C*a^3 - B*a^2*b)/(a^3*b^2 + a*b^4 + (a^2*b^3 + b^5)*tan(d*x +
 c)))/d
```

Fricas [B] time = 1.30591, size = 682, normalized size = 4.34

$$\frac{2Ca^3b^2-2Ba^2b^3-2\left(Ba^3b^2+2Ca^2b^3-Bab^4\right)dx+\left(Ca^5+3Ca^3b^2-2Ba^2b^3+\left(Ca^4b+3Ca^2b^3-2Bab^4\right)\tan(dx+c\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x, al
gorithm="fricas")
```

```
[Out] 1/2*(2*C*a^3*b^2 - 2*B*a^2*b^3 - 2*(B*a^3*b^2 + 2*C*a^2*b^3 - B*a*b^4)*d*x
+ (C*a^5 + 3*C*a^3*b^2 - 2*B*a^2*b^3 + (C*a^4*b + 3*C*a^2*b^3 - 2*B*a*b^4)*
tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x
```

```
+ c)^2 + 1)) - (C*a^5 + 2*C*a^3*b^2 + C*a*b^4 + (C*a^4*b + 2*C*a^2*b^3 + C*
b^5)*tan(d*x + c))*log(1/(tan(d*x + c)^2 + 1)) - 2*(C*a^4*b - B*a^3*b^2 + (
B*a^2*b^3 + 2*C*a*b^4 - B*b^5)*d*x)*tan(d*x + c))/((a^4*b^3 + 2*a^2*b^5 + b
^7)*d*tan(d*x + c) + (a^5*b^2 + 2*a^3*b^4 + a*b^6)*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.63448, size = 329, normalized size = 2.1

$$-\frac{\frac{2\left(Ba^2+2\,Cab-Bb^2\right)(dx+c)}{a^4+2\,a^2b^2+b^4}+\frac{\left(Ca^2-2\,Bab-Cb^2\right)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\left(Ca^4+3\,Ca^2b^2-2\,Bab^3\right)\log(|b\tan(dx+c)+a|)}{a^4b^2+2\,a^2b^4+b^6}+\frac{2\left(Ca^4\tan(dx+c)+3\,Ca^2b^2\tan\cdots\right.}{\left(a^4b+2\cdots\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x, al
gorithm="giac")
```

```
[Out] -1/2*(2*(B*a^2 + 2*C*a*b - B*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) + (C*a^
2 - 2*B*a*b - C*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2*(C
*a^4 + 3*C*a^2*b^2 - 2*B*a*b^3)*log(abs(b*tan(d*x + c) + a))/(a^4*b^2 + 2*a
^2*b^4 + b^6) + 2*(C*a^4*tan(d*x + c) + 3*C*a^2*b^2*tan(d*x + c) - 2*B*a*b^
3*tan(d*x + c) + B*a^4 + 2*C*a^3*b - B*a^2*b^2)/((a^4*b + 2*a^2*b^3 + b^5)*
(b*tan(d*x + c) + a)))/d
```

3.34 $\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=115

$$\frac{a(bB-aC)}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{\left(a^2B+2abC-b^2B\right)\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)^2}+\frac{x\left(a^2(-C)+2abB+b^2C\right)}{\left(a^2+b^2\right)^2}$$

```
[Out] ((2*a*b*B - a^2*C + b^2*C)*x)/(a^2 + b^2)^2 - ((a^2*B - b^2*B + 2*a*b*C)*Lo
g[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^2*d) + (a*(b*B - a*C))/(b*
(a^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

Rubi [A] time = 0.14657, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {3628, 3531, 3530}

$$\frac{a(bB-aC)}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{\left(a^2B+2abC-b^2B\right)\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)^2}+\frac{x\left(a^2(-C)+2abB+b^2C\right)}{\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Tan[c + d*x] + C*Tan[c + d*x]^2)/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((2*a*b*B - a^2*C + b^2*C)*x)/(a^2 + b^2)^2 - ((a^2*B - b^2*B + 2*a*b*C)*Lo
g[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^2*d) + (a*(b*B - a*C))/(b*
(a^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

Rule 3628

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2
- a*b*B + a^2*C)*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 + b^2)), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{a(bB-aC)}{b\left(a^2+b^2\right)d(a+b\tan(c+dx))} + \frac{\int \frac{bB-aC+(aB+bC)\tan(c+dx)}{a+b\tan(c+dx)}\,dx}{a^2+b^2}$$

$$= \frac{\left(2abB-a^2C+b^2C\right)x}{\left(a^2+b^2\right)^2} + \frac{a(bB-aC)}{b\left(a^2+b^2\right)d(a+b\tan(c+dx))} - \frac{\left(a^2B-b^2B+2ab\right.}{\left(a^2\right.}$$

$$= \frac{\left(2abB-a^2C+b^2C\right)x}{\left(a^2+b^2\right)^2} - \frac{\left(a^2B-b^2B+2abC\right)\log(a\cos(c+dx)+b\sin(c+d}{\left(a^2+b^2\right)^2 d}$$

Mathematica [C] time = 2.02736, size = 140, normalized size = 1.22

$$\frac{\frac{2\left(\left(a^2(-B)-2abC+b^2B\right)\log(a+b\tan(c+dx))-\frac{a\left(a^2+b^2\right)(aC-bB)}{b(a+b\tan(c+dx))}\right)}{\left(a^2+b^2\right)^2}+\frac{(B+iC)\log(-\tan(c+dx)+i)}{(a+ib)^2}+\frac{(B-iC)\log(\tan(c+dx)+i)}{(a-ib)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Tan[c + d*x] + C*Tan[c + d*x]^2)/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (((B + I*C)*Log[I - Tan[c + d*x]])/(a + I*b)^2 + ((B - I*C)*Log[I + Tan[c +
 d*x]])/(a - I*b)^2 + (2*((-(a^2*B) + b^2*B - 2*a*b*C)*Log[a + b*Tan[c + d*
x]] - (a*(a^2 + b^2)*(-(b*B) + a*C))/(b*(a + b*Tan[c + d*x]))))/(a^2 + b^2)
^2)/(2*d)
```

Maple [B] time = 0.042, size = 305, normalized size = 2.7

$$\frac{\ln\left(1+(\tan(dx+c))^2\right)a^2B}{2d\left(a^2+b^2\right)^2} - \frac{\ln\left(1+(\tan(dx+c))^2\right)b^2B}{2d\left(a^2+b^2\right)^2} + \frac{\ln\left(1+(\tan(dx+c))^2\right)Cab}{d\left(a^2+b^2\right)^2} + 2\frac{B\arctan(\tan(dx+c))}{d\left(a^2+b^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x)
```

```
[Out] 1/2/d/(a^2+b^2)^2*ln(1+tan(d*x+c)^2)*a^2*B-1/2/d/(a^2+b^2)^2*ln(1+tan(d*x+c
)^2)*b^2*B+1/d/(a^2+b^2)^2*ln(1+tan(d*x+c)^2)*C*a*b+2/d/(a^2+b^2)^2*B*arcta
n(tan(d*x+c))*a*b-1/d/(a^2+b^2)^2*C*arctan(tan(d*x+c))*a^2+1/d/(a^2+b^2)^2*
C*arctan(tan(d*x+c))*b^2+1/d*a/(a^2+b^2)/(a+b*tan(d*x+c))*B-1/d*a^2/(a^2+b^
2)/b/(a+b*tan(d*x+c))*C-1/d*a^2/(a^2+b^2)^2*ln(a+b*tan(d*x+c))*B+1/d/(a^2+b
^2)^2*ln(a+b*tan(d*x+c))*b^2*B-2/d/(a^2+b^2)^2*ln(a+b*tan(d*x+c))*C*a*b
```

Maxima [A] time = 1.72621, size = 250, normalized size = 2.17

$$-\frac{\frac{2\left(Ca^2-2Bab-Cb^2\right)(dx+c)}{a^4+2a^2b^2+b^4}+\frac{2\left(Ba^2+2Cab-Bb^2\right)\log(b\tan(dx+c)+a)}{a^4+2a^2b^2+b^4}-\frac{\left(Ba^2+2Cab-Bb^2\right)\log\left(\tan(dx+c)^2+1\right)}{a^4+2a^2b^2+b^4}+\frac{2\left(Ca^2-Bab\right)}{a^3b+ab^3+\left(a^2b^2+b^4\right)\tan(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/2*(2*(C*a^2 - 2*B*a*b - C*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) + 2*(B*
a^2 + 2*C*a*b - B*b^2)*log(b*tan(d*x + c) + a)/(a^4 + 2*a^2*b^2 + b^4) - (B
*a^2 + 2*C*a*b - B*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 2
*(C*a^2 - B*a*b)/(a^3*b + a*b^3 + (a^2*b^2 + b^4)*tan(d*x + c)))/d
```

Fricas [A] time = 1.13058, size = 490, normalized size = 4.26

$$-\frac{2\,Ca^2b-2\,Bab^2+2\left(Ca^3-2\,Ba^2b-Cab^2\right)dx+\left(Ba^3+2\,Ca^2b-Bab^2+\left(Ba^2b+2\,Cab^2-Bb^3\right)\tan\left(dx+c\right)\right)\log\left(\frac{b^2\tan\cdots}{\cdots}\right.\cdots}{2\left(\left(a^4b+2\,a^2b^3+b^5\right)d\tan\left(dx+c\right)+\left(a^5+2\,a^3b\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] -1/2*(2*C*a^2*b - 2*B*a*b^2 + 2*(C*a^3 - 2*B*a^2*b - C*a*b^2)*d*x + (B*a^3
+ 2*C*a^2*b - B*a*b^2 + (B*a^2*b + 2*C*a*b^2 - B*b^3)*tan(d*x + c))*log((b^
2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) - 2*(C*a
^3 - B*a^2*b - (C*a^2*b - 2*B*a*b^2 - C*b^3)*d*x)*tan(d*x + c))/((a^4*b + 2
*a^2*b^3 + b^5)*d*tan(d*x + c) + (a^5 + 2*a^3*b^2 + a*b^4)*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.65895, size = 325, normalized size = 2.83

$$-\frac{\frac{2\left(Ca^2-2\,Bab-Cb^2\right)(dx+c)}{a^4+2\,a^2b^2+b^4}-\frac{\left(Ba^2+2\,Cab-Bb^2\right)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{2\left(Ba^2b+2\,Cab^2-Bb^3\right)\log\left(|b\tan(dx+c)+a|\right)}{a^4b+2\,a^2b^3+b^5}-\frac{2\left(Ba^2b^2\tan(dx+c)+2\,Cab^3\tan(dx+c\cdots\right.}{\left(a^4b+2\,a^2b^3+b^5\cdots\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/2*(2*(C*a^2 - 2*B*a*b - C*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) - (B*a^
2 + 2*C*a*b - B*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 2*(B
*a^2*b + 2*C*a*b^2 - B*b^3)*log(abs(b*tan(d*x + c) + a))/(a^4*b + 2*a^2*b^3
 + b^5) - 2*(B*a^2*b^2*tan(d*x + c) + 2*C*a*b^3*tan(d*x + c) - B*b^4*tan(d*
x + c) - C*a^4 + 2*B*a^3*b + C*a^2*b^2)/((a^4*b + 2*a^2*b^3 + b^5)*(b*tan(d
*x + c) + a)))/d
```

3.35 $\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=111

$$-\frac{bB-aC}{d\left(a^2+b^2\right)(a+b\tan(c+dx))}+\frac{\left(a^2(-C)+2abB+b^2C\right)\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)^2}+\frac{x\left(a^2B+2abC-b^2B\right)}{\left(a^2+b^2\right)^2}$$

```
[Out] ((a^2*B - b^2*B + 2*a*b*C)*x)/(a^2 + b^2)^2 + ((2*a*b*B - a^2*C + b^2*C)*Lo
g[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^2*d) - (b*B - a*C)/((a^2 +
 b^2)*d*(a + b*Tan[c + d*x]))
```

Rubi [A] time = 0.207627, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3632, 3529, 3531, 3530}

$$-\frac{bB-aC}{d\left(a^2+b^2\right)(a+b\tan(c+dx))}+\frac{\left(a^2(-C)+2abB+b^2C\right)\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)^2}+\frac{x\left(a^2B+2abC-b^2B\right)}{\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x])
^2,x]
```

```
[Out] ((a^2*B - b^2*B + 2*a*b*C)*x)/(a^2 + b^2)^2 + ((2*a*b*B - a^2*C + b^2*C)*Lo
g[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^2*d) - (b*B - a*C)/((a^2 +
 b^2)*d*(a + b*Tan[c + d*x]))
```

Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx=\int \frac{B+C\tan(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$=-\frac{bB-aC}{\left(a^2+b^2\right)d(a+b\tan(c+dx))}+\frac{\int \frac{aB+bC-(bB-aC)\tan(c+dx)}{a+b\tan(c+dx)}\,dx}{a^2+b^2}$$
$$=\frac{\left(a^2B-b^2B+2abC\right)x}{\left(a^2+b^2\right)^2}-\frac{bB-aC}{\left(a^2+b^2\right)d(a+b\tan(c+dx))}+\frac{\left(2abB-\right.}{}$$
$$=\frac{\left(a^2B-b^2B+2abC\right)x}{\left(a^2+b^2\right)^2}+\frac{\left(2abB-a^2C+b^2C\right)\log(a\cos(c+dx)+}{\left(a^2+b^2\right)^2d}$$

Mathematica [C] time = 1.96289, size = 190, normalized size = 1.71

$$\frac{\frac{C((-b-ia)\log(-\tan(c+dx)+i)+i(a+ib)\log(\tan(c+dx)+i)+2b\log(a+b\tan(c+dx)))}{a^2+b^2}-(bB-aC)\left(\frac{2b\left(\frac{a^2+b^2}{a+b\tan(c+dx)}-2a\log(a+b\tan(c+dx))\right)}{\left(a^2+b^2\right)^2}+\frac{i\log(-\tan(c}{(a+ib)}\right.}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c +
 d*x])^2,x]
```

```
[Out] ((C*(((-I)*a - b)*Log[I - Tan[c + d*x]] + I*(a + I*b)*Log[I + Tan[c + d*x]]
 + 2*b*Log[a + b*Tan[c + d*x]]))/(a^2 + b^2) - (b*B - a*C)*((I*Log[I - Tan[
c + d*x]])/(a + I*b)^2 - (I*Log[I + Tan[c + d*x]])/(a - I*b)^2 + (2*b*(-2*a
*Log[a + b*Tan[c + d*x]] + (a^2 + b^2)/(a + b*Tan[c + d*x])))/(a^2 + b^2)^2
))/(2*b*d)
```

Maple [B] time = 0.135, size = 301, normalized size = 2.7

$$-\frac{\ln\left(1+(\tan(dx+c))^2\right)Bab}{d\left(a^2+b^2\right)^2}+\frac{\ln\left(1+(\tan(dx+c))^2\right)Ca^2}{2d\left(a^2+b^2\right)^2}-\frac{\ln\left(1+(\tan(dx+c))^2\right)b^2C}{2d\left(a^2+b^2\right)^2}+\frac{B\arctan(\tan(dx+c))a^2}{d\left(a^2+b^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x)
```

```
[Out] -1/d/(a^2+b^2)^2*ln(1+tan(d*x+c)^2)*B*a*b+1/2/d/(a^2+b^2)^2*ln(1+tan(d*x+c)
^2)*C*a^2-1/2/d/(a^2+b^2)^2*ln(1+tan(d*x+c)^2)*b^2*C+1/d/(a^2+b^2)^2*B*arct
an(tan(d*x+c))*a^2-1/d/(a^2+b^2)^2*B*arctan(tan(d*x+c))*b^2+2/d/(a^2+b^2)^2
*C*arctan(tan(d*x+c))*a*b-1/d/(a^2+b^2)/(a+b*tan(d*x+c))*B*b+1/d/(a^2+b^2)/
(a+b*tan(d*x+c))*C*a+2/d*a/(a^2+b^2)^2*b*ln(a+b*tan(d*x+c))*B-1/d*a^2/(a^2+
b^2)^2*ln(a+b*tan(d*x+c))*C+1/d/(a^2+b^2)^2*ln(a+b*tan(d*x+c))*b^2*C
```

Maxima [A] time = 1.81313, size = 239, normalized size = 2.15

$$\frac{\frac{2\left(Ba^2+2\,Cab-Bb^2\right)(dx+c)}{a^4+2\,a^2b^2+b^4}-\frac{2\left(Ca^2-2\,Bab-Cb^2\right)\log(b\tan(dx+c)+a)}{a^4+2\,a^2b^2+b^4}+\frac{\left(Ca^2-2\,Bab-Cb^2\right)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{2\,(Ca-Bb)}{a^3+ab^2+\left(a^2b+b^3\right)\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x, al
gorithm="maxima")
```

```
[Out] 1/2*(2*(B*a^2 + 2*C*a*b - B*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) - 2*(C*a
^2 - 2*B*a*b - C*b^2)*log(b*tan(d*x + c) + a)/(a^4 + 2*a^2*b^2 + b^4) + (C*
a^2 - 2*B*a*b - C*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 2*
(C*a - B*b)/(a^3 + a*b^2 + (a^2*b + b^3)*tan(d*x + c)))/d
```

Fricas [A] time = 1.14083, size = 489, normalized size = 4.41

$$\frac{2\,Cab^2-2\,Bb^3+2\left(Ba^3+2\,Ca^2b-Bab^2\right)dx-\left(Ca^3-2\,Ba^2b-Cab^2+\left(Ca^2b-2\,Bab^2-Cb^3\right)\tan\left(dx+c\right)\right)\log\left(\frac{b^2\tan\cdots}{\cdots}\right.\cdots}{2\left(\left(a^4b+2\,a^2b^3+b^5\right)d\tan\left(dx+c\right)+\left(a^5+2\,a\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x, al
gorithm="fricas")
```

```
[Out] 1/2*(2*C*a*b^2 - 2*B*b^3 + 2*(B*a^3 + 2*C*a^2*b - B*a*b^2)*d*x - (C*a^3 - 2
*B*a^2*b - C*a*b^2 + (C*a^2*b - 2*B*a*b^2 - C*b^3)*tan(d*x + c))*log((b^2*t
an(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) - 2*(C*a^2*
b - B*a*b^2 - (B*a^2*b + 2*C*a*b^2 - B*b^3)*d*x)*tan(d*x + c))/((a^4*b + 2*
a^2*b^3 + b^5)*d*tan(d*x + c) + (a^5 + 2*a^3*b^2 + a*b^4)*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.63085, size = 316, normalized size = 2.85

$$\frac{\frac{2\left(Ba^2+2\,Cab-Bb^2\right)(dx+c)}{a^4+2\,a^2b^2+b^4}+\frac{\left(Ca^2-2\,Bab-Cb^2\right)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\left(Ca^2b-2\,Bab^2-Cb^3\right)\log(|b\tan(dx+c)+a|)}{a^4b+2\,a^2b^3+b^5}+\frac{2\left(Ca^2b\tan(dx+c)-2\,Bab^2\tan(dx+\cdots\right.}{\left(a^4+2\,a^2b^2+b^{\cdots}\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x, al
gorithm="giac")
```

```
[Out] 1/2*(2*(B*a^2 + 2*C*a*b - B*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) + (C*a^2
 - 2*B*a*b - C*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2*(C*
a^2*b - 2*B*a*b^2 - C*b^3)*log(abs(b*tan(d*x + c) + a))/(a^4*b + 2*a^2*b^3
+ b^5) + 2*(C*a^2*b*tan(d*x + c) - 2*B*a*b^2*tan(d*x + c) - C*b^3*tan(d*x +
 c) + 2*C*a^3 - 3*B*a^2*b - B*b^3)/((a^4 + 2*a^2*b^2 + b^4)*(b*tan(d*x + c)
 + a)))/d
```

3.36 $\int \frac{\cot^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=137

$$\frac{b(bB-aC)}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{b\left(3a^2bB-2a^3C+b^3B\right)\log(a\cos(c+dx)+b\sin(c+dx))}{a^2d\left(a^2+b^2\right)^2}-\frac{x\left(a^2(-C)+2abB+b^{2}\right.}{\left(a^2+b^2\right)^2}$$

```
[Out] -((((2*a*b*B - a^2*C + b^2*C)*x)/(a^2 + b^2)^2) + (B*Log[Sin[c + d*x]])/(a^2
*d) - (b*(3*a^2*b*B + b^3*B - 2*a^3*C)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]]
)/(a^2*(a^2 + b^2)^2*d) + (b*(b*B - a*C))/(a*(a^2 + b^2)*d*(a + b*Tan[c + d
*x]))
```

Rubi [A] time = 0.402681, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3632, 3609, 3651, 3530, 3475}

$$\frac{b(bB-aC)}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{b\left(3a^2bB-2a^3C+b^3B\right)\log(a\cos(c+dx)+b\sin(c+dx))}{a^2d\left(a^2+b^2\right)^2}-\frac{x\left(a^2(-C)+2abB+b^{2}\right.}{\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x
])^2,x]
```

```
[Out] -((((2*a*b*B - a^2*C + b^2*C)*x)/(a^2 + b^2)^2) + (B*Log[Sin[c + d*x]])/(a^2
*d) - (b*(3*a^2*b*B + b^3*B - 2*a^3*C)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]]
)/(a^2*(a^2 + b^2)^2*d) + (b*(b*B - a*C))/(a*(a^2 + b^2)*d*(a + b*Tan[c + d
*x]))
```

Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1)
)/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B
*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)
) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n +
 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &
& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3651

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[((a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*x)
/((a^2 + b^2)*(c^2 + d^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx=\int \frac{\cot(c+dx)(B+C\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$

$$=\frac{b(bB-aC)}{a\left(a^2+b^2\right)d(a+b\tan(c+dx))}+\frac{\int \frac{\cot(c+dx)\left(\left(a^2+b^2\right)B-a(bB-aC)\tan(\cdots}{a+b\tan(c+\cdots}}{a\left(a^2+\cdots\right.}$$

$$=-\frac{\left(2abB-a^2C+b^2C\right)x}{\left(a^2+b^2\right)^2}+\frac{b(bB-aC)}{a\left(a^2+b^2\right)d(a+b\tan(c+dx))}+\frac{B\int\cdots}{\cdots}$$

$$=-\frac{\left(2abB-a^2C+b^2C\right)x}{\left(a^2+b^2\right)^2}+\frac{B\log(\sin(c+dx))}{a^2d}-\frac{b\left(3a^2bB+b^3B-\cdots\right.}{\cdots}$$

Mathematica [C] time = 2.36069, size = 159, normalized size = 1.16

$$-\frac{\frac{2b(aC-bB)}{a\left(a^2+b^2\right)(a+b\tan(c+dx))}+\frac{2b\left(3a^2bB-2a^3C+b^3B\right)\log(a+b\tan(c+dx))}{a^2\left(a^2+b^2\right)^2}-\frac{2B\log(\tan(c+dx))}{a^2}+\frac{(B+iC)\log(-\tan(c+dx)+i)}{(a+ib)^2}+\frac{(B-iC)\log(\tan(c+dx)+i)}{(a-ib)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c
 + d*x])^2,x]
```

```
[Out] -(((B + I*C)*Log[I - Tan[c + d*x]])/(a + I*b)^2 - (2*B*Log[Tan[c + d*x]])/a
^2 + ((B - I*C)*Log[I + Tan[c + d*x]])/(a - I*b)^2 + (2*b*(3*a^2*b*B + b^3*
B - 2*a^3*C)*Log[a + b*Tan[c + d*x]])/(a^2*(a^2 + b^2)^2) + (2*b*(-(b*B) +
a*C))/(a*(a^2 + b^2)*(a + b*Tan[c + d*x])))/(2*d)
```

Maple [B] time = 0.148, size = 325, normalized size = 2.4

$$-\frac{\ln\left(1+(\tan(dx+c))^2\right)a^2B}{2d\left(a^2+b^2\right)^2}+\frac{\ln\left(1+(\tan(dx+c))^2\right)b^2B}{2d\left(a^2+b^2\right)^2}-\frac{\ln\left(1+(\tan(dx+c))^2\right)Cab}{d\left(a^2+b^2\right)^2}-2\frac{B\arctan(\tan(dx+c))\,a\cdots}{d\left(a^2+b^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x)
```

```
[Out] -1/2/d/(a^2+b^2)^2*ln(1+tan(d*x+c)^2)*a^2*B+1/2/d/(a^2+b^2)^2*ln(1+tan(d*x+
c)^2)*b^2*B-1/d/(a^2+b^2)^2*ln(1+tan(d*x+c)^2)*C*a*b-2/d/(a^2+b^2)^2*B*arct
an(tan(d*x+c))*a*b+1/d/(a^2+b^2)^2*C*arctan(tan(d*x+c))*a^2-1/d/(a^2+b^2)^2
*C*arctan(tan(d*x+c))*b^2+1/d/a^2*B*ln(tan(d*x+c))+1/d*b^2/a/(a^2+b^2)/(a+b
*tan(d*x+c))*B-1/d*b/(a^2+b^2)/(a+b*tan(d*x+c))*C-3/d/(a^2+b^2)^2*ln(a+b*ta
n(d*x+c))*b^2*B-1/d*b^4/(a^2+b^2)^2/a^2*ln(a+b*tan(d*x+c))*B+2/d/(a^2+b^2)^
2*ln(a+b*tan(d*x+c))*C*a*b
```

Maxima [A] time = 1.63628, size = 281, normalized size = 2.05

$$\frac{\frac{2\left(Ca^2-2\,Bab-Cb^2\right)(dx+c)}{a^4+2\,a^2b^2+b^4}+\frac{2\left(2\,Ca^3b-3\,Ba^2b^2-Bb^4\right)\log(b\tan(dx+c)+a)}{a^6+2\,a^4b^2+a^2b^4}-\frac{\left(Ba^2+2\,Cab-Bb^2\right)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\left(Cab-Bb^2\right)}{a^4+a^2b^2+\left(a^3b+ab^3\right)\tan(dx+c)}+}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,
algorithm="maxima")
```

```
[Out] 1/2*(2*(C*a^2 - 2*B*a*b - C*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) + 2*(2*C
*a^3*b - 3*B*a^2*b^2 - B*b^4)*log(b*tan(d*x + c) + a)/(a^6 + 2*a^4*b^2 + a^
2*b^4) - (B*a^2 + 2*C*a*b - B*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2
 + b^4) - 2*(C*a*b - B*b^2)/(a^4 + a^2*b^2 + (a^3*b + a*b^3)*tan(d*x + c))
+ 2*B*log(tan(d*x + c))/a^2)/d
```

Fricas [B] time = 1.35152, size = 701, normalized size = 5.12

$$-\frac{2\,Ca^2b^3-2\,Bab^4-2\left(Ca^5-2\,Ba^4b-Ca^3b^2\right)dx-\left(Ba^5+2\,Ba^3b^2+Bab^4+\left(Ba^4b+2\,Ba^2b^3+Bb^5\right)\tan\left(dx+c\right)\right)\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,
algorithm="fricas")
```

```
[Out] -1/2*(2*C*a^2*b^3 - 2*B*a*b^4 - 2*(C*a^5 - 2*B*a^4*b - C*a^3*b^2)*d*x - (B*
a^5 + 2*B*a^3*b^2 + B*a*b^4 + (B*a^4*b + 2*B*a^2*b^3 + B*b^5)*tan(d*x + c))
*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1)) - (2*C*a^4*b - 3*B*a^3*b^2 - B*a*
b^4 + (2*C*a^3*b^2 - 3*B*a^2*b^3 - B*b^5)*tan(d*x + c))*log((b^2*tan(d*x +
c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) - 2*(C*a^3*b^2 - B*a
^2*b^3 + (C*a^4*b - 2*B*a^3*b^2 - C*a^2*b^3)*d*x)*tan(d*x + c))/((a^6*b + 2
*a^4*b^3 + a^2*b^5)*d*tan(d*x + c) + (a^7 + 2*a^5*b^2 + a^3*b^4)*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**2,
x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.8665, size = 377, normalized size = 2.75

$$\frac{\frac{2\left(Ca^2-2\,Bab-Cb^2\right)(dx+c)}{a^4+2\,a^2b^2+b^4}-\frac{\left(Ba^2+2\,Cab-Bb^2\right)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{2\left(2\,Ca^3b^2-3\,Ba^2b^3-Bb^5\right)\log(|b\tan(dx+c)+a|)}{a^6b+2\,a^4b^3+a^2b^5}+\frac{2\,B\log(|\tan(dx+c)|)}{a^2}-\frac{2\left(2\,Ca^3b^2\,\mathrm{t}\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,
algorithm="giac")
```

```
[Out] 1/2*(2*(C*a^2 - 2*B*a*b - C*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) - (B*a^2
 + 2*C*a*b - B*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 2*(2*
C*a^3*b^2 - 3*B*a^2*b^3 - B*b^5)*log(abs(b*tan(d*x + c) + a))/(a^6*b + 2*a^
4*b^3 + a^2*b^5) + 2*B*log(abs(tan(d*x + c)))/a^2 - 2*(2*C*a^3*b^2*tan(d*x
+ c) - 3*B*a^2*b^3*tan(d*x + c) - B*b^5*tan(d*x + c) + 3*C*a^4*b - 4*B*a^3*
b^2 + C*a^2*b^3 - 2*B*a*b^4)/((a^6 + 2*a^4*b^2 + a^2*b^4)*(b*tan(d*x + c) +
 a)))/d
```

3.37 $\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=192

$$-\frac{b\left(a^2B-abC+2b^2B\right)}{a^2d\left(a^2+b^2\right)(a+b\tan(c+dx))}+\frac{b^2\left(4a^2bB-3a^3C-ab^2C+2b^3B\right)\log(a\cos(c+dx)+b\sin(c+dx))}{a^3d\left(a^2+b^2\right)^2}-\frac{x\left(a^2B+\right.}{\left(a^2\right.}$$

```
[Out] -(((a^2*B - b^2*B + 2*a*b*C)*x)/(a^2 + b^2)^2) - ((2*b*B - a*C)*Log[Sin[c +
 d*x]])/(a^3*d) + (b^2*(4*a^2*b*B + 2*b^3*B - 3*a^3*C - a*b^2*C)*Log[a*Cos[
c + d*x] + b*Sin[c + d*x]])/(a^3*(a^2 + b^2)^2*d) - (b*(a^2*B + 2*b^2*B - a
*b*C))/(a^2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])) - (B*Cot[c + d*x])/(a*d*(a
+ b*Tan[c + d*x]))
```

Rubi [A] time = 0.607565, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {3632, 3609, 3649, 3651, 3530, 3475}

$$-\frac{b\left(a^2B-abC+2b^2B\right)}{a^2d\left(a^2+b^2\right)(a+b\tan(c+dx))}+\frac{b^2\left(4a^2bB-3a^3C-ab^2C+2b^3B\right)\log(a\cos(c+dx)+b\sin(c+dx))}{a^3d\left(a^2+b^2\right)^2}-\frac{x\left(a^2B+\right.}{\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x
])^2,x]
```


```
[Out] -(((a^2*B - b^2*B + 2*a*b*C)*x)/(a^2 + b^2)^2) - ((2*b*B - a*C)*Log[Sin[c +
 d*x]])/(a^3*d) + (b^2*(4*a^2*b*B + 2*b^3*B - 3*a^3*C - a*b^2*C)*Log[a*Cos[
c + d*x] + b*Sin[c + d*x]])/(a^3*(a^2 + b^2)^2*d) - (b*(a^2*B + 2*b^2*B - a
*b*C))/(a^2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])) - (B*Cot[c + d*x])/(a*d*(a
+ b*Tan[c + d*x]))
```


Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1)
)/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B
*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)
) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n +
 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &
& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
 b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3651

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[((a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*x)
/((a^2 + b^2)*(c^2 + d^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\cot^2(c+dx)(B+C\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$

$$= -\frac{B\cot(c+dx)}{ad(a+b\tan(c+dx))} - \frac{\int \frac{\cot(c+dx)\left(2bB-aC+aB\tan(c+dx)+2bB\tan^2(c+\ldots\right.}{(a+b\tan(c+dx))^2}}{a}$$

$$= -\frac{b\left(a^2B+2b^2B-abC\right)}{a^2\left(a^2+b^2\right)d(a+b\tan(c+dx))} - \frac{B\cot(c+dx)}{ad(a+b\tan(c+dx))} - \ldots$$

$$= -\frac{\left(a^2B-b^2B+2abC\right)x}{\left(a^2+b^2\right)^2} - \frac{b\left(a^2B+2b^2B-abC\right)}{a^2\left(a^2+b^2\right)d(a+b\tan(c+dx))} - \ldots$$

$$= -\frac{\left(a^2B-b^2B+2abC\right)x}{\left(a^2+b^2\right)^2} - \frac{(2bB-aC)\log(\sin(c+dx))}{a^3d} + \frac{b^2\left(4a^2\ldots\right.}{\ldots}$$

Mathematica [C] time = 3.43353, size = 193, normalized size = 1.01

$$\frac{\frac{2b^2(aC-bB)}{a^2(a^2+b^2)(a+b\tan(c+dx))}-\frac{2b^2(-4a^2bB+3a^3C+ab^2C-2b^3B)\log(a+b\tan(c+dx))}{a^3(a^2+b^2)^2}+\frac{2(aC-2bB)\log(\tan(c+dx))}{a^3}-\frac{2B\cot(c+dx)}{a^2}+\frac{i(B+iC)\log(-\tan(}{(a+ib)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c
 + d*x])^2,x]
```

```
[Out] ((-2*B*Cot[c + d*x])/a^2 + (I*(B + I*C)*Log[I - Tan[c + d*x]])/(a + I*b)^2
+ (2*(-2*b*B + a*C)*Log[Tan[c + d*x]])/a^3 - ((I*B + C)*Log[I + Tan[c + d*x
]])/(a - I*b)^2 - (2*b^2*(-4*a^2*b*B - 2*b^3*B + 3*a^3*C + a*b^2*C)*Log[a +
 b*Tan[c + d*x]])/(a^3*(a^2 + b^2)^2) + (2*b^2*(-(b*B) + a*C))/(a^2*(a^2 +
b^2)*(a + b*Tan[c + d*x])))/(2*d)
```

Maple [B] time = 0.137, size = 399, normalized size = 2.1

$$\frac{\ln\left(1+(\tan(dx+c))^2\right)Bab}{d\left(a^2+b^2\right)^2}-\frac{\ln\left(1+(\tan(dx+c))^2\right)Ca^2}{2d\left(a^2+b^2\right)^2}+\frac{\ln\left(1+(\tan(dx+c))^2\right)b^2C}{2d\left(a^2+b^2\right)^2}-\frac{B\arctan(\tan(dx+c))\,a}{d\left(a^2+b^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d/(a^2+b^2)^2*ln(1+tan(d*x+c)^2)*B*a*b-1/2/d/(a^2+b^2)^2*ln(1+tan(d*x+c)^
2)*C*a^2+1/2/d/(a^2+b^2)^2*ln(1+tan(d*x+c)^2)*b^2*C-1/d/(a^2+b^2)^2*B*arcta
n(tan(d*x+c))*a^2+1/d/(a^2+b^2)^2*B*arctan(tan(d*x+c))*b^2-2/d/(a^2+b^2)^2*
C*arctan(tan(d*x+c))*a*b-1/d/a^2/tan(d*x+c)*B-2/d/a^3*ln(tan(d*x+c))*B*b+1/
d/a^2*ln(tan(d*x+c))*C-1/d*b^3/(a^2+b^2)/a^2/(a+b*tan(d*x+c))*B+1/d*b^2/(a^
2+b^2)/a/(a+b*tan(d*x+c))*C+4/d*b^3/(a^2+b^2)^2/a*ln(a+b*tan(d*x+c))*B+2/d*
b^5/(a^2+b^2)^2/a^3*ln(a+b*tan(d*x+c))*B-3/d/(a^2+b^2)^2*ln(a+b*tan(d*x+c))
*b^2*C-1/d*b^4/(a^2+b^2)^2/a^2*ln(a+b*tan(d*x+c))*C
```

Maxima [A] time = 1.71552, size = 354, normalized size = 1.84

$$-\frac{\frac{2\left(Ba^2+2\,Cab-Bb^2\right)(dx+c)}{a^4+2\,a^2b^2+b^4}+\frac{2\left(3\,Ca^3b^2-4\,Ba^2b^3+Cab^4-2\,Bb^5\right)\log(b\tan(dx+c)+a)}{a^7+2\,a^5b^2+a^3b^4}+\frac{\left(Ca^2-2\,Bab-Cb^2\right)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{2\left(Ba^3+Bab^2+\left(Ba^2b-C\right.\right.}{\left(a^4b+a^2b^3\right)\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,
algorithm="maxima")
```

```
[Out] -1/2*(2*(B*a^2 + 2*C*a*b - B*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) + 2*(3*
C*a^3*b^2 - 4*B*a^2*b^3 + C*a*b^4 - 2*B*b^5)*log(b*tan(d*x + c) + a)/(a^7 +
 2*a^5*b^2 + a^3*b^4) + (C*a^2 - 2*B*a*b - C*b^2)*log(tan(d*x + c)^2 + 1)/(
a^4 + 2*a^2*b^2 + b^4) + 2*(B*a^3 + B*a*b^2 + (B*a^2*b - C*a*b^2 + 2*B*b^3)
*tan(d*x + c))/((a^4*b + a^2*b^3)*tan(d*x + c)^2 + (a^5 + a^3*b^2)*tan(d*x
+ c)) - 2*(C*a - 2*B*b)*log(tan(d*x + c))/a^3)/d
```

Fricas [B] time = 1.53073, size = 1017, normalized size = 5.3

$$-\frac{2\,Ba^6 + 4\,Ba^4b^2 + 2\,Ba^2b^4 + 2\left(Ca^3b^3 - Ba^2b^4 + \left(Ba^5b + 2\,Ca^4b^2 - Ba^3b^3\right)dx\right)\tan\left(dx+c\right)^2 - \left(\left(Ca^5b - 2\,Ba^4b^2 + 2\,Ca\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,
algorithm="fricas")
```

```
[Out] -1/2*(2*B*a^6 + 4*B*a^4*b^2 + 2*B*a^2*b^4 + 2*(C*a^3*b^3 - B*a^2*b^4 + (B*a
^5*b + 2*C*a^4*b^2 - B*a^3*b^3)*d*x)*tan(d*x + c)^2 - ((C*a^5*b - 2*B*a^4*b
^2 + 2*C*a^3*b^3 - 4*B*a^2*b^4 + C*a*b^5 - 2*B*b^6)*tan(d*x + c)^2 + (C*a^6
 - 2*B*a^5*b + 2*C*a^4*b^2 - 4*B*a^3*b^3 + C*a^2*b^4 - 2*B*a*b^5)*tan(d*x +
 c))*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1)) + ((3*C*a^3*b^3 - 4*B*a^2*b^4
 + C*a*b^5 - 2*B*b^6)*tan(d*x + c)^2 + (3*C*a^4*b^2 - 4*B*a^3*b^3 + C*a^2*b
^4 - 2*B*a*b^5)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c)
+ a^2)/(tan(d*x + c)^2 + 1)) + 2*(B*a^5*b + 2*B*a^3*b^3 - C*a^2*b^4 + 2*B*a
*b^5 + (B*a^6 + 2*C*a^5*b - B*a^4*b^2)*d*x)*tan(d*x + c))/((a^7*b + 2*a^5*b
^3 + a^3*b^5)*d*tan(d*x + c)^2 + (a^8 + 2*a^6*b^2 + a^4*b^4)*d*tan(d*x + c)
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**2,
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.81305, size = 489, normalized size = 2.55

$$-\frac{\frac{2\left(Ba^2+2\,Cab-Bb^2\right)(dx+c)}{a^4+2\,a^2b^2+b^4} + \frac{\left(Ca^2-2\,Bab-Cb^2\right)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4} + \frac{2\left(3\,Ca^3b^3-4\,Ba^2b^4+Cab^5-2\,Bb^6\right)\log\left(|b\tan(dx+c)+a|\right)}{a^7b+2\,a^5b^3+a^3b^5} + \frac{Ca^4b\tan(dx+c)^2-2\,Ba^3b^2}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,
algorithm="giac")
```

```
[Out] -1/2*(2*(B*a^2 + 2*C*a*b - B*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) + (C*a^
2 - 2*B*a*b - C*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 2*(3
*C*a^3*b^3 - 4*B*a^2*b^4 + C*a*b^5 - 2*B*b^6)*log(abs(b*tan(d*x + c) + a))/
(a^7*b + 2*a^5*b^3 + a^3*b^5) + (C*a^4*b*tan(d*x + c)^2 - 2*B*a^3*b^2*tan(d
*x + c)^2 - C*a^2*b^3*tan(d*x + c)^2 + C*a^5*tan(d*x + c) - 3*C*a^3*b^2*tan
(d*x + c) + 6*B*a^2*b^3*tan(d*x + c) - 2*C*a*b^4*tan(d*x + c) + 4*B*b^5*tan
(d*x + c) + 2*B*a^5 + 4*B*a^3*b^2 + 2*B*a*b^4)/((a^6 + 2*a^4*b^2 + a^2*b^4)
*(b*tan(d*x + c)^2 + a*tan(d*x + c))) - 2*(C*a - 2*B*b)*log(abs(tan(d*x + c
)))/a^3)/d
```

3.38 $\int \frac{\tan^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=331

$$\frac{a(bB-aC)\tan^3(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}+\frac{a\left(a^2bB-3a^3C-7ab^2C+5b^3B\right)\tan^2(c+dx)}{2b^2d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}-\frac{\left(-6a^2b^2C+a^3bB-3a^4C+3ab\right.}{b^3d\left(a^2+b^2\right.}$$

```
[Out] ((a^3*B - 3*a*b^2*B + 3*a^2*b*C - b^3*C)*x)/(a^2 + b^2)^3 + ((3*a^2*b*B - b
^3*B - a^3*C + 3*a*b^2*C)*Log[Cos[c + d*x]])/((a^2 + b^2)^3*d) + (a^2*(a^4*
b*B + 3*a^2*b^3*B + 6*b^5*B - 3*a^5*C - 9*a^3*b^2*C - 10*a*b^4*C)*Log[a + b
*Tan[c + d*x]])/(b^4*(a^2 + b^2)^3*d) - ((a^3*b*B + 3*a*b^3*B - 3*a^4*C - 6
*a^2*b^2*C - b^4*C)*Tan[c + d*x])/(b^3*(a^2 + b^2)^2*d) + (a*(b*B - a*C)*Ta
n[c + d*x]^3)/(2*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) + (a*(a^2*b*B + 5*
b^3*B - 3*a^3*C - 7*a*b^2*C)*Tan[c + d*x]^2)/(2*b^2*(a^2 + b^2)^2*d*(a + b*
Tan[c + d*x]))
```

Rubi [A] time = 0.860573, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3632, 3605, 3645, 3647, 3626, 3617, 31, 3475}

$$\frac{a(bB-aC)\tan^3(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}+\frac{a\left(a^2bB-3a^3C-7ab^2C+5b^3B\right)\tan^2(c+dx)}{2b^2d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}-\frac{\left(-6a^2b^2C+a^3bB-3a^4C+3ab\right.}{b^3d\left(a^2+b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Tan[c + d*x]^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x
])^3,x]
```

```
[Out] ((a^3*B - 3*a*b^2*B + 3*a^2*b*C - b^3*C)*x)/(a^2 + b^2)^3 + ((3*a^2*b*B - b
^3*B - a^3*C + 3*a*b^2*C)*Log[Cos[c + d*x]])/((a^2 + b^2)^3*d) + (a^2*(a^4*
b*B + 3*a^2*b^3*B + 6*b^5*B - 3*a^5*C - 9*a^3*b^2*C - 10*a*b^4*C)*Log[a + b
*Tan[c + d*x]])/(b^4*(a^2 + b^2)^3*d) - ((a^3*b*B + 3*a*b^3*B - 3*a^4*C - 6
*a^2*b^2*C - b^4*C)*Tan[c + d*x])/(b^3*(a^2 + b^2)^2*d) + (a*(b*B - a*C)*Ta
n[c + d*x]^3)/(2*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) + (a*(a^2*b*B + 5*
b^3*B - 3*a^3*C - 7*a*b^2*C)*Tan[c + d*x]^2)/(2*b^2*(a^2 + b^2)^2*d*(a + b*
Tan[c + d*x]))
```

Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3605

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x
])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)),
 Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(
b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
```

```
+ 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e
 + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&
 LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3645

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*d^2 + c*(c*C - B*d))*(a + b*Tan[e
+ f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3626

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((a*A + b*B -
a*C)*x)/(a^2 + b^2), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3617

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}dx = \int \frac{\tan^4(c+dx)(B+C\tan(c+dx))}{(a+b\tan(c+dx))^3}dx$$

$$= \frac{a(bB-aC)\tan^3(c+dx)}{2b\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} + \frac{\int \frac{\tan^2(c+dx)\left(-3a(bB-aC)+2b(\right.}{}}{}$$

$$= \frac{a(bB-aC)\tan^3(c+dx)}{2b\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} + \frac{a\left(a^2bB+5b^3B-3a^3C-\right.}{2b^2\left(a^2+b^2\right)^2 d(a}$$

$$= -\frac{\left(a^3bB+3ab^3B-3a^4C-6a^2b^2C-b^4C\right)\tan(c+dx)}{b^3\left(a^2+b^2\right)^2 d} + \frac{}{2b(a}$$

$$= \frac{\left(a^3B-3ab^2B+3a^2bC-b^3C\right)x}{\left(a^2+b^2\right)^3} - \frac{\left(a^3bB+3ab^3B-3a^4C-6\right.}{b^3\left(a^2+\right.}$$

$$= \frac{\left(a^3B-3ab^2B+3a^2bC-b^3C\right)x}{\left(a^2+b^2\right)^3} + \frac{\left(3a^2bB-b^3B-a^3C+3ab\right.}{\left(a^2+b^2\right)}$$

$$= \frac{\left(a^3B-3ab^2B+3a^2bC-b^3C\right)x}{\left(a^2+b^2\right)^3} + \frac{\left(3a^2bB-b^3B-a^3C+3ab\right.}{\left(a^2+b^2\right)}$$

<u>Mathematica</u> [C] time = 6.67839, size = 1146, normalized size = 3.46

$$\frac{(aC-bB)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))(B+C\tan(c+dx))a^4}{2(a-ib)^2(a+ib)^2b^2d(B\cos(c+dx)+C\sin(c+dx))(a+b\tan(c+dx))^3} + \frac{\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+d}{(a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c
 + d*x])^3,x]
```

```
[Out] (a^4*(-(b*B) + a*C)*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])*(B + C
*Tan[c + d*x]))/(2*(a - I*b)^2*(a + I*b)^2*b^2*d*(B*Cos[c + d*x] + C*Sin[c
+ d*x])*(a + b*Tan[c + d*x])^3) + ((a^3*B - 3*a*b^2*B + 3*a^2*b*C - b^3*C)*
(c + d*x)*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3*(B + C*Tan[c +
 d*x]))/((a - I*b)^3*(a + I*b)^3*d*(B*Cos[c + d*x] + C*Sin[c + d*x])*(a + b
*Tan[c + d*x])^3) + ((I*a^11*b^4*B + a^10*b^5*B + (5*I)*a^9*b^6*B + 5*a^8*b
^7*B + (13*I)*a^7*b^8*B + 13*a^6*b^9*B + (15*I)*a^5*b^10*B + 15*a^4*b^11*B
+ (6*I)*a^3*b^12*B + 6*a^2*b^13*B - (3*I)*a^12*b^3*C - 3*a^11*b^4*C - (15*I
)*a^10*b^5*C - 15*a^9*b^6*C - (31*I)*a^8*b^7*C - 31*a^7*b^8*C - (29*I)*a^6*
b^9*C - 29*a^5*b^10*C - (10*I)*a^4*b^11*C - 10*a^3*b^12*C)*(c + d*x)*Sec[c
+ d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3*(B + C*Tan[c + d*x]))/((a - I*
b)^6*(a + I*b)^5*b^7*d*(B*Cos[c + d*x] + C*Sin[c + d*x])*(a + b*Tan[c + d*x
])^3) - (I*(a^6*b*B + 3*a^4*b^3*B + 6*a^2*b^5*B - 3*a^7*C - 9*a^5*b^2*C - 1
0*a^3*b^4*C)*ArcTan[Tan[c + d*x]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c
+ d*x])^3*(B + C*Tan[c + d*x]))/(b^4*(a^2 + b^2)^3*d*(B*Cos[c + d*x] + C*Si
n[c + d*x])*(a + b*Tan[c + d*x])^3) + ((-(b*B) + 3*a*C)*Log[Cos[c + d*x]]*S
ec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3*(B + C*Tan[c + d*x]))/(b^
4*d*(B*Cos[c + d*x] + C*Sin[c + d*x])*(a + b*Tan[c + d*x])^3) + ((a^6*b*B +
 3*a^4*b^3*B + 6*a^2*b^5*B - 3*a^7*C - 9*a^5*b^2*C - 10*a^3*b^4*C)*Log[(a*C
os[c + d*x] + b*Sin[c + d*x])^2]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c +
 d*x])^3*(B + C*Tan[c + d*x]))/(2*b^4*(a^2 + b^2)^3*d*(B*Cos[c + d*x] + C*S
in[c + d*x])*(a + b*Tan[c + d*x])^3) + (Sec[c + d*x]^2*(a*Cos[c + d*x] + b*
```

```
Sin[c + d*x])^2*(-(a^4*b*B*Sin[c + d*x]) - 4*a^2*b^3*B*Sin[c + d*x] + 2*a^5
*C*Sin[c + d*x] + 5*a^3*b^2*C*Sin[c + d*x])*(B + C*Tan[c + d*x]))/((a - I*b
)^2*(a + I*b)^2*b^3*d*(B*Cos[c + d*x] + C*Sin[c + d*x])*(a + b*Tan[c + d*x]
)^3) + (C*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3*Tan[c + d*x]*(
B + C*Tan[c + d*x]))/(b^3*d*(B*Cos[c + d*x] + C*Sin[c + d*x])*(a + b*Tan[c
+ d*x])^3)
```

Maple [A] time = 0.048, size = 619, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x)
```

```
[Out] 1/d*C/b^3*tan(d*x+c)-3/2/d/(a^2+b^2)^3*ln(1+tan(d*x+c)^2)*B*a^2*b+1/2/d/(a^
2+b^2)^3*ln(1+tan(d*x+c)^2)*B*b^3+1/2/d/(a^2+b^2)^3*ln(1+tan(d*x+c)^2)*C*a^
3-3/2/d/(a^2+b^2)^3*ln(1+tan(d*x+c)^2)*C*a*b^2+1/d/(a^2+b^2)^3*B*arctan(tan
(d*x+c))*a^3-3/d/(a^2+b^2)^3*B*arctan(tan(d*x+c))*a*b^2+3/d/(a^2+b^2)^3*C*a
rctan(tan(d*x+c))*a^2*b-1/d/(a^2+b^2)^3*C*arctan(tan(d*x+c))*b^3+1/d/b^3*a^
6/(a^2+b^2)^3*ln(a+b*tan(d*x+c))*B+3/d/b*a^4/(a^2+b^2)^3*ln(a+b*tan(d*x+c))
*B+6/d*b*a^2/(a^2+b^2)^3*ln(a+b*tan(d*x+c))*B-3/d/b^4*a^7/(a^2+b^2)^3*ln(a+
b*tan(d*x+c))*C-9/d/b^2*a^5/(a^2+b^2)^3*ln(a+b*tan(d*x+c))*C-10/d*a^3/(a^2+
b^2)^3*ln(a+b*tan(d*x+c))*C-1/2/d/b^3*a^4/(a^2+b^2)/(a+b*tan(d*x+c))^2*B+1/
2/d/b^4*a^5/(a^2+b^2)/(a+b*tan(d*x+c))^2*C+2/d/b^3*a^5/(a^2+b^2)^2/(a+b*tan
(d*x+c))*B+4/d/b*a^3/(a^2+b^2)^2/(a+b*tan(d*x+c))*B-3/d/b^4*a^6/(a^2+b^2)^2
/(a+b*tan(d*x+c))*C-5/d/b^2*a^4/(a^2+b^2)^2/(a+b*tan(d*x+c))*C
```

Maxima [A] time = 1.7538, size = 525, normalized size = 1.59

$$\frac{\frac{2\left(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3\right)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{2\left(3\,Ca^7-Ba^6b+9\,Ca^5b^2-3\,Ba^4b^3+10\,Ca^3b^4-6\,Ba^2b^5\right)\log(b\tan(dx+c)+a)}{a^6b^4+3\,a^4b^6+3\,a^2b^8+b^{10}}+\frac{\left(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3\right)\log\left(\tan(dx+c)\right.}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,
algorithm="maxima")
```

```
[Out] 1/2*(2*(B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6) - 2*(3*C*a^7 - B*a^6*b + 9*C*a^5*b^2 - 3*B*a^4*b^3 + 10*C
*a^3*b^4 - 6*B*a^2*b^5)*log(b*tan(d*x + c) + a)/(a^6*b^4 + 3*a^4*b^6 + 3*a^
2*b^8 + b^10) + (C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*log(tan(d*x + c)^2
+ 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (5*C*a^7 - 3*B*a^6*b + 9*C*a^5*b
^2 - 7*B*a^4*b^3 + 2*(3*C*a^6*b - 2*B*a^5*b^2 + 5*C*a^4*b^3 - 4*B*a^3*b^4)*
tan(d*x + c))/(a^6*b^4 + 2*a^4*b^6 + a^2*b^8 + (a^4*b^6 + 2*a^2*b^8 + b^10)
*tan(d*x + c)^2 + 2*(a^5*b^5 + 2*a^3*b^7 + a*b^9)*tan(d*x + c)) + 2*C*tan(d
*x + c)/b^3)/d
```

Fricas [B] time = 1.85253, size = 1895, normalized size = 5.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,
algorithm="fricas")
```

```
[Out] -1/2*(3*C*a^7*b^2 - B*a^6*b^3 + 9*C*a^5*b^4 - 7*B*a^4*b^5 - 2*(C*a^6*b^3 +
3*C*a^4*b^5 + 3*C*a^2*b^7 + C*b^9)*tan(d*x + c)^3 - 2*(B*a^5*b^4 + 3*C*a^4*
b^5 - 3*B*a^3*b^6 - C*a^2*b^7)*d*x - (9*C*a^7*b^2 - 3*B*a^6*b^3 + 23*C*a^5*
b^4 - 9*B*a^4*b^5 + 12*C*a^3*b^6 + 4*C*a*b^8 + 2*(B*a^3*b^6 + 3*C*a^2*b^7 -
 3*B*a*b^8 - C*b^9)*d*x)*tan(d*x + c)^2 + (3*C*a^9 - B*a^8*b + 9*C*a^7*b^2
- 3*B*a^6*b^3 + 10*C*a^5*b^4 - 6*B*a^4*b^5 + (3*C*a^7*b^2 - B*a^6*b^3 + 9*C
*a^5*b^4 - 3*B*a^4*b^5 + 10*C*a^3*b^6 - 6*B*a^2*b^7)*tan(d*x + c)^2 + 2*(3*
C*a^8*b - B*a^7*b^2 + 9*C*a^6*b^3 - 3*B*a^5*b^4 + 10*C*a^4*b^5 - 6*B*a^3*b^
6)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d
*x + c)^2 + 1)) - (3*C*a^9 - B*a^8*b + 9*C*a^7*b^2 - 3*B*a^6*b^3 + 9*C*a^5*
b^4 - 3*B*a^4*b^5 + 3*C*a^3*b^6 - B*a^2*b^7 + (3*C*a^7*b^2 - B*a^6*b^3 + 9*
C*a^5*b^4 - 3*B*a^4*b^5 + 9*C*a^3*b^6 - 3*B*a^2*b^7 + 3*C*a*b^8 - B*b^9)*ta
n(d*x + c)^2 + 2*(3*C*a^8*b - B*a^7*b^2 + 9*C*a^6*b^3 - 3*B*a^5*b^4 + 9*C*a
^4*b^5 - 3*B*a^3*b^6 + 3*C*a^2*b^7 - B*a*b^8)*tan(d*x + c))*log(1/(tan(d*x
+ c)^2 + 1)) - 2*(3*C*a^8*b - B*a^7*b^2 + 6*C*a^6*b^3 - 3*B*a^5*b^4 - 2*C*a
^4*b^5 + 4*B*a^3*b^6 + C*a^2*b^7 + 2*(B*a^4*b^5 + 3*C*a^3*b^6 - 3*B*a^2*b^7
 - C*a*b^8)*d*x)*tan(d*x + c))/((a^6*b^6 + 3*a^4*b^8 + 3*a^2*b^10 + b^12)*d
*tan(d*x + c)^2 + 2*(a^7*b^5 + 3*a^5*b^7 + 3*a^3*b^9 + a*b^11)*d*tan(d*x +
c) + (a^8*b^4 + 3*a^6*b^6 + 3*a^4*b^8 + a^2*b^10)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**3,
x)
```

```
[Out] Timed out
```

Giac [A] time = 2.0753, size = 682, normalized size = 2.06

$$\frac{\frac{2\left(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3\right)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{\left(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3\right)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{2\left(3\,Ca^7-Ba^6b+9\,Ca^5b^2-3\,Ba^4b^3+10\,Ca^3b^4-6\,Ba^2b^5\right)\log(|b\tan(}{a^6b^4+3\,a^4b^6+3\,a^2b^8+b^{10}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,
algorithm="giac")
```

```
[Out] 1/2*(2*(B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6) + (C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*log(tan(d*x + c
)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 2*(3*C*a^7 - B*a^6*b + 9*C*a
^5*b^2 - 3*B*a^4*b^3 + 10*C*a^3*b^4 - 6*B*a^2*b^5)*log(abs(b*tan(d*x + c) +
 a))/(a^6*b^4 + 3*a^4*b^6 + 3*a^2*b^8 + b^10) + 2*C*tan(d*x + c)/b^3 + (9*C
*a^7*b^2*tan(d*x + c)^2 - 3*B*a^6*b^3*tan(d*x + c)^2 + 27*C*a^5*b^4*tan(d*x
 + c)^2 - 9*B*a^4*b^5*tan(d*x + c)^2 + 30*C*a^3*b^6*tan(d*x + c)^2 - 18*B*a
```

```
^2*b^7*tan(d*x + c)^2 + 12*C*a^8*b*tan(d*x + c) - 2*B*a^7*b^2*tan(d*x + c)
+ 38*C*a^6*b^3*tan(d*x + c) - 6*B*a^5*b^4*tan(d*x + c) + 50*C*a^4*b^5*tan(d
*x + c) - 28*B*a^3*b^6*tan(d*x + c) + 4*C*a^9 + 13*C*a^7*b^2 + B*a^6*b^3 +
21*C*a^5*b^4 - 11*B*a^4*b^5)/((a^6*b^4 + 3*a^4*b^6 + 3*a^2*b^8 + b^10)*(b*t
an(d*x + c) + a)^2))/d
```

3.39 $\int \frac{\tan^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=250

$$\frac{a(bB-aC)\tan^2(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{a^2\left(a^3(-C)-3ab^2C+2b^3B\right)}{b^3d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}+\frac{a\left(a^2b^3B+3a^3b^2C+a^5C+6ab^4C-3b^5B\right)\mathrm{l}}{b^3d\left(a^2+b^2\right)^3}$$

```
[Out] -(((3*a^2*b*B - b^3*B - a^3*C + 3*a*b^2*C)*x)/(a^2 + b^2)^3) + ((a^3*B - 3*
a*b^2*B + 3*a^2*b*C - b^3*C)*Log[Cos[c + d*x]])/((a^2 + b^2)^3*d) + (a*(a^2
*b^3*B - 3*b^5*B + a^5*C + 3*a^3*b^2*C + 6*a*b^4*C)*Log[a + b*Tan[c + d*x]]
)/(b^3*(a^2 + b^2)^3*d) + (a*(b*B - a*C)*Tan[c + d*x]^2)/(2*b*(a^2 + b^2)*d
*(a + b*Tan[c + d*x])^2) - (a^2*(2*b^3*B - a^3*C - 3*a*b^2*C))/(b^3*(a^2 +
b^2)^2*d*(a + b*Tan[c + d*x]))
```

Rubi [A] time = 0.581399, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {3632, 3605, 3635, 3626, 3617, 31, 3475}

$$\frac{a(bB-aC)\tan^2(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{a^2\left(a^3(-C)-3ab^2C+2b^3B\right)}{b^3d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}+\frac{a\left(a^2b^3B+3a^3b^2C+a^5C+6ab^4C-3b^5B\right)\mathrm{l}}{b^3d\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Tan[c + d*x]^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x
])^3,x]
```


```
[Out] -(((3*a^2*b*B - b^3*B - a^3*C + 3*a*b^2*C)*x)/(a^2 + b^2)^3) + ((a^3*B - 3*
a*b^2*B + 3*a^2*b*C - b^3*C)*Log[Cos[c + d*x]])/((a^2 + b^2)^3*d) + (a*(a^2
*b^3*B - 3*b^5*B + a^5*C + 3*a^3*b^2*C + 6*a*b^4*C)*Log[a + b*Tan[c + d*x]]
)/(b^3*(a^2 + b^2)^3*d) + (a*(b*B - a*C)*Tan[c + d*x]^2)/(2*b*(a^2 + b^2)*d
*(a + b*Tan[c + d*x])^2) - (a^2*(2*b^3*B - a^3*C - 3*a*b^2*C))/(b^3*(a^2 +
b^2)^2*d*(a + b*Tan[c + d*x]))
```


Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3605

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x
])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)),
 Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(
b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
+ 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e
 + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&
```

```
LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3635

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(c^2*C - B*c*d + A*d^2)*(c +
d*Tan[e + f*x])^(n + 1))/(d^2*f*(n + 1)*(c^2 + d^2)), x] + Dist[1/(d*(c^2 +
 d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(c^
2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*Ta
n[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -
1]
```

Rule 3626

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((a*A + b*B -
a*C)*x)/(a^2 + b^2), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3617

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\tan^3(c+dx)(B+C\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$

$$= \frac{a(bB-aC)\tan^2(c+dx)}{2b\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} + \frac{\int \frac{\tan(c+dx)\left(-2a(bB-aC)+2b(b\cdots\right.}{(a\cdots}}{2\cdots}$$

$$= \frac{a(bB-aC)\tan^2(c+dx)}{2b\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{a^2\left(2b^3B-a^3C-3ab\cdots\right.}{b^3\left(a^2+b^2\right)^2 d(a+b\tan\cdots}$$

$$= -\frac{\left(3a^2bB-b^3B-a^3C+3ab^2C\right)x}{\left(a^2+b^2\right)^3} + \frac{a(bB-aC)\tan^2(c+\cdots}{2b\left(a^2+b^2\right)d(a+b\tan(\cdots}$$

$$= -\frac{\left(3a^2bB-b^3B-a^3C+3ab^2C\right)x}{\left(a^2+b^2\right)^3} + \frac{\left(a^3B-3ab^2B+3a^2bC-\cdots\right.}{\left(a^2+b\cdots\right.}$$

$$= -\frac{\left(3a^2bB-b^3B-a^3C+3ab^2C\right)x}{\left(a^2+b^2\right)^3} + \frac{\left(a^3B-3ab^2B+3a^2bC-\cdots\right.}{\left(a^2+b\cdots\right.}$$

Mathematica [C] time = 4.5326, size = 462, normalized size = 1.85

$$\frac{\sec^2(c+dx)(B+C\tan(c+dx))(a\cos(c+dx)+b\sin(c+dx))\left(2ia(c+dx)\left(a^2b^3B+3a^3b^2C+a^5C+6ab^4C-3b^5B\right)\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c
 + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])*(a^3*b^2*(a^2 + b^2)*(b*B
 - a*C) - 2*a*b*(a^2 + b^2)*(-3*b^3*B + a^3*C + 4*a*b^2*C)*Sin[c + d*x]*(a*
Cos[c + d*x] + b*Sin[c + d*x]) + 2*b^3*(-3*a^2*b*B + b^3*B + a^3*C - 3*a*b^
2*C)*(c + d*x)*(a*Cos[c + d*x] + b*Sin[c + d*x])^2 + (2*I)*a*(a^2*b^3*B - 3
*b^5*B + a^5*C + 3*a^3*b^2*C + 6*a*b^4*C)*(c + d*x)*(a*Cos[c + d*x] + b*Sin
[c + d*x])^2 - (2*I)*a*(a^2*b^3*B - 3*b^5*B + a^5*C + 3*a^3*b^2*C + 6*a*b^4
*C)*ArcTan[Tan[c + d*x]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2 - 2*(a^2 + b^2
)^3*C*Log[Cos[c + d*x]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2 + a*(a^2*b^3*B
- 3*b^5*B + a^5*C + 3*a^3*b^2*C + 6*a*b^4*C)*Log[(a*Cos[c + d*x] + b*Sin[c
+ d*x])^2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)*(B + C*Tan[c + d*x]))/(2*b^
3*(a^2 + b^2)^3*d*(B*Cos[c + d*x] + C*Sin[c + d*x])*(a + b*Tan[c + d*x])^3)
```

Maple [B] time = 0.054, size = 566, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x)
```

```
[Out] -1/2/d/(a^2+b^2)^3*ln(1+tan(d*x+c)^2)*B*a^3+3/2/d/(a^2+b^2)^3*ln(1+tan(d*x+
c)^2)*B*a*b^2-3/2/d/(a^2+b^2)^3*ln(1+tan(d*x+c)^2)*C*a^2*b+1/2/d/(a^2+b^2)^
```

```
3*ln(1+tan(d*x+c)^2)*C*b^3-3/d/(a^2+b^2)^3*B*arctan(tan(d*x+c))*a^2*b+1/d/(
a^2+b^2)^3*B*arctan(tan(d*x+c))*b^3+1/d/(a^2+b^2)^3*C*arctan(tan(d*x+c))*a^
3-3/d/(a^2+b^2)^3*C*arctan(tan(d*x+c))*a*b^2+1/d*a^3/(a^2+b^2)^3*ln(a+b*tan
(d*x+c))*B-3/d*a/(a^2+b^2)^3*b^2*ln(a+b*tan(d*x+c))*B+1/d*a^6/(a^2+b^2)^3/b
^3*ln(a+b*tan(d*x+c))*C+3/d*a^4/(a^2+b^2)^3/b*ln(a+b*tan(d*x+c))*C+6/d*a^2/
(a^2+b^2)^3*b*ln(a+b*tan(d*x+c))*C-1/d*a^4/b^2/(a^2+b^2)^2/(a+b*tan(d*x+c))
*B-3/d*a^2/(a^2+b^2)^2/(a+b*tan(d*x+c))*B+2/d*a^5/b^3/(a^2+b^2)^2/(a+b*tan(
d*x+c))*C+4/d*a^3/b/(a^2+b^2)^2/(a+b*tan(d*x+c))*C+1/2/d*a^3/b^2/(a^2+b^2)/
(a+b*tan(d*x+c))^2*B-1/2/d*a^4/b^3/(a^2+b^2)/(a+b*tan(d*x+c))^2*C
```

Maxima [A] time = 1.89238, size = 494, normalized size = 1.98

$$\frac{\frac{2\left(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3\right)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\left(Ca^6+3\,Ca^4b^2+Ba^3b^3+6\,Ca^2b^4-3\,Bab^5\right)\log(b\tan(dx+c)+a)}{a^6b^3+3\,a^4b^5+3\,a^2b^7+b^9}-\frac{\left(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3\right)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{}{a^6}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,
algorithm="maxima")
```

```
[Out] 1/2*(2*(C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6) + 2*(C*a^6 + 3*C*a^4*b^2 + B*a^3*b^3 + 6*C*a^2*b^4 - 3*B*
a*b^5)*log(b*tan(d*x + c) + a)/(a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9) - (B
*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*
b^2 + 3*a^2*b^4 + b^6) + (3*C*a^6 - B*a^5*b + 7*C*a^4*b^2 - 5*B*a^3*b^3 + 2
*(2*C*a^5*b - B*a^4*b^2 + 4*C*a^3*b^3 - 3*B*a^2*b^4)*tan(d*x + c))/(a^6*b^3
 + 2*a^4*b^5 + a^2*b^7 + (a^4*b^5 + 2*a^2*b^7 + b^9)*tan(d*x + c)^2 + 2*(a^
5*b^4 + 2*a^3*b^6 + a*b^8)*tan(d*x + c)))/d
```

Fricas [B] time = 1.60578, size = 1432, normalized size = 5.73

$$\frac{Ca^6b^2+Ba^5b^3+7\,Ca^4b^4-5\,Ba^3b^5+2\left(Ca^5b^3-3\,Ba^4b^4-3\,Ca^3b^5+Ba^2b^6\right)dx-\left(3\,Ca^6b^2-Ba^5b^3+9\,Ca^4b^4-7\,Ba^3b^5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,
algorithm="fricas")
```

```
[Out] 1/2*(C*a^6*b^2 + B*a^5*b^3 + 7*C*a^4*b^4 - 5*B*a^3*b^5 + 2*(C*a^5*b^3 - 3*B
*a^4*b^4 - 3*C*a^3*b^5 + B*a^2*b^6)*d*x - (3*C*a^6*b^2 - B*a^5*b^3 + 9*C*a^
4*b^4 - 7*B*a^3*b^5 - 2*(C*a^3*b^5 - 3*B*a^2*b^6 - 3*C*a*b^7 + B*b^8)*d*x)*
tan(d*x + c)^2 + (C*a^8 + 3*C*a^6*b^2 + B*a^5*b^3 + 6*C*a^4*b^4 - 3*B*a^3*b
^5 + (C*a^6*b^2 + 3*C*a^4*b^4 + B*a^3*b^5 + 6*C*a^2*b^6 - 3*B*a*b^7)*tan(d*
x + c)^2 + 2*(C*a^7*b + 3*C*a^5*b^3 + B*a^4*b^4 + 6*C*a^3*b^5 - 3*B*a^2*b^6
)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*
x + c)^2 + 1)) - (C*a^8 + 3*C*a^6*b^2 + 3*C*a^4*b^4 + C*a^2*b^6 + (C*a^6*b^
2 + 3*C*a^4*b^4 + 3*C*a^2*b^6 + C*b^8)*tan(d*x + c)^2 + 2*(C*a^7*b + 3*C*a^
5*b^3 + 3*C*a^3*b^5 + C*a*b^7)*tan(d*x + c))*log(1/(tan(d*x + c)^2 + 1)) -
2*(C*a^7*b + 3*C*a^5*b^3 - 3*B*a^4*b^4 - 4*C*a^3*b^5 + 3*B*a^2*b^6 - 2*(C*a
^4*b^4 - 3*B*a^3*b^5 - 3*C*a^2*b^6 + B*a*b^7)*d*x)*tan(d*x + c))/((a^6*b^5
+ 3*a^4*b^7 + 3*a^2*b^9 + b^11)*d*tan(d*x + c)^2 + 2*(a^7*b^4 + 3*a^5*b^6 +
 3*a^3*b^8 + a*b^10)*d*tan(d*x + c) + (a^8*b^3 + 3*a^6*b^5 + 3*a^4*b^7 + a^
```

```
2*b^9)*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**3,
x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.93239, size = 618, normalized size = 2.47

$$\frac{\frac{2\left(Ca^{3}-3\,Ba^{2}b-3\,Cab^{2}+Bb^{3}\right)(dx+c)}{a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}}-\frac{\left(Ba^{3}+3\,Ca^{2}b-3\,Bab^{2}-Cb^{3}\right)\log\left(\tan(dx+c)^{2}+1\right)}{a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}}+\frac{2\left(Ca^{6}+3\,Ca^{4}b^{2}+Ba^{3}b^{3}+6\,Ca^{2}b^{4}-3\,Bab^{5}\right)\log(|b\tan(dx+c)+a|)}{a^{6}b^{3}+3\,a^{4}b^{5}+3\,a^{2}b^{7}+b^{9}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,
algorithm="giac")
```

```
[Out] 1/2*(2*(C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6) - (B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*log(tan(d*x + c
)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(C*a^6 + 3*C*a^4*b^2 + B*a
^3*b^3 + 6*C*a^2*b^4 - 3*B*a*b^5)*log(abs(b*tan(d*x + c) + a))/(a^6*b^3 + 3
*a^4*b^5 + 3*a^2*b^7 + b^9) - (3*C*a^6*b*tan(d*x + c)^2 + 9*C*a^4*b^3*tan(d
*x + c)^2 + 3*B*a^3*b^4*tan(d*x + c)^2 + 18*C*a^2*b^5*tan(d*x + c)^2 - 9*B*
a*b^6*tan(d*x + c)^2 + 2*C*a^7*tan(d*x + c) + 2*B*a^6*b*tan(d*x + c) + 6*C*
a^5*b^2*tan(d*x + c) + 14*B*a^4*b^3*tan(d*x + c) + 28*C*a^3*b^4*tan(d*x + c
) - 12*B*a^2*b^5*tan(d*x + c) + B*a^7 - C*a^6*b + 9*B*a^5*b^2 + 11*C*a^4*b^
3 - 4*B*a^3*b^4)/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*(b*tan(d*x + c) +
 a)^2))/d
```

3.40 $\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=189

$$-\frac{a^2(bB-aC)}{2b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}+\frac{a\left(a^3(-C)-3ab^2C+2b^3B\right)}{b^2d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}-\frac{\left(3a^2bB+a^3(-C)+3ab^2C-b^3B\right)\log(a\cos(c$$

```
[Out] -(((a^3*B - 3*a*b^2*B + 3*a^2*b*C - b^3*C)*x)/(a^2 + b^2)^3) - ((3*a^2*b*B
- b^3*B - a^3*C + 3*a*b^2*C)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 +
b^2)^3*d) - (a^2*(b*B - a*C))/(2*b^2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2)
+ (a*(2*b^3*B - a^3*C - 3*a*b^2*C))/(b^2*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x
]))
```

Rubi [A] time = 0.427431, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {3632, 3604, 3628, 3531, 3530}

$$-\frac{a^2(bB-aC)}{2b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}+\frac{a\left(a^3(-C)-3ab^2C+2b^3B\right)}{b^2d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}-\frac{\left(3a^2bB+a^3(-C)+3ab^2C-b^3B\right)\log(a\cos(c$$

Antiderivative was successfully verified.

```
[In] Int[(Tan[c + d*x]*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x])
^3,x]
```

```
[Out] -(((a^3*B - 3*a*b^2*B + 3*a^2*b*C - b^3*C)*x)/(a^2 + b^2)^3) - ((3*a^2*b*B
- b^3*B - a^3*C + 3*a*b^2*C)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 +
b^2)^3*d) - (a^2*(b*B - a*C))/(2*b^2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2)
+ (a*(2*b^3*B - a^3*C - 3*a*b^2*C))/(b^2*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x
]))
```

Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3604

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp
[((B*c - A*d)*(b*c - a*d)^2*(c + d*Tan[e + f*x])^(n + 1))/(f*d^2*(n + 1)*(c
^2 + d^2)), x] + Dist[1/(d*(c^2 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*S
imp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*c +
 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2)*T
an[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

Rule 3628

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2
```

```
- a*b*B + a^2*C)*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 + b^2)), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\tan^2(c+dx)(B+C\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$

$$= -\frac{a^2(bB-aC)}{2b^2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} + \frac{\int \frac{-a(bB-aC)+b(bB-aC)\tan(\cdots}{(a+b\tan\cdots}}{b\left(a^2\cdots\right.}$$

$$= -\frac{a^2(bB-aC)}{2b^2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} + \frac{a\left(2b^3B-a^3C-3a\cdots\right.}{b^2\left(a^2+b^2\right)^2 d(a+b\tan\cdots}$$

$$= -\frac{\left(a^3B-3ab^2B+3a^2bC-b^3C\right)x}{\left(a^2+b^2\right)^3} - \frac{a^2(bB-aC)}{2b^2\left(a^2+b^2\right)d(a+b\tan(\cdots}$$

$$= -\frac{\left(a^3B-3ab^2B+3a^2bC-b^3C\right)x}{\left(a^2+b^2\right)^3} - \frac{\left(3a^2bB-b^3B-a^3C+3ab\cdots\right.}{}$$

Mathematica [C] time = 4.6474, size = 288, normalized size = 1.52

$$\frac{(bB-aC)\left(\frac{b\left(\frac{\left(a^2+b^2\right)\left(5a^2+4ab\tan(c+dx)+b^2\right)}{(a+b\tan(c+dx))^2}+\left(2b^2-6a^2\right)\log(a+b\tan(c+dx))\right)}{\left(a^2+b^2\right)^3}+\frac{i\log(-\tan(c+dx)+i)}{(a+ib)^3}-\frac{\log(\tan(c+dx)+i)}{(b+ia)^3}\right)+C\left(\frac{2b\left(\frac{a^2+b^2}{a+b\tan(c+dx)}-2\cdots\right.}{\left(a^2\cdots\right.}\right.}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c +
 d*x])^3,x]
```

```
[Out] (-((b*B + a*C)/(b*(a + b*Tan[c + d*x])^2)) - (2*C*Tan[c + d*x])/(a + b*Tan[
c + d*x])^2 + C*((I*Log[I - Tan[c + d*x]])/(a + I*b)^2 - (I*Log[I + Tan[c +
 d*x]])/(a - I*b)^2 + (2*b*(-2*a*Log[a + b*Tan[c + d*x]] + (a^2 + b^2)/(a +
 b*Tan[c + d*x])))/(a^2 + b^2)^2) + (b*B - a*C)*((I*Log[I - Tan[c + d*x]])/
(a + I*b)^3 - Log[I + Tan[c + d*x]]/(I*a + b)^3 + (b*((-6*a^2 + 2*b^2)*Log[
a + b*Tan[c + d*x]] + ((a^2 + b^2)*(5*a^2 + b^2 + 4*a*b*Tan[c + d*x]))/(a +
```

```
b*Tan[c + d*x])^2))/(a^2 + b^2)^3))/(2*b*d)
```

Maple [B] time = 0.051, size = 495, normalized size = 2.6

$$\frac{3\ln\left(1+(\tan(dx+c))^2\right)Ba^2b}{2d\left(a^2+b^2\right)^3}-\frac{\ln\left(1+(\tan(dx+c))^2\right)Bb^3}{2d\left(a^2+b^2\right)^3}-\frac{\ln\left(1+(\tan(dx+c))^2\right)Ca^3}{2d\left(a^2+b^2\right)^3}+\frac{3\ln\left(1+(\tan(dx+c))^2\right)}{2d\left(a^2+b^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x)
```

```
[Out] 3/2/d/(a^2+b^2)^3*ln(1+tan(d*x+c)^2)*B*a^2*b-1/2/d/(a^2+b^2)^3*ln(1+tan(d*x
+c)^2)*B*b^3-1/2/d/(a^2+b^2)^3*ln(1+tan(d*x+c)^2)*C*a^3+3/2/d/(a^2+b^2)^3*l
n(1+tan(d*x+c)^2)*C*a*b^2-1/d/(a^2+b^2)^3*B*arctan(tan(d*x+c))*a^3+3/d/(a^2
+b^2)^3*B*arctan(tan(d*x+c))*a*b^2-3/d/(a^2+b^2)^3*C*arctan(tan(d*x+c))*a^2
*b+1/d/(a^2+b^2)^3*C*arctan(tan(d*x+c))*b^3-1/2/d*a^2/b/(a^2+b^2)/(a+b*tan(
d*x+c))^2*B+1/2/d*a^3/b^2/(a^2+b^2)/(a+b*tan(d*x+c))^2*C-3/d*b*a^2/(a^2+b^2
)^3*ln(a+b*tan(d*x+c))*B+1/d/(a^2+b^2)^3*ln(a+b*tan(d*x+c))*B*b^3+1/d*a^3/(
a^2+b^2)^3*ln(a+b*tan(d*x+c))*C-3/d/(a^2+b^2)^3*ln(a+b*tan(d*x+c))*C*a*b^2+
2/d*a/(a^2+b^2)^2*b/(a+b*tan(d*x+c))*B-1/d/b^2*a^4/(a^2+b^2)^2/(a+b*tan(d*x
+c))*C-3/d*a^2/(a^2+b^2)^2/(a+b*tan(d*x+c))*C
```

Maxima [A] time = 1.81723, size = 450, normalized size = 2.38

$$-\frac{\frac{2\left(Ba^3+3Ca^2b-3Bab^2-Cb^3\right)(dx+c)}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{2\left(Ca^3-3Ba^2b-3Cab^2+Bb^3\right)\log(b\tan(dx+c)+a)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{\left(Ca^3-3Ba^2b-3Cab^2+Bb^3\right)\log\left(\tan(dx+c)^2+1\right)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{Ca^5+}{a^6b^2+2a^4b^4+}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x, al
gorithm="maxima")
```

```
[Out] -1/2*(2*(B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6) - 2*(C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*log(b*tan(d*
x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (C*a^3 - 3*B*a^2*b - 3*C*
a*b^2 + B*b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
+ (C*a^5 + B*a^4*b + 5*C*a^3*b^2 - 3*B*a^2*b^3 + 2*(C*a^4*b + 3*C*a^2*b^3 -
 2*B*a*b^4)*tan(d*x + c))/(a^6*b^2 + 2*a^4*b^4 + a^2*b^6 + (a^4*b^4 + 2*a^2
*b^6 + b^8)*tan(d*x + c)^2 + 2*(a^5*b^3 + 2*a^3*b^5 + a*b^7)*tan(d*x + c)))
/d
```

Fricas [B] time = 1.1764, size = 1038, normalized size = 5.49

$$\frac{Ca^5-3Ba^4b-5Ca^3b^2+3Ba^2b^3-2\left(Ba^5+3Ca^4b-3Ba^3b^2-Ca^2b^3\right)dx+\left(Ca^5+Ba^4b+7Ca^3b^2-5Ba^2b^3-2\left(Ba^3b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x, al
gorithm="fricas")
```

```
[Out] 1/2*(C*a^5 - 3*B*a^4*b - 5*C*a^3*b^2 + 3*B*a^2*b^3 - 2*(B*a^5 + 3*C*a^4*b -
 3*B*a^3*b^2 - C*a^2*b^3)*d*x + (C*a^5 + B*a^4*b + 7*C*a^3*b^2 - 5*B*a^2*b^
3 - 2*(B*a^3*b^2 + 3*C*a^2*b^3 - 3*B*a*b^4 - C*b^5)*d*x)*tan(d*x + c)^2 + (
C*a^5 - 3*B*a^4*b - 3*C*a^3*b^2 + B*a^2*b^3 + (C*a^3*b^2 - 3*B*a^2*b^3 - 3*
C*a*b^4 + B*b^5)*tan(d*x + c)^2 + 2*(C*a^4*b - 3*B*a^3*b^2 - 3*C*a^2*b^3 +
B*a*b^4)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/
(tan(d*x + c)^2 + 1)) + 2*(B*a^5 + 3*C*a^4*b - 3*B*a^3*b^2 - 3*C*a^2*b^3 +
2*B*a*b^4 - 2*(B*a^4*b + 3*C*a^3*b^2 - 3*B*a^2*b^3 - C*a*b^4)*d*x)*tan(d*x
+ c))/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d*tan(d*x + c)^2 + 2*(a^7*b
+ 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*tan(d*x + c) + (a^8 + 3*a^6*b^2 + 3*a^4*
b^4 + a^2*b^6)*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.62453, size = 554, normalized size = 2.93

$$-\frac{\frac{2\left(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3\right)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{\left(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3\right)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{2\left(Ca^3b-3\,Ba^2b^2-3\,Cab^3+Bb^4\right)\log(|b\tan(dx+c)+a|)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7}+\frac{3\,Ca^3\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x, al
gorithm="giac")
```

```
[Out] -1/2*(2*(B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6) + (C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*log(tan(d*x +
c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 2*(C*a^3*b - 3*B*a^2*b^2 -
3*C*a*b^3 + B*b^4)*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*
b^5 + b^7) + (3*C*a^3*b^4*tan(d*x + c)^2 - 9*B*a^2*b^5*tan(d*x + c)^2 - 9*C
*a*b^6*tan(d*x + c)^2 + 3*B*b^7*tan(d*x + c)^2 + 2*C*a^6*b*tan(d*x + c) + 1
4*C*a^4*b^3*tan(d*x + c) - 22*B*a^3*b^4*tan(d*x + c) - 12*C*a^2*b^5*tan(d*x
 + c) + 2*B*a*b^6*tan(d*x + c) + C*a^7 + B*a^6*b + 9*C*a^5*b^2 - 11*B*a^4*b
^3 - 4*C*a^3*b^4)/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*(b*tan(d*x + c)
+ a)^2))/d
```

3.41 $\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=179

$$\frac{a(bB-aC)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}+\frac{a^2B+2abC-b^2B}{d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}-\frac{\left(3a^2bC+a^3B-3ab^2B-b^3C\right)\log(a\cos(c+dx)+}{d\left(a^2+b^2\right)^3}$$

```
[Out] ((3*a^2*b*B - b^3*B - a^3*C + 3*a*b^2*C)*x)/(a^2 + b^2)^3 - ((a^3*B - 3*a*b
^2*B + 3*a^2*b*C - b^3*C)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2
)^3*d) + (a*(b*B - a*C))/(2*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) + (a^2*
B - b^2*B + 2*a*b*C)/((a^2 + b^2)^2*d*(a + b*Tan[c + d*x]))
```

Rubi [A] time = 0.254986, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3628, 3529, 3531, 3530}

$$\frac{a(bB-aC)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}+\frac{a^2B+2abC-b^2B}{d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}-\frac{\left(3a^2bC+a^3B-3ab^2B-b^3C\right)\log(a\cos(c+dx)+}{d\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Tan[c + d*x] + C*Tan[c + d*x]^2)/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((3*a^2*b*B - b^3*B - a^3*C + 3*a*b^2*C)*x)/(a^2 + b^2)^3 - ((a^3*B - 3*a*b
^2*B + 3*a^2*b*C - b^3*C)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2
)^3*d) + (a*(b*B - a*C))/(2*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) + (a^2*
B - b^2*B + 2*a*b*C)/((a^2 + b^2)^2*d*(a + b*Tan[c + d*x]))
```

Rule 3628

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2
- a*b*B + a^2*C)*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 + b^2)), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{a(bB-aC)}{2b\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} + \frac{\int \frac{bB-aC+(aB+bC)\tan(c+dx)}{(a+b\tan(c+dx))^2}\,dx}{a^2+b^2}$$

$$= \frac{a(bB-aC)}{2b\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} + \frac{a^2B-b^2B+2abC}{\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))} + \int \frac{2abB}{\cdots}$$

$$= \frac{\left(3a^2bB-b^3B-a^3C+3ab^2C\right)x}{\left(a^2+b^2\right)^3} + \frac{a(bB-aC)}{2b\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} + \frac{\cdots}{\left(a^2+\cdots\right.}$$

$$= \frac{\left(3a^2bB-b^3B-a^3C+3ab^2C\right)x}{\left(a^2+b^2\right)^3} - \frac{\left(a^3B-3ab^2B+3a^2bC-b^3C\right)\log(a\cos(c\cdots}{\left(a^2+b^2\right)^3 d}$$

Mathematica [C] time = 3.7464, size = 188, normalized size = 1.05

$$\frac{\frac{a(bB-aC)}{b\left(a^2+b^2\right)(a+b\tan(c+dx))^2} + \frac{2\left(a^2B+2abC-b^2B\right)}{\left(a^2+b^2\right)^2(a+b\tan(c+dx))} - \frac{2\left(3a^2bC+a^3B-3ab^2B-b^3C\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^3} + \frac{(B+iC)\log(-\tan(c+dx)+i)}{(a+ib)^3} + \frac{(B-iC)\log(\cdots}{(a\cdots}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Tan[c + d*x] + C*Tan[c + d*x]^2)/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (((B + I*C)*Log[I - Tan[c + d*x]])/(a + I*b)^3 + ((B - I*C)*Log[I + Tan[c +
 d*x]])/(a - I*b)^3 - (2*(a^3*B - 3*a*b^2*B + 3*a^2*b*C - b^3*C)*Log[a + b*
Tan[c + d*x]])/(a^2 + b^2)^3 + (a*(b*B - a*C))/(b*(a^2 + b^2)*(a + b*Tan[c
+ d*x])^2) + (2*(a^2*B - b^2*B + 2*a*b*C))/((a^2 + b^2)^2*(a + b*Tan[c + d*
x])))/(2*d)
```

Maple [B] time = 0.047, size = 488, normalized size = 2.7

$$\frac{\ln\left(1+(\tan(dx+c))^2\right)Ba^3}{2d\left(a^2+b^2\right)^3} - \frac{3\ln\left(1+(\tan(dx+c))^2\right)Bab^2}{2d\left(a^2+b^2\right)^3} + \frac{3\ln\left(1+(\tan(dx+c))^2\right)Ca^2b}{2d\left(a^2+b^2\right)^3} - \frac{\ln\left(1+(\tan(dx+\cdots\right.}{2d\left(a^2+b^2\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x)
```

```
[Out] 1/2/d/(a^2+b^2)^3*ln(1+tan(d*x+c)^2)*B*a^3-3/2/d/(a^2+b^2)^3*ln(1+tan(d*x+c
)^2)*B*a*b^2+3/2/d/(a^2+b^2)^3*ln(1+tan(d*x+c)^2)*C*a^2*b-1/2/d/(a^2+b^2)^3
*ln(1+tan(d*x+c)^2)*C*b^3+3/d/(a^2+b^2)^3*B*arctan(tan(d*x+c))*a^2*b-1/d/(a
^2+b^2)^3*B*arctan(tan(d*x+c))*b^3-1/d/(a^2+b^2)^3*C*arctan(tan(d*x+c))*a^3
+3/d/(a^2+b^2)^3*C*arctan(tan(d*x+c))*a*b^2+1/2/d*a/(a^2+b^2)/(a+b*tan(d*x+
c))^2*B-1/2/d*a^2/(a^2+b^2)/b/(a+b*tan(d*x+c))^2*C+1/d*a^2/(a^2+b^2)^2/(a+b
*tan(d*x+c))*B-1/d/(a^2+b^2)^2/(a+b*tan(d*x+c))*b^2*B+2/d/(a^2+b^2)^2/(a+b*
```

```
tan(d*x+c))*C*a*b-1/d*a^3/(a^2+b^2)^3*ln(a+b*tan(d*x+c))*B+3/d*a/(a^2+b^2)^
3*b^2*ln(a+b*tan(d*x+c))*B-3/d*a^2/(a^2+b^2)^3*b*ln(a+b*tan(d*x+c))*C+1/d/(
a^2+b^2)^3*ln(a+b*tan(d*x+c))*C*b^3
```

Maxima [A] time = 1.87509, size = 446, normalized size = 2.49

$$-\frac{\frac{2\left(Ca^3-3Ba^2b-3Cab^2+Bb^3\right)(dx+c)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{2\left(Ba^3+3Ca^2b-3Bab^2-Cb^3\right)\log(b\tan(dx+c)+a)}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{\left(Ba^3+3Ca^2b-3Bab^2-Cb^3\right)\log\left(\tan(dx+c)^2+1\right)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{Ca^4}{a^6b+2a^4b^3+}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] -1/2*(2*(C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6) + 2*(B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*log(b*tan(d*
x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (B*a^3 + 3*C*a^2*b - 3*B*
a*b^2 - C*b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
+ (C*a^4 - 3*B*a^3*b - 3*C*a^2*b^2 + B*a*b^3 - 2*(B*a^2*b^2 + 2*C*a*b^3 - B
*b^4)*tan(d*x + c))/(a^6*b + 2*a^4*b^3 + a^2*b^5 + (a^4*b^3 + 2*a^2*b^5 + b
^7)*tan(d*x + c)^2 + 2*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*tan(d*x + c)))/d
```

Fricas [B] time = 1.15363, size = 1058, normalized size = 5.91

$$-\frac{3Ca^4b-5Ba^3b^2-3Ca^2b^3+Bab^4+2\left(Ca^5-3Ba^4b-3Ca^3b^2+Ba^2b^3\right)dx-\left(Ca^4b-3Ba^3b^2-5Ca^2b^3+3Bab^4-2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] -1/2*(3*C*a^4*b - 5*B*a^3*b^2 - 3*C*a^2*b^3 + B*a*b^4 + 2*(C*a^5 - 3*B*a^4*
b - 3*C*a^3*b^2 + B*a^2*b^3)*d*x - (C*a^4*b - 3*B*a^3*b^2 - 5*C*a^2*b^3 + 3
*B*a*b^4 - 2*(C*a^3*b^2 - 3*B*a^2*b^3 - 3*C*a*b^4 + B*b^5)*d*x)*tan(d*x + c
)^2 + (B*a^5 + 3*C*a^4*b - 3*B*a^3*b^2 - C*a^2*b^3 + (B*a^3*b^2 + 3*C*a^2*b
^3 - 3*B*a*b^4 - C*b^5)*tan(d*x + c)^2 + 2*(B*a^4*b + 3*C*a^3*b^2 - 3*B*a^2
*b^3 - C*a*b^4)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c)
+ a^2)/(tan(d*x + c)^2 + 1)) - 2*(C*a^5 - 2*B*a^4*b - 3*C*a^3*b^2 + 3*B*a^2
*b^3 + 2*C*a*b^4 - B*b^5 - 2*(C*a^4*b - 3*B*a^3*b^2 - 3*C*a^2*b^3 + B*a*b^4
)*d*x)*tan(d*x + c))/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d*tan(d*x + c
)^2 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*tan(d*x + c) + (a^8 + 3*a
^6*b^2 + 3*a^4*b^4 + a^2*b^6)*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.53292, size = 554, normalized size = 3.09

$$-\frac{\frac{2\left(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3\right)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{\left(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3\right)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\left(Ba^3b+3\,Ca^2b^2-3\,Bab^3-Cb^4\right)\log(|b\tan(dx+c)+a|)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7}-\frac{3\,Ba^3b}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/2*(2*(C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6) - (B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*log(tan(d*x +
c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(B*a^3*b + 3*C*a^2*b^2 -
3*B*a*b^3 - C*b^4)*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*
b^5 + b^7) - (3*B*a^3*b^3*tan(d*x + c)^2 + 9*C*a^2*b^4*tan(d*x + c)^2 - 9*B
*a*b^5*tan(d*x + c)^2 - 3*C*b^6*tan(d*x + c)^2 + 8*B*a^4*b^2*tan(d*x + c) +
 22*C*a^3*b^3*tan(d*x + c) - 18*B*a^2*b^4*tan(d*x + c) - 2*C*a*b^5*tan(d*x
+ c) - 2*B*b^6*tan(d*x + c) - C*a^6 + 6*B*a^5*b + 11*C*a^4*b^2 - 7*B*a^3*b^
3 - B*a*b^5)/((a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*(b*tan(d*x + c) + a)^2)
)/d
```

3.42 $\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=175

$$-\frac{bB-aC}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{a^2(-C)+2abB+b^2C}{d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}+\frac{\left(3a^2bB+a^3(-C)+3ab^2C-b^3B\right)\log(a\cos(c+d\ldots}{d\left(a^2+b^2\right)^3}$$

```
[Out] ((a^3*B - 3*a*b^2*B + 3*a^2*b*C - b^3*C)*x)/(a^2 + b^2)^3 + ((3*a^2*b*B - b
^3*B - a^3*C + 3*a*b^2*C)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2
)^3*d) - (b*B - a*C)/(2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) - (2*a*b*B -
a^2*C + b^2*C)/((a^2 + b^2)^2*d*(a + b*Tan[c + d*x]))
```

Rubi [A] time = 0.315501, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3632, 3529, 3531, 3530}

$$-\frac{bB-aC}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{a^2(-C)+2abB+b^2C}{d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}+\frac{\left(3a^2bB+a^3(-C)+3ab^2C-b^3B\right)\log(a\cos(c+d\ldots}{d\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x])
^3,x]
```

```
[Out] ((a^3*B - 3*a*b^2*B + 3*a^2*b*C - b^3*C)*x)/(a^2 + b^2)^3 + ((3*a^2*b*B - b
^3*B - a^3*C + 3*a*b^2*C)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2
)^3*d) - (b*B - a*C)/(2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) - (2*a*b*B -
a^2*C + b^2*C)/((a^2 + b^2)^2*d*(a + b*Tan[c + d*x]))
```

Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx=\int \frac{B+C\tan(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

$$=-\frac{bB-aC}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2}+\frac{\int \frac{aB+bC-(bB-aC)\tan(c+dx)}{(a+b\tan(c+dx))^2}\,d}{a^2+b^2}$$

$$=-\frac{bB-aC}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2}-\frac{2abB-a^2C+b^2C}{\left(a^2+b^2\right)^2d(a+b\tan(c+}$$

$$=\frac{\left(a^3B-3ab^2B+3a^2bC-b^3C\right)x}{\left(a^2+b^2\right)^3}-\frac{bB-aC}{2\left(a^2+b^2\right)d(a+b\tan(c+}$$

$$=\frac{\left(a^3B-3ab^2B+3a^2bC-b^3C\right)x}{\left(a^2+b^2\right)^3}+\frac{\left(3a^2bB-b^3B-a^3C+3ab^2\right.}{(}$$

Mathematica [C] time = 3.80715, size = 243, normalized size = 1.39

$$-\frac{(bB-aC)\left(\frac{b\left(\frac{\left(a^2+b^2\right)\left(5a^2+4ab\tan(c+dx)+b^2\right)}{(a+b\tan(c+dx))^2}+\left(2b^2-6a^2\right)\log(a+b\tan(c+dx))\right)}{\left(a^2+b^2\right)^3}+\frac{i\log(-\tan(c+dx)+i)}{(a+ib)^3}-\frac{\log(\tan(c+dx)+i)}{(b+ia)^3}\right)+C\left(\frac{2b\left(\frac{a^2+b^2}{a+b\tan(c+dx)}\right.}{(}\right.}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c +
 d*x])^3,x]
```

```
[Out] -(C*((I*Log[I - Tan[c + d*x]])/(a + I*b)^2 - (I*Log[I + Tan[c + d*x]])/(a -
 I*b)^2 + (2*b*(-2*a*Log[a + b*Tan[c + d*x]] + (a^2 + b^2)/(a + b*Tan[c + d
*x])))/(a^2 + b^2)^2) + (b*B - a*C)*((I*Log[I - Tan[c + d*x]])/(a + I*b)^3
- Log[I + Tan[c + d*x]]/(I*a + b)^3 + (b*((-6*a^2 + 2*b^2)*Log[a + b*Tan[c
+ d*x]] + ((a^2 + b^2)*(5*a^2 + b^2 + 4*a*b*Tan[c + d*x]))/(a + b*Tan[c + d
*x])^2))/(a^2 + b^2)^3))/(2*b*d)
```

Maple [B] time = 0.154, size = 483, normalized size = 2.8

$$-\frac{3\ln\left(1+(\tan(dx+c))^2\right)Ba^2b}{2d\left(a^2+b^2\right)^3}+\frac{\ln\left(1+(\tan(dx+c))^2\right)Bb^3}{2d\left(a^2+b^2\right)^3}+\frac{\ln\left(1+(\tan(dx+c))^2\right)Ca^3}{2d\left(a^2+b^2\right)^3}-\frac{3\ln\left(1+(\tan(dx+\right.}{2d\left(a^2+b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x)
```

```
[Out] -3/2/d/(a^2+b^2)^3*ln(1+tan(d*x+c)^2)*B*a^2*b+1/2/d/(a^2+b^2)^3*ln(1+tan(d*
x+c)^2)*B*b^3+1/2/d/(a^2+b^2)^3*ln(1+tan(d*x+c)^2)*C*a^3-3/2/d/(a^2+b^2)^3*
ln(1+tan(d*x+c)^2)*C*a*b^2+1/d/(a^2+b^2)^3*B*arctan(tan(d*x+c))*a^3-3/d/(a^
2+b^2)^3*B*arctan(tan(d*x+c))*a*b^2+3/d/(a^2+b^2)^3*C*arctan(tan(d*x+c))*a^
2*b-1/d/(a^2+b^2)^3*C*arctan(tan(d*x+c))*b^3-1/2/d/(a^2+b^2)/(a+b*tan(d*x+c
))^2*B*b+1/2/d/(a^2+b^2)/(a+b*tan(d*x+c))^2*C*a-2/d*a/(a^2+b^2)^2*b/(a+b*ta
n(d*x+c))*B+1/d*a^2/(a^2+b^2)^2/(a+b*tan(d*x+c))*C-1/d/(a^2+b^2)^2/(a+b*tan
(d*x+c))*b^2*C+3/d*b*a^2/(a^2+b^2)^3*ln(a+b*tan(d*x+c))*B-1/d/(a^2+b^2)^3*l
n(a+b*tan(d*x+c))*B*b^3-1/d*a^3/(a^2+b^2)^3*ln(a+b*tan(d*x+c))*C+3/d/(a^2+b
^2)^3*ln(a+b*tan(d*x+c))*C*a*b^2
```

Maxima [A] time = 1.62061, size = 433, normalized size = 2.47

$$\frac{\frac{2\left(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3\right)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{2\left(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3\right)\log(b\tan(dx+c)+a)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{\left(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3\right)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{3\,Ca^3 \cdots}{a^6+2\,a^4b^2+a^2b^4 \cdots}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x, al
gorithm="maxima")
```

```
[Out] 1/2*(2*(B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6) - 2*(C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*log(b*tan(d*x
 + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (C*a^3 - 3*B*a^2*b - 3*C*a
*b^2 + B*b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) +
 (3*C*a^3 - 5*B*a^2*b - C*a*b^2 - B*b^3 + 2*(C*a^2*b - 2*B*a*b^2 - C*b^3)*t
an(d*x + c))/(a^6 + 2*a^4*b^2 + a^2*b^4 + (a^4*b^2 + 2*a^2*b^4 + b^6)*tan(d
*x + c)^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*tan(d*x + c)))/d
```

Fricas [B] time = 1.19085, size = 1038, normalized size = 5.93

$$\frac{5\,Ca^3b^2-7\,Ba^2b^3-Cab^4-Bb^5+2\left(Ba^5+3\,Ca^4b-3\,Ba^3b^2-Ca^2b^3\right)dx-\left(3\,Ca^3b^2-5\,Ba^2b^3-3\,Cab^4+Bb^5-2\left(Ba^3b \cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x, al
gorithm="fricas")
```

```
[Out] 1/2*(5*C*a^3*b^2 - 7*B*a^2*b^3 - C*a*b^4 - B*b^5 + 2*(B*a^5 + 3*C*a^4*b - 3
*B*a^3*b^2 - C*a^2*b^3)*d*x - (3*C*a^3*b^2 - 5*B*a^2*b^3 - 3*C*a*b^4 + B*b^
5 - 2*(B*a^3*b^2 + 3*C*a^2*b^3 - 3*B*a*b^4 - C*b^5)*d*x)*tan(d*x + c)^2 - (
C*a^5 - 3*B*a^4*b - 3*C*a^3*b^2 + B*a^2*b^3 + (C*a^3*b^2 - 3*B*a^2*b^3 - 3*
C*a*b^4 + B*b^5)*tan(d*x + c)^2 + 2*(C*a^4*b - 3*B*a^3*b^2 - 3*C*a^2*b^3 +
B*a*b^4)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/
(tan(d*x + c)^2 + 1)) - 2*(2*C*a^4*b - 3*B*a^3*b^2 - 3*C*a^2*b^3 + 3*B*a*b^
4 + C*b^5 - 2*(B*a^4*b + 3*C*a^3*b^2 - 3*B*a^2*b^3 - C*a*b^4)*d*x)*tan(d*x
+ c))/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d*tan(d*x + c)^2 + 2*(a^7*b
+ 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*tan(d*x + c) + (a^8 + 3*a^6*b^2 + 3*a^4*
b^4 + a^2*b^6)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.50252, size = 552, normalized size = 3.15

$$\frac{\frac{2\left(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3\right)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{\left(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3\right)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{2\left(Ca^3b-3\,Ba^2b^2-3\,Cab^3+Bb^4\right)\log(|b\tan(dx+c)+a|)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7}+\frac{3\,Ca^3b^2}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x, al
gorithm="giac")
```

```
[Out] 1/2*(2*(B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6) + (C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*log(tan(d*x + c
)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 2*(C*a^3*b - 3*B*a^2*b^2 - 3
*C*a*b^3 + B*b^4)*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b
^5 + b^7) + (3*C*a^3*b^2*tan(d*x + c)^2 - 9*B*a^2*b^3*tan(d*x + c)^2 - 9*C*
a*b^4*tan(d*x + c)^2 + 3*B*b^5*tan(d*x + c)^2 + 8*C*a^4*b*tan(d*x + c) - 22
*B*a^3*b^2*tan(d*x + c) - 18*C*a^2*b^3*tan(d*x + c) + 2*B*a*b^4*tan(d*x + c
) - 2*C*b^5*tan(d*x + c) + 6*C*a^5 - 14*B*a^4*b - 7*C*a^3*b^2 - 3*B*a^2*b^3
 - C*a*b^4 - B*b^5)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(b*tan(d*x + c) +
a)^2))/d
```

3.43 $\int \frac{\cot^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=215

$$\frac{b(bB-aC)}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}+\frac{b\left(3a^2bB-2a^3C+b^3B\right)}{a^2d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}-\frac{b\left(3a^2b^3B+a^3b^2C+6a^4bB-3a^5C+b^5B\right)\log(}{a^3d\left(a^2+b^2\right)^3}$$

```
[Out] -(((3*a^2*b*B - b^3*B - a^3*C + 3*a*b^2*C)*x)/(a^2 + b^2)^3) + (B*Log[Sin[c
 + d*x]])/(a^3*d) - (b*(6*a^4*b*B + 3*a^2*b^3*B + b^5*B - 3*a^5*C + a^3*b^2
*C)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/(a^3*(a^2 + b^2)^3*d) + (b*(b*B -
 a*C))/(2*a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) + (b*(3*a^2*b*B + b^3*B -
 2*a^3*C))/(a^2*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x]))
```

Rubi [A] time = 0.679883, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {3632, 3609, 3649, 3651, 3530, 3475}

$$\frac{b(bB-aC)}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}+\frac{b\left(3a^2bB-2a^3C+b^3B\right)}{a^2d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}-\frac{b\left(3a^2b^3B+a^3b^2C+6a^4bB-3a^5C+b^5B\right)\log(}{a^3d\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x
])^3,x]
```

```
[Out] -(((3*a^2*b*B - b^3*B - a^3*C + 3*a*b^2*C)*x)/(a^2 + b^2)^3) + (B*Log[Sin[c
 + d*x]])/(a^3*d) - (b*(6*a^4*b*B + 3*a^2*b^3*B + b^5*B - 3*a^5*C + a^3*b^2
*C)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/(a^3*(a^2 + b^2)^3*d) + (b*(b*B -
 a*C))/(2*a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) + (b*(3*a^2*b*B + b^3*B -
 2*a^3*C))/(a^2*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x]))
```

Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1)
)/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B
*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)
) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n +
 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &
& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
 b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3651

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[((a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*x)
/((a^2 + b^2)*(c^2 + d^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx=\int \frac{\cot(c+dx)(B+C\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$

$$=\frac{b(bB-aC)}{2a\left(a^2+b^2\right)d(a+b\tan(c+dx))^2}+\frac{\int \frac{\cot(c+dx)\left(2\left(a^2+b^2\right)B-2a(bB-}{(a+}}{2}$$

$$=\frac{b(bB-aC)}{2a\left(a^2+b^2\right)d(a+b\tan(c+dx))^2}+\frac{b\left(3a^2bB+b^3B-2a\right.}{a^2\left(a^2+b^2\right)^2d(a+b\tan(}$$

$$=-\frac{\left(3a^2bB-b^3B-a^3C+3ab^2C\right)x}{\left(a^2+b^2\right)^3}+\frac{b(bB-aC)}{2a\left(a^2+b^2\right)d(a+b\tan(}$$

$$=-\frac{\left(3a^2bB-b^3B-a^3C+3ab^2C\right)x}{\left(a^2+b^2\right)^3}+\frac{B\log(\sin(c+dx))}{a^3d}-\frac{b\left(6\right.}{}$$

Mathematica [C] time = 2.87756, size = 223, normalized size = 1.04

$$\frac{\frac{b(bB-aC)}{a(a^2+b^2)(a+b\tan(c+dx))^2}+\frac{2b(3a^2bB-2a^3C+b^3B)}{a^2(a^2+b^2)^2(a+b\tan(c+dx))}-\frac{2b(3a^2b^3B+a^3b^2C+6a^4bB-3a^5C+b^5B)\log(a+b\tan(c+dx))}{a^3(a^2+b^2)^3}+\frac{2B\log(\tan(c+dx))}{a^3}-\frac{(B+iC)\log \cdots}{(\cdots}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c
 + d*x])^3,x]
```

```
[Out] (-(((B + I*C)*Log[I - Tan[c + d*x]])/(a + I*b)^3) + (2*B*Log[Tan[c + d*x]])
/a^3 - ((B - I*C)*Log[I + Tan[c + d*x]])/(a - I*b)^3 - (2*b*(6*a^4*b*B + 3*
a^2*b^3*B + b^5*B - 3*a^5*C + a^3*b^2*C)*Log[a + b*Tan[c + d*x]])/(a^3*(a^2
 + b^2)^3) + (b*(b*B - a*C))/(a*(a^2 + b^2)*(a + b*Tan[c + d*x])^2) + (2*b*
(3*a^2*b*B + b^3*B - 2*a^3*C))/(a^2*(a^2 + b^2)^2*(a + b*Tan[c + d*x])))/(2
*d)
```

Maple [B] time = 0.179, size = 540, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x)
```

```
[Out] -1/2/d/(a^2+b^2)^3*ln(1+tan(d*x+c)^2)*B*a^3+3/2/d/(a^2+b^2)^3*ln(1+tan(d*x+
c)^2)*B*a*b^2-3/2/d/(a^2+b^2)^3*ln(1+tan(d*x+c)^2)*C*a^2*b+1/2/d/(a^2+b^2)^
3*ln(1+tan(d*x+c)^2)*C*b^3-3/d/(a^2+b^2)^3*B*arctan(tan(d*x+c))*a^2*b+1/d/(
a^2+b^2)^3*B*arctan(tan(d*x+c))*b^3+1/d/(a^2+b^2)^3*C*arctan(tan(d*x+c))*a^
3-3/d/(a^2+b^2)^3*C*arctan(tan(d*x+c))*a*b^2+1/d/a^3*B*ln(tan(d*x+c))+1/2/d
*b^2/a/(a^2+b^2)/(a+b*tan(d*x+c))^2*B-1/2/d*b/(a^2+b^2)/(a+b*tan(d*x+c))^2*
C+3/d/(a^2+b^2)^2/(a+b*tan(d*x+c))*b^2*B+1/d*b^4/(a^2+b^2)^2/a^2/(a+b*tan(d
*x+c))*B-2/d/(a^2+b^2)^2/(a+b*tan(d*x+c))*C*a*b-6/d*a/(a^2+b^2)^3*b^2*ln(a+
b*tan(d*x+c))*B-3/d*b^4/(a^2+b^2)^3/a*ln(a+b*tan(d*x+c))*B-1/d*b^6/(a^2+b^2
)^3/a^3*ln(a+b*tan(d*x+c))*B+3/d*a^2/(a^2+b^2)^3*b*ln(a+b*tan(d*x+c))*C-1/d
/(a^2+b^2)^3*ln(a+b*tan(d*x+c))*C*b^3
```

Maxima [A] time = 1.83939, size = 502, normalized size = 2.33

$$\frac{\frac{2(Ca^3-3Ba^2b-3Cab^2+Bb^3)(dx+c)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{2(3Ca^5b-6Ba^4b^2-Ca^3b^3-3Ba^2b^4-Bb^6)\log(b\tan(dx+c)+a)}{a^9+3a^7b^2+3a^5b^4+a^3b^6}-\frac{(Ba^3+3Ca^2b-3Bab^2-Cb^3)\log(\tan(dx+c)^2+1)}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{\cdots}{a^8+\cdots}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,
algorithm="maxima")
```

```
[Out] 1/2*(2*(C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6) + 2*(3*C*a^5*b - 6*B*a^4*b^2 - C*a^3*b^3 - 3*B*a^2*b^4 -
B*b^6)*log(b*tan(d*x + c) + a)/(a^9 + 3*a^7*b^2 + 3*a^5*b^4 + a^3*b^6) - (B
*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*
b^2 + 3*a^2*b^4 + b^6) - (5*C*a^4*b - 7*B*a^3*b^2 + C*a^2*b^3 - 3*B*a*b^4 +
```

```
2*(2*C*a^3*b^2 - 3*B*a^2*b^3 - B*b^5)*tan(d*x + c))/(a^8 + 2*a^6*b^2 + a^4
*b^4 + (a^6*b^2 + 2*a^4*b^4 + a^2*b^6)*tan(d*x + c)^2 + 2*(a^7*b + 2*a^5*b^
3 + a^3*b^5)*tan(d*x + c)) + 2*B*log(tan(d*x + c))/a^3)/d
```

Fricas [B] time = 1.61426, size = 1451, normalized size = 6.75

$$-\frac{7\,Ca^5b^3 - 9\,Ba^4b^4 + Ca^3b^5 - 3\,Ba^2b^6 - 2\left(Ca^8 - 3\,Ba^7b - 3\,Ca^6b^2 + Ba^5b^3\right)dx - \left(5\,Ca^5b^3 - 7\,Ba^4b^4 - Ca^3b^5 - Ba^2b^6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,
algorithm="fricas")
```

```
[Out] -1/2*(7*C*a^5*b^3 - 9*B*a^4*b^4 + C*a^3*b^5 - 3*B*a^2*b^6 - 2*(C*a^8 - 3*B*
a^7*b - 3*C*a^6*b^2 + B*a^5*b^3)*d*x - (5*C*a^5*b^3 - 7*B*a^4*b^4 - C*a^3*b
^5 - B*a^2*b^6 + 2*(C*a^6*b^2 - 3*B*a^5*b^3 - 3*C*a^4*b^4 + B*a^3*b^5)*d*x)
*tan(d*x + c)^2 - (B*a^8 + 3*B*a^6*b^2 + 3*B*a^4*b^4 + B*a^2*b^6 + (B*a^6*b
^2 + 3*B*a^4*b^4 + 3*B*a^2*b^6 + B*b^8)*tan(d*x + c)^2 + 2*(B*a^7*b + 3*B*a
^5*b^3 + 3*B*a^3*b^5 + B*a*b^7)*tan(d*x + c))*log(tan(d*x + c)^2/(tan(d*x +
 c)^2 + 1)) - (3*C*a^7*b - 6*B*a^6*b^2 - C*a^5*b^3 - 3*B*a^4*b^4 - B*a^2*b^
6 + (3*C*a^5*b^3 - 6*B*a^4*b^4 - C*a^3*b^5 - 3*B*a^2*b^6 - B*b^8)*tan(d*x +
 c)^2 + 2*(3*C*a^6*b^2 - 6*B*a^5*b^3 - C*a^4*b^4 - 3*B*a^3*b^5 - B*a*b^7)*t
an(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x +
 c)^2 + 1)) - 2*(3*C*a^6*b^2 - 4*B*a^5*b^3 - 3*C*a^4*b^4 + 3*B*a^3*b^5 + B*
a*b^7 + 2*(C*a^7*b - 3*B*a^6*b^2 - 3*C*a^5*b^3 + B*a^4*b^4)*d*x)*tan(d*x +
c))/((a^9*b^2 + 3*a^7*b^4 + 3*a^5*b^6 + a^3*b^8)*d*tan(d*x + c)^2 + 2*(a^10
*b + 3*a^8*b^3 + 3*a^6*b^5 + a^4*b^7)*d*tan(d*x + c) + (a^11 + 3*a^9*b^2 +
3*a^7*b^4 + a^5*b^6)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**3,
x)
```

```
[Out] Timed out
```

Giac [B] time = 1.58297, size = 647, normalized size = 3.01

$$\frac{\frac{2\left(Ca^3 - 3\,Ba^2b - 3\,Cab^2 + Bb^3\right)(dx+c)}{a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6} - \frac{\left(Ba^3 + 3\,Ca^2b - 3\,Bab^2 - Cb^3\right)\log\left(\tan(dx+c)^2 + 1\right)}{a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6} + \frac{2\left(3\,Ca^5b^2 - 6\,Ba^4b^3 - Ca^3b^4 - 3\,Ba^2b^5 - Bb^7\right)\log(|b\tan(dx+c) + a|)}{a^9b + 3\,a^7b^3 + 3\,a^5b^5 + a^3b^7}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,
algorithm="giac")
```

```
[Out] 1/2*(2*(C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6) - (B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*log(tan(d*x + c
)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(3*C*a^5*b^2 - 6*B*a^4*b^3
 - C*a^3*b^4 - 3*B*a^2*b^5 - B*b^7)*log(abs(b*tan(d*x + c) + a))/(a^9*b + 3
*a^7*b^3 + 3*a^5*b^5 + a^3*b^7) + 2*B*log(abs(tan(d*x + c)))/a^3 - (9*C*a^5
*b^3*tan(d*x + c)^2 - 18*B*a^4*b^4*tan(d*x + c)^2 - 3*C*a^3*b^5*tan(d*x + c
)^2 - 9*B*a^2*b^6*tan(d*x + c)^2 - 3*B*b^8*tan(d*x + c)^2 + 22*C*a^6*b^2*ta
n(d*x + c) - 42*B*a^5*b^3*tan(d*x + c) - 2*C*a^4*b^4*tan(d*x + c) - 26*B*a^
3*b^5*tan(d*x + c) - 8*B*a*b^7*tan(d*x + c) + 14*C*a^7*b - 25*B*a^6*b^2 + 3
*C*a^5*b^3 - 19*B*a^4*b^4 + C*a^3*b^5 - 6*B*a^2*b^6)/((a^9 + 3*a^7*b^2 + 3*
a^5*b^4 + a^3*b^6)*(b*tan(d*x + c) + a)^2))/d
```

3.44 $\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=287

$$-\frac{b\left(6a^2b^2B-3a^3bC+a^4B-ab^3C+3b^4B\right)}{a^3d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}-\frac{b\left(2a^2B-abC+3b^2B\right)}{2a^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}+\frac{b^2\left(9a^2b^3B-3a^3b^2C+10a^4bB-\right.}{}$$

```
[Out] -(((a^3*B - 3*a*b^2*B + 3*a^2*b*C - b^3*C)*x)/(a^2 + b^2)^3) - ((3*b*B - a*
C)*Log[Sin[c + d*x]])/(a^4*d) + (b^2*(10*a^4*b*B + 9*a^2*b^3*B + 3*b^5*B -
6*a^5*C - 3*a^3*b^2*C - a*b^4*C)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/(a^4
*(a^2 + b^2)^3*d) - (b*(2*a^2*B + 3*b^2*B - a*b*C))/(2*a^2*(a^2 + b^2)*d*(a
 + b*Tan[c + d*x])^2) - (B*Cot[c + d*x])/(a*d*(a + b*Tan[c + d*x])^2) - (b*
(a^4*B + 6*a^2*b^2*B + 3*b^4*B - 3*a^3*b*C - a*b^3*C))/(a^3*(a^2 + b^2)^2*d
*(a + b*Tan[c + d*x]))
```

Rubi [A] time = 0.9412, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {3632, 3609, 3649, 3651, 3530, 3475}

$$-\frac{b\left(6a^2b^2B-3a^3bC+a^4B-ab^3C+3b^4B\right)}{a^3d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}-\frac{b\left(2a^2B-abC+3b^2B\right)}{2a^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}+\frac{b^2\left(9a^2b^3B-3a^3b^2C+10a^4bB-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x
])^3,x]
```

```
[Out] -(((a^3*B - 3*a*b^2*B + 3*a^2*b*C - b^3*C)*x)/(a^2 + b^2)^3) - ((3*b*B - a*
C)*Log[Sin[c + d*x]])/(a^4*d) + (b^2*(10*a^4*b*B + 9*a^2*b^3*B + 3*b^5*B -
6*a^5*C - 3*a^3*b^2*C - a*b^4*C)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/(a^4
*(a^2 + b^2)^3*d) - (b*(2*a^2*B + 3*b^2*B - a*b*C))/(2*a^2*(a^2 + b^2)*d*(a
 + b*Tan[c + d*x])^2) - (B*Cot[c + d*x])/(a*d*(a + b*Tan[c + d*x])^2) - (b*
(a^4*B + 6*a^2*b^2*B + 3*b^4*B - 3*a^3*b*C - a*b^3*C))/(a^3*(a^2 + b^2)^2*d
*(a + b*Tan[c + d*x]))
```

Rule 3632

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1)
)/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B
*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)
) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n +
 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
```

```
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &
& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
 b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3651

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[((a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*x)
/((a^2 + b^2)*(c^2 + d^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\cot^2(c+dx)(B+C\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$

$$= -\frac{B\cot(c+dx)}{ad(a+b\tan(c+dx))^2} - \frac{\int \frac{\cot(c+dx)\left(3bB-aC+aB\tan(c+dx)+3bB\tan\right.}{(a+b\tan(c+dx))^3}}{a}$$

$$= -\frac{b\left(2a^2B+3b^2B-abC\right)}{2a^2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{B\cot(c+dx)}{ad(a+b\tan(c+dx))^2}$$

$$= -\frac{b\left(2a^2B+3b^2B-abC\right)}{2a^2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{B\cot(c+dx)}{ad(a+b\tan(c+dx))^2}$$

$$= -\frac{\left(a^3B-3ab^2B+3a^2bC-b^3C\right)x}{\left(a^2+b^2\right)^3} - \frac{b\left(2a^2B+3b^2B-al\right.}{2a^2\left(a^2+b^2\right)d(a+b\tan(\right.}$$

$$= -\frac{\left(a^3B-3ab^2B+3a^2bC-b^3C\right)x}{\left(a^2+b^2\right)^3} - \frac{(3bB-aC)\log(\sin(c+dx}{a^4d}$$

Mathematica [C] time = 6.40237, size = 288, normalized size = 1.

$$-\frac{b^2\left(4a^2bB-3a^3C-ab^2C+2b^3B\right)}{a^3d\left(a^2+b^2\right)^2(a+b\tan(c+dx))} - \frac{b^2(bB-aC)}{2a^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2} + \frac{b^2\left(9a^2b^3B-3a^3b^2C+10a^4bB-6a^5C-\right.}{a^4d\left(a^2+\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c
 + d*x])^3,x]
```

```
[Out] -((B*Cot[c + d*x])/(a^3*d)) + ((B + I*C)*Log[I - Tan[c + d*x]])/(2*(I*a - b
)^3*d) - ((3*b*B - a*C)*Log[Tan[c + d*x]])/(a^4*d) - ((I*B + C)*Log[I + Tan
[c + d*x]])/(2*(a - I*b)^3*d) + (b^2*(10*a^4*b*B + 9*a^2*b^3*B + 3*b^5*B -
6*a^5*C - 3*a^3*b^2*C - a*b^4*C)*Log[a + b*Tan[c + d*x]])/(a^4*(a^2 + b^2)^
3*d) - (b^2*(b*B - a*C))/(2*a^2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) - (b^
2*(4*a^2*b*B + 2*b^3*B - 3*a^3*C - a*b^2*C))/(a^3*(a^2 + b^2)^2*d*(a + b*Ta
n[c + d*x]))
```

Maple [B] time = 0.174, size = 651, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x)
```

```
[Out] 3/2/d/(a^2+b^2)^3*ln(1+tan(d*x+c)^2)*B*a^2*b-1/2/d/(a^2+b^2)^3*ln(1+tan(d*x
+c)^2)*B*b^3-1/2/d/(a^2+b^2)^3*ln(1+tan(d*x+c)^2)*C*a^3+3/2/d/(a^2+b^2)^3*l
n(1+tan(d*x+c)^2)*C*a*b^2-1/d/(a^2+b^2)^3*B*arctan(tan(d*x+c))*a^3+3/d/(a^2
+b^2)^3*B*arctan(tan(d*x+c))*a*b^2-3/d/(a^2+b^2)^3*C*arctan(tan(d*x+c))*a^2
*b+1/d/(a^2+b^2)^3*C*arctan(tan(d*x+c))*b^3-1/d/a^3/tan(d*x+c)*B-3/d/a^4*ln
(tan(d*x+c))*B*b+1/d/a^3*ln(tan(d*x+c))*C-1/2/d*b^3/(a^2+b^2)/a^2/(a+b*tan(
d*x+c))^2*B+1/2/d*b^2/(a^2+b^2)/a/(a+b*tan(d*x+c))^2*C-4/d*b^3/(a^2+b^2)^2/
```

```
a/(a+b*tan(d*x+c))*B-2/d*b^5/(a^2+b^2)^2/a^3/(a+b*tan(d*x+c))*B+3/d/(a^2+b^
2)^2/(a+b*tan(d*x+c))*b^2*C+1/d*b^4/(a^2+b^2)^2/a^2/(a+b*tan(d*x+c))*C+10/d
/(a^2+b^2)^3*ln(a+b*tan(d*x+c))*B*b^3+9/d*b^5/(a^2+b^2)^3/a^2*ln(a+b*tan(d*
x+c))*B+3/d*b^7/(a^2+b^2)^3/a^4*ln(a+b*tan(d*x+c))*B-6/d/(a^2+b^2)^3*ln(a+b
*tan(d*x+c))*C*a*b^2-3/d*b^4/(a^2+b^2)^3/a*ln(a+b*tan(d*x+c))*C-1/d*b^6/(a^
2+b^2)^3/a^3*ln(a+b*tan(d*x+c))*C
```

Maxima [A] time = 1.80384, size = 613, normalized size = 2.14

$$-\frac{\frac{2\left(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3\right)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\left(6\,Ca^5b^2-10\,Ba^4b^3+3\,Ca^3b^4-9\,Ba^2b^5+Cab^6-3\,Bb^7\right)\log(b\tan(dx+c)+a)}{a^{10}+3\,a^8b^2+3\,a^6b^4+a^4b^6}+\frac{\left(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3\right)\log\left(\tan(dx+\cdots\right.}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,
algorithm="maxima")
```

```
[Out] -1/2*(2*(B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6) + 2*(6*C*a^5*b^2 - 10*B*a^4*b^3 + 3*C*a^3*b^4 - 9*B*a^2*
b^5 + C*a*b^6 - 3*B*b^7)*log(b*tan(d*x + c) + a)/(a^10 + 3*a^8*b^2 + 3*a^6*
b^4 + a^4*b^6) + (C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*log(tan(d*x + c)^2
 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (2*B*a^6 + 4*B*a^4*b^2 + 2*B*a^
2*b^4 + 2*(B*a^4*b^2 - 3*C*a^3*b^3 + 6*B*a^2*b^4 - C*a*b^5 + 3*B*b^6)*tan(d
*x + c)^2 + (4*B*a^5*b - 7*C*a^4*b^2 + 17*B*a^3*b^3 - 3*C*a^2*b^4 + 9*B*a*b
^5)*tan(d*x + c))/((a^7*b^2 + 2*a^5*b^4 + a^3*b^6)*tan(d*x + c)^3 + 2*(a^8*
b + 2*a^6*b^3 + a^4*b^5)*tan(d*x + c)^2 + (a^9 + 2*a^7*b^2 + a^5*b^4)*tan(d
*x + c)) - 2*(C*a - 3*B*b)*log(tan(d*x + c))/a^4)/d
```

Fricas [B] time = 1.82269, size = 1982, normalized size = 6.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,
algorithm="fricas")
```

```
[Out] -1/2*(2*B*a^9 + 6*B*a^7*b^2 + 6*B*a^5*b^4 + 2*B*a^3*b^6 + (7*C*a^5*b^4 - 9*
B*a^4*b^5 + C*a^3*b^6 - 3*B*a^2*b^7 + 2*(B*a^7*b^2 + 3*C*a^6*b^3 - 3*B*a^5*
b^4 - C*a^4*b^5)*d*x)*tan(d*x + c)^3 + 2*(B*a^7*b^2 + 4*C*a^6*b^3 - 2*B*a^5
*b^4 - 3*C*a^4*b^5 + 6*B*a^3*b^6 - C*a^2*b^7 + 3*B*a*b^8 + 2*(B*a^8*b + 3*C
*a^7*b^2 - 3*B*a^6*b^3 - C*a^5*b^4)*d*x)*tan(d*x + c)^2 - ((C*a^7*b^2 - 3*B
*a^6*b^3 + 3*C*a^5*b^4 - 9*B*a^4*b^5 + 3*C*a^3*b^6 - 9*B*a^2*b^7 + C*a*b^8
- 3*B*b^9)*tan(d*x + c)^3 + 2*(C*a^8*b - 3*B*a^7*b^2 + 3*C*a^6*b^3 - 9*B*a^
5*b^4 + 3*C*a^4*b^5 - 9*B*a^3*b^6 + C*a^2*b^7 - 3*B*a*b^8)*tan(d*x + c)^2 +
 (C*a^9 - 3*B*a^8*b + 3*C*a^7*b^2 - 9*B*a^6*b^3 + 3*C*a^5*b^4 - 9*B*a^4*b^5
 + C*a^3*b^6 - 3*B*a^2*b^7)*tan(d*x + c))*log(tan(d*x + c)^2/(tan(d*x + c)^
2 + 1)) + ((6*C*a^5*b^4 - 10*B*a^4*b^5 + 3*C*a^3*b^6 - 9*B*a^2*b^7 + C*a*b^
8 - 3*B*b^9)*tan(d*x + c)^3 + 2*(6*C*a^6*b^3 - 10*B*a^5*b^4 + 3*C*a^4*b^5 -
 9*B*a^3*b^6 + C*a^2*b^7 - 3*B*a*b^8)*tan(d*x + c)^2 + (6*C*a^7*b^2 - 10*B*
a^6*b^3 + 3*C*a^5*b^4 - 9*B*a^4*b^5 + C*a^3*b^6 - 3*B*a^2*b^7)*tan(d*x + c)
)*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1))
 + (4*B*a^8*b + 12*B*a^6*b^3 - 9*C*a^5*b^4 + 23*B*a^4*b^5 - 3*C*a^3*b^6 + 9
*B*a^2*b^7 + 2*(B*a^9 + 3*C*a^8*b - 3*B*a^7*b^2 - C*a^6*b^3)*d*x)*tan(d*x +
```

```
c))/((a^10*b^2 + 3*a^8*b^4 + 3*a^6*b^6 + a^4*b^8)*d*tan(d*x + c)^3 + 2*(a^
11*b + 3*a^9*b^3 + 3*a^7*b^5 + a^5*b^7)*d*tan(d*x + c)^2 + (a^12 + 3*a^10*b
^2 + 3*a^8*b^4 + a^6*b^6)*d*tan(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**3,
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.6371, size = 756, normalized size = 2.63

$$-\frac{\frac{2\left(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3\right)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{\left(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3\right)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\left(6\,Ca^5b^3-10\,Ba^4b^4+3\,Ca^3b^5-9\,Ba^2b^6+Cab^7-3\,Bb^8\right)\log(|b\,\mathrm{ta}}{a^{10}b+3\,a^8b^3+3\,a^6b^5+a^4b^7}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,
algorithm="giac")
```

```
[Out] -1/2*(2*(B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6) + (C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*log(tan(d*x +
c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(6*C*a^5*b^3 - 10*B*a^4*b
^4 + 3*C*a^3*b^5 - 9*B*a^2*b^6 + C*a*b^7 - 3*B*b^8)*log(abs(b*tan(d*x + c)
+ a))/(a^10*b + 3*a^8*b^3 + 3*a^6*b^5 + a^4*b^7) - (18*C*a^5*b^4*tan(d*x +
c)^2 - 30*B*a^4*b^5*tan(d*x + c)^2 + 9*C*a^3*b^6*tan(d*x + c)^2 - 27*B*a^2*
b^7*tan(d*x + c)^2 + 3*C*a*b^8*tan(d*x + c)^2 - 9*B*b^9*tan(d*x + c)^2 + 42
*C*a^6*b^3*tan(d*x + c) - 68*B*a^5*b^4*tan(d*x + c) + 26*C*a^4*b^5*tan(d*x
+ c) - 66*B*a^3*b^6*tan(d*x + c) + 8*C*a^2*b^7*tan(d*x + c) - 22*B*a*b^8*ta
n(d*x + c) + 25*C*a^7*b^2 - 39*B*a^6*b^3 + 19*C*a^5*b^4 - 41*B*a^4*b^5 + 6*
C*a^3*b^6 - 14*B*a^2*b^7)/((a^10 + 3*a^8*b^2 + 3*a^6*b^4 + a^4*b^6)*(b*tan(
d*x + c) + a)^2) - 2*(C*a - 3*B*b)*log(abs(tan(d*x + c)))/a^4 + 2*(C*a*tan(
d*x + c) - 3*B*b*tan(d*x + c) + B*a)/(a^4*tan(d*x + c)))/d
```

3.45 $\int \tan^2(c+dx)(b\tan(c+dx))^n \left(A + B\tan(c+dx) + C\tan^2(c\right.$

Optimal. Leaf size=132

$$\frac{(A-C)(b\tan(c+dx))^{n+3}\text{Hypergeometric2F1}\left(1,\frac{n+3}{2},\frac{n+5}{2},-\tan^2(c+dx)\right)}{b^3d(n+3)}+\frac{B(b\tan(c+dx))^{n+4}\text{Hypergeometric2F}}{b^4d(n+4}$$

```
[Out] (C*(b*Tan[c + d*x])^(3 + n))/(b^3*d*(3 + n)) + ((A - C)*Hypergeometric2F1[1
, (3 + n)/2, (5 + n)/2, -Tan[c + d*x]^2]*(b*Tan[c + d*x])^(3 + n))/(b^3*d*(
3 + n)) + (B*Hypergeometric2F1[1, (4 + n)/2, (6 + n)/2, -Tan[c + d*x]^2]*(b
*Tan[c + d*x])^(4 + n))/(b^4*d*(4 + n))
```

Rubi [A] time = 0.155361, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {16, 3630, 3538, 3476, 364}

$$\frac{(A-C)(b\tan(c+dx))^{n+3}\,{}_2F_1\left(1,\frac{n+3}{2};\frac{n+5}{2};-\tan^2(c+dx)\right)}{b^3d(n+3)}+\frac{B(b\tan(c+dx))^{n+4}\,{}_2F_1\left(1,\frac{n+4}{2};\frac{n+6}{2};-\tan^2(c+dx)\right)}{b^4d(n+4)}+\frac{C(b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2*(b*Tan[c + d*x])^n*(A + B*Tan[c + d*x] + C*Tan[c + d*x]^
2),x]
```

```
[Out] (C*(b*Tan[c + d*x])^(3 + n))/(b^3*d*(3 + n)) + ((A - C)*Hypergeometric2F1[1
, (3 + n)/2, (5 + n)/2, -Tan[c + d*x]^2]*(b*Tan[c + d*x])^(3 + n))/(b^3*d*(
3 + n)) + (B*Hypergeometric2F1[1, (4 + n)/2, (6 + n)/2, -Tan[c + d*x]^2]*(b
*Tan[c + d*x])^(4 + n))/(b^4*d*(4 + n))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3538

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \tan^2(c+dx)(b\tan(c+dx))^n \left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)dx = \frac{\int (b\tan(c+dx))^{2+n}\left(A+B\tan(c+dx)\right.}{b^2}$$

$$= \frac{C(b\tan(c+dx))^{3+n}}{b^3d(3+n)} + \frac{\int (b\tan(c+dx))^2}{}$$

$$= \frac{C(b\tan(c+dx))^{3+n}}{b^3d(3+n)} + \frac{B\int (b\tan(c+dx)}{b^3}$$

$$= \frac{C(b\tan(c+dx))^{3+n}}{b^3d(3+n)} + \frac{B\,\mathrm{Subst}\left(\int \frac{x^{3+n}}{b^2+x^2}\right.}{b}$$

$$= \frac{C(b\tan(c+dx))^{3+n}}{b^3d(3+n)} + \frac{(A-C)\,{}_2F_1\left(1,\frac{3+}{2}\right.}{}$$

Mathematica [A] time = 0.401928, size = 110, normalized size = 0.83

$$\frac{\tan^3(c+dx)(b\tan(c+dx))^n\left((n+4)(A-C)\text{Hypergeometric2F1}\left(1,\frac{n+3}{2},\frac{n+5}{2},-\tan^2(c+dx)\right)+B(n+3)\tan(c+\right.}{d(n+3)(n+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2*(b*Tan[c + d*x])^n*(A + B*Tan[c + d*x] + C*Tan[c +
 d*x]^2),x]
```

```
[Out] (Tan[c + d*x]^3*(b*Tan[c + d*x])^n*(C*(4 + n) + (A - C)*(4 + n)*Hypergeomet
ric2F1[1, (3 + n)/2, (5 + n)/2, -Tan[c + d*x]^2] + B*(3 + n)*Hypergeometric
2F1[1, (4 + n)/2, (6 + n)/2, -Tan[c + d*x]^2]*Tan[c + d*x]))/(d*(3 + n)*(4
+ n))
```

Maple [F] time = 0.326, size = 0, normalized size = 0.

$$\int (\tan(dx+c))^2 (b\tan(dx+c))^n \left(A+B\tan(dx+c)+C(\tan(dx+c))^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2*(b*tan(d*x+c))^n*(A+B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

```
[Out] int(tan(d*x+c)^2*(b*tan(d*x+c))^n*(A+B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\tan(dx+c)^2+B\tan(dx+c)+A\right)(b\tan(dx+c))^n\tan(dx+c)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(b*tan(d*x+c))^n*(A+B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] integrate((C*tan(d*x + c)^2 + B*tan(d*x + c) + A)*(b*tan(d*x + c))^n*tan(d*
x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\tan\left(dx+c\right)^4+B\tan\left(dx+c\right)^3+A\tan\left(dx+c\right)^2\right)\left(b\tan\left(dx+c\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(b*tan(d*x+c))^n*(A+B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] integral((C*tan(d*x + c)^4 + B*tan(d*x + c)^3 + A*tan(d*x + c)^2)*(b*tan(d*
x + c))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\tan\left(c+dx\right)\right)^n\left(A+B\tan\left(c+dx\right)+C\tan^2\left(c+dx\right)\right)\tan^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2*(b*tan(d*x+c))**n*(A+B*tan(d*x+c)+C*tan(d*x+c)**2),
x)
```

```
[Out] Integral((b*tan(c + d*x))**n*(A + B*tan(c + d*x) + C*tan(c + d*x)**2)*tan(c
 + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\tan\left(dx+c\right)^2+B\tan\left(dx+c\right)+A\right)\left(b\tan\left(dx+c\right)\right)^n\tan\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(b*tan(d*x+c))^n*(A+B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] integrate((C*tan(d*x + c)^2 + B*tan(d*x + c) + A)*(b*tan(d*x + c))^n*tan(d*
x + c)^2, x)
```

3.46 $\int \tan^m(c+dx)(b\tan(c+dx))^n \left(A + B\tan(c+dx) + C\tan^2\right.$

Optimal. Leaf size=154

$$\frac{(A-C)\tan^{m+1}(c+dx)(b\tan(c+dx))^n\text{Hypergeometric2F1}\left(1,\frac{1}{2}(m+n+1),\frac{1}{2}(m+n+3),-\tan^2(c+dx)\right)}{d(m+n+1)}+\frac{B\,\text{ta}}{}$$

```
[Out] (C*Tan[c + d*x]^(1 + m)*(b*Tan[c + d*x])^n)/(d*(1 + m + n)) + ((A - C)*Hype
rgeometric2F1[1, (1 + m + n)/2, (3 + m + n)/2, -Tan[c + d*x]^2]*Tan[c + d*x
]^(1 + m)*(b*Tan[c + d*x])^n)/(d*(1 + m + n)) + (B*Hypergeometric2F1[1, (2
+ m + n)/2, (4 + m + n)/2, -Tan[c + d*x]^2]*Tan[c + d*x]^(2 + m)*(b*Tan[c +
 d*x])^n)/(d*(2 + m + n))
```

Rubi [A] time = 0.137775, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {20, 3630, 3538, 3476, 364}

$$\frac{(A-C)\tan^{m+1}(c+dx)(b\tan(c+dx))^n\,{}_2F_1\left(1,\frac{1}{2}(m+n+1);\frac{1}{2}(m+n+3);-\tan^2(c+dx)\right)}{d(m+n+1)}+\frac{B\tan^{m+2}(c+dx)(b\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^m*(b*Tan[c + d*x])^n*(A + B*Tan[c + d*x] + C*Tan[c + d*x]^
2),x]
```

```
[Out] (C*Tan[c + d*x]^(1 + m)*(b*Tan[c + d*x])^n)/(d*(1 + m + n)) + ((A - C)*Hype
rgeometric2F1[1, (1 + m + n)/2, (3 + m + n)/2, -Tan[c + d*x]^2]*Tan[c + d*x
]^(1 + m)*(b*Tan[c + d*x])^n)/(d*(1 + m + n)) + (B*Hypergeometric2F1[1, (2
+ m + n)/2, (4 + m + n)/2, -Tan[c + d*x]^2]*Tan[c + d*x]^(2 + m)*(b*Tan[c +
 d*x])^n)/(d*(2 + m + n))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && !LeQ[m, -1]
```

Rule 3538

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] && !IntegerQ[2*m]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \tan^m(c+dx)(b\tan(c+dx))^n \left(A+B\tan(c+dx)+C\tan^2(c+dx)\right) dx = (\tan^{-n}(c+dx)(b\tan(c+dx))^n) \int \tan^{m+n}(c$$
$$= \frac{C\tan^{1+m}(c+dx)(b\tan(c+dx))^n}{d(1+m+n)} + (\tan^{-n}$$
$$= \frac{C\tan^{1+m}(c+dx)(b\tan(c+dx))^n}{d(1+m+n)} + (B\tan$$
$$= \frac{C\tan^{1+m}(c+dx)(b\tan(c+dx))^n}{d(1+m+n)} + \frac{(B\tan}{}$$
$$= \frac{C\tan^{1+m}(c+dx)(b\tan(c+dx))^n}{d(1+m+n)} + \frac{(A-C}{}$$

Mathematica [A] time = 0.376412, size = 115, normalized size = 0.75

$$\frac{\tan^{m+1}(c+dx)(b\tan(c+dx))^n \left(\frac{(A-C)\text{Hypergeometric2F1}\left(1,\frac{1}{2}(m+n+1),\frac{1}{2}(m+n+3),-\tan^2(c+dx)\right)}{m+n+1} + \frac{B\tan(c+dx)\text{Hypergeometric2F1}\left(1,\frac{1}{2}(m+\right.}{m+n+2}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^m*(b*Tan[c + d*x])^n*(A + B*Tan[c + d*x] + C*Tan[c +
 d*x]^2),x]
```

```
[Out] (Tan[c + d*x]^(1 + m)*(b*Tan[c + d*x])^n*(C/(1 + m + n) + ((A - C)*Hypergeo
metric2F1[1, (1 + m + n)/2, (3 + m + n)/2, -Tan[c + d*x]^2])/(1 + m + n) +
(B*Hypergeometric2F1[1, (2 + m + n)/2, (4 + m + n)/2, -Tan[c + d*x]^2]*Tan[
c + d*x])/(2 + m + n)))/d
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int (\tan(dx+c))^m (b\tan(dx+c))^n \left(A+B\tan(dx+c)+C(\tan(dx+c))^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^m*(b*tan(d*x+c))^n*(A+B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

```
[Out] int(tan(d*x+c)^m*(b*tan(d*x+c))^n*(A+B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \tan \left(dx+c\right)^2 + B \tan \left(dx+c\right) + A\right) \left(b \tan \left(dx+c\right)\right)^n \tan \left(dx+c\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(b*tan(d*x+c))^n*(A+B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] integrate((C*tan(d*x + c)^2 + B*tan(d*x + c) + A)*(b*tan(d*x + c))^n*tan(d*
x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \tan \left(dx+c\right)^2 + B \tan \left(dx+c\right) + A\right) \left(b \tan \left(dx+c\right)\right)^n \tan \left(dx+c\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(b*tan(d*x+c))^n*(A+B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] integral((C*tan(d*x + c)^2 + B*tan(d*x + c) + A)*(b*tan(d*x + c))^n*tan(d*x
 + c)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \tan \left(c+dx\right)\right)^n \left(A + B \tan \left(c+dx\right) + C \tan^2 \left(c+dx\right)\right) \tan^m \left(c+dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**m*(b*tan(d*x+c))**n*(A+B*tan(d*x+c)+C*tan(d*x+c)**2),
x)
```

```
[Out] Integral((b*tan(c + d*x))**n*(A + B*tan(c + d*x) + C*tan(c + d*x)**2)*tan(c
 + d*x)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \tan \left(dx+c\right)^2 + B \tan \left(dx+c\right) + A\right) \left(b \tan \left(dx+c\right)\right)^n \tan \left(dx+c\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(b*tan(d*x+c))^n*(A+B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] integrate((C*tan(d*x + c)^2 + B*tan(d*x + c) + A)*(b*tan(d*x + c))^n*tan(d*
x + c)^m, x)
```

3.47 $\int \tan^m(c+dx)\sqrt{b\tan(c+dx)}\left(A+B\tan(c+dx)+C\tan^2(c\right.$

Optimal. Leaf size=170

$$\frac{2(A-C)\sqrt{b\tan(c+dx)}\tan^{m+1}(c+dx)\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2m+3),\frac{1}{4}(2m+7),-\tan^2(c+dx)\right)}{d(2m+3)}+\frac{2B\sqrt{b\tan(c}}{}$$

```
[Out] (2*C*Tan[c + d*x]^(1 + m)*Sqrt[b*Tan[c + d*x]])/(d*(3 + 2*m)) + (2*(A - C)*
Hypergeometric2F1[1, (3 + 2*m)/4, (7 + 2*m)/4, -Tan[c + d*x]^2]*Tan[c + d*x
]^(1 + m)*Sqrt[b*Tan[c + d*x]])/(d*(3 + 2*m)) + (2*B*Hypergeometric2F1[1, (
5 + 2*m)/4, (9 + 2*m)/4, -Tan[c + d*x]^2]*Tan[c + d*x]^(2 + m)*Sqrt[b*Tan[c
 + d*x]])/(d*(5 + 2*m))
```

Rubi [A] time = 0.143555, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {20, 3630, 3538, 3476, 364}

$$\frac{2(A-C)\sqrt{b\tan(c+dx)}\tan^{m+1}(c+dx)\,{}_2F_1\left(1,\frac{1}{4}(2m+3);\frac{1}{4}(2m+7);-\tan^2(c+dx)\right)}{d(2m+3)}+\frac{2B\sqrt{b\tan(c+dx)}\tan^{m+2}(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^m*Sqrt[b*Tan[c + d*x]]*(A + B*Tan[c + d*x] + C*Tan[c + d*x
]^2),x]
```

```
[Out] (2*C*Tan[c + d*x]^(1 + m)*Sqrt[b*Tan[c + d*x]])/(d*(3 + 2*m)) + (2*(A - C)*
Hypergeometric2F1[1, (3 + 2*m)/4, (7 + 2*m)/4, -Tan[c + d*x]^2]*Tan[c + d*x
]^(1 + m)*Sqrt[b*Tan[c + d*x]])/(d*(3 + 2*m)) + (2*B*Hypergeometric2F1[1, (
5 + 2*m)/4, (9 + 2*m)/4, -Tan[c + d*x]^2]*Tan[c + d*x]^(2 + m)*Sqrt[b*Tan[c
 + d*x]])/(d*(5 + 2*m))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && !LeQ[m, -1]
```

Rule 3538

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] && !IntegerQ[2*m]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \tan^m(c+dx)\sqrt{b\tan(c+dx)}\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)dx = \frac{\sqrt{b\tan(c+dx)}\int \tan^{\frac{1}{2}+m}(c+dx)\left(A+B\right.}{\sqrt{\tan(c+}}$$

$$= \frac{2C\tan^{1+m}(c+dx)\sqrt{b\tan(c+dx)}}{d(3+2m)} + \frac{\sqrt{b\,t}}{}$$

$$= \frac{2C\tan^{1+m}(c+dx)\sqrt{b\tan(c+dx)}}{d(3+2m)} + \frac{\left(B\sqrt{\right.}}{}$$

$$= \frac{2C\tan^{1+m}(c+dx)\sqrt{b\tan(c+dx)}}{d(3+2m)} + \frac{\left(B\sqrt{\right.}}{}$$

$$= \frac{2C\tan^{1+m}(c+dx)\sqrt{b\tan(c+dx)}}{d(3+2m)} + \frac{2(A}{}$$

Mathematica [A] time = 0.512908, size = 133, normalized size = 0.78

$$\frac{2\sqrt{b\tan(c+dx)}\tan^{m+1}(c+dx)\left((2m+5)(A-C)\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2m+3),\frac{1}{4}(2m+7),-\tan^2(c+dx)\right)+\right.}{d(2m+3)(2m+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^m*Sqrt[b*Tan[c + d*x]]*(A + B*Tan[c + d*x] + C*Tan[c
 + d*x]^2),x]
```

```
[Out] (2*Tan[c + d*x]^(1 + m)*Sqrt[b*Tan[c + d*x]]*(C*(5 + 2*m) + (A - C)*(5 + 2*
m)*Hypergeometric2F1[1, (3 + 2*m)/4, (7 + 2*m)/4, -Tan[c + d*x]^2] + B*(3 +
 2*m)*Hypergeometric2F1[1, (5 + 2*m)/4, (9 + 2*m)/4, -Tan[c + d*x]^2]*Tan[c
 + d*x]))/(d*(3 + 2*m)*(5 + 2*m))
```

Maple [F] time = 0.435, size = 0, normalized size = 0.

$$\int (\tan(dx+c))^m\sqrt{b\tan(dx+c)}\left(A+B\tan(dx+c)+C(\tan(dx+c))^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^m*(b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

```
[Out] int(tan(d*x+c)^m*(b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)+C*tan(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\tan\left(dx+c\right)^{2}+B\tan\left(dx+c\right)+A\right)\sqrt{b\tan\left(dx+c\right)}\tan\left(dx+c\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)+C*tan(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*tan(d*x + c)^2 + B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c))*tan(d
*x + c)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\tan\left(c+dx\right)}\left(A+B\tan\left(c+dx\right)+C\tan^{2}\left(c+dx\right)\right)\tan^{m}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**m*(b*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)+C*tan(d*x+c)*
*2),x)
```

```
[Out] Integral(sqrt(b*tan(c + d*x))*(A + B*tan(c + d*x) + C*tan(c + d*x)**2)*tan(
c + d*x)**m, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)+C*tan(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.48 $\int \frac{\tan^m(c+dx)\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)}{\sqrt{b\tan(c+dx)}}\,dx$

Optimal. Leaf size=170

$$\frac{2(A-C)\tan^{m+1}(c+dx)\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2m+1),\frac{1}{4}(2m+5),-\tan^2(c+dx)\right)}{d(2m+1)\sqrt{b\tan(c+dx)}}+\frac{2B\tan^{m+2}(c+dx)\text{Hype}}{}$$

```
[Out] (2*C*Tan[c + d*x]^(1 + m))/(d*(1 + 2*m)*Sqrt[b*Tan[c + d*x]]) + (2*(A - C)*
Hypergeometric2F1[1, (1 + 2*m)/4, (5 + 2*m)/4, -Tan[c + d*x]^2]*Tan[c + d*x
]^(1 + m))/(d*(1 + 2*m)*Sqrt[b*Tan[c + d*x]]) + (2*B*Hypergeometric2F1[1, (
3 + 2*m)/4, (7 + 2*m)/4, -Tan[c + d*x]^2]*Tan[c + d*x]^(2 + m))/(d*(3 + 2*m
)*Sqrt[b*Tan[c + d*x]])
```

Rubi [A] time = 0.136535, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {20, 3630, 3538, 3476, 364}

$$\frac{2(A-C)\tan^{m+1}(c+dx)\,{}_2F_1\left(1,\frac{1}{4}(2m+1);\frac{1}{4}(2m+5);-\tan^2(c+dx)\right)}{d(2m+1)\sqrt{b\tan(c+dx)}}+\frac{2B\tan^{m+2}(c+dx)\,{}_2F_1\left(1,\frac{1}{4}(2m+3);\frac{1}{4}(2m\right.}{d(2m+3)\sqrt{b\tan(c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(Tan[c + d*x]^m*(A + B*Tan[c + d*x] + C*Tan[c + d*x]^2))/Sqrt[b*Tan[c +
 d*x]],x]
```

```
[Out] (2*C*Tan[c + d*x]^(1 + m))/(d*(1 + 2*m)*Sqrt[b*Tan[c + d*x]]) + (2*(A - C)*
Hypergeometric2F1[1, (1 + 2*m)/4, (5 + 2*m)/4, -Tan[c + d*x]^2]*Tan[c + d*x
]^(1 + m))/(d*(1 + 2*m)*Sqrt[b*Tan[c + d*x]]) + (2*B*Hypergeometric2F1[1, (
3 + 2*m)/4, (7 + 2*m)/4, -Tan[c + d*x]^2]*Tan[c + d*x]^(2 + m))/(d*(3 + 2*m
)*Sqrt[b*Tan[c + d*x]])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && !LeQ[m, -1]
```

Rule 3538

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] && !IntegerQ[2*m]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\tan^m(c+dx)\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)}{\sqrt{b\tan(c+dx)}}\,dx = \frac{\sqrt{\tan(c+dx)}\int \tan^{-\frac{1}{2}+m}(c+dx)\left(A+B\tan(c+dx)+C\,\mathrm{ta}\right.}{\sqrt{b\tan(c+dx)}}$$

$$= \frac{2C\tan^{1+m}(c+dx)}{d(1+2m)\sqrt{b\tan(c+dx)}} + \frac{\sqrt{\tan(c+dx)}\int \tan^{-\frac{1}{2}+m}(c+d}{\sqrt{b\tan(}}$$

$$= \frac{2C\tan^{1+m}(c+dx)}{d(1+2m)\sqrt{b\tan(c+dx)}} + \frac{\left(B\sqrt{\tan(c+dx)}\right)\int \tan^{\frac{1}{2}+m}(c+}{\sqrt{b\tan(c+dx)}}$$

$$= \frac{2C\tan^{1+m}(c+dx)}{d(1+2m)\sqrt{b\tan(c+dx)}} + \frac{\left(B\sqrt{\tan(c+dx)}\right)\mathrm{Subst}\left(\int \frac{x^{\frac{1}{2}+}}{1+x}\right.}{d\sqrt{b\tan(c+}}$$

$$= \frac{2C\tan^{1+m}(c+dx)}{d(1+2m)\sqrt{b\tan(c+dx)}} + \frac{2(A-C)\,{}_2F_1\left(1,\frac{1}{4}(1+2m);\frac{1}{4}(5\right.}{d(1+2m}$$

Mathematica [A] time = 0.503705, size = 133, normalized size = 0.78

$$\frac{2\tan^{m+1}(c+dx)\left((2m+3)(A-C)\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2m+1),\frac{1}{4}(2m+5),-\tan^2(c+dx)\right)+B(2m+1)\tan(c+\right.}{d(2m+1)(2m+3)\sqrt{b\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]^m*(A + B*Tan[c + d*x] + C*Tan[c + d*x]^2))/Sqrt[b*T
an[c + d*x]],x]
```

```
[Out] (2*Tan[c + d*x]^(1 + m)*(C*(3 + 2*m) + (A - C)*(3 + 2*m)*Hypergeometric2F1[
1, (1 + 2*m)/4, (5 + 2*m)/4, -Tan[c + d*x]^2] + B*(1 + 2*m)*Hypergeometric2
F1[1, (3 + 2*m)/4, (7 + 2*m)/4, -Tan[c + d*x]^2]*Tan[c + d*x]))/(d*(1 + 2*m
)*(3 + 2*m)*Sqrt[b*Tan[c + d*x]])
```

Maple [F] time = 0.411, size = 0, normalized size = 0.

$$\int (\tan(dx+c))^m \left(A+B\tan(dx+c)+C(\tan(dx+c))^2\right)\frac{1}{\sqrt{b\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^m*(A+B*tan(d*x+c)+C*tan(d*x+c)^2)/(b*tan(d*x+c))^(1/2),x)
```

```
[Out] int(tan(d*x+c)^m*(A+B*tan(d*x+c)+C*tan(d*x+c)^2)/(b*tan(d*x+c))^(1/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(A+B*tan(d*x+c)+C*tan(d*x+c)^2)/(b*tan(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\tan\left(dx+c\right)^{2}+B\tan\left(dx+c\right)+A\right)\sqrt{b\tan\left(dx+c\right)}\tan\left(dx+c\right)^{m}}{b\tan\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(A+B*tan(d*x+c)+C*tan(d*x+c)^2)/(b*tan(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*tan(d*x + c)^2 + B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c))*tan(d
*x + c)^m/(b*tan(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(A+B\tan\left(c+dx\right)+C\tan^{2}\left(c+dx\right)\right)\tan^{m}\left(c+dx\right)}{\sqrt{b\tan\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**m*(A+B*tan(d*x+c)+C*tan(d*x+c)**2)/(b*tan(d*x+c))**(1
/2),x)
```

```
[Out] Integral((A + B*tan(c + d*x) + C*tan(c + d*x)**2)*tan(c + d*x)**m/sqrt(b*ta
n(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\tan\left(dx+c\right)^{2}+B\tan\left(dx+c\right)+A\right)\tan\left(dx+c\right)^{m}}{\sqrt{b\tan\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(A+B*tan(d*x+c)+C*tan(d*x+c)^2)/(b*tan(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*tan(d*x + c)^2 + B*tan(d*x + c) + A)*tan(d*x + c)^m/sqrt(b*tan
(d*x + c)), x)
```

3.49 $\int \frac{\tan^m(c+dx)\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal. Leaf size=328

$$\frac{2C\tan^m(c+dx)\sqrt{a+b\tan(c+dx)}\left(-\frac{b\tan(c+dx)}{a}\right)^{-m}\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},\frac{b\tan(c+dx)}{a}+1\right)}{bd}-\frac{\left(\sqrt{-b^2}(A-C)+\right.}{}$$

```
[Out] -(((b*B + Sqrt[-b^2]*(A - C))*AppellF1[1/2, 1, -m, 3/2, (a + b*Tan[c + d*x]
)/(a - Sqrt[-b^2]), 1 + (b*Tan[c + d*x])/a]*Tan[c + d*x]^m*Sqrt[a + b*Tan[c
 + d*x]])/(b*(a - Sqrt[-b^2])*d*(-((b*Tan[c + d*x])/a))^m)) - ((b*B - Sqrt[
-b^2]*(A - C))*AppellF1[1/2, 1, -m, 3/2, (a + b*Tan[c + d*x])/(a + Sqrt[-b^
2]), 1 + (b*Tan[c + d*x])/a]*Tan[c + d*x]^m*Sqrt[a + b*Tan[c + d*x]])/(b*(a
 + Sqrt[-b^2])*d*(-((b*Tan[c + d*x])/a))^m) + (2*C*Hypergeometric2F1[1/2, -
m, 3/2, 1 + (b*Tan[c + d*x])/a]*Tan[c + d*x]^m*Sqrt[a + b*Tan[c + d*x]])/(b
*d*(-((b*Tan[c + d*x])/a))^m)
```

Rubi [A] time = 1.56086, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3655, 6720, 1692, 246, 245, 430, 429}

$$-\frac{\left(\sqrt{-b^2}(A-C)+bB\right)\tan^m(c+dx)\sqrt{a+b\tan(c+dx)}\left(-\frac{b\tan(c+dx)}{a}\right)^{-m}F_1\left(\frac{1}{2};1,-m;\frac{3}{2};\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{b\tan(c+dx)}{a}+1\right)}{bd\left(a-\sqrt{-b^2}\right)}-\frac{(bl}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Tan[c + d*x]^m*(A + B*Tan[c + d*x] + C*Tan[c + d*x]^2))/Sqrt[a + b*Tan
[c + d*x]],x]
```

```
[Out] -(((b*B + Sqrt[-b^2]*(A - C))*AppellF1[1/2, 1, -m, 3/2, (a + b*Tan[c + d*x]
)/(a - Sqrt[-b^2]), 1 + (b*Tan[c + d*x])/a]*Tan[c + d*x]^m*Sqrt[a + b*Tan[c
 + d*x]])/(b*(a - Sqrt[-b^2])*d*(-((b*Tan[c + d*x])/a))^m)) - ((b*B - Sqrt[
-b^2]*(A - C))*AppellF1[1/2, 1, -m, 3/2, (a + b*Tan[c + d*x])/(a + Sqrt[-b^
2]), 1 + (b*Tan[c + d*x])/a]*Tan[c + d*x]^m*Sqrt[a + b*Tan[c + d*x]])/(b*(a
 + Sqrt[-b^2])*d*(-((b*Tan[c + d*x])/a))^m) + (2*C*Hypergeometric2F1[1/2, -
m, 3/2, 1 + (b*Tan[c + d*x])/a]*Tan[c + d*x]^m*Sqrt[a + b*Tan[c + d*x]])/(b
*d*(-((b*Tan[c + d*x])/a))^m)
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[((a + b*ff*x)^m*(c + d*ff*x)^n*(A + B*ff*x + C*ff^2*x^2
))/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
```

```
[v, x] && EqQ[m, 1])
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\tan^m(c+dx)\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^m(A+Bx+Cx^2)}{\sqrt{a+bx}(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{\left(\frac{-a+x^2}{b}\right)^m \left(Ab^2+(a-x^2)(-bB+C(a-x^2))\right)}{a^2+b^2-2ax^2+x^4}\,dx, x, \sqrt{a+b\tan\right.}{bd}$$

$$= \frac{(2\tan^m(c+dx)(b\tan(c+dx))^{-m})\,\text{Subst}\left(\int \frac{(-a+x^2)^m(Ab^2+(a-}{a^2+b^2-}\right.}{bd}$$

$$= \frac{(2\tan^m(c+dx)(b\tan(c+dx))^{-m})\,\text{Subst}\left(\int\left(C\left(-a+x^2\right)^m\right.\right.}{bd}$$

$$= \frac{(2\tan^m(c+dx)(b\tan(c+dx))^{-m})\,\text{Subst}\left(\int \frac{(-a+x^2)^m(b(Ab-aB}{a^2+b^2-2ax}\right.}{bd}$$

$$= \frac{(2\tan^m(c+dx)(b\tan(c+dx))^{-m})\,\text{Subst}\left(\int\left(\frac{\left(bB-\sqrt{-b^2}(A-C)\right)}{-2a-2\sqrt{-b^2}}\right.\right.}{b}$$

$$= \frac{2C\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{a+b\tan(c+dx)}{a}\right)\tan^m(c+dx)\left(-\frac{b\tan(c+dx)}{a}\right)^{-m}}{bd}$$

$$= \frac{2C\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{a+b\tan(c+dx)}{a}\right)\tan^m(c+dx)\left(-\frac{b\tan(c+dx)}{a}\right)^{-m}}{bd}$$

$$- \frac{\left(bB+\sqrt{-b^2}(A-C)\right)F_1\left(\frac{1}{2};1,-m;\frac{3}{2};\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+}{a}\right.}{b\left(a-\sqrt{-\right.}}$$

Mathematica [F] time = 27.1726, size = 0, normalized size = 0.

$$\int \frac{\tan^m(c+dx)\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Tan[c + d*x]^m*(A + B*Tan[c + d*x] + C*Tan[c + d*x]^2))/Sqrt[a +
 b*Tan[c + d*x]],x]
```

```
[Out] Integrate[(Tan[c + d*x]^m*(A + B*Tan[c + d*x] + C*Tan[c + d*x]^2))/Sqrt[a +
 b*Tan[c + d*x]], x]
```

Maple [F] time = 0.6, size = 0, normalized size = 0.

$$\int (\tan(dx+c))^m \left(A+B\tan(dx+c)+C(\tan(dx+c))^2\right)\frac{1}{\sqrt{a+b\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^m*(A+B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^(1/2),x)
```

```
[Out] int(tan(d*x+c)^m*(A+B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^(1/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(A+B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\tan\left(dx+c\right)^2+B\tan\left(dx+c\right)+A\right)\tan\left(dx+c\right)^m}{\sqrt{b\tan\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(A+B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*tan(d*x + c)^2 + B*tan(d*x + c) + A)*tan(d*x + c)^m/sqrt(b*tan(
d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(A+B\tan\left(c+dx\right)+C\tan^2\left(c+dx\right)\right)\tan^m\left(c+dx\right)}{\sqrt{a+b\tan\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**m*(A+B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**
(1/2),x)
```

```
[Out] Integral((A + B*tan(c + d*x) + C*tan(c + d*x)**2)*tan(c + d*x)**m/sqrt(a +
b*tan(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\tan\left(dx+c\right)^2+B\tan\left(dx+c\right)+A\right)\tan\left(dx+c\right)^m}{\sqrt{b\tan\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(A+B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*tan(d*x + c)^2 + B*tan(d*x + c) + A)*tan(d*x + c)^m/sqrt(b*tan
(d*x + c) + a), x)
```

3.50 $\int (a+b\tan(e+fx))^3(c+d\tan(e+fx)) \left(A + B\tan(e + fx) + C\right.$

Optimal. Leaf size=353

$$\frac{b\tan(e+fx)\left(a^2(d(A-C)+Bc)+2ab(Ac-Bd-cC)-b^2(d(A-C)+Bc)\right)}{f}-\frac{\log(\cos(e+fx))\left(3a^2b(Ac-Bd-cC)+\right.}{}$$

```
[Out] (a^3*(A*c - c*C - B*d) - 3*a*b^2*(A*c - c*C - B*d) - 3*a^2*b*(B*c + (A - C)
*d) + b^3*(B*c + (A - C)*d))*x - ((3*a^2*b*(A*c - c*C - B*d) - b^3*(A*c - c
*C - B*d) + a^3*(B*c + (A - C)*d) - 3*a*b^2*(B*c + (A - C)*d))*Log[Cos[e +
f*x]])/f + (b*(2*a*b*(A*c - c*C - B*d) + a^2*(B*c + (A - C)*d) - b^2*(B*c +
 (A - C)*d))*Tan[e + f*x])/f + ((A*b*c + a*B*c - b*c*C + a*A*d - b*B*d - a*
C*d)*(a + b*Tan[e + f*x])^2)/(2*f) + ((B*c + (A - C)*d)*(a + b*Tan[e + f*x]
)^3)/(3*f) - ((a*C*d - 5*b*(c*C + B*d))*(a + b*Tan[e + f*x])^4)/(20*b^2*f)
+ (C*d*Tan[e + f*x]*(a + b*Tan[e + f*x])^4)/(5*b*f)
```

Rubi [A] time = 0.785304, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {3637, 3630, 3528, 3525, 3475}

$$\frac{b\tan(e+fx)\left(a^2(d(A-C)+Bc)+2ab(Ac-Bd-cC)-b^2(d(A-C)+Bc)\right)}{f}-\frac{\log(\cos(e+fx))\left(3a^2b(Ac-Bd-cC)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan
[e + f*x]^2),x]
```


```
[Out] (a^3*(A*c - c*C - B*d) - 3*a*b^2*(A*c - c*C - B*d) - 3*a^2*b*(B*c + (A - C)
*d) + b^3*(B*c + (A - C)*d))*x - ((3*a^2*b*(A*c - c*C - B*d) - b^3*(A*c - c
*C - B*d) + a^3*(B*c + (A - C)*d) - 3*a*b^2*(B*c + (A - C)*d))*Log[Cos[e +
f*x]])/f + (b*(2*a*b*(A*c - c*C - B*d) + a^2*(B*c + (A - C)*d) - b^2*(B*c +
 (A - C)*d))*Tan[e + f*x])/f + ((A*b*c + a*B*c - b*c*C + a*A*d - b*B*d - a*
C*d)*(a + b*Tan[e + f*x])^2)/(2*f) + ((B*c + (A - C)*d)*(a + b*Tan[e + f*x]
)^3)/(3*f) - ((a*C*d - 5*b*(c*C + B*d))*(a + b*Tan[e + f*x])^4)/(20*b^2*f)
+ (C*d*Tan[e + f*x]*(a + b*Tan[e + f*x])^4)/(5*b*f)
```


Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx = \frac{Cd\tan(e+fx)(a+b\tan(e+f}{5bf}$$

$$= -\frac{(aCd-5b(cC+Bd))(a+b\,\mathrm{ta}}{20b^2f}$$

$$= \frac{(Bc+(A-C)d)(a+b\tan(e+}{3f}$$

$$= \frac{(Abc+aBc-bcC+aAd-bBa}{2f}$$

$$= \left(a^3(Ac-cC-Bd)-3ab^2(Ac-\right.$$

$$= \left(a^3(Ac-cC-Bd)-3ab^2(Ac-\right.$$

Mathematica [C] time = 6.36469, size = 300, normalized size = 0.85

$$\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^4}{5bf} - \frac{\frac{(aCd-5b(Bd+cC))(a+b\tan(e+fx))^4}{4bf} - \frac{5\left(3(-aAd-aBc+aCd+Abc-bBd-bcC)\left(6ab^2\tan(e+fx)+(-b+ia)^3\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] +
 C*Tan[e + f*x]^2),x]
```

```
[Out] (C*d*Tan[e + f*x]*(a + b*Tan[e + f*x])^4)/(5*b*f) - (((a*C*d - 5*b*(c*C + B
*d))*(a + b*Tan[e + f*x])^4)/(4*b*f) - (5*(3*(A*b*c - a*B*c - b*c*C - a*A*d
 - b*B*d + a*C*d)*((I*a - b)^3*Log[I - Tan[e + f*x]] - (I*a + b)^3*Log[I +
Tan[e + f*x]] + 6*a*b^2*Tan[e + f*x] + b^3*Tan[e + f*x]^2) - (B*c + (A - C)
*d)*((3*I)*(a + I*b)^4*Log[I - Tan[e + f*x]] - (3*I)*(a - I*b)^4*Log[I + Ta
n[e + f*x]] - 6*b^2*(6*a^2 - b^2)*Tan[e + f*x] - 12*a*b^3*Tan[e + f*x]^2 -
2*b^4*Tan[e + f*x]^3)))/(6*f))/(5*b)
```

Maple [B] time = 0.018, size = 994, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x)
```

```
[Out] 1/4/f*B*tan(f*x+e)^4*b^3*d+1/4/f*C*tan(f*x+e)^4*b^3*c+1/3/f*A*tan(f*x+e)^3*
b^3*d-1/f*C*arctan(tan(f*x+e))*b^3*d+1/2/f*ln(1+tan(f*x+e)^2)*A*a^3*d-1/2/f
*ln(1+tan(f*x+e)^2)*A*b^3*c+1/2/f*ln(1+tan(f*x+e)^2)*B*a^3*c+1/2/f*ln(1+tan
(f*x+e)^2)*B*b^3*d+1/2/f*ln(1+tan(f*x+e)^2)*C*b^3*c+1/f*C*a^3*c*tan(f*x+e)+
3/f*C*arctan(tan(f*x+e))*a*b^2*c+3/f*A*a^2*b*d*tan(f*x+e)+3/2/f*ln(1+tan(f*
x+e)^2)*C*a*b^2*d+1/f*C*tan(f*x+e)^3*a*b^2*c+1/f*B*tan(f*x+e)^3*a*b^2*d+1/f
*A*arctan(tan(f*x+e))*a^3*c+1/f*A*arctan(tan(f*x+e))*b^3*d-1/f*B*arctan(tan
(f*x+e))*a^3*d+1/f*B*arctan(tan(f*x+e))*b^3*c-1/f*C*arctan(tan(f*x+e))*a^3*
c+1/f*B*a^3*d*tan(f*x+e)-1/f*B*b^3*c*tan(f*x+e)+1/3/f*B*tan(f*x+e)^3*b^3*c-
1/3/f*C*tan(f*x+e)^3*b^3*d+1/f*C*b^3*d*tan(f*x+e)-1/f*A*b^3*d*tan(f*x+e)+1/
2/f*C*tan(f*x+e)^2*a^3*d-1/2/f*C*tan(f*x+e)^2*b^3*c+1/2/f*A*tan(f*x+e)^2*b^
3*c-1/2/f*B*tan(f*x+e)^2*b^3*d+1/5/f*C*b^3*d*tan(f*x+e)^5-1/2/f*ln(1+tan(f*
x+e)^2)*a^3*C*d-3/f*A*arctan(tan(f*x+e))*a^2*b*d-3/f*C*a^2*b*d*tan(f*x+e)+3
/2/f*B*tan(f*x+e)^2*a*b^2*c+3/2/f*A*tan(f*x+e)^2*a*b^2*d+3/f*A*a*b^2*c*tan(
f*x+e)+3/f*B*a^2*b*c*tan(f*x+e)-3/f*B*a*b^2*d*tan(f*x+e)-3/f*A*arctan(tan(f
*x+e))*a*b^2*c-3/f*B*arctan(tan(f*x+e))*a^2*b*c+3/f*B*arctan(tan(f*x+e))*a*
b^2*d+1/f*C*tan(f*x+e)^3*a^2*b*d-3/2/f*ln(1+tan(f*x+e)^2)*B*a*b^2*c-3/2/f*l
n(1+tan(f*x+e)^2)*C*a^2*b*c+3/4/f*C*tan(f*x+e)^4*a*b^2*d+3/f*C*arctan(tan(f
*x+e))*a^2*b*d-3/f*C*a*b^2*c*tan(f*x+e)+3/2/f*B*tan(f*x+e)^2*a^2*b*d-3/2/f*
C*tan(f*x+e)^2*a*b^2*d+3/2/f*C*tan(f*x+e)^2*a^2*b*c+3/2/f*ln(1+tan(f*x+e)^2
)*A*a^2*b*c-3/2/f*ln(1+tan(f*x+e)^2)*A*a*b^2*d-3/2/f*ln(1+tan(f*x+e)^2)*B*a
^2*b*d
```

Maxima [A] time = 1.50679, size = 562, normalized size = 1.59

$$\frac{12\,Cb^3d\tan\left(fx+e\right)^5+15\left(Cb^3c+\left(3\,Cab^2+Bb^3\right)d\right)\tan\left(fx+e\right)^4+20\left(\left(3\,Cab^2+Bb^3\right)c+\left(3\,Ca^2b+3\,Bab^2+(A-C\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x, algorithm="maxima")
```

```
[Out] 1/60*(12*C*b^3*d*tan(f*x + e)^5 + 15*(C*b^3*c + (3*C*a*b^2 + B*b^3)*d)*tan(
f*x + e)^4 + 20*((3*C*a*b^2 + B*b^3)*c + (3*C*a^2*b + 3*B*a*b^2 + (A - C)*b
^3)*d)*tan(f*x + e)^3 + 30*((3*C*a^2*b + 3*B*a*b^2 + (A - C)*b^3)*c + (C*a^
3 + 3*B*a^2*b + 3*(A - C)*a*b^2 - B*b^3)*d)*tan(f*x + e)^2 + 60*(((A - C)*a
^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*c - (B*a^3 + 3*(A - C)*a^2*b - 3*
B*a*b^2 - (A - C)*b^3)*d)*(f*x + e) + 30*((B*a^3 + 3*(A - C)*a^2*b - 3*B*a*
b^2 - (A - C)*b^3)*c + ((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*
d)*log(tan(f*x + e)^2 + 1) + 60*((C*a^3 + 3*B*a^2*b + 3*(A - C)*a*b^2 - B*b
^3)*c + (B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*d)*tan(f*x + e)
)/f
```

Fricas [A] time = 1.22339, size = 915, normalized size = 2.59

$$12\,Cb^3d\tan\left(fx+e\right)^5+15\left(Cb^3c+\left(3\,Cab^2+Bb^3\right)d\right)\tan\left(fx+e\right)^4+20\left(\left(3\,Cab^2+Bb^3\right)c+\left(3\,Ca^2b+3\,Bab^2+(A-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x, algorithm="fricas")
```

```
[Out] 1/60*(12*C*b^3*d*tan(f*x + e)^5 + 15*(C*b^3*c + (3*C*a*b^2 + B*b^3)*d)*tan(
f*x + e)^4 + 20*((3*C*a*b^2 + B*b^3)*c + (3*C*a^2*b + 3*B*a*b^2 + (A - C)*b
^3)*d)*tan(f*x + e)^3 + 60*(((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*
b^3)*c - (B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*d)*f*x + 30*((
3*C*a^2*b + 3*B*a*b^2 + (A - C)*b^3)*c + (C*a^3 + 3*B*a^2*b + 3*(A - C)*a*b
^2 - B*b^3)*d)*tan(f*x + e)^2 - 30*((B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 -
(A - C)*b^3)*c + ((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*d)*log
(1/(tan(f*x + e)^2 + 1)) + 60*((C*a^3 + 3*B*a^2*b + 3*(A - C)*a*b^2 - B*b^3
)*c + (B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*d)*tan(f*x + e))/
f
```

Sympy [A] time = 5.48655, size = 1001, normalized size = 2.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)
**2),x)
```

```
[Out] Piecewise((A*a**3*c*x + A*a**3*d*log(tan(e + f*x)**2 + 1)/(2*f) + 3*A*a**2*
b*c*log(tan(e + f*x)**2 + 1)/(2*f) - 3*A*a**2*b*d*x + 3*A*a**2*b*d*tan(e +
f*x)/f - 3*A*a*b**2*c*x + 3*A*a*b**2*c*tan(e + f*x)/f - 3*A*a*b**2*d*log(ta
n(e + f*x)**2 + 1)/(2*f) + 3*A*a*b**2*d*tan(e + f*x)**2/(2*f) - A*b**3*c*lo
g(tan(e + f*x)**2 + 1)/(2*f) + A*b**3*c*tan(e + f*x)**2/(2*f) + A*b**3*d*x
+ A*b**3*d*tan(e + f*x)**3/(3*f) - A*b**3*d*tan(e + f*x)/f + B*a**3*c*log(t
an(e + f*x)**2 + 1)/(2*f) - B*a**3*d*x + B*a**3*d*tan(e + f*x)/f - 3*B*a**2
*b*c*x + 3*B*a**2*b*c*tan(e + f*x)/f - 3*B*a**2*b*d*log(tan(e + f*x)**2 + 1
)/(2*f) + 3*B*a**2*b*d*tan(e + f*x)**2/(2*f) - 3*B*a*b**2*c*log(tan(e + f*x
)**2 + 1)/(2*f) + 3*B*a*b**2*c*tan(e + f*x)**2/(2*f) + 3*B*a*b**2*d*x + B*a
*b**2*d*tan(e + f*x)**3/f - 3*B*a*b**2*d*tan(e + f*x)/f + B*b**3*c*x + B*b*
*3*c*tan(e + f*x)**3/(3*f) - B*b**3*c*tan(e + f*x)/f + B*b**3*d*log(tan(e +
 f*x)**2 + 1)/(2*f) + B*b**3*d*tan(e + f*x)**4/(4*f) - B*b**3*d*tan(e + f*x
)**2/(2*f) - C*a**3*c*x + C*a**3*c*tan(e + f*x)/f - C*a**3*d*log(tan(e + f*
x)**2 + 1)/(2*f) + C*a**3*d*tan(e + f*x)**2/(2*f) - 3*C*a**2*b*c*log(tan(e
+ f*x)**2 + 1)/(2*f) + 3*C*a**2*b*c*tan(e + f*x)**2/(2*f) + 3*C*a**2*b*d*x
+ C*a**2*b*d*tan(e + f*x)**3/f - 3*C*a**2*b*d*tan(e + f*x)/f + 3*C*a*b**2*c
*x + C*a*b**2*c*tan(e + f*x)**3/f - 3*C*a*b**2*c*tan(e + f*x)/f + 3*C*a*b**
2*d*log(tan(e + f*x)**2 + 1)/(2*f) + 3*C*a*b**2*d*tan(e + f*x)**4/(4*f) - 3
*C*a*b**2*d*tan(e + f*x)**2/(2*f) + C*b**3*c*log(tan(e + f*x)**2 + 1)/(2*f)
 + C*b**3*c*tan(e + f*x)**4/(4*f) - C*b**3*c*tan(e + f*x)**2/(2*f) - C*b**3
*d*x + C*b**3*d*tan(e + f*x)**5/(5*f) - C*b**3*d*tan(e + f*x)**3/(3*f) + C*
b**3*d*tan(e + f*x)/f, Ne(f, 0)), (x*(a + b*tan(e))**3*(c + d*tan(e))*(A +
B*tan(e) + C*tan(e)**2), True))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.51 $\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))\left(A+B\tan(e+fx)+\right.$

Optimal. Leaf size=248

$$-\frac{\log(\cos(e+fx))\left(a^2(d(A-C)+Bc)+2ab(Ac-Bd-cC)-b^2(d(A-C)+Bc)\right)}{f}+x\left(a^2(Ac-Bd-cC)-2ab(d(A-\right.$$

```
[Out] (a^2*(A*c - c*C - B*d) - b^2*(A*c - c*C - B*d) - 2*a*b*(B*c + (A - C)*d))*x
 - ((2*a*b*(A*c - c*C - B*d) + a^2*(B*c + (A - C)*d) - b^2*(B*c + (A - C)*d
))*Log[Cos[e + f*x]])/f + (b*(A*b*c + a*B*c - b*c*C + a*A*d - b*B*d - a*C*d
)*Tan[e + f*x])/f + ((B*c + (A - C)*d)*(a + b*Tan[e + f*x])^2)/(2*f) - ((a*
C*d - 4*b*(c*C + B*d))*(a + b*Tan[e + f*x])^3)/(12*b^2*f) + (C*d*Tan[e + f*
x]*(a + b*Tan[e + f*x])^3)/(4*b*f)
```

Rubi [A] time = 0.451264, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {3637, 3630, 3528, 3525, 3475}

$$-\frac{\log(\cos(e+fx))\left(a^2(d(A-C)+Bc)+2ab(Ac-Bd-cC)-b^2(d(A-C)+Bc)\right)}{f}+x\left(a^2(Ac-Bd-cC)-2ab(d(A-\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan
[e + f*x]^2),x]
```


```
[Out] (a^2*(A*c - c*C - B*d) - b^2*(A*c - c*C - B*d) - 2*a*b*(B*c + (A - C)*d))*x
 - ((2*a*b*(A*c - c*C - B*d) + a^2*(B*c + (A - C)*d) - b^2*(B*c + (A - C)*d
))*Log[Cos[e + f*x]])/f + (b*(A*b*c + a*B*c - b*c*C + a*A*d - b*B*d - a*C*d
)*Tan[e + f*x])/f + ((B*c + (A - C)*d)*(a + b*Tan[e + f*x])^2)/(2*f) - ((a*
C*d - 4*b*(c*C + B*d))*(a + b*Tan[e + f*x])^3)/(12*b^2*f) + (C*d*Tan[e + f*
x]*(a + b*Tan[e + f*x])^3)/(4*b*f)
```


Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
```

```
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx=\frac{Cd\tan(e+fx)(a+b\tan(e+fx))}{4bf}$$

$$=-\frac{(aCd-4b(cC+Bd))(a+b\tan(e}{12b^2f}$$

$$=\frac{(Bc+(A-C)d)(a+b\tan(e+fx)}{2f}$$

$$=\left(a^2(Ac-cC-Bd)-b^2(Ac-cC-\right.$$

$$=\left(a^2(Ac-cC-Bd)-b^2(Ac-cC-\right.$$

Mathematica [C] time = 3.17545, size = 243, normalized size = 0.98

$$\frac{6(d(A-C)+Bc)\left(6ab^2\tan(e+fx)+(-b+ia)^3\log(-\tan(e+fx)+i)-(b+ia)^3\log(\tan(e+fx)+i)+b^3\tan^2(e+fx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] +
 C*Tan[e + f*x]^2),x]
```

```
[Out] (((-(a*C*d) + 4*b*(c*C + B*d))*(a + b*Tan[e + f*x])^3)/b + 3*C*d*Tan[e + f*
x]*(a + b*Tan[e + f*x])^3 - 6*(A*b*c - a*B*c - b*c*C - a*A*d - b*B*d + a*C*
d)*(I*((a + I*b)^2*Log[I - Tan[e + f*x]] - (a - I*b)^2*Log[I + Tan[e + f*x]
]) - 2*b^2*Tan[e + f*x]) + 6*(B*c + (A - C)*d)*((I*a - b)^3*Log[I - Tan[e +
 f*x]] - (I*a + b)^3*Log[I + Tan[e + f*x]] + 6*a*b^2*Tan[e + f*x] + b^3*Tan
[e + f*x]^2))/(12*b*f)
```

Maple [B] time = 0.017, size = 631, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x)
```

```
[Out] 1/2/f*ln(1+tan(f*x+e)^2)*C*b^2*d+1/f*A*arctan(tan(f*x+e))*a^2*c-1/2/f*C*tan
(f*x+e)^2*b^2*d+1/f*A*b^2*c*tan(f*x+e)+1/2/f*A*tan(f*x+e)^2*b^2*d-1/2/f*ln(
1+tan(f*x+e)^2)*B*b^2*c+1/f*B*a^2*d*tan(f*x+e)+1/4/f*C*b^2*d*tan(f*x+e)^4+1
/3/f*B*tan(f*x+e)^3*b^2*d+1/3/f*C*tan(f*x+e)^3*b^2*c+1/2/f*B*tan(f*x+e)^2*b
^2*c+1/2/f*C*tan(f*x+e)^2*a^2*d+1/f*B*arctan(tan(f*x+e))*b^2*d-1/f*C*arctan
(tan(f*x+e))*a^2*c+1/f*C*arctan(tan(f*x+e))*b^2*c-1/2/f*ln(1+tan(f*x+e)^2)*
C*a^2*d-1/f*C*b^2*c*tan(f*x+e)+1/2/f*ln(1+tan(f*x+e)^2)*A*a^2*d-1/2/f*ln(1+
tan(f*x+e)^2)*A*b^2*d+1/2/f*ln(1+tan(f*x+e)^2)*B*a^2*c-1/f*B*arctan(tan(f*x
+e))*a^2*d-1/f*B*b^2*d*tan(f*x+e)+1/f*C*a^2*c*tan(f*x+e)+1/f*C*tan(f*x+e)^2
*a*b*c-2/f*A*arctan(tan(f*x+e))*a*b*d+1/f*ln(1+tan(f*x+e)^2)*A*a*b*c+2/f*C*
arctan(tan(f*x+e))*a*b*d-1/f*ln(1+tan(f*x+e)^2)*C*a*b*c+1/f*B*tan(f*x+e)^2*
a*b*d+2/f*A*a*b*d*tan(f*x+e)-2/f*B*arctan(tan(f*x+e))*a*b*c-1/f*ln(1+tan(f*
x+e)^2)*B*a*b*d-1/f*A*arctan(tan(f*x+e))*b^2*c+2/f*B*a*b*c*tan(f*x+e)-2/f*C
*a*b*d*tan(f*x+e)+2/3/f*C*tan(f*x+e)^3*a*b*d
```

Maxima [A] time = 1.49138, size = 370, normalized size = 1.49

$$3\,Cb^2d\tan\left(fx+e\right)^4+4\left(Cb^2c+\left(2\,Cab+Bb^2\right)d\right)\tan\left(fx+e\right)^3+6\left(\left(2\,Cab+Bb^2\right)c+\left(Ca^2+2\,Bab+(A-C)b^2\right)d\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x, algorithm="maxima")
```

```
[Out] 1/12*(3*C*b^2*d*tan(f*x + e)^4 + 4*(C*b^2*c + (2*C*a*b + B*b^2)*d)*tan(f*x
 + e)^3 + 6*((2*C*a*b + B*b^2)*c + (C*a^2 + 2*B*a*b + (A - C)*b^2)*d)*tan(f*
x + e)^2 + 12*(((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c - (B*a^2 + 2*(A - C)
*a*b - B*b^2)*d)*(f*x + e) + 6*((B*a^2 + 2*(A - C)*a*b - B*b^2)*c + ((A - C
)*a^2 - 2*B*a*b - (A - C)*b^2)*d)*log(tan(f*x + e)^2 + 1) + 12*((C*a^2 + 2*
B*a*b + (A - C)*b^2)*c + (B*a^2 + 2*(A - C)*a*b - B*b^2)*d)*tan(f*x + e))/f
```

Fricas [A] time = 1.21043, size = 608, normalized size = 2.45

$$3\,Cb^2d\tan\left(fx+e\right)^4+4\left(Cb^2c+\left(2\,Cab+Bb^2\right)d\right)\tan\left(fx+e\right)^3+12\left(\left((A-C)a^2-2\,Bab-(A-C)b^2\right)c-\left(Ba^2+2\,(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*C*b^2*d*tan(f*x + e)^4 + 4*(C*b^2*c + (2*C*a*b + B*b^2)*d)*tan(f*x
 + e)^3 + 12*(((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c - (B*a^2 + 2*(A - C)*a
*b - B*b^2)*d)*f*x + 6*((2*C*a*b + B*b^2)*c + (C*a^2 + 2*B*a*b + (A - C)*b^
2)*d)*tan(f*x + e)^2 - 6*((B*a^2 + 2*(A - C)*a*b - B*b^2)*c + ((A - C)*a^2
 - 2*B*a*b - (A - C)*b^2)*d)*log(1/(tan(f*x + e)^2 + 1)) + 12*((C*a^2 + 2*B*
a*b + (A - C)*b^2)*c + (B*a^2 + 2*(A - C)*a*b - B*b^2)*d)*tan(f*x + e))/f
```

Sympy [A] time = 1.88886, size = 617, normalized size = 2.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)
**2),x)
```

```
[Out] Piecewise((A*a**2*c*x + A*a**2*d*log(tan(e + f*x)**2 + 1)/(2*f) + A*a*b*c*l
og(tan(e + f*x)**2 + 1)/f - 2*A*a*b*d*x + 2*A*a*b*d*tan(e + f*x)/f - A*b**2
*c*x + A*b**2*c*tan(e + f*x)/f - A*b**2*d*log(tan(e + f*x)**2 + 1)/(2*f) +
A*b**2*d*tan(e + f*x)**2/(2*f) + B*a**2*c*log(tan(e + f*x)**2 + 1)/(2*f) -
B*a**2*d*x + B*a**2*d*tan(e + f*x)/f - 2*B*a*b*c*x + 2*B*a*b*c*tan(e + f*x)
/f - B*a*b*d*log(tan(e + f*x)**2 + 1)/f + B*a*b*d*tan(e + f*x)**2/f - B*b**
2*c*log(tan(e + f*x)**2 + 1)/(2*f) + B*b**2*c*tan(e + f*x)**2/(2*f) + B*b**
2*d*x + B*b**2*d*tan(e + f*x)**3/(3*f) - B*b**2*d*tan(e + f*x)/f - C*a**2*c
*x + C*a**2*c*tan(e + f*x)/f - C*a**2*d*log(tan(e + f*x)**2 + 1)/(2*f) + C*
a**2*d*tan(e + f*x)**2/(2*f) - C*a*b*c*log(tan(e + f*x)**2 + 1)/f + C*a*b*c
*tan(e + f*x)**2/f + 2*C*a*b*d*x + 2*C*a*b*d*tan(e + f*x)**3/(3*f) - 2*C*a*
b*d*tan(e + f*x)/f + C*b**2*c*x + C*b**2*c*tan(e + f*x)**3/(3*f) - C*b**2*c
*tan(e + f*x)/f + C*b**2*d*log(tan(e + f*x)**2 + 1)/(2*f) + C*b**2*d*tan(e
+ f*x)**4/(4*f) - C*b**2*d*tan(e + f*x)**2/(2*f), Ne(f, 0)), (x*(a + b*tan(
e))**2*(c + d*tan(e))*(A + B*tan(e) + C*tan(e)**2), True))
```

Giac [B] time = 8.41758, size = 8778, normalized size = 35.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x, algorithm="giac")
```

```
[Out] 1/12*(12*A*a^2*c*f*x*tan(f*x)^4*tan(e)^4 - 12*C*a^2*c*f*x*tan(f*x)^4*tan(e)
^4 - 24*B*a*b*c*f*x*tan(f*x)^4*tan(e)^4 - 12*A*b^2*c*f*x*tan(f*x)^4*tan(e)^
4 + 12*C*b^2*c*f*x*tan(f*x)^4*tan(e)^4 - 12*B*a^2*d*f*x*tan(f*x)^4*tan(e)^4
 - 24*A*a*b*d*f*x*tan(f*x)^4*tan(e)^4 + 24*C*a*b*d*f*x*tan(f*x)^4*tan(e)^4
+ 12*B*b^2*d*f*x*tan(f*x)^4*tan(e)^4 - 6*B*a^2*c*log(4*(tan(e)^2 + 1)/(tan(
f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 -
2*tan(f*x)*tan(e) + 1))*tan(f*x)^4*tan(e)^4 - 12*A*a*b*c*log(4*(tan(e)^2 +
1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f
*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^4*tan(e)^4 + 12*C*a*b*c*log(4*(tan
(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2
 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^4*tan(e)^4 + 6*B*b^2*c*log
(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*t
an(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^4*tan(e)^4 - 6*A*a^
2*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f
*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^4*tan(e)^4 +
 6*C*a^2*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e)
+ tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^4*tan
(e)^4 + 12*B*a*b*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3
*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*
x)^4*tan(e)^4 + 6*A*b^2*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan
(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))
*tan(f*x)^4*tan(e)^4 - 6*C*b^2*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2
```

```
- 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e
) + 1))*tan(f*x)^4*tan(e)^4 - 48*A*a^2*c*f*x*tan(f*x)^3*tan(e)^3 + 48*C*a^2
*c*f*x*tan(f*x)^3*tan(e)^3 + 96*B*a*b*c*f*x*tan(f*x)^3*tan(e)^3 + 48*A*b^2*
c*f*x*tan(f*x)^3*tan(e)^3 - 48*C*b^2*c*f*x*tan(f*x)^3*tan(e)^3 + 48*B*a^2*d
*f*x*tan(f*x)^3*tan(e)^3 + 96*A*a*b*d*f*x*tan(f*x)^3*tan(e)^3 - 96*C*a*b*d*
f*x*tan(f*x)^3*tan(e)^3 - 48*B*b^2*d*f*x*tan(f*x)^3*tan(e)^3 + 12*C*a*b*c*t
an(f*x)^4*tan(e)^4 + 6*B*b^2*c*tan(f*x)^4*tan(e)^4 + 6*C*a^2*d*tan(f*x)^4*t
an(e)^4 + 12*B*a*b*d*tan(f*x)^4*tan(e)^4 + 6*A*b^2*d*tan(f*x)^4*tan(e)^4 -
9*C*b^2*d*tan(f*x)^4*tan(e)^4 + 24*B*a^2*c*log(4*(tan(e)^2 + 1)/(tan(f*x)^4
*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(
f*x)*tan(e) + 1))*tan(f*x)^3*tan(e)^3 + 48*A*a*b*c*log(4*(tan(e)^2 + 1)/(ta
n(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2
- 2*tan(f*x)*tan(e) + 1))*tan(f*x)^3*tan(e)^3 - 48*C*a*b*c*log(4*(tan(e)^2
+ 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan
(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^3*tan(e)^3 - 24*B*b^2*c*log(4*(t
an(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)
^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^3*tan(e)^3 + 24*A*a^2*d*
log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^
2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^3*tan(e)^3 - 24*
C*a^2*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + t
an(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^3*tan(e)
^3 - 48*B*a*b*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*ta
n(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^
3*tan(e)^3 - 24*A*b^2*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f
*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*t
an(f*x)^3*tan(e)^3 + 24*C*b^2*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 -
 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e)
 + 1))*tan(f*x)^3*tan(e)^3 - 12*C*a^2*c*tan(f*x)^4*tan(e)^3 - 24*B*a*b*c*ta
n(f*x)^4*tan(e)^3 - 12*A*b^2*c*tan(f*x)^4*tan(e)^3 + 12*C*b^2*c*tan(f*x)^4*
tan(e)^3 - 12*B*a^2*d*tan(f*x)^4*tan(e)^3 - 24*A*a*b*d*tan(f*x)^4*tan(e)^3
+ 24*C*a*b*d*tan(f*x)^4*tan(e)^3 + 12*B*b^2*d*tan(f*x)^4*tan(e)^3 - 12*C*a^
2*c*tan(f*x)^3*tan(e)^4 - 24*B*a*b*c*tan(f*x)^3*tan(e)^4 - 12*A*b^2*c*tan(f
*x)^3*tan(e)^4 + 12*C*b^2*c*tan(f*x)^3*tan(e)^4 - 12*B*a^2*d*tan(f*x)^3*tan
(e)^4 - 24*A*a*b*d*tan(f*x)^3*tan(e)^4 + 24*C*a*b*d*tan(f*x)^3*tan(e)^4 + 1
2*B*b^2*d*tan(f*x)^3*tan(e)^4 + 72*A*a^2*c*f*x*tan(f*x)^2*tan(e)^2 - 72*C*a
^2*c*f*x*tan(f*x)^2*tan(e)^2 - 144*B*a*b*c*f*x*tan(f*x)^2*tan(e)^2 - 72*A*b
^2*c*f*x*tan(f*x)^2*tan(e)^2 + 72*C*b^2*c*f*x*tan(f*x)^2*tan(e)^2 - 72*B*a^
2*d*f*x*tan(f*x)^2*tan(e)^2 - 144*A*a*b*d*f*x*tan(f*x)^2*tan(e)^2 + 144*C*a
*b*d*f*x*tan(f*x)^2*tan(e)^2 + 72*B*b^2*d*f*x*tan(f*x)^2*tan(e)^2 + 12*C*a*
b*c*tan(f*x)^4*tan(e)^2 + 6*B*b^2*c*tan(f*x)^4*tan(e)^2 + 6*C*a^2*d*tan(f*x
)^4*tan(e)^2 + 12*B*a*b*d*tan(f*x)^4*tan(e)^2 + 6*A*b^2*d*tan(f*x)^4*tan(e)
^2 - 6*C*b^2*d*tan(f*x)^4*tan(e)^2 - 24*C*a*b*c*tan(f*x)^3*tan(e)^3 - 12*B*
b^2*c*tan(f*x)^3*tan(e)^3 - 12*C*a^2*d*tan(f*x)^3*tan(e)^3 - 24*B*a*b*d*tan
(f*x)^3*tan(e)^3 - 12*A*b^2*d*tan(f*x)^3*tan(e)^3 + 24*C*b^2*d*tan(f*x)^3*t
an(e)^3 + 12*C*a*b*c*tan(f*x)^2*tan(e)^4 + 6*B*b^2*c*tan(f*x)^2*tan(e)^4 +
6*C*a^2*d*tan(f*x)^2*tan(e)^4 + 12*B*a*b*d*tan(f*x)^2*tan(e)^4 + 6*A*b^2*d*
tan(f*x)^2*tan(e)^4 - 6*C*b^2*d*tan(f*x)^2*tan(e)^4 - 4*C*b^2*c*tan(f*x)^4*
tan(e) - 8*C*a*b*d*tan(f*x)^4*tan(e) - 4*B*b^2*d*tan(f*x)^4*tan(e) - 36*B*a
^2*c*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(
f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^2*tan(e)^2
- 72*A*a*b*c*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e
) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^2*t
an(e)^2 + 72*C*a*b*c*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)
^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(
f*x)^2*tan(e)^2 + 36*B*b^2*c*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*
tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) +
1))*tan(f*x)^2*tan(e)^2 - 36*A*a^2*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e
)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*t
an(e) + 1))*tan(f*x)^2*tan(e)^2 + 36*C*a^2*d*log(4*(tan(e)^2 + 1)/(tan(f*x)
```

```
^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*ta
n(f*x)*tan(e) + 1))*tan(f*x)^2*tan(e)^2 + 72*B*a*b*d*log(4*(tan(e)^2 + 1)/(
tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^
2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^2*tan(e)^2 + 36*A*b^2*d*log(4*(tan(e)^
2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + t
an(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^2*tan(e)^2 - 36*C*b^2*d*log(4*
(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(
e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^2*tan(e)^2 + 36*C*a^2*
c*tan(f*x)^3*tan(e)^2 + 72*B*a*b*c*tan(f*x)^3*tan(e)^2 + 36*A*b^2*c*tan(f*x
)^3*tan(e)^2 - 48*C*b^2*c*tan(f*x)^3*tan(e)^2 + 36*B*a^2*d*tan(f*x)^3*tan(e
)^2 + 72*A*a*b*d*tan(f*x)^3*tan(e)^2 - 96*C*a*b*d*tan(f*x)^3*tan(e)^2 - 48*
B*b^2*d*tan(f*x)^3*tan(e)^2 + 36*C*a^2*c*tan(f*x)^2*tan(e)^3 + 72*B*a*b*c*t
an(f*x)^2*tan(e)^3 + 36*A*b^2*c*tan(f*x)^2*tan(e)^3 - 48*C*b^2*c*tan(f*x)^2
*tan(e)^3 + 36*B*a^2*d*tan(f*x)^2*tan(e)^3 + 72*A*a*b*d*tan(f*x)^2*tan(e)^3
 - 96*C*a*b*d*tan(f*x)^2*tan(e)^3 - 48*B*b^2*d*tan(f*x)^2*tan(e)^3 - 4*C*b^
2*c*tan(f*x)*tan(e)^4 - 8*C*a*b*d*tan(f*x)*tan(e)^4 - 4*B*b^2*d*tan(f*x)*ta
n(e)^4 + 3*C*b^2*d*tan(f*x)^4 - 48*A*a^2*c*f*x*tan(f*x)*tan(e) + 48*C*a^2*c
*f*x*tan(f*x)*tan(e) + 96*B*a*b*c*f*x*tan(f*x)*tan(e) + 48*A*b^2*c*f*x*tan(
f*x)*tan(e) - 48*C*b^2*c*f*x*tan(f*x)*tan(e) + 48*B*a^2*d*f*x*tan(f*x)*tan(
e) + 96*A*a*b*d*f*x*tan(f*x)*tan(e) - 96*C*a*b*d*f*x*tan(f*x)*tan(e) - 48*B
*b^2*d*f*x*tan(f*x)*tan(e) - 24*C*a*b*c*tan(f*x)^3*tan(e) - 12*B*b^2*c*tan(
f*x)^3*tan(e) - 12*C*a^2*d*tan(f*x)^3*tan(e) - 24*B*a*b*d*tan(f*x)^3*tan(e)
 - 12*A*b^2*d*tan(f*x)^3*tan(e) + 24*C*b^2*d*tan(f*x)^3*tan(e) + 24*C*a*b*c
*tan(f*x)^2*tan(e)^2 + 12*B*b^2*c*tan(f*x)^2*tan(e)^2 + 12*C*a^2*d*tan(f*x)
^2*tan(e)^2 + 24*B*a*b*d*tan(f*x)^2*tan(e)^2 + 12*A*b^2*d*tan(f*x)^2*tan(e)
^2 - 12*C*b^2*d*tan(f*x)^2*tan(e)^2 - 24*C*a*b*c*tan(f*x)*tan(e)^3 - 12*B*b
^2*c*tan(f*x)*tan(e)^3 - 12*C*a^2*d*tan(f*x)*tan(e)^3 - 24*B*a*b*d*tan(f*x)
*tan(e)^3 - 12*A*b^2*d*tan(f*x)*tan(e)^3 + 24*C*b^2*d*tan(f*x)*tan(e)^3 + 3
*C*b^2*d*tan(e)^4 + 4*C*b^2*c*tan(f*x)^3 + 8*C*a*b*d*tan(f*x)^3 + 4*B*b^2*d
*tan(f*x)^3 + 24*B*a^2*c*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(
f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*
tan(f*x)*tan(e) + 48*A*a*b*c*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*
tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) +
1))*tan(f*x)*tan(e) - 48*C*a*b*c*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2
 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e
) + 1))*tan(f*x)*tan(e) - 24*B*b^2*c*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e
)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*t
an(e) + 1))*tan(f*x)*tan(e) + 24*A*a^2*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*t
an(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*
x)*tan(e) + 1))*tan(f*x)*tan(e) - 24*C*a^2*d*log(4*(tan(e)^2 + 1)/(tan(f*x)
^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*ta
n(f*x)*tan(e) + 1))*tan(f*x)*tan(e) - 48*B*a*b*d*log(4*(tan(e)^2 + 1)/(tan(
f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 -
2*tan(f*x)*tan(e) + 1))*tan(f*x)*tan(e) - 24*A*b^2*d*log(4*(tan(e)^2 + 1)/(
tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^
2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)*tan(e) + 24*C*b^2*d*log(4*(tan(e)^2 +
1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f
*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)*tan(e) - 36*C*a^2*c*tan(f*x)^2*tan
(e) - 72*B*a*b*c*tan(f*x)^2*tan(e) - 36*A*b^2*c*tan(f*x)^2*tan(e) + 48*C*b^
2*c*tan(f*x)^2*tan(e) - 36*B*a^2*d*tan(f*x)^2*tan(e) - 72*A*a*b*d*tan(f*x)^
2*tan(e) + 96*C*a*b*d*tan(f*x)^2*tan(e) + 48*B*b^2*d*tan(f*x)^2*tan(e) - 36
*C*a^2*c*tan(f*x)*tan(e)^2 - 72*B*a*b*c*tan(f*x)*tan(e)^2 - 36*A*b^2*c*tan(
f*x)*tan(e)^2 + 48*C*b^2*c*tan(f*x)*tan(e)^2 - 36*B*a^2*d*tan(f*x)*tan(e)^2
 - 72*A*a*b*d*tan(f*x)*tan(e)^2 + 96*C*a*b*d*tan(f*x)*tan(e)^2 + 48*B*b^2*d
*tan(f*x)*tan(e)^2 + 4*C*b^2*c*tan(e)^3 + 8*C*a*b*d*tan(e)^3 + 4*B*b^2*d*ta
n(e)^3 + 12*A*a^2*c*f*x - 12*C*a^2*c*f*x - 24*B*a*b*c*f*x - 12*A*b^2*c*f*x
+ 12*C*b^2*c*f*x - 12*B*a^2*d*f*x - 24*A*a*b*d*f*x + 24*C*a*b*d*f*x + 12*B*
b^2*d*f*x + 12*C*a*b*c*tan(f*x)^2 + 6*B*b^2*c*tan(f*x)^2 + 6*C*a^2*d*tan(f*
x)^2 + 12*B*a*b*d*tan(f*x)^2 + 6*A*b^2*d*tan(f*x)^2 - 6*C*b^2*d*tan(f*x)^2
```

```
- 24*C*a*b*c*tan(f*x)*tan(e) - 12*B*b^2*c*tan(f*x)*tan(e) - 12*C*a^2*d*tan(
f*x)*tan(e) - 24*B*a*b*d*tan(f*x)*tan(e) - 12*A*b^2*d*tan(f*x)*tan(e) + 24*
C*b^2*d*tan(f*x)*tan(e) + 12*C*a*b*c*tan(e)^2 + 6*B*b^2*c*tan(e)^2 + 6*C*a^
2*d*tan(e)^2 + 12*B*a*b*d*tan(e)^2 + 6*A*b^2*d*tan(e)^2 - 6*C*b^2*d*tan(e)^
2 - 6*B*a^2*c*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(
e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)) - 12*A*a*b*
c*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x
)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)) + 12*C*a*b*c*log(4*(tan
(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2
 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)) + 6*B*b^2*c*log(4*(tan(e)^2 + 1)/(t
an(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2
 - 2*tan(f*x)*tan(e) + 1)) - 6*A*a^2*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan
(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)
*tan(e) + 1)) + 6*C*a^2*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan
(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))
 + 12*B*a*b*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(
e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)) + 6*A*b^2*d
*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)
^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)) - 6*C*b^2*d*log(4*(tan(e
)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 +
 tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)) + 12*C*a^2*c*tan(f*x) + 24*B*a*b*c*ta
n(f*x) + 12*A*b^2*c*tan(f*x) - 12*C*b^2*c*tan(f*x) + 12*B*a^2*d*tan(f*x) +
24*A*a*b*d*tan(f*x) - 24*C*a*b*d*tan(f*x) - 12*B*b^2*d*tan(f*x) + 12*C*a^2*
c*tan(e) + 24*B*a*b*c*tan(e) + 12*A*b^2*c*tan(e) - 12*C*b^2*c*tan(e) + 12*B
*a^2*d*tan(e) + 24*A*a*b*d*tan(e) - 24*C*a*b*d*tan(e) - 12*B*b^2*d*tan(e) +
 12*C*a*b*c + 6*B*b^2*c + 6*C*a^2*d + 12*B*a*b*d + 6*A*b^2*d - 9*C*b^2*d)/(
f*tan(f*x)^4*tan(e)^4 - 4*f*tan(f*x)^3*tan(e)^3 + 6*f*tan(f*x)^2*tan(e)^2 -
 4*f*tan(f*x)*tan(e) + f)
```

3.52 $\int (a+b\tan(e+fx))(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\right.$

Optimal. Leaf size=161

$$-\frac{\log(\cos(e+fx))(aAd+aBc-aCd+Abc-bBd-bcC)}{f}+x(a(Ac-Bd-cC)-b(d(A-C)+Bc))+\frac{d\tan(e+fx)(aB}{f}$$

```
[Out] (a*(A*c - c*C - B*d) - b*(B*c + (A - C)*d))*x - ((A*b*c + a*B*c - b*c*C + a
*A*d - b*B*d - a*C*d)*Log[Cos[e + f*x]])/f + ((A*b + a*B - b*C)*d*Tan[e + f
*x])/f - ((b*c*C - 3*b*B*d - 3*a*C*d)*(c + d*Tan[e + f*x])^2)/(6*d^2*f) + (
b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^2)/(3*d*f)
```

Rubi [A] time = 0.241394, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {3637, 3630, 3525, 3475}

$$-\frac{\log(\cos(e+fx))(aAd+aBc-aCd+Abc-bBd-bcC)}{f}-x(-a(Ac-Bd-cC)+bd(A-C)+bBc)+\frac{d\tan(e+fx)(aB}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e
 + f*x]^2),x]
```

```
[Out] -((b*B*c + b*(A - C)*d - a*(A*c - c*C - B*d))*x) - ((A*b*c + a*B*c - b*c*C
+ a*A*d - b*B*d - a*C*d)*Log[Cos[e + f*x]])/f + ((A*b + a*B - b*C)*d*Tan[e
+ f*x])/f - ((b*c*C - 3*b*B*d - 3*a*C*d)*(c + d*Tan[e + f*x])^2)/(6*d^2*f)
+ (b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^2)/(3*d*f)
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \frac{bC\tan(e+fx)(c+d\tan(e+f x}{3df}$$

$$= -\frac{(bcC-3bBd-3aCd)(c+d\tan}{6d^2f}$$

$$= -(bBc+b(A-C)d-a(Ac-cC$$

$$= -(bBc+b(A-C)d-a(Ac-cC$$

Mathematica [C] time = 1.5388, size = 161, normalized size = 1.

$$\frac{3(a+ib)(d-ic)(A+iB-C)\log(-\tan(e+fx)+i)+3(a-ib)(d+ic)(A-iB-C)\log(\tan(e+fx)+i)+6d\tan(e+}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C
*Tan[e + f*x]^2),x]
```

```
[Out] (3*(a + I*b)*(A + I*B - C)*((-I)*c + d)*Log[I - Tan[e + f*x]] + 3*(a - I*b)
*(A - I*B - C)*(I*c + d)*Log[I + Tan[e + f*x]] + 6*(A*b + a*B - b*C)*d*Tan[
e + f*x] + ((-(b*c*C) + 3*b*B*d + 3*a*C*d)*(c + d*Tan[e + f*x])^2)/d^2 + (2
*b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^2)/d)/(6*f)
```

Maple [B] time = 0.014, size = 334, normalized size = 2.1

$$\frac{C\left(\tan\left(fx+e\right)\right)^{3}bd}{3f}+\frac{B\left(\tan\left(fx+e\right)\right)^{2}bd}{2f}+\frac{C\left(\tan\left(fx+e\right)\right)^{2}ad}{2f}+\frac{C\left(\tan\left(fx+e\right)\right)^{2}bc}{2f}+\frac{A\tan\left(fx+e\right)bd}{f}+\frac{B}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x)
```

```
[Out] 1/3/f*C*b*d*tan(f*x+e)^3+1/2/f*B*tan(f*x+e)^2*b*d+1/2/f*C*tan(f*x+e)^2*a*d+
1/2/f*C*tan(f*x+e)^2*b*c+1/f*A*b*d*tan(f*x+e)+1/f*B*a*d*tan(f*x+e)+1/f*B*b*
c*tan(f*x+e)+1/f*C*a*c*tan(f*x+e)-1/f*C*b*d*tan(f*x+e)+1/2/f*ln(1+tan(f*x+e
)^2)*A*a*d+1/2/f*ln(1+tan(f*x+e)^2)*A*b*c+1/2/f*ln(1+tan(f*x+e)^2)*B*a*c-1/
2/f*ln(1+tan(f*x+e)^2)*B*b*d-1/2/f*ln(1+tan(f*x+e)^2)*a*C*d-1/2/f*ln(1+tan(
f*x+e)^2)*C*b*c+1/f*A*arctan(tan(f*x+e))*a*c-1/f*A*arctan(tan(f*x+e))*b*d-1
/f*B*arctan(tan(f*x+e))*a*d-1/f*B*arctan(tan(f*x+e))*b*c-1/f*C*arctan(tan(f
*x+e))*a*c+1/f*C*arctan(tan(f*x+e))*b*d
```

Maxima [A] time = 1.45627, size = 204, normalized size = 1.27

$$\frac{2\,Cbd\tan\left(fx+e\right)^{3}+3\,(Cbc+(Ca+Bb)d)\tan\left(fx+e\right)^{2}+6\,(((A-C)a-Bb)c-(Ba+(A-C)b)d)\left(fx+e\right)+3\,((B}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)
,x, algorithm="maxima")
```

```
[Out] 1/6*(2*C*b*d*tan(f*x + e)^3 + 3*(C*b*c + (C*a + B*b)*d)*tan(f*x + e)^2 + 6*
(((A - C)*a - B*b)*c - (B*a + (A - C)*b)*d)*(f*x + e) + 3*((B*a + (A - C)*b
)*c + ((A - C)*a - B*b)*d)*log(tan(f*x + e)^2 + 1) + 6*((C*a + B*b)*c + (B*
a + (A - C)*b)*d)*tan(f*x + e))/f
```

Fricas [A] time = 1.15453, size = 348, normalized size = 2.16

$$\frac{2\,Cbd\tan\left(fx+e\right)^{3}+6\,(((A-C)a-Bb)c-(Ba+(A-C)b)d)fx+3\,(Cbc+(Ca+Bb)d)\tan\left(fx+e\right)^{2}-3\,((Ba+(A-}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)
,x, algorithm="fricas")
```

```
[Out] 1/6*(2*C*b*d*tan(f*x + e)^3 + 6*(((A - C)*a - B*b)*c - (B*a + (A - C)*b)*d)
*f*x + 3*(C*b*c + (C*a + B*b)*d)*tan(f*x + e)^2 - 3*((B*a + (A - C)*b)*c +
((A - C)*a - B*b)*d)*log(1/(tan(f*x + e)^2 + 1)) + 6*((C*a + B*b)*c + (B*a
+ (A - C)*b)*d)*tan(f*x + e))/f
```

Sympy [A] time = 0.834445, size = 326, normalized size = 2.02

$$\begin{cases} Aacx+\frac{Aad\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2f}+\frac{Abc\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2f}-Abdx+\frac{Abd\tan\left(e+fx\right)}{f}+\frac{Bac\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2f}-Badx+\frac{Bad\tan\left(e+fx\right)}{f} \\ x\left(a+b\tan\left(e\right)\right)\left(c+d\tan\left(e\right)\right)\left(A+B\tan\left(e\right)+C\tan^{2}\left(e\right)\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)**2
),x)
```

```
[Out] Piecewise((A*a*c*x + A*a*d*log(tan(e + f*x)**2 + 1)/(2*f) + A*b*c*log(tan(e
 + f*x)**2 + 1)/(2*f) - A*b*d*x + A*b*d*tan(e + f*x)/f + B*a*c*log(tan(e +
f*x)**2 + 1)/(2*f) - B*a*d*x + B*a*d*tan(e + f*x)/f - B*b*c*x + B*b*c*tan(e
 + f*x)/f - B*b*d*log(tan(e + f*x)**2 + 1)/(2*f) + B*b*d*tan(e + f*x)**2/(2
*f) - C*a*c*x + C*a*c*tan(e + f*x)/f - C*a*d*log(tan(e + f*x)**2 + 1)/(2*f)
 + C*a*d*tan(e + f*x)**2/(2*f) - C*b*c*log(tan(e + f*x)**2 + 1)/(2*f) + C*b
*c*tan(e + f*x)**2/(2*f) + C*b*d*x + C*b*d*tan(e + f*x)**3/(3*f) - C*b*d*ta
n(e + f*x)/f, Ne(f, 0)), (x*(a + b*tan(e))*(c + d*tan(e))*(A + B*tan(e) + C
*tan(e)**2), True))
```

Giac [B] time = 3.87224, size = 3939, normalized size = 24.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)
,x, algorithm="giac")
```

```
[Out] 1/6*(6*A*a*c*f*x*tan(f*x)^3*tan(e)^3 - 6*C*a*c*f*x*tan(f*x)^3*tan(e)^3 - 6*
B*b*c*f*x*tan(f*x)^3*tan(e)^3 - 6*B*a*d*f*x*tan(f*x)^3*tan(e)^3 - 6*A*b*d*f
*x*tan(f*x)^3*tan(e)^3 + 6*C*b*d*f*x*tan(f*x)^3*tan(e)^3 - 3*B*a*c*log(4*(t
an(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)
^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^3*tan(e)^3 - 3*A*b*c*log
(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*t
an(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^3*tan(e)^3 + 3*C*b*
c*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x
)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^3*tan(e)^3 - 3
*A*a*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + ta
n(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^3*tan(e)^
3 + 3*C*a*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e)
 + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^3*ta
n(e)^3 + 3*B*b*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*t
an(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)
^3*tan(e)^3 - 18*A*a*c*f*x*tan(f*x)^2*tan(e)^2 + 18*C*a*c*f*x*tan(f*x)^2*ta
n(e)^2 + 18*B*b*c*f*x*tan(f*x)^2*tan(e)^2 + 18*B*a*d*f*x*tan(f*x)^2*tan(e)^
2 + 18*A*b*d*f*x*tan(f*x)^2*tan(e)^2 - 18*C*b*d*f*x*tan(f*x)^2*tan(e)^2 + 3
*C*b*c*tan(f*x)^3*tan(e)^3 + 3*C*a*d*tan(f*x)^3*tan(e)^3 + 3*B*b*d*tan(f*x)
^3*tan(e)^3 + 9*B*a*c*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x
)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan
(f*x)^2*tan(e)^2 + 9*A*b*c*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*ta
n(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)
)*tan(f*x)^2*tan(e)^2 - 9*C*b*c*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 -
 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e)
 + 1))*tan(f*x)^2*tan(e)^2 + 9*A*a*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e
)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*t
an(e) + 1))*tan(f*x)^2*tan(e)^2 - 9*C*a*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*
tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f
*x)*tan(e) + 1))*tan(f*x)^2*tan(e)^2 - 9*B*b*d*log(4*(tan(e)^2 + 1)/(tan(f*
x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*
tan(f*x)*tan(e) + 1))*tan(f*x)^2*tan(e)^2 - 6*C*a*c*tan(f*x)^3*tan(e)^2 - 6
*B*b*c*tan(f*x)^3*tan(e)^2 - 6*B*a*d*tan(f*x)^3*tan(e)^2 - 6*A*b*d*tan(f*x)
^3*tan(e)^2 + 6*C*b*d*tan(f*x)^3*tan(e)^2 - 6*C*a*c*tan(f*x)^2*tan(e)^3 - 6
*B*b*c*tan(f*x)^2*tan(e)^3 - 6*B*a*d*tan(f*x)^2*tan(e)^3 - 6*A*b*d*tan(f*x)
^2*tan(e)^3 + 6*C*b*d*tan(f*x)^2*tan(e)^3 + 18*A*a*c*f*x*tan(f*x)*tan(e) -
18*C*a*c*f*x*tan(f*x)*tan(e) - 18*B*b*c*f*x*tan(f*x)*tan(e) - 18*B*a*d*f*x*
tan(f*x)*tan(e) - 18*A*b*d*f*x*tan(f*x)*tan(e) + 18*C*b*d*f*x*tan(f*x)*tan(
e) + 3*C*b*c*tan(f*x)^3*tan(e) + 3*C*a*d*tan(f*x)^3*tan(e) + 3*B*b*d*tan(f*
x)^3*tan(e) - 3*C*b*c*tan(f*x)^2*tan(e)^2 - 3*C*a*d*tan(f*x)^2*tan(e)^2 - 3
*B*b*d*tan(f*x)^2*tan(e)^2 + 3*C*b*c*tan(f*x)*tan(e)^3 + 3*C*a*d*tan(f*x)*t
an(e)^3 + 3*B*b*d*tan(f*x)*tan(e)^3 - 2*C*b*d*tan(f*x)^3 - 9*B*a*c*log(4*(t
an(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)
^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)*tan(e) - 9*A*b*c*log(4*(
tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e
)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)*tan(e) + 9*C*b*c*log(4*
(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(
e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)*tan(e) - 9*A*a*d*log(4
*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan
(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)*tan(e) + 9*C*a*d*log(
4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*ta
n(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)*tan(e) + 9*B*b*d*log
(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*t
an(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)*tan(e) + 12*C*a*c*t
an(f*x)^2*tan(e) + 12*B*b*c*tan(f*x)^2*tan(e) + 12*B*a*d*tan(f*x)^2*tan(e)
+ 12*A*b*d*tan(f*x)^2*tan(e) - 18*C*b*d*tan(f*x)^2*tan(e) + 12*C*a*c*tan(f*
```

```
x)*tan(e)^2 + 12*B*b*c*tan(f*x)*tan(e)^2 + 12*B*a*d*tan(f*x)*tan(e)^2 + 12*
A*b*d*tan(f*x)*tan(e)^2 - 18*C*b*d*tan(f*x)*tan(e)^2 - 2*C*b*d*tan(e)^3 - 6
*A*a*c*f*x + 6*C*a*c*f*x + 6*B*b*c*f*x + 6*B*a*d*f*x + 6*A*b*d*f*x - 6*C*b*
d*f*x - 3*C*b*c*tan(f*x)^2 - 3*C*a*d*tan(f*x)^2 - 3*B*b*d*tan(f*x)^2 + 3*C*
b*c*tan(f*x)*tan(e) + 3*C*a*d*tan(f*x)*tan(e) + 3*B*b*d*tan(f*x)*tan(e) - 3
*C*b*c*tan(e)^2 - 3*C*a*d*tan(e)^2 - 3*B*b*d*tan(e)^2 + 3*B*a*c*log(4*(tan(
e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2
+ tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)) + 3*A*b*c*log(4*(tan(e)^2 + 1)/(tan(
f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 -
2*tan(f*x)*tan(e) + 1)) - 3*C*b*c*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2
 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(
e) + 1)) + 3*A*a*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3
*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)) - 3*C*
a*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f
*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)) - 3*B*b*d*log(4*(tan(
e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2
+ tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)) - 6*C*a*c*tan(f*x) - 6*B*b*c*tan(f*x
) - 6*B*a*d*tan(f*x) - 6*A*b*d*tan(f*x) + 6*C*b*d*tan(f*x) - 6*C*a*c*tan(e)
 - 6*B*b*c*tan(e) - 6*B*a*d*tan(e) - 6*A*b*d*tan(e) + 6*C*b*d*tan(e) - 3*C*
b*c - 3*C*a*d - 3*B*b*d)/(f*tan(f*x)^3*tan(e)^3 - 3*f*tan(f*x)^2*tan(e)^2 +
 3*f*tan(f*x)*tan(e) - f)
```

3.53 $\int (c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)$ d

Optimal. Leaf size=73

$$-\frac{(d(A-C)+Bc)\log(\cos(e+fx))}{f}+x(Ac-Bd-cC)+\frac{Bd\tan(e+fx)}{f}+\frac{C(c+d\tan(e+fx))^2}{2df}$$

```
[Out] (A*c - c*C - B*d)*x - ((B*c + (A - C)*d)*Log[Cos[e + f*x]])/f + (B*d*Tan[e
+ f*x])/f + (C*(c + d*Tan[e + f*x])^2)/(2*d*f)
```

Rubi [A] time = 0.0606425, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {3630, 3525, 3475}

$$-\frac{(d(A-C)+Bc)\log(\cos(e+fx))}{f}+x(Ac-Bd-cC)+\frac{Bd\tan(e+fx)}{f}+\frac{C(c+d\tan(e+fx))^2}{2df}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] (A*c - c*C - B*d)*x - ((B*c + (A - C)*d)*Log[Cos[e + f*x]])/f + (B*d*Tan[e
+ f*x])/f + (C*(c + d*Tan[e + f*x])^2)/(2*d*f)
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx &= \frac{C(c+d\tan(e+fx))^2}{2df}+\int (A-C+B\tan(e+fx)\\ &= (Ac-cC-Bd)x+\frac{Bd\tan(e+fx)}{f}+\frac{C(c+d\tan(e\\ &= (Ac-cC-Bd)x-\frac{(Bc+(A-C)d)\log(\cos(e+fx}{f}\end{aligned}$$

Mathematica [A] time = 0.446251, size = 76, normalized size = 1.04

$$\frac{-2(d(A-C)+Bc)\log(\cos(e+fx))+2Acfx-2(Bd+cC)\tan^{-1}(\tan(e+fx))+2(Bd+cC)\tan(e+fx)+Cd\tan^2(e+}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] (2*A*c*f*x - 2*(c*C + B*d)*ArcTan[Tan[e + f*x]] - 2*(B*c + (A - C)*d)*Log[C
os[e + f*x]] + 2*(c*C + B*d)*Tan[e + f*x] + C*d*Tan[e + f*x]^2)/(2*f)
```

Maple [A] time = 0.014, size = 136, normalized size = 1.9

$$\frac{C\left(\tan\left(fx+e\right)\right)^2 d}{2f}+\frac{B\tan\left(fx+e\right)d}{f}+\frac{C\tan\left(fx+e\right)c}{f}+\frac{\ln\left(1+\left(\tan\left(fx+e\right)\right)^2\right)Ad}{2f}+\frac{\ln\left(1+\left(\tan\left(fx+e\right)\right)^2\right)Bc}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x)
```

```
[Out] 1/2/f*C*d*tan(f*x+e)^2+B*d*tan(f*x+e)/f+1/f*c*C*tan(f*x+e)+1/2/f*ln(1+tan(f
*x+e)^2)*A*d+1/2/f*ln(1+tan(f*x+e)^2)*B*c-1/2/f*ln(1+tan(f*x+e)^2)*C*d+1/f*
A*arctan(tan(f*x+e))*c-1/f*B*arctan(tan(f*x+e))*d-1/f*C*arctan(tan(f*x+e))*
c
```

Maxima [A] time = 1.47783, size = 100, normalized size = 1.37

$$\frac{Cd\tan\left(fx+e\right)^2+2\left((A-C)c-Bd\right)\left(fx+e\right)+\left(Bc+(A-C)d\right)\log\left(\tan\left(fx+e\right)^2+1\right)+2\left(Cc+Bd\right)\tan\left(fx+e\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm="ma
xima")
```

```
[Out] 1/2*(C*d*tan(f*x + e)^2 + 2*((A - C)*c - B*d)*(f*x + e) + (B*c + (A - C)*d)
*log(tan(f*x + e)^2 + 1) + 2*(C*c + B*d)*tan(f*x + e))/f
```

Fricas [A] time = 1.06793, size = 177, normalized size = 2.42

$$\frac{Cd\tan\left(fx+e\right)^2+2\left((A-C)c-Bd\right)fx-\left(Bc+(A-C)d\right)\log\left(\frac{1}{\tan\left(fx+e\right)^2+1}\right)+2\left(Cc+Bd\right)\tan\left(fx+e\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm="fr
icas")
```

```
[Out] 1/2*(C*d*tan(f*x + e)^2 + 2*((A - C)*c - B*d)*f*x - (B*c + (A - C)*d)*log(1
/(tan(f*x + e)^2 + 1)) + 2*(C*c + B*d)*tan(f*x + e))/f
```

Sympy [A] time = 0.705185, size = 131, normalized size = 1.79

$$\begin{cases} Acx + \frac{Ad\log\left(\tan^2\left(e+fx\right)+1\right)}{2f} + \frac{Bc\log\left(\tan^2\left(e+fx\right)+1\right)}{2f} - Bdx + \frac{Bd\tan\left(e+fx\right)}{f} - Ccx + \frac{Cc\tan\left(e+fx\right)}{f} - \frac{Cd\log\left(\tan^2\left(e+fx\right)+1\right)}{2f} + \frac{Cd\,\mathrm{t}}{} \\ x\left(c+d\tan\left(e\right)\right)\left(A+B\tan\left(e\right)+C\tan^2\left(e\right)\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((A*c*x + A*d*log(tan(e + f*x)**2 + 1)/(2*f) + B*c*log(tan(e + f*x
)**2 + 1)/(2*f) - B*d*x + B*d*tan(e + f*x)/f - C*c*x + C*c*tan(e + f*x)/f -
 C*d*log(tan(e + f*x)**2 + 1)/(2*f) + C*d*tan(e + f*x)**2/(2*f), Ne(f, 0)),
 (x*(c + d*tan(e))*(A + B*tan(e) + C*tan(e)**2), True))
```

Giac [B] time = 1.81836, size = 1239, normalized size = 16.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm="gi
ac")
```

```
[Out] 1/2*(2*A*c*f*x*tan(f*x)^2*tan(e)^2 - 2*C*c*f*x*tan(f*x)^2*tan(e)^2 - 2*B*d*
f*x*tan(f*x)^2*tan(e)^2 - B*c*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2
*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) +
 1))*tan(f*x)^2*tan(e)^2 - A*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 -
2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e)
+ 1))*tan(f*x)^2*tan(e)^2 + C*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 -
 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e)
 + 1))*tan(f*x)^2*tan(e)^2 - 4*A*c*f*x*tan(f*x)*tan(e) + 4*C*c*f*x*tan(f*x)
*tan(e) + 4*B*d*f*x*tan(f*x)*tan(e) + C*d*tan(f*x)^2*tan(e)^2 + 2*B*c*log(4
*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan
(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)*tan(e) + 2*A*d*log(4*
(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(
e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)*tan(e) - 2*C*d*log(4*(
tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e
)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)*tan(e) - 2*C*c*tan(f*x)
^2*tan(e) - 2*B*d*tan(f*x)^2*tan(e) - 2*C*c*tan(f*x)*tan(e)^2 - 2*B*d*tan(f
*x)*tan(e)^2 + 2*A*c*f*x - 2*C*c*f*x - 2*B*d*f*x + C*d*tan(f*x)^2 + C*d*tan
(e)^2 - B*c*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e)
 + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)) - A*d*log(4*(
tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e
)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)) + C*d*log(4*(tan(e)^2 + 1)/(tan(
f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 -
2*tan(f*x)*tan(e) + 1)) + 2*C*c*tan(f*x) + 2*B*d*tan(f*x) + 2*C*c*tan(e) +
2*B*d*tan(e) + C*d)/(f*tan(f*x)^2*tan(e)^2 - 2*f*tan(f*x)*tan(e) + f)
```

3.54 $\int \frac{(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=156

$$\frac{(bc-ad)\left(Ab^2-a(bB-aC)\right)\log(a+b\tan(e+fx))}{b^2f\left(a^2+b^2\right)}+\frac{\log(\cos(e+fx))(-aAd-aBc+aCd+Abc-bBd-bcC)}{f\left(a^2+b^2\right)}+\frac{x(a(A$$

```
[Out] ((a*(A*c - c*C - B*d) + b*(B*c + (A - C)*d))*x)/(a^2 + b^2) + ((A*b*c - a*B
*c - b*c*C - a*A*d - b*B*d + a*C*d)*Log[Cos[e + f*x]])/((a^2 + b^2)*f) + ((
A*b^2 - a*(b*B - a*C))*(b*c - a*d)*Log[a + b*Tan[e + f*x]])/(b^2*(a^2 + b^2
)*f) + (C*d*Tan[e + f*x])/(b*f)
```

Rubi [A] time = 0.349431, antiderivative size = 155, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {3637, 3626, 3617, 31, 3475}

$$\frac{(bc-ad)\left(Ab^2-a(bB-aC)\right)\log(a+b\tan(e+fx))}{b^2f\left(a^2+b^2\right)}+\frac{\log(\cos(e+fx))(-aAd-aBc+aCd+Abc-bBd-bcC)}{f\left(a^2+b^2\right)}+\frac{x(a(A$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a + b*T
an[e + f*x]),x]
```

```
[Out] ((b*B*c + b*(A - C)*d + a*(A*c - c*C - B*d))*x)/(a^2 + b^2) + ((A*b*c - a*B
*c - b*c*C - a*A*d - b*B*d + a*C*d)*Log[Cos[e + f*x]])/((a^2 + b^2)*f) + ((
A*b^2 - a*(b*B - a*C))*(b*c - a*d)*Log[a + b*Tan[e + f*x]])/(b^2*(a^2 + b^2
)*f) + (C*d*Tan[e + f*x])/(b*f)
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3626

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((a*A + b*B -
a*C)*x)/(a^2 + b^2), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3617

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
```

```
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx = \frac{Cd\tan(e+fx)}{bf} - \frac{\int \frac{-Abc+aCd-b(Bc+(A-C)d)\tan(e+fx)-}{a+b\tan(e+f}}{b}$$

$$= \frac{(bBc+b(A-C)d+a(Ac-cC-Bd))x}{a^2+b^2} + \frac{Cd\tan(}{bj}$$

$$= \frac{(bBc+b(A-C)d+a(Ac-cC-Bd))x}{a^2+b^2} + \frac{(Abc-a}{}$$

$$= \frac{(bBc+b(A-C)d+a(Ac-cC-Bd))x}{a^2+b^2} + \frac{(Abc-a}{}$$

Mathematica [C] time = 1.11109, size = 148, normalized size = 0.95

$$\frac{\frac{2(bc-ad)\left(a(aC-bB)+Ab^2\right)\log(a+b\tan(e+fx))}{b^2\left(a^2+b^2\right)}+\frac{(d-ic)(A+iB-C)\log(-\tan(e+fx)+i)}{a+ib}+\frac{(d+ic)(A-iB-C)\log(\tan(e+fx)+i)}{a-ib}+\frac{2Cd\tan(e+fx)}{b}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
 + b*Tan[e + f*x]),x]
```

```
[Out] (((A + I*B - C)*((-I)*c + d)*Log[I - Tan[e + f*x]])/(a + I*b) + ((A - I*B -
 C)*(I*c + d)*Log[I + Tan[e + f*x]])/(a - I*b) + (2*(A*b^2 + a*(-(b*B) + a*
C))*(b*c - a*d)*Log[a + b*Tan[e + f*x]])/(b^2*(a^2 + b^2)) + (2*C*d*Tan[e +
 f*x])/b)/(2*f)
```

Maple [B] time = 0.041, size = 506, normalized size = 3.2

$$\frac{Cd\tan\left(fx+e\right)}{bf} + \frac{\ln\left(1+\left(\tan\left(fx+e\right)\right)^2\right)Aad}{2f\left(a^2+b^2\right)} - \frac{\ln\left(1+\left(\tan\left(fx+e\right)\right)^2\right)Abc}{2f\left(a^2+b^2\right)} + \frac{\ln\left(1+\left(\tan\left(fx+e\right)\right)^2\right)Bac}{2f\left(a^2+b^2\right)} + \frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)),x)
```

```
[Out] C*d*tan(f*x+e)/b/f+1/2/f/(a^2+b^2)*ln(1+tan(f*x+e)^2)*A*a*d-1/2/f/(a^2+b^2)
*ln(1+tan(f*x+e)^2)*A*b*c+1/2/f/(a^2+b^2)*ln(1+tan(f*x+e)^2)*B*a*c+1/2/f/(a
^2+b^2)*ln(1+tan(f*x+e)^2)*B*b*d-1/2/f/(a^2+b^2)*ln(1+tan(f*x+e)^2)*a*C*d+1
/2/f/(a^2+b^2)*ln(1+tan(f*x+e)^2)*C*b*c+1/f/(a^2+b^2)*A*arctan(tan(f*x+e))*
a*c+1/f/(a^2+b^2)*A*arctan(tan(f*x+e))*b*d-1/f/(a^2+b^2)*B*arctan(tan(f*x+e
))*a*d+1/f/(a^2+b^2)*B*arctan(tan(f*x+e))*b*c-1/f/(a^2+b^2)*C*arctan(tan(f*
x+e))*a*c-1/f/(a^2+b^2)*C*arctan(tan(f*x+e))*b*d-1/f/(a^2+b^2)*ln(a+b*tan(f
*x+e))*A*a*d+1/f*b/(a^2+b^2)*ln(a+b*tan(f*x+e))*A*c+1/f/b/(a^2+b^2)*ln(a+b*
tan(f*x+e))*B*a^2*d-1/f/(a^2+b^2)*ln(a+b*tan(f*x+e))*B*a*c-1/f/b^2/(a^2+b^2
)*ln(a+b*tan(f*x+e))*a^3*C*d+1/f/b/(a^2+b^2)*ln(a+b*tan(f*x+e))*C*a^2*c
```

Maxima [A] time = 1.46592, size = 247, normalized size = 1.58

$$\frac{\frac{2\,Cd\tan\left(fx+e\right)}{b} + \frac{2\,(((A-C)a+Bb)c-(Ba-(A-C)b)d)\left(fx+e\right)}{a^2+b^2} + \frac{2\left(\left(Ca^2b-Bab^2+Ab^3\right)c-\left(Ca^3-Ba^2b+Aab^2\right)d\right)\log\left(b\tan\left(fx+e\right)+a\right)}{a^2b^2+b^4} + \frac{((Ba-(A-C)b)c+((A-C)}{}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
,x, algorithm="maxima")
```

```
[Out] 1/2*(2*C*d*tan(f*x + e)/b + 2*(((A - C)*a + B*b)*c - (B*a - (A - C)*b)*d)*(
f*x + e)/(a^2 + b^2) + 2*((C*a^2*b - B*a*b^2 + A*b^3)*c - (C*a^3 - B*a^2*b
+ A*a*b^2)*d)*log(b*tan(f*x + e) + a)/(a^2*b^2 + b^4) + ((B*a - (A - C)*b)*
c + ((A - C)*a + B*b)*d)*log(tan(f*x + e)^2 + 1)/(a^2 + b^2))/f
```

Fricas [A] time = 1.96562, size = 483, normalized size = 3.1

$$\frac{2\left(\left(\left(A-C\right)ab^2+Bb^3\right)c-\left(Bab^2-\left(A-C\right)b^3\right)d\right)fx+2\left(Ca^2b+Cb^3\right)d\tan\left(fx+e\right)+\left(\left(Ca^2b-Bab^2+Ab^3\right)c-\left(Ca^3-B\right.\right.}{2\left(a^2b^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(((A - C)*a*b^2 + B*b^3)*c - (B*a*b^2 - (A - C)*b^3)*d)*f*x + 2*(C*a
^2*b + C*b^3)*d*tan(f*x + e) + ((C*a^2*b - B*a*b^2 + A*b^3)*c - (C*a^3 - B*
a^2*b + A*a*b^2)*d)*log((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)/(ta
n(f*x + e)^2 + 1)) - ((C*a^2*b + C*b^3)*c - (C*a^3 - B*a^2*b + C*a*b^2 - B*
b^3)*d)*log(1/(tan(f*x + e)^2 + 1)))/((a^2*b^2 + b^4)*f)
```

Sympy [A] time = 23.0993, size = 2387, normalized size = 15.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e)
),x)
```

```
[Out] Piecewise((zoo*x*(c + d*tan(e))*(A + B*tan(e) + C*tan(e)**2)/tan(e), Eq(a,
0) & Eq(b, 0) & Eq(f, 0)), ((A*c*x + A*d*log(tan(e + f*x)**2 + 1)/(2*f) + B
*c*log(tan(e + f*x)**2 + 1)/(2*f) - B*d*x + B*d*tan(e + f*x)/f - C*c*x + C*
c*tan(e + f*x)/f - C*d*log(tan(e + f*x)**2 + 1)/(2*f) + C*d*tan(e + f*x)**2
/(2*f))/a, Eq(b, 0)), (-I*A*c*f*x*tan(e + f*x)/(-2*b*f*tan(e + f*x) + 2*I*b
*f) - A*c*f*x/(-2*b*f*tan(e + f*x) + 2*I*b*f) - I*A*c/(-2*b*f*tan(e + f*x)
+ 2*I*b*f) - A*d*f*x*tan(e + f*x)/(-2*b*f*tan(e + f*x) + 2*I*b*f) + I*A*d*f
*x/(-2*b*f*tan(e + f*x) + 2*I*b*f) + A*d/(-2*b*f*tan(e + f*x) + 2*I*b*f) -
B*c*f*x*tan(e + f*x)/(-2*b*f*tan(e + f*x) + 2*I*b*f) + I*B*c*f*x/(-2*b*f*ta
n(e + f*x) + 2*I*b*f) + B*c/(-2*b*f*tan(e + f*x) + 2*I*b*f) - I*B*d*f*x*tan
(e + f*x)/(-2*b*f*tan(e + f*x) + 2*I*b*f) - B*d*f*x/(-2*b*f*tan(e + f*x) +
2*I*b*f) - B*d*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(-2*b*f*tan(e + f*x) +
 2*I*b*f) + I*B*d*log(tan(e + f*x)**2 + 1)/(-2*b*f*tan(e + f*x) + 2*I*b*f)
+ I*B*d/(-2*b*f*tan(e + f*x) + 2*I*b*f) - I*C*c*f*x*tan(e + f*x)/(-2*b*f*ta
n(e + f*x) + 2*I*b*f) - C*c*f*x/(-2*b*f*tan(e + f*x) + 2*I*b*f) - C*c*log(t
an(e + f*x)**2 + 1)*tan(e + f*x)/(-2*b*f*tan(e + f*x) + 2*I*b*f) + I*C*c*lo
g(tan(e + f*x)**2 + 1)/(-2*b*f*tan(e + f*x) + 2*I*b*f) + I*C*c/(-2*b*f*tan(
e + f*x) + 2*I*b*f) + 3*C*d*f*x*tan(e + f*x)/(-2*b*f*tan(e + f*x) + 2*I*b*f
) - 3*I*C*d*f*x/(-2*b*f*tan(e + f*x) + 2*I*b*f) - I*C*d*log(tan(e + f*x)**2
 + 1)*tan(e + f*x)/(-2*b*f*tan(e + f*x) + 2*I*b*f) - C*d*log(tan(e + f*x)**
2 + 1)/(-2*b*f*tan(e + f*x) + 2*I*b*f) - 2*C*d*tan(e + f*x)**2/(-2*b*f*tan(
e + f*x) + 2*I*b*f) - 3*C*d/(-2*b*f*tan(e + f*x) + 2*I*b*f), Eq(a, -I*b)),
(-I*A*c*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x) + 2*I*b*f) + A*c*f*x/(2*b*f*ta
n(e + f*x) + 2*I*b*f) - I*A*c/(2*b*f*tan(e + f*x) + 2*I*b*f) + A*d*f*x*tan(
e + f*x)/(2*b*f*tan(e + f*x) + 2*I*b*f) + I*A*d*f*x/(2*b*f*tan(e + f*x) + 2
*I*b*f) - A*d/(2*b*f*tan(e + f*x) + 2*I*b*f) + B*c*f*x*tan(e + f*x)/(2*b*f*
tan(e + f*x) + 2*I*b*f) + I*B*c*f*x/(2*b*f*tan(e + f*x) + 2*I*b*f) - B*c/(2
*b*f*tan(e + f*x) + 2*I*b*f) - I*B*d*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x) +
 2*I*b*f) + B*d*f*x/(2*b*f*tan(e + f*x) + 2*I*b*f) + B*d*log(tan(e + f*x)**
2 + 1)*tan(e + f*x)/(2*b*f*tan(e + f*x) + 2*I*b*f) + I*B*d*log(tan(e + f*x)
**2 + 1)/(2*b*f*tan(e + f*x) + 2*I*b*f) + I*B*d/(2*b*f*tan(e + f*x) + 2*I*b
*f) - I*C*c*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x) + 2*I*b*f) + C*c*f*x/(2*b*
f*tan(e + f*x) + 2*I*b*f) + C*c*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*b*
f*tan(e + f*x) + 2*I*b*f) + I*C*c*log(tan(e + f*x)**2 + 1)/(2*b*f*tan(e + f
*x) + 2*I*b*f) + I*C*c/(2*b*f*tan(e + f*x) + 2*I*b*f) - 3*C*d*f*x*tan(e + f
*x)/(2*b*f*tan(e + f*x) + 2*I*b*f) - 3*I*C*d*f*x/(2*b*f*tan(e + f*x) + 2*I*
b*f) - I*C*d*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*b*f*tan(e + f*x) + 2*
I*b*f) + C*d*log(tan(e + f*x)**2 + 1)/(2*b*f*tan(e + f*x) + 2*I*b*f) + 2*C*
d*tan(e + f*x)**2/(2*b*f*tan(e + f*x) + 2*I*b*f) + 3*C*d/(2*b*f*tan(e + f*x
) + 2*I*b*f), Eq(a, I*b)), (x*(c + d*tan(e))*(A + B*tan(e) + C*tan(e)**2)/(
a + b*tan(e)), Eq(f, 0)), (2*A*a*b**2*c*f*x/(2*a**2*b**2*f + 2*b**4*f) - 2*
A*a*b**2*d*log(a/b + tan(e + f*x))/(2*a**2*b**2*f + 2*b**4*f) + A*a*b**2*d*
log(tan(e + f*x)**2 + 1)/(2*a**2*b**2*f + 2*b**4*f) + 2*A*b**3*c*log(a/b +
tan(e + f*x))/(2*a**2*b**2*f + 2*b**4*f) - A*b**3*c*log(tan(e + f*x)**2 + 1
)/(2*a**2*b**2*f + 2*b**4*f) + 2*A*b**3*d*f*x/(2*a**2*b**2*f + 2*b**4*f) +
2*B*a**2*b*d*log(a/b + tan(e + f*x))/(2*a**2*b**2*f + 2*b**4*f) - 2*B*a*b**
2*c*log(a/b + tan(e + f*x))/(2*a**2*b**2*f + 2*b**4*f) + B*a*b**2*c*log(tan
(e + f*x)**2 + 1)/(2*a**2*b**2*f + 2*b**4*f) - 2*B*a*b**2*d*f*x/(2*a**2*b**
2*f + 2*b**4*f) + 2*B*b**3*c*f*x/(2*a**2*b**2*f + 2*b**4*f) + B*b**3*d*log(
tan(e + f*x)**2 + 1)/(2*a**2*b**2*f + 2*b**4*f) - 2*C*a**3*d*log(a/b + tan(
e + f*x))/(2*a**2*b**2*f + 2*b**4*f) + 2*C*a**2*b*c*log(a/b + tan(e + f*x))
/(2*a**2*b**2*f + 2*b**4*f) + 2*C*a**2*b*d*tan(e + f*x)/(2*a**2*b**2*f + 2*
b**4*f) - 2*C*a*b**2*c*f*x/(2*a**2*b**2*f + 2*b**4*f) - C*a*b**2*d*log(tan(
e + f*x)**2 + 1)/(2*a**2*b**2*f + 2*b**4*f) + C*b**3*c*log(tan(e + f*x)**2
+ 1)/(2*a**2*b**2*f + 2*b**4*f) - 2*C*b**3*d*f*x/(2*a**2*b**2*f + 2*b**4*f)
 + 2*C*b**3*d*tan(e + f*x)/(2*a**2*b**2*f + 2*b**4*f), True))
```

Giac [A] time = 1.40056, size = 251, normalized size = 1.61

$$\frac{\frac{2\,Cd\tan\left(fx+e\right)}{b}+\frac{2\,(Aac-Cac+Bbc-Bad+Abd-Cbd)\left(fx+e\right)}{a^2+b^2}+\frac{(Bac-Abc+Cbc+Aad-Cad+Bbd)\log\left(\tan\left(fx+e\right)^2+1\right)}{a^2+b^2}+\frac{2\left(Ca^2bc-Bab^2c+Ab^3c-Ca^3d+Ba^2b\right.}{a^2b^2+}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
,x, algorithm="giac")
```

```
[Out] 1/2*(2*C*d*tan(f*x + e)/b + 2*(A*a*c - C*a*c + B*b*c - B*a*d + A*b*d - C*b*
d)*(f*x + e)/(a^2 + b^2) + (B*a*c - A*b*c + C*b*c + A*a*d - C*a*d + B*b*d)*
log(tan(f*x + e)^2 + 1)/(a^2 + b^2) + 2*(C*a^2*b*c - B*a*b^2*c + A*b^3*c -
C*a^3*d + B*a^2*b*d - A*a*b^2*d)*log(abs(b*tan(f*x + e) + a))/(a^2*b^2 + b^
4))/f
```

3.55 $\int \frac{(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=265

$$-\frac{(bc-ad)\left(Ab^2-a(bB-aC)\right)}{b^2f\left(a^2+b^2\right)(a+b\tan(e+fx))}+\frac{\left(-a^2b^2(d(A-3C)+Bc)+a^4Cd+2ab^3(Ac-Bd-cC)+b^4(Ad+Bc)\right)\log(a+b}{b^2f\left(a^2+b^2\right)^2}$$

```
[Out] ((a^2*(A*c - c*C - B*d) - b^2*(A*c - c*C - B*d) + 2*a*b*(B*c + (A - C)*d))*
x)/(a^2 + b^2)^2 + ((2*a*b*(A*c - c*C - B*d) - a^2*(B*c + (A - C)*d) + b^2*
(B*c + (A - C)*d))*Log[Cos[e + f*x]])/((a^2 + b^2)^2*f) + ((a^4*C*d + b^4*(
B*c + A*d) + 2*a*b^3*(A*c - c*C - B*d) - a^2*b^2*(B*c + (A - 3*C)*d))*Log[a
 + b*Tan[e + f*x]])/(b^2*(a^2 + b^2)^2*f) - ((A*b^2 - a*(b*B - a*C))*(b*c -
 a*d))/(b^2*(a^2 + b^2)*f*(a + b*Tan[e + f*x]))
```

Rubi [A] time = 0.473547, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {3635, 3626, 3617, 31, 3475}

$$-\frac{(bc-ad)\left(Ab^2-a(bB-aC)\right)}{b^2f\left(a^2+b^2\right)(a+b\tan(e+fx))}+\frac{\left(-a^2b^2(d(A-3C)+Bc)+a^4Cd+2ab^3(Ac-Bd-cC)+b^4(Ad+Bc)\right)\log(a+b}{b^2f\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a + b*T
an[e + f*x])^2,x]
```

```
[Out] ((a^2*(A*c - c*C - B*d) - b^2*(A*c - c*C - B*d) + 2*a*b*(B*c + (A - C)*d))*
x)/(a^2 + b^2)^2 + ((2*a*b*(A*c - c*C - B*d) - a^2*(B*c + (A - C)*d) + b^2*
(B*c + (A - C)*d))*Log[Cos[e + f*x]])/((a^2 + b^2)^2*f) + ((a^4*C*d + b^4*(
B*c + A*d) + 2*a*b^3*(A*c - c*C - B*d) - a^2*b^2*(B*c + (A - 3*C)*d))*Log[a
 + b*Tan[e + f*x]])/(b^2*(a^2 + b^2)^2*f) - ((A*b^2 - a*(b*B - a*C))*(b*c -
 a*d))/(b^2*(a^2 + b^2)*f*(a + b*Tan[e + f*x]))
```

Rule 3635

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(c^2*C - B*c*d + A*d^2)*(c +
d*Tan[e + f*x])^(n + 1))/(d^2*f*(n + 1)*(c^2 + d^2)), x] + Dist[1/(d*(c^2 +
 d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(c^
2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*Ta
n[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -
1]
```

Rule 3626

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((a*A + b*B -
a*C)*x)/(a^2 + b^2), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3617

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}dx = -\frac{\left(Ab^2-a(bB-aC)\right)(bc-ad)}{b^2\left(a^2+b^2\right)f(a+b\tan(e+fx))}+\int\frac{a^2Cd+b^2(Bc+Ad)\cdots}{\cdots}$$

$$= \frac{\left(a^2(Ac-cC-Bd)-b^2(Ac-cC-Bd)+2ab(Bc+(A\cdots\right.}{\left(a^2+b^2\right)^2}$$

$$= \frac{\left(a^2(Ac-cC-Bd)-b^2(Ac-cC-Bd)+2ab(Bc+(A\cdots\right.}{\left(a^2+b^2\right)^2}$$

$$= \frac{\left(a^2(Ac-cC-Bd)-b^2(Ac-cC-Bd)+2ab(Bc+(A\cdots\right.}{\left(a^2+b^2\right)^2}$$

Mathematica [C] time = 6.48375, size = 589, normalized size = 2.22

$$\frac{-2ia\tan^{-1}(\tan(e+fx))(a+b\tan(e+fx))\left(-a^2b^2(d(A-3C)+Bc)+a^4Cd+2ab^3(Ac-Bd-cC)+b^4(Ad+Bc)\right)+a^2\left(\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
 + b*Tan[e + f*x])^2,x]
```

```
[Out] (a^2*(2*(a + I*b)^2*(A*b^2*(c - I*d) + I*a^2*C*d + 2*a*b*C*d + b^2*((-I)*B*
c - c*C - B*d))*(e + f*x) - 2*(a^2 + b^2)^2*C*d*Log[Cos[e + f*x]] + (a^4*C*
d + b^4*(B*c + A*d) + 2*a*b^3*(A*c - c*C - B*d) - a^2*b^2*(B*c + (A - 3*C)*
d))*Log[(a*Cos[e + f*x] + b*Sin[e + f*x])^2]) + b*(2*(a + I*b)*((-I)*A*b^4*
c + I*a^4*C*d*(I + e + f*x) + a*b^3*(A*c*(1 + I*e + I*f*x) - I*c*C*(e + f*x
) - I*B*d*(e + f*x) + B*c*(I + e + f*x) + A*d*(I + e + f*x)) - I*a^2*b^2*(I
*A*c*(e + f*x) - 2*C*d*(e + f*x) + B*c*(-I + e + f*x) + A*d*(-I + e + f*x)
- I*c*C*(I + e + f*x) - I*B*d*(I + e + f*x)) + a^3*b*(c*C + d*(B + C*(I + e
 + f*x)))) - 2*a*(a^2 + b^2)^2*C*d*Log[Cos[e + f*x]] + a*(a^4*C*d + b^4*(B*
c + A*d) + 2*a*b^3*(A*c - c*C - B*d) - a^2*b^2*(B*c + (A - 3*C)*d))*Log[(a*
Cos[e + f*x] + b*Sin[e + f*x])^2])*Tan[e + f*x] - (2*I)*a*(a^4*C*d + b^4*(B
*c + A*d) + 2*a*b^3*(A*c - c*C - B*d) - a^2*b^2*(B*c + (A - 3*C)*d))*ArcTan
[Tan[e + f*x]]*(a + b*Tan[e + f*x]))/(2*a*b^2*(a^2 + b^2)^2*f*(a + b*Tan[e
```

```
+ f*x]))
```

Maple [B] time = 0.056, size = 948, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2,x)
```

```
[Out] 1/f/(a^2+b^2)^2*ln(1+tan(f*x+e)^2)*C*a*b*c-1/f/(a^2+b^2)^2*ln(1+tan(f*x+e)^
2)*A*a*b*c+1/f/(a^2+b^2)^2*ln(1+tan(f*x+e)^2)*B*a*b*d-2/f/(a^2+b^2)^2*b*ln(
a+b*tan(f*x+e))*B*a*d+1/f/(a^2+b^2)^2/b^2*ln(a+b*tan(f*x+e))*a^4*C*d-2/f/(a
^2+b^2)^2*C*arctan(tan(f*x+e))*a*b*d-1/f/b/(a^2+b^2)/(a+b*tan(f*x+e))*B*a^2
*d+1/f/b^2/(a^2+b^2)/(a+b*tan(f*x+e))*a^3*C*d-1/f/b/(a^2+b^2)/(a+b*tan(f*x+
e))*C*a^2*c-2/f/(a^2+b^2)^2*b*ln(a+b*tan(f*x+e))*C*a*c+2/f/(a^2+b^2)^2*A*ar
ctan(tan(f*x+e))*a*b*d+2/f/(a^2+b^2)^2*B*arctan(tan(f*x+e))*a*b*c+2/f/(a^2+
b^2)^2*b*ln(a+b*tan(f*x+e))*A*a*c-1/f/(a^2+b^2)^2*A*arctan(tan(f*x+e))*b^2*
c-1/f/(a^2+b^2)^2*B*arctan(tan(f*x+e))*a^2*d+1/f/(a^2+b^2)^2*B*arctan(tan(f
*x+e))*b^2*d+1/f/(a^2+b^2)^2*b^2*ln(a+b*tan(f*x+e))*A*d+1/f/(a^2+b^2)/(a+b*
tan(f*x+e))*B*a*c+1/f/(a^2+b^2)^2*b^2*ln(a+b*tan(f*x+e))*B*c+1/2/f/(a^2+b^2
)^2*ln(1+tan(f*x+e)^2)*A*a^2*d-1/f*b/(a^2+b^2)/(a+b*tan(f*x+e))*A*c-1/f/(a^
2+b^2)^2*ln(a+b*tan(f*x+e))*A*a^2*d-1/f/(a^2+b^2)^2*ln(a+b*tan(f*x+e))*B*a^
2*c+3/f/(a^2+b^2)^2*ln(a+b*tan(f*x+e))*C*a^2*d+1/f/(a^2+b^2)/(a+b*tan(f*x+e
))*A*a*d-1/2/f/(a^2+b^2)^2*ln(1+tan(f*x+e)^2)*A*b^2*d+1/2/f/(a^2+b^2)^2*ln(
1+tan(f*x+e)^2)*B*a^2*c-1/2/f/(a^2+b^2)^2*ln(1+tan(f*x+e)^2)*B*b^2*c-1/2/f/
(a^2+b^2)^2*ln(1+tan(f*x+e)^2)*C*a^2*d+1/2/f/(a^2+b^2)^2*ln(1+tan(f*x+e)^2)
*C*b^2*d+1/f/(a^2+b^2)^2*A*arctan(tan(f*x+e))*a^2*c-1/f/(a^2+b^2)^2*C*arcta
n(tan(f*x+e))*a^2*c+1/f/(a^2+b^2)^2*C*arctan(tan(f*x+e))*b^2*c
```

Maxima [A] time = 1.48858, size = 456, normalized size = 1.72

$$\frac{\frac{2\left(\left((A-C)a^2+2\,Bab-(A-C)b^2\right)c-\left(Ba^2-2\,(A-C)ab-Bb^2\right)d\right)(fx+e)}{a^4+2\,a^2b^2+b^4}-\frac{2\left(\left(Ba^2b^2-2\,(A-C)ab^3-Bb^4\right)c-\left(Ca^4-(A-3\,C)a^2b^2-2\,Bab^3+Ab^4\right)d\right)\log\left(b\tan\left(fx+e\right)+a\right)}{a^4b^2+2\,a^2b^4+b^6}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*c - (B*a^2 - 2*(A - C)*a*b -
B*b^2)*d)*(f*x + e)/(a^4 + 2*a^2*b^2 + b^4) - 2*((B*a^2*b^2 - 2*(A - C)*a*b
^3 - B*b^4)*c - (C*a^4 - (A - 3*C)*a^2*b^2 - 2*B*a*b^3 + A*b^4)*d)*log(b*ta
n(f*x + e) + a)/(a^4*b^2 + 2*a^2*b^4 + b^6) + ((B*a^2 - 2*(A - C)*a*b - B*b
^2)*c + ((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*d)*log(tan(f*x + e)^2 + 1)/(a
^4 + 2*a^2*b^2 + b^4) - 2*((C*a^2*b - B*a*b^2 + A*b^3)*c - (C*a^3 - B*a^2*b
 + A*a*b^2)*d)/(a^3*b^2 + a*b^4 + (a^2*b^3 + b^5)*tan(f*x + e)))/f
```

Fricas [B] time = 2.32446, size = 1160, normalized size = 4.38

$$2\left(\left((A-C)a^3b^2+2\,Ba^2b^3-(A-C)ab^4\right)c-\left(Ba^3b^2-2\,(A-C)a^2b^3-Bab^4\right)d\right)fx-2\left(Ca^2b^3-Bab^4+Ab^5\right)c+2\left(Ca\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(((A - C)*a^3*b^2 + 2*B*a^2*b^3 - (A - C)*a*b^4)*c - (B*a^3*b^2 - 2*
(A - C)*a^2*b^3 - B*a*b^4)*d)*f*x - 2*(C*a^2*b^3 - B*a*b^4 + A*b^5)*c + 2*(
C*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*d - ((B*a^3*b^2 - 2*(A - C)*a^2*b^3 - B*a*
b^4)*c - (C*a^5 - (A - 3*C)*a^3*b^2 - 2*B*a^2*b^3 + A*a*b^4)*d + ((B*a^2*b^
3 - 2*(A - C)*a*b^4 - B*b^5)*c - (C*a^4*b - (A - 3*C)*a^2*b^3 - 2*B*a*b^4 +
 A*b^5)*d)*tan(f*x + e))*log((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2
)/(tan(f*x + e)^2 + 1)) - ((C*a^4*b + 2*C*a^2*b^3 + C*b^5)*d*tan(f*x + e) +
 (C*a^5 + 2*C*a^3*b^2 + C*a*b^4)*d)*log(1/(tan(f*x + e)^2 + 1)) + 2*((((A -
 C)*a^2*b^3 + 2*B*a*b^4 - (A - C)*b^5)*c - (B*a^2*b^3 - 2*(A - C)*a*b^4 - B
*b^5)*d)*f*x + (C*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*c - (C*a^4*b - B*a^3*b^2 +
 A*a^2*b^3)*d)*tan(f*x + e))/((a^4*b^3 + 2*a^2*b^5 + b^7)*f*tan(f*x + e) +
(a^5*b^2 + 2*a^3*b^4 + a*b^6)*f)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e)
)**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.39674, size = 717, normalized size = 2.71

$$\frac{\frac{2\left(Aa^2c-Ca^2c+2\,Babc-Ab^2c+Cb^2c-Ba^2d+2\,Aabd-2\,Cabd+Bb^2d\right)\left(fx+e\right)}{a^4+2\,a^2b^2+b^4}+\frac{\left(Ba^2c-2\,Aabc+2\,Cabc-Bb^2c+Aa^2d-Ca^2d+2\,Babd-Ab^2d+Cb^2d\right)\log\left(\tan\left(fx+e\right)^2+\right.}{a^4+2\,a^2b^2+b^4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a^2*c - C*a^2*c + 2*B*a*b*c - A*b^2*c + C*b^2*c - B*a^2*d + 2*A*a
*b*d - 2*C*a*b*d + B*b^2*d)*(f*x + e)/(a^4 + 2*a^2*b^2 + b^4) + (B*a^2*c -
2*A*a*b*c + 2*C*a*b*c - B*b^2*c + A*a^2*d - C*a^2*d + 2*B*a*b*d - A*b^2*d +
 C*b^2*d)*log(tan(f*x + e)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2*(B*a^2*b^2*c
- 2*A*a*b^3*c + 2*C*a*b^3*c - B*b^4*c - C*a^4*d + A*a^2*b^2*d - 3*C*a^2*b^2
*d + 2*B*a*b^3*d - A*b^4*d)*log(abs(b*tan(f*x + e) + a))/(a^4*b^2 + 2*a^2*b
^4 + b^6) + 2*(B*a^2*b^2*c*tan(f*x + e) - 2*A*a*b^3*c*tan(f*x + e) + 2*C*a*
b^3*c*tan(f*x + e) - B*b^4*c*tan(f*x + e) - C*a^4*d*tan(f*x + e) + A*a^2*b^
2*d*tan(f*x + e) - 3*C*a^2*b^2*d*tan(f*x + e) + 2*B*a*b^3*d*tan(f*x + e) -
A*b^4*d*tan(f*x + e) - C*a^4*c + 2*B*a^3*b*c - 3*A*a^2*b^2*c + C*a^2*b^2*c
- A*b^4*c - B*a^4*d + 2*A*a^3*b*d - 2*C*a^3*b*d + B*a^2*b^2*d)/((a^4*b + 2*
a^2*b^3 + b^5)*(b*tan(f*x + e) + a)))/f
```

3.56 $\int \frac{(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx$

Optimal. Leaf size=320

$$-\frac{(bc-ad)\left(Ab^2-a(bB-aC)\right)}{2b^2f\left(a^2+b^2\right)(a+b\tan(e+fx))^2}-\frac{-a^2b^2(d(A-3C)+Bc)+a^4Cd+2ab^3(Ac-Bd-cC)+b^4(Ad+Bc)}{b^2f\left(a^2+b^2\right)^2(a+b\tan(e+fx))}+\frac{\left(3a^2b\right.}{}$$

```
[Out] ((a^3*(A*c - c*C - B*d) - 3*a*b^2*(A*c - c*C - B*d) + 3*a^2*b*(B*c + (A - C
)*d) - b^3*(B*c + (A - C)*d))*x)/(a^2 + b^2)^3 + ((3*a^2*b*(A*c - c*C - B*d
) - b^3*(A*c - c*C - B*d) - a^3*(B*c + (A - C)*d) + 3*a*b^2*(B*c + (A - C)*
d))*Log[a*Cos[e + f*x] + b*Sin[e + f*x]])/((a^2 + b^2)^3*f) - ((A*b^2 - a*(
b*B - a*C))*(b*c - a*d))/(2*b^2*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^2) - (a^
4*C*d + b^4*(B*c + A*d) + 2*a*b^3*(A*c - c*C - B*d) - a^2*b^2*(B*c + (A - 3
*C)*d))/(b^2*(a^2 + b^2)^2*f*(a + b*Tan[e + f*x]))
```

Rubi [A] time = 0.703237, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {3635, 3628, 3531, 3530}

$$-\frac{(bc-ad)\left(Ab^2-a(bB-aC)\right)}{2b^2f\left(a^2+b^2\right)(a+b\tan(e+fx))^2}-\frac{-a^2b^2(d(A-3C)+Bc)+a^4Cd+2ab^3(Ac-Bd-cC)+b^4(Ad+Bc)}{b^2f\left(a^2+b^2\right)^2(a+b\tan(e+fx))}+\frac{\left(3a^2b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a + b*T
an[e + f*x])^3,x]
```


```
[Out] ((a^3*(A*c - c*C - B*d) - 3*a*b^2*(A*c - c*C - B*d) + 3*a^2*b*(B*c + (A - C
)*d) - b^3*(B*c + (A - C)*d))*x)/(a^2 + b^2)^3 + ((3*a^2*b*(A*c - c*C - B*d
) - b^3*(A*c - c*C - B*d) - a^3*(B*c + (A - C)*d) + 3*a*b^2*(B*c + (A - C)*
d))*Log[a*Cos[e + f*x] + b*Sin[e + f*x]])/((a^2 + b^2)^3*f) - ((A*b^2 - a*(
b*B - a*C))*(b*c - a*d))/(2*b^2*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^2) - (a^
4*C*d + b^4*(B*c + A*d) + 2*a*b^3*(A*c - c*C - B*d) - a^2*b^2*(B*c + (A - 3
*C)*d))/(b^2*(a^2 + b^2)^2*f*(a + b*Tan[e + f*x]))
```


Rule 3635

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(c^2*C - B*c*d + A*d^2)*(c +
d*Tan[e + f*x])^(n + 1))/(d^2*f*(n + 1)*(c^2 + d^2)), x] + Dist[1/(d*(c^2 +
 d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(c^
2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*Ta
n[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -
1]
```

Rule 3628

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2
- a*b*B + a^2*C)*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 + b^2)), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
```

```
C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)(bc-ad)}{2b^2\left(a^2+b^2\right)f(a+b\tan(e+fx))^2} + \int \frac{a^2Cd+b^2(Bc+A \cdots}{\cdots}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)(bc-ad)}{2b^2\left(a^2+b^2\right)f(a+b\tan(e+fx))^2} - \frac{a^4Cd+b^4(Bc \cdots}{\cdots}$$

$$= \frac{\left(a^3(Ac-cC-Bd)-3ab^2(Ac-cC-Bd)+3a^2b(Bc+ \cdots\right.}{\left(a^2+b^2\right)^3}$$

$$= \frac{\left(a^3(Ac-cC-Bd)-3ab^2(Ac-cC-Bd)+3a^2b(Bc+ \cdots\right.}{\left(a^2+b^2\right)^3}$$

Mathematica [C] time = 6.22806, size = 331, normalized size = 1.03

$$\frac{2b(d(A-C)+Bc)\left(\frac{b\left(2a\log(a+b\tan(e+fx))-\frac{a^2+b^2}{a+b\tan(e+fx)}\right)}{\left(a^2+b^2\right)^2}-\frac{i\log(-\tan(e+fx)+i)}{2(a+ib)^2}+\frac{i\log(\tan(e+fx)+i)}{2(a-ib)^2}\right)-b(aAd+aBc-aCd-Abc+ \cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
 + b*Tan[e + f*x])^3,x]
```

```
[Out] ((b*c*C - b*B*d - a*C*d)/(a + b*Tan[e + f*x])^2 - (2*b*C*(c + d*Tan[e + f*x
]))/(a + b*Tan[e + f*x])^2 + 2*b*(B*c + (A - C)*d)*(((-I/2)*Log[I - Tan[e +
 f*x]])/(a + I*b)^2 + ((I/2)*Log[I + Tan[e + f*x]])/(a - I*b)^2 + (b*(2*a*L
og[a + b*Tan[e + f*x]] - (a^2 + b^2)/(a + b*Tan[e + f*x])))/(a^2 + b^2)^2)
- b*(-(A*b*c) + a*B*c + b*c*C + a*A*d + b*B*d - a*C*d)*(Log[I - Tan[e + f*x
]]/((-I)*a + b)^3 + Log[I + Tan[e + f*x]]/(I*a + b)^3 + (b*((6*a^2 - 2*b^2)
*Log[a + b*Tan[e + f*x]] - ((a^2 + b^2)*(5*a^2 + b^2 + 4*a*b*Tan[e + f*x]))
/(a + b*Tan[e + f*x])^2))/(a^2 + b^2)^3))/(2*b^2*f)
```

Maple [B] time = 0.068, size = 1513, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^3,x)
```

```
[Out] -1/f/(a^2+b^2)^2*b^2/(a+b*tan(f*x+e))*B*c-1/2/f/(a^2+b^2)^3*ln(1+tan(f*x+e)
^2)*a^3*C*d-1/2/f/(a^2+b^2)^3*ln(1+tan(f*x+e)^2)*C*b^3*c+1/f/(a^2+b^2)^3*A*
arctan(tan(f*x+e))*a^3*c-1/f/(a^2+b^2)^3*A*arctan(tan(f*x+e))*b^3*d-1/f/(a^
2+b^2)^3*ln(a+b*tan(f*x+e))*A*a^3*d-1/f/(a^2+b^2)^3*ln(a+b*tan(f*x+e))*A*b^
3*c-1/f/(a^2+b^2)^3*ln(a+b*tan(f*x+e))*B*a^3*c+1/f/(a^2+b^2)^3*ln(a+b*tan(f
*x+e))*B*b^3*d+1/2/f/(a^2+b^2)/(a+b*tan(f*x+e))^2*B*a*c+1/f/(a^2+b^2)^3*ln(
a+b*tan(f*x+e))*a^3*C*d+1/f/(a^2+b^2)^3*ln(a+b*tan(f*x+e))*C*b^3*c+1/f/(a^2
+b^2)^2/(a+b*tan(f*x+e))*A*a^2*d+1/f/(a^2+b^2)^2/(a+b*tan(f*x+e))*B*a^2*c+3
/f/(a^2+b^2)^3*C*arctan(tan(f*x+e))*a*b^2*c-3/2/f/(a^2+b^2)^3*ln(1+tan(f*x+
e)^2)*A*a^2*b*c-3/2/f/(a^2+b^2)^3*ln(1+tan(f*x+e)^2)*B*a*b^2*c+3/2/f/(a^2+b
^2)^3*ln(1+tan(f*x+e)^2)*C*a^2*b*c-3/f/(a^2+b^2)^2/(a+b*tan(f*x+e))*C*a^2*d
+1/2/f/(a^2+b^2)/(a+b*tan(f*x+e))^2*A*a*d-1/f/(a^2+b^2)^3*B*arctan(tan(f*x+
e))*a^3*d-1/f/(a^2+b^2)^3*B*arctan(tan(f*x+e))*b^3*c-1/f/(a^2+b^2)^3*C*arct
an(tan(f*x+e))*a^3*c+1/f/(a^2+b^2)^3*C*arctan(tan(f*x+e))*b^3*d+1/2/f/(a^2+
b^2)^3*ln(1+tan(f*x+e)^2)*A*a^3*d+1/2/f/(a^2+b^2)^3*ln(1+tan(f*x+e)^2)*A*b^
3*c+1/2/f/(a^2+b^2)^3*ln(1+tan(f*x+e)^2)*B*a^3*c-1/2/f/(a^2+b^2)^3*ln(1+tan
(f*x+e)^2)*B*b^3*d-1/2/f*b/(a^2+b^2)/(a+b*tan(f*x+e))^2*A*c-1/f/(a^2+b^2)^2
*b^2/(a+b*tan(f*x+e))*A*d+3/2/f/(a^2+b^2)^3*ln(1+tan(f*x+e)^2)*C*a*b^2*d+3/
f/(a^2+b^2)^3*A*arctan(tan(f*x+e))*a^2*b*d-3/f/(a^2+b^2)^3*A*arctan(tan(f*x
+e))*a*b^2*c+3/f/(a^2+b^2)^3*B*arctan(tan(f*x+e))*a^2*b*c-1/2/f/b/(a^2+b^2)
/(a+b*tan(f*x+e))^2*B*a^2*d+1/2/f/b^2/(a^2+b^2)/(a+b*tan(f*x+e))^2*a^3*C*d-
1/2/f/b/(a^2+b^2)/(a+b*tan(f*x+e))^2*C*a^2*c-2/f/(a^2+b^2)^2*b/(a+b*tan(f*x
+e))*A*a*c-1/f/(a^2+b^2)^2/b^2/(a+b*tan(f*x+e))*a^4*C*d-3/f/(a^2+b^2)^3*ln(
a+b*tan(f*x+e))*B*a^2*b*d+3/f/(a^2+b^2)^3*ln(a+b*tan(f*x+e))*B*a*b^2*c-3/f/
(a^2+b^2)^3*ln(a+b*tan(f*x+e))*C*a^2*b*c-3/f/(a^2+b^2)^3*ln(a+b*tan(f*x+e))
*C*a*b^2*d-3/f/(a^2+b^2)^3*C*arctan(tan(f*x+e))*a^2*b*d+2/f/(a^2+b^2)^2*b/(
a+b*tan(f*x+e))*B*a*d+3/f/(a^2+b^2)^3*B*arctan(tan(f*x+e))*a*b^2*d-3/2/f/(a
^2+b^2)^3*ln(1+tan(f*x+e)^2)*A*a*b^2*d+3/2/f/(a^2+b^2)^3*ln(1+tan(f*x+e)^2)
*B*a^2*b*d+2/f/(a^2+b^2)^2*b/(a+b*tan(f*x+e))*C*a*c+3/f/(a^2+b^2)^3*ln(a+b*
tan(f*x+e))*A*a^2*b*c+3/f/(a^2+b^2)^3*ln(a+b*tan(f*x+e))*A*a*b^2*d
```

Maxima [A] time = 1.52346, size = 775, normalized size = 2.42

$$\frac{\frac{2\left(\left(\left((A-C)a^3+3\,Ba^2b-3\,(A-C)ab^2-Bb^3\right)c-\left(Ba^3-3\,(A-C)a^2b-3\,Bab^2+(A-C)b^3\right)d\right)\left(fx+e\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{2\left(\left(Ba^3-3\,(A-C)a^2b-3\,Bab^2+(A-C)b^3\right)c+\left((A-C)a^3+3\,Ba^2b-3\right.\right.}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(((A - C)*a^3 + 3*B*a^2*b - 3*(A - C)*a*b^2 - B*b^3)*c - (B*a^3 - 3*
(A - C)*a^2*b - 3*B*a*b^2 + (A - C)*b^3)*d)*(f*x + e)/(a^6 + 3*a^4*b^2 + 3*
a^2*b^4 + b^6) - 2*((B*a^3 - 3*(A - C)*a^2*b - 3*B*a*b^2 + (A - C)*b^3)*c +
 ((A - C)*a^3 + 3*B*a^2*b - 3*(A - C)*a*b^2 - B*b^3)*d)*log(b*tan(f*x + e)
+ a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + ((B*a^3 - 3*(A - C)*a^2*b - 3*B*
a*b^2 + (A - C)*b^3)*c + ((A - C)*a^3 + 3*B*a^2*b - 3*(A - C)*a*b^2 - B*b^3
```

```
)*d)*log(tan(f*x + e)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - ((C*a^4*
b - 3*B*a^3*b^2 + (5*A - 3*C)*a^2*b^3 + B*a*b^4 + A*b^5)*c + (C*a^5 + B*a^4
*b - (3*A - 5*C)*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*d - 2*((B*a^2*b^3 - 2*(A
- C)*a*b^4 - B*b^5)*c - (C*a^4*b - (A - 3*C)*a^2*b^3 - 2*B*a*b^4 + A*b^5)*d
)*tan(f*x + e))/(a^6*b^2 + 2*a^4*b^4 + a^2*b^6 + (a^4*b^4 + 2*a^2*b^6 + b^8
)*tan(f*x + e)^2 + 2*(a^5*b^3 + 2*a^3*b^5 + a*b^7)*tan(f*x + e)))/f
```

Fricas [B] time = 1.32655, size = 2072, normalized size = 6.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(((A - C)*a^5 + 3*B*a^4*b - 3*(A - C)*a^3*b^2 - B*a^2*b^3)*c - (B*a^
5 - 3*(A - C)*a^4*b - 3*B*a^3*b^2 + (A - C)*a^2*b^3)*d)*f*x + (2*(((A - C)*
a^3*b^2 + 3*B*a^2*b^3 - 3*(A - C)*a*b^4 - B*b^5)*c - (B*a^3*b^2 - 3*(A - C)
*a^2*b^3 - 3*B*a*b^4 + (A - C)*b^5)*d)*f*x + (C*a^4*b - 3*B*a^3*b^2 + 5*(A
- C)*a^2*b^3 + 3*B*a*b^4 - A*b^5)*c + (C*a^5 + B*a^4*b - (3*A - 7*C)*a^3*b^
2 - 5*B*a^2*b^3 + 3*A*a*b^4)*d)*tan(f*x + e)^2 - (3*C*a^4*b - 5*B*a^3*b^2 +
 (7*A - 3*C)*a^2*b^3 + B*a*b^4 + A*b^5)*c + (C*a^5 - 3*B*a^4*b + 5*(A - C)*
a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4)*d - (((B*a^3*b^2 - 3*(A - C)*a^2*b^3 - 3*B
*a*b^4 + (A - C)*b^5)*c + ((A - C)*a^3*b^2 + 3*B*a^2*b^3 - 3*(A - C)*a*b^4
- B*b^5)*d)*tan(f*x + e)^2 + (B*a^5 - 3*(A - C)*a^4*b - 3*B*a^3*b^2 + (A -
C)*a^2*b^3)*c + ((A - C)*a^5 + 3*B*a^4*b - 3*(A - C)*a^3*b^2 - B*a^2*b^3)*d
 + 2*((B*a^4*b - 3*(A - C)*a^3*b^2 - 3*B*a^2*b^3 + (A - C)*a*b^4)*c + ((A -
 C)*a^4*b + 3*B*a^3*b^2 - 3*(A - C)*a^2*b^3 - B*a*b^4)*d)*tan(f*x + e))*log
((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)/(tan(f*x + e)^2 + 1)) + 2*
(2*(((A - C)*a^4*b + 3*B*a^3*b^2 - 3*(A - C)*a^2*b^3 - B*a*b^4)*c - (B*a^4*
b - 3*(A - C)*a^3*b^2 - 3*B*a^2*b^3 + (A - C)*a*b^4)*d)*f*x + (C*a^5 - 2*B*
a^4*b + 3*(A - C)*a^3*b^2 + 3*B*a^2*b^3 - (3*A - 2*C)*a*b^4 - B*b^5)*c + (B
*a^5 - (2*A - 3*C)*a^4*b - 3*B*a^3*b^2 + 3*(A - C)*a^2*b^3 + 2*B*a*b^4 - A*
b^5)*d)*tan(f*x + e))/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*f*tan(f*x +
e)^2 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*f*tan(f*x + e) + (a^8 + 3*
a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*f)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e)
)**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.49648, size = 1400, normalized size = 4.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a^3*c - C*a^3*c + 3*B*a^2*b*c - 3*A*a*b^2*c + 3*C*a*b^2*c - B*b^3
*c - B*a^3*d + 3*A*a^2*b*d - 3*C*a^2*b*d + 3*B*a*b^2*d - A*b^3*d + C*b^3*d)
*(f*x + e)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (B*a^3*c - 3*A*a^2*b*c + 3
*C*a^2*b*c - 3*B*a*b^2*c + A*b^3*c - C*b^3*c + A*a^3*d - C*a^3*d + 3*B*a^2*
b*d - 3*A*a*b^2*d + 3*C*a*b^2*d - B*b^3*d)*log(tan(f*x + e)^2 + 1)/(a^6 + 3
*a^4*b^2 + 3*a^2*b^4 + b^6) - 2*(B*a^3*b*c - 3*A*a^2*b^2*c + 3*C*a^2*b^2*c
- 3*B*a*b^3*c + A*b^4*c - C*b^4*c + A*a^3*b*d - C*a^3*b*d + 3*B*a^2*b^2*d -
 3*A*a*b^3*d + 3*C*a*b^3*d - B*b^4*d)*log(abs(b*tan(f*x + e) + a))/(a^6*b +
 3*a^4*b^3 + 3*a^2*b^5 + b^7) + (3*B*a^3*b^4*c*tan(f*x + e)^2 - 9*A*a^2*b^5
*c*tan(f*x + e)^2 + 9*C*a^2*b^5*c*tan(f*x + e)^2 - 9*B*a*b^6*c*tan(f*x + e)
^2 + 3*A*b^7*c*tan(f*x + e)^2 - 3*C*b^7*c*tan(f*x + e)^2 + 3*A*a^3*b^4*d*ta
n(f*x + e)^2 - 3*C*a^3*b^4*d*tan(f*x + e)^2 + 9*B*a^2*b^5*d*tan(f*x + e)^2
- 9*A*a*b^6*d*tan(f*x + e)^2 + 9*C*a*b^6*d*tan(f*x + e)^2 - 3*B*b^7*d*tan(f
*x + e)^2 + 8*B*a^4*b^3*c*tan(f*x + e) - 22*A*a^3*b^4*c*tan(f*x + e) + 22*C
*a^3*b^4*c*tan(f*x + e) - 18*B*a^2*b^5*c*tan(f*x + e) + 2*A*a*b^6*c*tan(f*x
 + e) - 2*C*a*b^6*c*tan(f*x + e) - 2*B*b^7*c*tan(f*x + e) - 2*C*a^6*b*d*tan
(f*x + e) + 8*A*a^4*b^3*d*tan(f*x + e) - 14*C*a^4*b^3*d*tan(f*x + e) + 22*B
*a^3*b^4*d*tan(f*x + e) - 18*A*a^2*b^5*d*tan(f*x + e) + 12*C*a^2*b^5*d*tan(
f*x + e) - 2*B*a*b^6*d*tan(f*x + e) - 2*A*b^7*d*tan(f*x + e) - C*a^6*b*c +
6*B*a^5*b^2*c - 14*A*a^4*b^3*c + 11*C*a^4*b^3*c - 7*B*a^3*b^4*c - 3*A*a^2*b
^5*c - B*a*b^6*c - A*b^7*c - C*a^7*d - B*a^6*b*d + 6*A*a^5*b^2*d - 9*C*a^5*
b^2*d + 11*B*a^4*b^3*d - 7*A*a^3*b^4*d + 4*C*a^3*b^4*d - A*a*b^6*d)/((a^6*b
^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*(b*tan(f*x + e) + a)^2))/f
```

3.57 $\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^2 \left(A + B\tan(e + fx) + \right.$

Optimal. Leaf size=661

$$\frac{(c+d\tan(e+fx))^3 \left(-3a^2bd^2(3cC-16Bd)+4a^3Cd^3+3ab^2d\left(20d^2(A-C)-5Bcd+2c^2C\right)+b^3\left(-\left(5cd^2(A-C)-2Bc\right.\right.\right.}{60d^4f}$$

```
[Out] -((a^3*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - 3*a*b^2*(c^2*C + 2*B*c*d
 - C*d^2 - A*(c^2 - d^2)) + 3*a^2*b*(2*c*(A - C)*d + B*(c^2 - d^2)) - b^3*(
2*c*(A - C)*d + B*(c^2 - d^2)))*x) + ((3*a^2*b*(c^2*C + 2*B*c*d - C*d^2 - A
*(c^2 - d^2)) - b^3*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - a^3*(2*c*(A
 - C)*d + B*(c^2 - d^2)) + 3*a*b^2*(2*c*(A - C)*d + B*(c^2 - d^2)))*Log[Cos
[e + f*x]])/f + (d*(3*a^2*b*(A*c - c*C - B*d) - b^3*(A*c - c*C - B*d) + a^3
*(B*c + (A - C)*d) - 3*a*b^2*(B*c + (A - C)*d))*Tan[e + f*x])/f + ((a^3*B -
 3*a*b^2*B + 3*a^2*b*(A - C) - b^3*(A - C))*(c + d*Tan[e + f*x])^2)/(2*f) +
 ((4*a^3*C*d^3 - 3*a^2*b*d^2*(3*c*C - 16*B*d) + 3*a*b^2*d*(2*c^2*C - 5*B*c*
d + 20*(A - C)*d^2) - b^3*(c^3*C - 2*B*c^2*d + 5*c*(A - C)*d^2 + 20*B*d^3))
*(c + d*Tan[e + f*x])^3)/(60*d^4*f) + (b*(5*b*(A*b + a*B - b*C)*d^2 + (b*c
- a*d)*(b*c*C - 2*b*B*d - a*C*d))*Tan[e + f*x]*(c + d*Tan[e + f*x])^3)/(20*
d^3*f) - ((b*c*C - 2*b*B*d - a*C*d)*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f
*x])^3)/(10*d^2*f) + (C*(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^3)/(6*d
*f)
```

Rubi [A] time = 2.38359, antiderivative size = 661, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3647, 3637, 3630, 3528, 3525, 3475}

$$\frac{(c+d\tan(e+fx))^3 \left(-3a^2bd^2(3cC-16Bd)+4a^3Cd^3+3ab^2d\left(20d^2(A-C)-5Bcd+2c^2C\right)+b^3\left(-\left(5cd^2(A-C)-2Bc\right.\right.\right.}{60d^4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*T
an[e + f*x]^2),x]
```

```
[Out] -((a^3*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - 3*a*b^2*(c^2*C + 2*B*c*d
 - C*d^2 - A*(c^2 - d^2)) + 3*a^2*b*(2*c*(A - C)*d + B*(c^2 - d^2)) - b^3*(
2*c*(A - C)*d + B*(c^2 - d^2)))*x) + ((3*a^2*b*(c^2*C + 2*B*c*d - C*d^2 - A
*(c^2 - d^2)) - b^3*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - a^3*(2*c*(A
 - C)*d + B*(c^2 - d^2)) + 3*a*b^2*(2*c*(A - C)*d + B*(c^2 - d^2)))*Log[Cos
[e + f*x]])/f + (d*(3*a^2*b*(A*c - c*C - B*d) - b^3*(A*c - c*C - B*d) + a^3
*(B*c + (A - C)*d) - 3*a*b^2*(B*c + (A - C)*d))*Tan[e + f*x])/f + ((a^3*B -
 3*a*b^2*B + 3*a^2*b*(A - C) - b^3*(A - C))*(c + d*Tan[e + f*x])^2)/(2*f) +
 ((4*a^3*C*d^3 - 3*a^2*b*d^2*(3*c*C - 16*B*d) + 3*a*b^2*d*(2*c^2*C - 5*B*c*
d + 20*(A - C)*d^2) - b^3*(c^3*C - 2*B*c^2*d + 5*c*(A - C)*d^2 + 20*B*d^3))
*(c + d*Tan[e + f*x])^3)/(60*d^4*f) + (b*(5*b*(A*b + a*B - b*C)*d^2 + (b*c
- a*d)*(b*c*C - 2*b*B*d - a*C*d))*Tan[e + f*x]*(c + d*Tan[e + f*x])^3)/(20*
d^3*f) - ((b*c*C - 2*b*B*d - a*C*d)*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f
*x])^3)/(10*d^2*f) + (C*(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^3)/(6*d
*f)
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
```

```
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx = \frac{C(a+b\tan(e+fx))^3(c+d\tan(\ldots}{6df}$$

$$= -\frac{(bcC-2bBd-aCd)(a+b\tan(\ldots}{10d^2\ldots}$$

$$= \frac{b\left(5b(Ab+aB-bC)d^2+(bc-a\ldots\right.}{\ldots}$$

$$= \frac{\left(4a^3Cd^3-3a^2bd^2(3cC-16Bd)+\ldots\right.}{\ldots}$$

$$= \frac{\left(a^3B-3ab^2B+3a^2b(A-C)-b^3\ldots\right.}{2f\ldots}$$

$$= -\left(a^3\left(c^2C+2Bcd-Cd^2-A\left(c^2\ldots\right.\right.\right.$$

$$= -\left(a^3\left(c^2C+2Bcd-Cd^2-A\left(c^2\ldots\right.\right.\right.$$

Mathematica [C] time = 6.64793, size = 573, normalized size = 0.87

$$\frac{C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^3}{6df}+\frac{-\frac{3(-aCd-2bBd+bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}{5df}+\frac{3b\tan(e+fx)(c+d\tan(e+fx))^3\left(5bd^2(aB+Ab\ldots\right.}{2df}\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x]
 + C*Tan[e + f*x]^2),x]
```

```
[Out] (C*(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^3)/(6*d*f) + ((-3*(b*c*C - 2
*b*B*d - a*C*d)*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^3)/(5*d*f) + ((
3*b*(5*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(b*c*C - 2*b*B*d - a*C*d))*Tan
[e + f*x]*(c + d*Tan[e + f*x])^3)/(2*d*f) - (((-24*a*d*(5*b*(A*b + a*B - b*
C)*d^2 + (b*c - a*d)*(b*c*C - 2*b*B*d - a*C*d)) + b*(-120*(a^2*B - b^2*B +
2*a*b*(A - C))*d^3 + 6*c*(5*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(b*c*C -
2*b*B*d - a*C*d))))*(c + d*Tan[e + f*x])^3)/(3*d*f) - (60*(d^2*(3*a^2*b*(A*
c - c*C + B*d) - b^3*(A*c - c*C + B*d) + a^3*(B*c - (A - C)*d) - 3*a*b^2*(B
*c - (A - C)*d))*(I*(c + I*d)^2*Log[I - Tan[e + f*x]] - I*(c - I*d)^2*Log[I
 + Tan[e + f*x]] - 2*d^2*Tan[e + f*x]) + (a^3*B - 3*a*b^2*B + 3*a^2*b*(A -
C) - b^3*(A - C))*d^2*((I*c - d)^3*Log[I - Tan[e + f*x]] - (I*c + d)^3*Log[
I + Tan[e + f*x]] + 6*c*d^2*Tan[e + f*x] + d^3*Tan[e + f*x]^2)))/f)/(4*d))/
(5*d))/(6*d)
```

Maple [B] time = 0.024, size = 1807, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x
)
```

```
[Out] 1/2/f*B*tan(f*x+e)^4*b^3*c*d+3/2/f*ln(1+tan(f*x+e)^2)*B*a*b^2*d^2+1/f*ln(1+
tan(f*x+e)^2)*B*b^3*c*d+1/f*ln(1+tan(f*x+e)^2)*A*a^3*c*d+1/f*A*tan(f*x+e)^3
*a*b^2*d^2+1/f*C*tan(f*x+e)^2*a^3*c*d-2/f*C*arctan(tan(f*x+e))*b^3*c*d+3/f*
C*arctan(tan(f*x+e))*a*b^2*c^2+1/f*B*tan(f*x+e)^3*a^2*b*d^2+3/5/f*C*tan(f*x
+e)^5*a*b^2*d^2+1/f*C*arctan(tan(f*x+e))*a^3*d^2-1/f*B*b^3*c^2*tan(f*x+e)+1
/f*B*b^3*d^2*tan(f*x+e)-1/2/f*ln(1+tan(f*x+e)^2)*A*b^3*c^2+1/2/f*ln(1+tan(f
*x+e)^2)*A*b^3*d^2+1/2/f*ln(1+tan(f*x+e)^2)*B*a^3*c^2-1/2/f*ln(1+tan(f*x+e)
^2)*B*a^3*d^2+1/2/f*ln(1+tan(f*x+e)^2)*C*b^3*c^2-1/2/f*ln(1+tan(f*x+e)^2)*C
*b^3*d^2+1/f*C*a^3*c^2*tan(f*x+e)+1/2/f*A*tan(f*x+e)^2*b^3*c^2-1/2/f*A*tan(
f*x+e)^2*b^3*d^2+1/6/f*C*b^3*d^2*tan(f*x+e)^6+1/f*A*a^3*d^2*tan(f*x+e)+1/f*
A*arctan(tan(f*x+e))*a^3*c^2-1/f*A*arctan(tan(f*x+e))*a^3*d^2+1/f*B*arctan(
tan(f*x+e))*b^3*c^2-3/2/f*ln(1+tan(f*x+e)^2)*A*a^2*b*d^2-3/2/f*ln(1+tan(f*x
+e)^2)*B*a*b^2*c^2+2/f*C*tan(f*x+e)^3*a^2*b*c*d+6/f*A*a^2*b*c*d*tan(f*x+e)+
2/f*B*tan(f*x+e)^3*a*b^2*c*d-6/f*C*a^2*b*c*d*tan(f*x+e)+3/2/f*C*tan(f*x+e)^
4*a*b^2*c*d+3/f*B*tan(f*x+e)^2*a^2*b*c*d-3/f*C*tan(f*x+e)^2*a*b^2*c*d-3/f*l
n(1+tan(f*x+e)^2)*A*a*b^2*c*d+3/f*A*tan(f*x+e)^2*a*b^2*c*d-6/f*A*arctan(tan
(f*x+e))*a^2*b*c*d+6/f*B*arctan(tan(f*x+e))*a*b^2*c*d+6/f*C*arctan(tan(f*x+
e))*a^2*b*c*d-6/f*B*a*b^2*c*d*tan(f*x+e)-3/f*ln(1+tan(f*x+e)^2)*B*a^2*b*c*d
+3/f*ln(1+tan(f*x+e)^2)*C*a*b^2*c*d+2/f*A*arctan(tan(f*x+e))*b^3*c*d-3/2/f*
C*tan(f*x+e)^2*a^2*b*d^2+1/2/f*B*tan(f*x+e)^2*a^3*d^2+1/4/f*A*tan(f*x+e)^4*
b^3*d^2-1/2/f*C*tan(f*x+e)^2*b^3*c^2+1/2/f*C*tan(f*x+e)^2*b^3*d^2+1/3/f*C*t
an(f*x+e)^3*a^3*d^2+1/5/f*B*tan(f*x+e)^5*b^3*d^2+1/3/f*B*tan(f*x+e)^3*b^3*c
^2-1/f*B*arctan(tan(f*x+e))*b^3*d^2-1/f*C*arctan(tan(f*x+e))*a^3*c^2-1/f*C*
a^3*d^2*tan(f*x+e)-1/3/f*B*tan(f*x+e)^3*b^3*d^2+1/4/f*C*tan(f*x+e)^4*b^3*c^
2-1/4/f*C*tan(f*x+e)^4*b^3*d^2+2/5/f*C*tan(f*x+e)^5*b^3*c*d+1/f*C*tan(f*x+e
)^3*a*b^2*c^2+3/2/f*C*tan(f*x+e)^2*a^2*b*c^2-3/f*C*arctan(tan(f*x+e))*a*b^2
*d^2+3/4/f*C*tan(f*x+e)^4*a^2*b*d^2+2/3/f*A*tan(f*x+e)^3*b^3*c*d+2/f*C*b^3*
c*d*tan(f*x+e)-1/f*C*tan(f*x+e)^3*a*b^2*d^2-2/3/f*C*tan(f*x+e)^3*b^3*c*d-3/
f*C*a*b^2*c^2*tan(f*x+e)-2/f*B*arctan(tan(f*x+e))*a^3*c*d-3/f*B*arctan(tan(
f*x+e))*a^2*b*c^2+2/f*B*a^3*c*d*tan(f*x+e)+3/4/f*B*tan(f*x+e)^4*a*b^2*d^2+3
/2/f*A*tan(f*x+e)^2*a^2*b*d^2-3/f*A*arctan(tan(f*x+e))*a*b^2*c^2+3/f*A*arct
an(tan(f*x+e))*a*b^2*d^2+3/f*B*arctan(tan(f*x+e))*a^2*b*d^2+3/2/f*B*tan(f*x
+e)^2*a*b^2*c^2+3/f*B*a^2*b*c^2*tan(f*x+e)-3/2/f*B*tan(f*x+e)^2*a*b^2*d^2+3
/f*A*a*b^2*c^2*tan(f*x+e)-3/f*A*a*b^2*d^2*tan(f*x+e)-2/f*A*b^3*c*d*tan(f*x+
e)+3/2/f*ln(1+tan(f*x+e)^2)*A*a^2*b*c^2-1/f*ln(1+tan(f*x+e)^2)*C*a^3*c*d-3/
2/f*ln(1+tan(f*x+e)^2)*C*a^2*b*c^2+3/2/f*ln(1+tan(f*x+e)^2)*C*a^2*b*d^2+3/f
*C*a*b^2*d^2*tan(f*x+e)-3/f*B*a^2*b*d^2*tan(f*x+e)-1/f*B*tan(f*x+e)^2*b^3*c
*d
```

<u>Maxima</u> [A] time = 1.48842, size = 933, normalized size = 1.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e
)^2),x, algorithm="maxima")
```

```
[Out] 1/60*(10*C*b^3*d^2*tan(f*x + e)^6 + 12*(2*C*b^3*c*d + (3*C*a*b^2 + B*b^3)*d
^2)*tan(f*x + e)^5 + 15*(C*b^3*c^2 + 2*(3*C*a*b^2 + B*b^3)*c*d + (3*C*a^2*b
 + 3*B*a*b^2 + (A - C)*b^3)*d^2)*tan(f*x + e)^4 + 20*((3*C*a*b^2 + B*b^3)*c
^2 + 2*(3*C*a^2*b + 3*B*a*b^2 + (A - C)*b^3)*c*d + (C*a^3 + 3*B*a^2*b + 3*(
A - C)*a*b^2 - B*b^3)*d^2)*tan(f*x + e)^3 + 30*((3*C*a^2*b + 3*B*a*b^2 + (A
 - C)*b^3)*c^2 + 2*(C*a^3 + 3*B*a^2*b + 3*(A - C)*a*b^2 - B*b^3)*c*d + (B*a
^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*d^2)*tan(f*x + e)^2 + 60*((
(A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*c^2 - 2*(B*a^3 + 3*(A -
C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*c*d - ((A - C)*a^3 - 3*B*a^2*b - 3*(A -
```

```
C)*a*b^2 + B*b^3)*d^2)*(f*x + e) + 30*((B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^
2 - (A - C)*b^3)*c^2 + 2*((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3
)*c*d - (B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*d^2)*log(tan(f*
x + e)^2 + 1) + 60*((C*a^3 + 3*B*a^2*b + 3*(A - C)*a*b^2 - B*b^3)*c^2 + 2*(
B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*c*d + ((A - C)*a^3 - 3*B
*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*d^2)*tan(f*x + e))/f
```

Fricas [A] time = 1.28231, size = 1490, normalized size = 2.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e
)^2),x, algorithm="fricas")
```

```
[Out] 1/60*(10*C*b^3*d^2*tan(f*x + e)^6 + 12*(2*C*b^3*c*d + (3*C*a*b^2 + B*b^3)*d
^2)*tan(f*x + e)^5 + 15*(C*b^3*c^2 + 2*(3*C*a*b^2 + B*b^3)*c*d + (3*C*a^2*b
 + 3*B*a*b^2 + (A - C)*b^3)*d^2)*tan(f*x + e)^4 + 20*((3*C*a*b^2 + B*b^3)*c
^2 + 2*(3*C*a^2*b + 3*B*a*b^2 + (A - C)*b^3)*c*d + (C*a^3 + 3*B*a^2*b + 3*(
A - C)*a*b^2 - B*b^3)*d^2)*tan(f*x + e)^3 + 60*(((A - C)*a^3 - 3*B*a^2*b -
3*(A - C)*a*b^2 + B*b^3)*c^2 - 2*(B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A
- C)*b^3)*c*d - ((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*d^2)*f*
x + 30*((3*C*a^2*b + 3*B*a*b^2 + (A - C)*b^3)*c^2 + 2*(C*a^3 + 3*B*a^2*b +
3*(A - C)*a*b^2 - B*b^3)*c*d + (B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A -
C)*b^3)*d^2)*tan(f*x + e)^2 - 30*((B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A
 - C)*b^3)*c^2 + 2*((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*c*d
- (B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*d^2)*log(1/(tan(f*x +
 e)^2 + 1)) + 60*((C*a^3 + 3*B*a^2*b + 3*(A - C)*a*b^2 - B*b^3)*c^2 + 2*(B*
a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*c*d + ((A - C)*a^3 - 3*B*a
^2*b - 3*(A - C)*a*b^2 + B*b^3)*d^2)*tan(f*x + e))/f
```

Sympy [A] time = 6.12102, size = 1819, normalized size = 2.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3*(c+d*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x
+e)**2),x)
```

```
[Out] Piecewise((A*a**3*c**2*x + A*a**3*c*d*log(tan(e + f*x)**2 + 1)/f - A*a**3*d
**2*x + A*a**3*d**2*tan(e + f*x)/f + 3*A*a**2*b*c**2*log(tan(e + f*x)**2 +
1)/(2*f) - 6*A*a**2*b*c*d*x + 6*A*a**2*b*c*d*tan(e + f*x)/f - 3*A*a**2*b*d*
*2*log(tan(e + f*x)**2 + 1)/(2*f) + 3*A*a**2*b*d**2*tan(e + f*x)**2/(2*f) -
 3*A*a*b**2*c**2*x + 3*A*a*b**2*c**2*tan(e + f*x)/f - 3*A*a*b**2*c*d*log(ta
n(e + f*x)**2 + 1)/f + 3*A*a*b**2*c*d*tan(e + f*x)**2/f + 3*A*a*b**2*d**2*x
 + A*a*b**2*d**2*tan(e + f*x)**3/f - 3*A*a*b**2*d**2*tan(e + f*x)/f - A*b**
3*c**2*log(tan(e + f*x)**2 + 1)/(2*f) + A*b**3*c**2*tan(e + f*x)**2/(2*f) +
 2*A*b**3*c*d*x + 2*A*b**3*c*d*tan(e + f*x)**3/(3*f) - 2*A*b**3*c*d*tan(e +
 f*x)/f + A*b**3*d**2*log(tan(e + f*x)**2 + 1)/(2*f) + A*b**3*d**2*tan(e +
f*x)**4/(4*f) - A*b**3*d**2*tan(e + f*x)**2/(2*f) + B*a**3*c**2*log(tan(e +
 f*x)**2 + 1)/(2*f) - 2*B*a**3*c*d*x + 2*B*a**3*c*d*tan(e + f*x)/f - B*a**3
*d**2*log(tan(e + f*x)**2 + 1)/(2*f) + B*a**3*d**2*tan(e + f*x)**2/(2*f) -
```

```
3*B*a**2*b*c**2*x + 3*B*a**2*b*c**2*tan(e + f*x)/f - 3*B*a**2*b*c*d*log(tan
(e + f*x)**2 + 1)/f + 3*B*a**2*b*c*d*tan(e + f*x)**2/f + 3*B*a**2*b*d**2*x
+ B*a**2*b*d**2*tan(e + f*x)**3/f - 3*B*a**2*b*d**2*tan(e + f*x)/f - 3*B*a*
b**2*c**2*log(tan(e + f*x)**2 + 1)/(2*f) + 3*B*a*b**2*c**2*tan(e + f*x)**2/
(2*f) + 6*B*a*b**2*c*d*x + 2*B*a*b**2*c*d*tan(e + f*x)**3/f - 6*B*a*b**2*c*
d*tan(e + f*x)/f + 3*B*a*b**2*d**2*log(tan(e + f*x)**2 + 1)/(2*f) + 3*B*a*b
**2*d**2*tan(e + f*x)**4/(4*f) - 3*B*a*b**2*d**2*tan(e + f*x)**2/(2*f) + B*
b**3*c**2*x + B*b**3*c**2*tan(e + f*x)**3/(3*f) - B*b**3*c**2*tan(e + f*x)/
f + B*b**3*c*d*log(tan(e + f*x)**2 + 1)/f + B*b**3*c*d*tan(e + f*x)**4/(2*f
) - B*b**3*c*d*tan(e + f*x)**2/f - B*b**3*d**2*x + B*b**3*d**2*tan(e + f*x)
**5/(5*f) - B*b**3*d**2*tan(e + f*x)**3/(3*f) + B*b**3*d**2*tan(e + f*x)/f
- C*a**3*c**2*x + C*a**3*c**2*tan(e + f*x)/f - C*a**3*c*d*log(tan(e + f*x)*
*2 + 1)/f + C*a**3*c*d*tan(e + f*x)**2/f + C*a**3*d**2*x + C*a**3*d**2*tan(
e + f*x)**3/(3*f) - C*a**3*d**2*tan(e + f*x)/f - 3*C*a**2*b*c**2*log(tan(e
+ f*x)**2 + 1)/(2*f) + 3*C*a**2*b*c**2*tan(e + f*x)**2/(2*f) + 6*C*a**2*b*c
*d*x + 2*C*a**2*b*c*d*tan(e + f*x)**3/f - 6*C*a**2*b*c*d*tan(e + f*x)/f + 3
*C*a**2*b*d**2*log(tan(e + f*x)**2 + 1)/(2*f) + 3*C*a**2*b*d**2*tan(e + f*x
)**4/(4*f) - 3*C*a**2*b*d**2*tan(e + f*x)**2/(2*f) + 3*C*a*b**2*c**2*x + C*
a*b**2*c**2*tan(e + f*x)**3/f - 3*C*a*b**2*c**2*tan(e + f*x)/f + 3*C*a*b**2
*c*d*log(tan(e + f*x)**2 + 1)/f + 3*C*a*b**2*c*d*tan(e + f*x)**4/(2*f) - 3*
C*a*b**2*c*d*tan(e + f*x)**2/f - 3*C*a*b**2*d**2*x + 3*C*a*b**2*d**2*tan(e
+ f*x)**5/(5*f) - C*a*b**2*d**2*tan(e + f*x)**3/f + 3*C*a*b**2*d**2*tan(e +
 f*x)/f + C*b**3*c**2*log(tan(e + f*x)**2 + 1)/(2*f) + C*b**3*c**2*tan(e +
f*x)**4/(4*f) - C*b**3*c**2*tan(e + f*x)**2/(2*f) - 2*C*b**3*c*d*x + 2*C*b*
*3*c*d*tan(e + f*x)**5/(5*f) - 2*C*b**3*c*d*tan(e + f*x)**3/(3*f) + 2*C*b**
3*c*d*tan(e + f*x)/f - C*b**3*d**2*log(tan(e + f*x)**2 + 1)/(2*f) + C*b**3*
d**2*tan(e + f*x)**6/(6*f) - C*b**3*d**2*tan(e + f*x)**4/(4*f) + C*b**3*d**
2*tan(e + f*x)**2/(2*f), Ne(f, 0)), (x*(a + b*tan(e))**3*(c + d*tan(e))**2*
(A + B*tan(e) + C*tan(e)**2), True))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e
)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.58 $\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\right.$

Optimal. Leaf size=443

$$\frac{(c+d\tan(e+fx))^3\left(8a^2Cd^2-10abd(cC-4Bd)+b^2\left(20d^2(A-C)-5Bcd+2c^2C\right)\right)}{60d^3f}+\frac{\log(\cos(e+fx))\left(a^2\left(-\left(2cd(A-\right.\right.\right.}{}$$

```
[Out] -((a^2*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - b^2*(c^2*C + 2*B*c*d - C
*d^2 - A*(c^2 - d^2)) + 2*a*b*(2*c*(A - C)*d + B*(c^2 - d^2)))*x) + ((2*a*b
*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - a^2*(2*c*(A - C)*d + B*(c^2 -
d^2)) + b^2*(2*c*(A - C)*d + B*(c^2 - d^2)))*Log[Cos[e + f*x]])/f + (d*(2*a
*b*(A*c - c*C - B*d) + a^2*(B*c + (A - C)*d) - b^2*(B*c + (A - C)*d))*Tan[e
 + f*x])/f + ((a^2*B - b^2*B + 2*a*b*(A - C))*(c + d*Tan[e + f*x])^2)/(2*f)
 + ((8*a^2*C*d^2 - 10*a*b*d*(c*C - 4*B*d) + b^2*(2*c^2*C - 5*B*c*d + 20*(A
- C)*d^2))*(c + d*Tan[e + f*x])^3)/(60*d^3*f) - (b*(2*b*c*C - 5*b*B*d - 2*a
*C*d)*Tan[e + f*x]*(c + d*Tan[e + f*x])^3)/(20*d^2*f) + (C*(a + b*Tan[e + f
*x])^2*(c + d*Tan[e + f*x])^3)/(5*d*f)
```

Rubi [A] time = 1.27812, antiderivative size = 443, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3647, 3637, 3630, 3528, 3525, 3475}

$$\frac{(c+d\tan(e+fx))^3\left(8a^2Cd^2-10abd(cC-4Bd)+b^2\left(20d^2(A-C)-5Bcd+2c^2C\right)\right)}{60d^3f}+\frac{\log(\cos(e+fx))\left(a^2\left(-\left(2cd(A-\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*T
an[e + f*x]^2),x]
```


```
[Out] -((a^2*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - b^2*(c^2*C + 2*B*c*d - C
*d^2 - A*(c^2 - d^2)) + 2*a*b*(2*c*(A - C)*d + B*(c^2 - d^2)))*x) + ((2*a*b
*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - a^2*(2*c*(A - C)*d + B*(c^2 -
d^2)) + b^2*(2*c*(A - C)*d + B*(c^2 - d^2)))*Log[Cos[e + f*x]])/f + (d*(2*a
*b*(A*c - c*C - B*d) + a^2*(B*c + (A - C)*d) - b^2*(B*c + (A - C)*d))*Tan[e
 + f*x])/f + ((a^2*B - b^2*B + 2*a*b*(A - C))*(c + d*Tan[e + f*x])^2)/(2*f)
 + ((8*a^2*C*d^2 - 10*a*b*d*(c*C - 4*B*d) + b^2*(2*c^2*C - 5*B*c*d + 20*(A
- C)*d^2))*(c + d*Tan[e + f*x])^3)/(60*d^3*f) - (b*(2*b*c*C - 5*b*B*d - 2*a
*C*d)*Tan[e + f*x]*(c + d*Tan[e + f*x])^3)/(20*d^2*f) + (C*(a + b*Tan[e + f
*x])^2*(c + d*Tan[e + f*x])^3)/(5*d*f)
```


Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx = \frac{C(a+b\tan(e+fx))^2(c+d\,\text{ta}}{5df}$$

$$= -\frac{b(2bcC-5bBd-2aCd)\tan(}{20a}$$

$$= \frac{\left(8a^2Cd^2-10abd(cC-4Bd)+\right.}{}$$

$$= \frac{\left(a^2B-b^2B+2ab(A-C)\right)(c-}{2f}$$

$$= -\left(a^2\left(c^2C+2Bcd-Cd^2-A\right.\right.$$

$$= -\left(a^2\left(c^2C+2Bcd-Cd^2-A\right.\right.$$

Mathematica [C] time = 6.5013, size = 383, normalized size = 0.86

$$\frac{C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}{5df}+\frac{\frac{b\tan(e+fx)(2aCd+5bBd-2bcC)(c+d\tan(e+fx))^3}{4df}-\frac{(c+d\tan(e+fx))^3\left(-8a^2Cd^2+10abd(cC-4Bd)+b^2\left(-(20\cdots\right.\right.}{3df}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x]
 + C*Tan[e + f*x]^2),x]
```

```
[Out] (C*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^3)/(5*d*f) + ((b*(-2*b*c*C +
 5*b*B*d + 2*a*C*d)*Tan[e + f*x]*(c + d*Tan[e + f*x])^3)/(4*d*f) - (((-8*a^
2*C*d^2 + 10*a*b*d*(c*C - 4*B*d) - b^2*(2*c^2*C - 5*B*c*d + 20*(A - C)*d^2)
)*(c + d*Tan[e + f*x])^3)/(3*d*f) - (10*(d*(2*a*b*(A*c - c*C + B*d) + a^2*(
B*c - (A - C)*d) - b^2*(B*c - (A - C)*d))*(I*(c + I*d)^2*Log[I - Tan[e + f*
x]] - I*(c - I*d)^2*Log[I + Tan[e + f*x]] - 2*d^2*Tan[e + f*x]) + (a^2*B -
b^2*B + 2*a*b*(A - C))*d*((I*c - d)^3*Log[I - Tan[e + f*x]] - (I*c + d)^3*L
og[I + Tan[e + f*x]] + 6*c*d^2*Tan[e + f*x] + d^3*Tan[e + f*x]^2)))/f)/(4*d
))/(5*d)
```

Maple [B] time = 0.017, size = 1165, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x
)
```

```
[Out] 4/f*C*arctan(tan(f*x+e))*a*b*c*d-2/f*ln(1+tan(f*x+e)^2)*B*a*b*c*d-4/f*C*a*b
*c*d*tan(f*x+e)+4/3/f*C*tan(f*x+e)^3*a*b*c*d+2/f*B*tan(f*x+e)^2*a*b*c*d+4/f
*A*a*b*c*d*tan(f*x+e)-4/f*A*arctan(tan(f*x+e))*a*b*c*d+1/3/f*A*tan(f*x+e)^3
*b^2*d^2-1/2/f*B*tan(f*x+e)^2*b^2*d^2+1/2/f*ln(1+tan(f*x+e)^2)*B*a^2*c^2+1/
4/f*B*tan(f*x+e)^4*b^2*d^2-1/f*C*b^2*c^2*tan(f*x+e)+1/f*A*b^2*c^2*tan(f*x+e
)-1/f*A*b^2*d^2*tan(f*x+e)-1/f*a^2*C*d^2*tan(f*x+e)+1/5/f*C*b^2*d^2*tan(f*x
+e)^5+1/3/f*C*tan(f*x+e)^3*b^2*c^2-1/3/f*C*tan(f*x+e)^3*b^2*d^2+1/f*A*arcta
n(tan(f*x+e))*a^2*c^2-1/f*A*arctan(tan(f*x+e))*a^2*d^2-1/f*A*arctan(tan(f*x
+e))*b^2*c^2+1/f*A*arctan(tan(f*x+e))*b^2*d^2-1/f*C*arctan(tan(f*x+e))*a^2*
c^2-1/f*C*arctan(tan(f*x+e))*b^2*d^2+1/f*C*b^2*d^2*tan(f*x+e)+1/f*A*a^2*d^2
*tan(f*x+e)+1/f*C*arctan(tan(f*x+e))*a^2*d^2+1/f*C*arctan(tan(f*x+e))*b^2*c
^2-1/2/f*ln(1+tan(f*x+e)^2)*B*a^2*d^2-1/2/f*ln(1+tan(f*x+e)^2)*B*b^2*c^2+1/
3/f*C*tan(f*x+e)^3*a^2*d^2+1/f*C*a^2*c^2*tan(f*x+e)+1/2/f*B*tan(f*x+e)^2*a^
2*d^2+1/2/f*B*tan(f*x+e)^2*b^2*c^2+1/2/f*ln(1+tan(f*x+e)^2)*B*b^2*d^2+1/f*A
*tan(f*x+e)^2*b^2*c*d+1/2/f*C*tan(f*x+e)^4*a*b*d^2+1/f*A*tan(f*x+e)^2*a*b*d
^2+1/f*C*tan(f*x+e)^2*a*b*c^2-1/f*C*tan(f*x+e)^2*a*b*d^2-1/f*ln(1+tan(f*x+e
)^2)*A*a*b*d^2-1/f*ln(1+tan(f*x+e)^2)*A*b^2*c*d-1/f*ln(1+tan(f*x+e)^2)*C*a^
2*c*d-2/f*B*a*b*d^2*tan(f*x+e)-2/f*B*b^2*c*d*tan(f*x+e)+1/2/f*C*tan(f*x+e)^
4*b^2*c*d-1/f*ln(1+tan(f*x+e)^2)*C*a*b*c^2+1/f*ln(1+tan(f*x+e)^2)*C*a*b*d^2
+1/f*ln(1+tan(f*x+e)^2)*C*b^2*c*d-2/f*B*arctan(tan(f*x+e))*a^2*c*d+2/3/f*B*
tan(f*x+e)^3*a*b*d^2+2/3/f*B*tan(f*x+e)^3*b^2*c*d-1/f*C*tan(f*x+e)^2*b^2*c*
d+1/f*C*tan(f*x+e)^2*a^2*c*d+1/f*ln(1+tan(f*x+e)^2)*A*a^2*c*d+1/f*ln(1+tan(
f*x+e)^2)*A*a*b*c^2+2/f*B*a^2*c*d*tan(f*x+e)-2/f*B*arctan(tan(f*x+e))*a*b*c
^2+2/f*B*arctan(tan(f*x+e))*a*b*d^2+2/f*B*arctan(tan(f*x+e))*b^2*c*d+2/f*B*
a*b*c^2*tan(f*x+e)
```

Maxima [A] time = 1.48102, size = 625, normalized size = 1.41

$$12\,Cb^2d^2\tan\left(fx+e\right)^5+15\left(2\,Cb^2cd+\left(2\,Cab+Bb^2\right)d^2\right)\tan\left(fx+e\right)^4+20\left(Cb^2c^2+2\left(2\,Cab+Bb^2\right)cd+\left(Ca^2+2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e
)^2),x, algorithm="maxima")
```

```
[Out] 1/60*(12*C*b^2*d^2*tan(f*x + e)^5 + 15*(2*C*b^2*c*d + (2*C*a*b + B*b^2)*d^2
)*tan(f*x + e)^4 + 20*(C*b^2*c^2 + 2*(2*C*a*b + B*b^2)*c*d + (C*a^2 + 2*B*a
*b + (A - C)*b^2)*d^2)*tan(f*x + e)^3 + 30*((2*C*a*b + B*b^2)*c^2 + 2*(C*a^
2 + 2*B*a*b + (A - C)*b^2)*c*d + (B*a^2 + 2*(A - C)*a*b - B*b^2)*d^2)*tan(f
*x + e)^2 + 60*(((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c^2 - 2*(B*a^2 + 2*(A
 - C)*a*b - B*b^2)*c*d - ((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*d^2)*(f*x +
e) + 30*((B*a^2 + 2*(A - C)*a*b - B*b^2)*c^2 + 2*((A - C)*a^2 - 2*B*a*b - (
A - C)*b^2)*c*d - (B*a^2 + 2*(A - C)*a*b - B*b^2)*d^2)*log(tan(f*x + e)^2 +
 1) + 60*((C*a^2 + 2*B*a*b + (A - C)*b^2)*c^2 + 2*(B*a^2 + 2*(A - C)*a*b -
B*b^2)*c*d + ((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*d^2)*tan(f*x + e))/f
```

Fricas [A] time = 1.22046, size = 1007, normalized size = 2.27

$$12\,Cb^2d^2\tan\left(fx+e\right)^5+15\left(2\,Cb^2cd+\left(2\,Cab+Bb^2\right)d^2\right)\tan\left(fx+e\right)^4+20\left(Cb^2c^2+2\left(2\,Cab+Bb^2\right)cd+\left(Ca^2+2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e
)^2),x, algorithm="fricas")
```

```
[Out] 1/60*(12*C*b^2*d^2*tan(f*x + e)^5 + 15*(2*C*b^2*c*d + (2*C*a*b + B*b^2)*d^2
)*tan(f*x + e)^4 + 20*(C*b^2*c^2 + 2*(2*C*a*b + B*b^2)*c*d + (C*a^2 + 2*B*a
*b + (A - C)*b^2)*d^2)*tan(f*x + e)^3 + 60*(((A - C)*a^2 - 2*B*a*b - (A - C
)*b^2)*c^2 - 2*(B*a^2 + 2*(A - C)*a*b - B*b^2)*c*d - ((A - C)*a^2 - 2*B*a*b
 - (A - C)*b^2)*d^2)*f*x + 30*((2*C*a*b + B*b^2)*c^2 + 2*(C*a^2 + 2*B*a*b +
 (A - C)*b^2)*c*d + (B*a^2 + 2*(A - C)*a*b - B*b^2)*d^2)*tan(f*x + e)^2 - 3
0*((B*a^2 + 2*(A - C)*a*b - B*b^2)*c^2 + 2*((A - C)*a^2 - 2*B*a*b - (A - C)
*b^2)*c*d - (B*a^2 + 2*(A - C)*a*b - B*b^2)*d^2)*log(1/(tan(f*x + e)^2 + 1)
) + 60*((C*a^2 + 2*B*a*b + (A - C)*b^2)*c^2 + 2*(B*a^2 + 2*(A - C)*a*b - B*
b^2)*c*d + ((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*d^2)*tan(f*x + e))/f
```

Sympy [A] time = 4.66021, size = 1134, normalized size = 2.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2*(c+d*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x
+e)**2),x)
```

```
[Out] Piecewise((A*a**2*c**2*x + A*a**2*c*d*log(tan(e + f*x)**2 + 1)/f - A*a**2*d
**2*x + A*a**2*d**2*tan(e + f*x)/f + A*a*b*c**2*log(tan(e + f*x)**2 + 1)/f
- 4*A*a*b*c*d*x + 4*A*a*b*c*d*tan(e + f*x)/f - A*a*b*d**2*log(tan(e + f*x)*
*2 + 1)/f + A*a*b*d**2*tan(e + f*x)**2/f - A*b**2*c**2*x + A*b**2*c**2*tan(
e + f*x)/f - A*b**2*c*d*log(tan(e + f*x)**2 + 1)/f + A*b**2*c*d*tan(e + f*x
)**2/f + A*b**2*d**2*x + A*b**2*d**2*tan(e + f*x)**3/(3*f) - A*b**2*d**2*ta
n(e + f*x)/f + B*a**2*c**2*log(tan(e + f*x)**2 + 1)/(2*f) - 2*B*a**2*c*d*x
+ 2*B*a**2*c*d*tan(e + f*x)/f - B*a**2*d**2*log(tan(e + f*x)**2 + 1)/(2*f)
+ B*a**2*d**2*tan(e + f*x)**2/(2*f) - 2*B*a*b*c**2*x + 2*B*a*b*c**2*tan(e +
 f*x)/f - 2*B*a*b*c*d*log(tan(e + f*x)**2 + 1)/f + 2*B*a*b*c*d*tan(e + f*x)
**2/f + 2*B*a*b*d**2*x + 2*B*a*b*d**2*tan(e + f*x)**3/(3*f) - 2*B*a*b*d**2*
tan(e + f*x)/f - B*b**2*c**2*log(tan(e + f*x)**2 + 1)/(2*f) + B*b**2*c**2*t
an(e + f*x)**2/(2*f) + 2*B*b**2*c*d*x + 2*B*b**2*c*d*tan(e + f*x)**3/(3*f)
- 2*B*b**2*c*d*tan(e + f*x)/f + B*b**2*d**2*log(tan(e + f*x)**2 + 1)/(2*f)
+ B*b**2*d**2*tan(e + f*x)**4/(4*f) - B*b**2*d**2*tan(e + f*x)**2/(2*f) - C
*a**2*c**2*x + C*a**2*c**2*tan(e + f*x)/f - C*a**2*c*d*log(tan(e + f*x)**2
+ 1)/f + C*a**2*c*d*tan(e + f*x)**2/f + C*a**2*d**2*x + C*a**2*d**2*tan(e +
 f*x)**3/(3*f) - C*a**2*d**2*tan(e + f*x)/f - C*a*b*c**2*log(tan(e + f*x)**
2 + 1)/f + C*a*b*c**2*tan(e + f*x)**2/f + 4*C*a*b*c*d*x + 4*C*a*b*c*d*tan(e
 + f*x)**3/(3*f) - 4*C*a*b*c*d*tan(e + f*x)/f + C*a*b*d**2*log(tan(e + f*x)
**2 + 1)/f + C*a*b*d**2*tan(e + f*x)**4/(2*f) - C*a*b*d**2*tan(e + f*x)**2/
f + C*b**2*c**2*x + C*b**2*c**2*tan(e + f*x)**3/(3*f) - C*b**2*c**2*tan(e +
 f*x)/f + C*b**2*c*d*log(tan(e + f*x)**2 + 1)/f + C*b**2*c*d*tan(e + f*x)**
4/(2*f) - C*b**2*c*d*tan(e + f*x)**2/f - C*b**2*d**2*x + C*b**2*d**2*tan(e
+ f*x)**5/(5*f) - C*b**2*d**2*tan(e + f*x)**3/(3*f) + C*b**2*d**2*tan(e + f
*x)/f, Ne(f, 0)), (x*(a + b*tan(e))**2*(c + d*tan(e))**2*(A + B*tan(e) + C*
tan(e)**2), True))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e
)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.59 $\int (a+b\tan(e+fx))(c+d\tan(e+fx))^2 \left(A+B\tan(e+fx)+\right.$

Optimal. Leaf size=266

$$-\frac{\log(\cos(e+fx))\left(A\left(2acd+b\left(c^2-d^2\right)\right)+a\left(Bc^2-Bd^2-2cCd\right)-b\left(2Bcd+c^2C-Cd^2\right)\right)}{f}-x\left(a\left(-A\left(c^2-d^2\right)+2B\right.\right.$$

```
[Out] -((a*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) + b*(2*c*(A - C)*d + B*(c^2
- d^2)))*x) - ((a*(B*c^2 - 2*c*C*d - B*d^2) - b*(c^2*C + 2*B*c*d - C*d^2) +
 A*(2*a*c*d + b*(c^2 - d^2)))*Log[Cos[e + f*x]])/f + (d*(A*b*c + a*B*c - b*
c*C + a*A*d - b*B*d - a*C*d)*Tan[e + f*x])/f + ((A*b + a*B - b*C)*(c + d*Ta
n[e + f*x])^2)/(2*f) - ((b*c*C - 4*b*B*d - 4*a*C*d)*(c + d*Tan[e + f*x])^3)
/(12*d^2*f) + (b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^3)/(4*d*f)
```

Rubi [A] time = 0.472202, antiderivative size = 264, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {3637, 3630, 3528, 3525, 3475}

$$-\frac{\log(\cos(e+fx))\left(2aAcd+aB\left(c^2-d^2\right)-2acCd+Ab\left(c^2-d^2\right)-b\left(2Bcd+c^2C-Cd^2\right)\right)}{f}-x\left(a\left(-A\left(c^2-d^2\right)+2Bc\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan
[e + f*x]^2),x]
```

```
[Out] -((a*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) + b*(2*c*(A - C)*d + B*(c^2
- d^2)))*x) - ((2*a*A*c*d - 2*a*c*C*d + A*b*(c^2 - d^2) + a*B*(c^2 - d^2) -
 b*(c^2*C + 2*B*c*d - C*d^2))*Log[Cos[e + f*x]])/f + (d*(A*b*c + a*B*c - b*
c*C + a*A*d - b*B*d - a*C*d)*Tan[e + f*x])/f + ((A*b + a*B - b*C)*(c + d*Ta
n[e + f*x])^2)/(2*f) - ((b*c*C - 4*b*B*d - 4*a*C*d)*(c + d*Tan[e + f*x])^3)
/(12*d^2*f) + (b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^3)/(4*d*f)
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
```

```
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx = \frac{bC\tan(e+fx)(c+d\tan(e+fx))}{4df}$$

$$= -\frac{(bcC-4bBd-4aCd)(c+d\tan(}{12d^2f}$$

$$= \frac{(Ab+aB-bC)(c+d\tan(e+fx)}{2f}$$

$$= -\left(a\left(c^2C+2Bcd-Cd^2-A\left(c^2-\right.\right.\right.$$

$$= -\left(a\left(c^2C+2Bcd-Cd^2-A\left(c^2-\right.\right.\right.$$

Mathematica [C] time = 2.6466, size = 241, normalized size = 0.91

$$\frac{6(aB+Ab-bC)\left(6cd^2\tan(e+fx)+(-d+ic)^3\log(-\tan(e+fx)+i)-(d+ic)^3\log(\tan(e+fx)+i)+d^3\tan^2(e+fx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] +
 C*Tan[e + f*x]^2),x]
```

```
[Out] (((-(b*c*C) + 4*b*B*d + 4*a*C*d)*(c + d*Tan[e + f*x])^3)/d + 3*b*C*Tan[e +
f*x]*(c + d*Tan[e + f*x])^3 + 6*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*
C*d)*(I*((c + I*d)^2*Log[I - Tan[e + f*x]] - (c - I*d)^2*Log[I + Tan[e + f*
x]]) - 2*d^2*Tan[e + f*x]) + 6*(A*b + a*B - b*C)*((I*c - d)^3*Log[I - Tan[e
 + f*x]] - (I*c + d)^3*Log[I + Tan[e + f*x]] + 6*c*d^2*Tan[e + f*x] + d^3*T
an[e + f*x]^2))/(12*d*f)
```

Maple [B] time = 0.016, size = 631, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x)
```

```
[Out] -2/f*B*arctan(tan(f*x+e))*a*c*d+2/f*A*b*c*d*tan(f*x+e)+2/f*B*a*c*d*tan(f*x+
e)+1/f*B*tan(f*x+e)^2*b*c*d-1/f*ln(1+tan(f*x+e)^2)*B*b*c*d+1/f*C*tan(f*x+e)
^2*a*c*d-2/f*A*arctan(tan(f*x+e))*b*c*d-1/2/f*ln(1+tan(f*x+e)^2)*A*b*d^2+1/
f*ln(1+tan(f*x+e)^2)*A*a*c*d+1/f*A*arctan(tan(f*x+e))*a*c^2-1/f*A*arctan(ta
n(f*x+e))*a*d^2-1/f*B*arctan(tan(f*x+e))*b*c^2+1/4/f*C*b*d^2*tan(f*x+e)^4-1
/2/f*ln(1+tan(f*x+e)^2)*B*a*d^2-1/2/f*ln(1+tan(f*x+e)^2)*C*b*c^2-1/2/f*C*ta
n(f*x+e)^2*b*d^2+1/f*A*a*d^2*tan(f*x+e)+1/3/f*C*tan(f*x+e)^3*a*d^2+1/2/f*A*
tan(f*x+e)^2*b*d^2+1/2/f*B*tan(f*x+e)^2*a*d^2+1/2/f*ln(1+tan(f*x+e)^2)*C*b*
d^2-1/f*B*b*d^2*tan(f*x+e)+1/f*C*a*c^2*tan(f*x+e)-1/f*C*a*d^2*tan(f*x+e)+1/
2/f*ln(1+tan(f*x+e)^2)*A*b*c^2+1/3/f*B*tan(f*x+e)^3*b*d^2+1/f*B*b*c^2*tan(f
*x+e)+1/2/f*C*tan(f*x+e)^2*b*c^2+1/2/f*ln(1+tan(f*x+e)^2)*B*a*c^2+1/f*B*arc
tan(tan(f*x+e))*b*d^2-1/f*C*arctan(tan(f*x+e))*a*c^2+1/f*C*arctan(tan(f*x+e
))*a*d^2-1/f*ln(1+tan(f*x+e)^2)*C*a*c*d+2/f*C*arctan(tan(f*x+e))*b*c*d+2/3/
f*C*tan(f*x+e)^3*b*c*d-2/f*C*b*c*d*tan(f*x+e)
```

Maxima [A] time = 1.47412, size = 351, normalized size = 1.32

$$\frac{3\,Cbd^2\tan\left(fx+e\right)^4 + 4\left(2\,Cbcd + (Ca+Bb)d^2\right)\tan\left(fx+e\right)^3 + 6\left(Cbc^2 + 2\,(Ca+Bb)cd + (Ba+(A-C)b)d^2\right)\tan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x, algorithm="maxima")
```

```
[Out] 1/12*(3*C*b*d^2*tan(f*x + e)^4 + 4*(2*C*b*c*d + (C*a + B*b)*d^2)*tan(f*x +
e)^3 + 6*(C*b*c^2 + 2*(C*a + B*b)*c*d + (B*a + (A - C)*b)*d^2)*tan(f*x + e)
^2 + 12*(((A - C)*a - B*b)*c^2 - 2*(B*a + (A - C)*b)*c*d - ((A - C)*a - B*b
)*d^2)*(f*x + e) + 6*((B*a + (A - C)*b)*c^2 + 2*((A - C)*a - B*b)*c*d - (B*
a + (A - C)*b)*d^2)*log(tan(f*x + e)^2 + 1) + 12*((C*a + B*b)*c^2 + 2*(B*a
+ (A - C)*b)*c*d + ((A - C)*a - B*b)*d^2)*tan(f*x + e))/f
```

Fricas [A] time = 1.12755, size = 583, normalized size = 2.19

$$\frac{3\,Cbd^2\tan\left(fx+e\right)^4 + 4\left(2\,Cbcd + (Ca+Bb)d^2\right)\tan\left(fx+e\right)^3 + 12\left(((A-C)a-Bb)c^2 - 2\,(Ba+(A-C)b)cd - ((A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*C*b*d^2*tan(f*x + e)^4 + 4*(2*C*b*c*d + (C*a + B*b)*d^2)*tan(f*x +
e)^3 + 12*(((A - C)*a - B*b)*c^2 - 2*(B*a + (A - C)*b)*c*d - ((A - C)*a - B
*b)*d^2)*f*x + 6*(C*b*c^2 + 2*(C*a + B*b)*c*d + (B*a + (A - C)*b)*d^2)*tan(
f*x + e)^2 - 6*((B*a + (A - C)*b)*c^2 + 2*((A - C)*a - B*b)*c*d - (B*a + (A
 - C)*b)*d^2)*log(1/(tan(f*x + e)^2 + 1)) + 12*((C*a + B*b)*c^2 + 2*(B*a +
(A - C)*b)*c*d + ((A - C)*a - B*b)*d^2)*tan(f*x + e))/f
```

Sympy [A] time = 3.39541, size = 617, normalized size = 2.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x+e)
**2),x)
```

```
[Out] Piecewise((A*a*c**2*x + A*a*c*d*log(tan(e + f*x)**2 + 1)/f - A*a*d**2*x + A
*a*d**2*tan(e + f*x)/f + A*b*c**2*log(tan(e + f*x)**2 + 1)/(2*f) - 2*A*b*c*
d*x + 2*A*b*c*d*tan(e + f*x)/f - A*b*d**2*log(tan(e + f*x)**2 + 1)/(2*f) +
A*b*d**2*tan(e + f*x)**2/(2*f) + B*a*c**2*log(tan(e + f*x)**2 + 1)/(2*f) -
2*B*a*c*d*x + 2*B*a*c*d*tan(e + f*x)/f - B*a*d**2*log(tan(e + f*x)**2 + 1)/
(2*f) + B*a*d**2*tan(e + f*x)**2/(2*f) - B*b*c**2*x + B*b*c**2*tan(e + f*x)
/f - B*b*c*d*log(tan(e + f*x)**2 + 1)/f + B*b*c*d*tan(e + f*x)**2/f + B*b*d
**2*x + B*b*d**2*tan(e + f*x)**3/(3*f) - B*b*d**2*tan(e + f*x)/f - C*a*c**2
*x + C*a*c**2*tan(e + f*x)/f - C*a*c*d*log(tan(e + f*x)**2 + 1)/f + C*a*c*d
*tan(e + f*x)**2/f + C*a*d**2*x + C*a*d**2*tan(e + f*x)**3/(3*f) - C*a*d**2
*tan(e + f*x)/f - C*b*c**2*log(tan(e + f*x)**2 + 1)/(2*f) + C*b*c**2*tan(e
+ f*x)**2/(2*f) + 2*C*b*c*d*x + 2*C*b*c*d*tan(e + f*x)**3/(3*f) - 2*C*b*c*d
*tan(e + f*x)/f + C*b*d**2*log(tan(e + f*x)**2 + 1)/(2*f) + C*b*d**2*tan(e
+ f*x)**4/(4*f) - C*b*d**2*tan(e + f*x)**2/(2*f), Ne(f, 0)), (x*(a + b*tan(
e))*(c + d*tan(e))**2*(A + B*tan(e) + C*tan(e)**2), True))
```

Giac [B] time = 9.2471, size = 8778, normalized size = 33.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x, algorithm="giac")
```

```
[Out] 1/12*(12*A*a*c^2*f*x*tan(f*x)^4*tan(e)^4 - 12*C*a*c^2*f*x*tan(f*x)^4*tan(e)
^4 - 12*B*b*c^2*f*x*tan(f*x)^4*tan(e)^4 - 24*B*a*c*d*f*x*tan(f*x)^4*tan(e)^
4 - 24*A*b*c*d*f*x*tan(f*x)^4*tan(e)^4 + 24*C*b*c*d*f*x*tan(f*x)^4*tan(e)^4
 - 12*A*a*d^2*f*x*tan(f*x)^4*tan(e)^4 + 12*C*a*d^2*f*x*tan(f*x)^4*tan(e)^4
+ 12*B*b*d^2*f*x*tan(f*x)^4*tan(e)^4 - 6*B*a*c^2*log(4*(tan(e)^2 + 1)/(tan(
f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 -
2*tan(f*x)*tan(e) + 1))*tan(f*x)^4*tan(e)^4 - 6*A*b*c^2*log(4*(tan(e)^2 + 1
)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*
x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^4*tan(e)^4 + 6*C*b*c^2*log(4*(tan(e
)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 +
 tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^4*tan(e)^4 - 12*A*a*c*d*log(
4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*ta
n(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^4*tan(e)^4 + 12*C*a*
c*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f
*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^4*tan(e)^4 +
 12*B*b*c*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e)
 + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^4*ta
n(e)^4 + 6*B*a*d^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3
*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*
x)^4*tan(e)^4 + 6*A*b*d^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan
(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))
*tan(f*x)^4*tan(e)^4 - 6*C*b*d^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2
```

```
- 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e
) + 1))*tan(f*x)^4*tan(e)^4 - 48*A*a*c^2*f*x*tan(f*x)^3*tan(e)^3 + 48*C*a*c
^2*f*x*tan(f*x)^3*tan(e)^3 + 48*B*b*c^2*f*x*tan(f*x)^3*tan(e)^3 + 96*B*a*c*
d*f*x*tan(f*x)^3*tan(e)^3 + 96*A*b*c*d*f*x*tan(f*x)^3*tan(e)^3 - 96*C*b*c*d
*f*x*tan(f*x)^3*tan(e)^3 + 48*A*a*d^2*f*x*tan(f*x)^3*tan(e)^3 - 48*C*a*d^2*
f*x*tan(f*x)^3*tan(e)^3 - 48*B*b*d^2*f*x*tan(f*x)^3*tan(e)^3 + 6*C*b*c^2*ta
n(f*x)^4*tan(e)^4 + 12*C*a*c*d*tan(f*x)^4*tan(e)^4 + 12*B*b*c*d*tan(f*x)^4*
tan(e)^4 + 6*B*a*d^2*tan(f*x)^4*tan(e)^4 + 6*A*b*d^2*tan(f*x)^4*tan(e)^4 -
9*C*b*d^2*tan(f*x)^4*tan(e)^4 + 24*B*a*c^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4
*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(
f*x)*tan(e) + 1))*tan(f*x)^3*tan(e)^3 + 24*A*b*c^2*log(4*(tan(e)^2 + 1)/(ta
n(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2
- 2*tan(f*x)*tan(e) + 1))*tan(f*x)^3*tan(e)^3 - 24*C*b*c^2*log(4*(tan(e)^2
+ 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan
(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^3*tan(e)^3 + 48*A*a*c*d*log(4*(t
an(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)
^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^3*tan(e)^3 - 48*C*a*c*d*
log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^
2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^3*tan(e)^3 - 48*
B*b*c*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + t
an(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^3*tan(e)
^3 - 24*B*a*d^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*ta
n(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^
3*tan(e)^3 - 24*A*b*d^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f
*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*t
an(f*x)^3*tan(e)^3 + 24*C*b*d^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 -
 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e)
 + 1))*tan(f*x)^3*tan(e)^3 - 12*C*a*c^2*tan(f*x)^4*tan(e)^3 - 12*B*b*c^2*ta
n(f*x)^4*tan(e)^3 - 24*B*a*c*d*tan(f*x)^4*tan(e)^3 - 24*A*b*c*d*tan(f*x)^4*
tan(e)^3 + 24*C*b*c*d*tan(f*x)^4*tan(e)^3 - 12*A*a*d^2*tan(f*x)^4*tan(e)^3
+ 12*C*a*d^2*tan(f*x)^4*tan(e)^3 + 12*B*b*d^2*tan(f*x)^4*tan(e)^3 - 12*C*a*
c^2*tan(f*x)^3*tan(e)^4 - 12*B*b*c^2*tan(f*x)^3*tan(e)^4 - 24*B*a*c*d*tan(f
*x)^3*tan(e)^4 - 24*A*b*c*d*tan(f*x)^3*tan(e)^4 + 24*C*b*c*d*tan(f*x)^3*tan
(e)^4 - 12*A*a*d^2*tan(f*x)^3*tan(e)^4 + 12*C*a*d^2*tan(f*x)^3*tan(e)^4 + 1
2*B*b*d^2*tan(f*x)^3*tan(e)^4 + 72*A*a*c^2*f*x*tan(f*x)^2*tan(e)^2 - 72*C*a
*c^2*f*x*tan(f*x)^2*tan(e)^2 - 72*B*b*c^2*f*x*tan(f*x)^2*tan(e)^2 - 144*B*a
*c*d*f*x*tan(f*x)^2*tan(e)^2 - 144*A*b*c*d*f*x*tan(f*x)^2*tan(e)^2 + 144*C*
b*c*d*f*x*tan(f*x)^2*tan(e)^2 - 72*A*a*d^2*f*x*tan(f*x)^2*tan(e)^2 + 72*C*a
*d^2*f*x*tan(f*x)^2*tan(e)^2 + 72*B*b*d^2*f*x*tan(f*x)^2*tan(e)^2 + 6*C*b*c
^2*tan(f*x)^4*tan(e)^2 + 12*C*a*c*d*tan(f*x)^4*tan(e)^2 + 12*B*b*c*d*tan(f*
x)^4*tan(e)^2 + 6*B*a*d^2*tan(f*x)^4*tan(e)^2 + 6*A*b*d^2*tan(f*x)^4*tan(e)
^2 - 6*C*b*d^2*tan(f*x)^4*tan(e)^2 - 12*C*b*c^2*tan(f*x)^3*tan(e)^3 - 24*C*
a*c*d*tan(f*x)^3*tan(e)^3 - 24*B*b*c*d*tan(f*x)^3*tan(e)^3 - 12*B*a*d^2*tan
(f*x)^3*tan(e)^3 - 12*A*b*d^2*tan(f*x)^3*tan(e)^3 + 24*C*b*d^2*tan(f*x)^3*t
an(e)^3 + 6*C*b*c^2*tan(f*x)^2*tan(e)^4 + 12*C*a*c*d*tan(f*x)^2*tan(e)^4 +
12*B*b*c*d*tan(f*x)^2*tan(e)^4 + 6*B*a*d^2*tan(f*x)^2*tan(e)^4 + 6*A*b*d^2*
tan(f*x)^2*tan(e)^4 - 6*C*b*d^2*tan(f*x)^2*tan(e)^4 - 8*C*b*c*d*tan(f*x)^4*
tan(e) - 4*C*a*d^2*tan(f*x)^4*tan(e) - 4*B*b*d^2*tan(f*x)^4*tan(e) - 36*B*a
*c^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(
f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^2*tan(e)^2
- 36*A*b*c^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e
) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^2*t
an(e)^2 + 36*C*b*c^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)
^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(
f*x)^2*tan(e)^2 - 72*A*a*c*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*
tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) +
1))*tan(f*x)^2*tan(e)^2 + 72*C*a*c*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e
)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*t
an(e) + 1))*tan(f*x)^2*tan(e)^2 + 72*B*b*c*d*log(4*(tan(e)^2 + 1)/(tan(f*x)
```

```
^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*ta
n(f*x)*tan(e) + 1))*tan(f*x)^2*tan(e)^2 + 36*B*a*d^2*log(4*(tan(e)^2 + 1)/(
tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^
2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^2*tan(e)^2 + 36*A*b*d^2*log(4*(tan(e)^
2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + t
an(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^2*tan(e)^2 - 36*C*b*d^2*log(4*
(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(
e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^2*tan(e)^2 + 36*C*a*c^
2*tan(f*x)^3*tan(e)^2 + 36*B*b*c^2*tan(f*x)^3*tan(e)^2 + 72*B*a*c*d*tan(f*x
)^3*tan(e)^2 + 72*A*b*c*d*tan(f*x)^3*tan(e)^2 - 96*C*b*c*d*tan(f*x)^3*tan(e
)^2 + 36*A*a*d^2*tan(f*x)^3*tan(e)^2 - 48*C*a*d^2*tan(f*x)^3*tan(e)^2 - 48*
B*b*d^2*tan(f*x)^3*tan(e)^2 + 36*C*a*c^2*tan(f*x)^2*tan(e)^3 + 36*B*b*c^2*t
an(f*x)^2*tan(e)^3 + 72*B*a*c*d*tan(f*x)^2*tan(e)^3 + 72*A*b*c*d*tan(f*x)^2
*tan(e)^3 - 96*C*b*c*d*tan(f*x)^2*tan(e)^3 + 36*A*a*d^2*tan(f*x)^2*tan(e)^3
 - 48*C*a*d^2*tan(f*x)^2*tan(e)^3 - 48*B*b*d^2*tan(f*x)^2*tan(e)^3 - 8*C*b*
c*d*tan(f*x)*tan(e)^4 - 4*C*a*d^2*tan(f*x)*tan(e)^4 - 4*B*b*d^2*tan(f*x)*ta
n(e)^4 + 3*C*b*d^2*tan(f*x)^4 - 48*A*a*c^2*f*x*tan(f*x)*tan(e) + 48*C*a*c^2
*f*x*tan(f*x)*tan(e) + 48*B*b*c^2*f*x*tan(f*x)*tan(e) + 96*B*a*c*d*f*x*tan(
f*x)*tan(e) + 96*A*b*c*d*f*x*tan(f*x)*tan(e) - 96*C*b*c*d*f*x*tan(f*x)*tan(
e) + 48*A*a*d^2*f*x*tan(f*x)*tan(e) - 48*C*a*d^2*f*x*tan(f*x)*tan(e) - 48*B
*b*d^2*f*x*tan(f*x)*tan(e) - 12*C*b*c^2*tan(f*x)^3*tan(e) - 24*C*a*c*d*tan(
f*x)^3*tan(e) - 24*B*b*c*d*tan(f*x)^3*tan(e) - 12*B*a*d^2*tan(f*x)^3*tan(e)
 - 12*A*b*d^2*tan(f*x)^3*tan(e) + 24*C*b*d^2*tan(f*x)^3*tan(e) + 12*C*b*c^2
*tan(f*x)^2*tan(e)^2 + 24*C*a*c*d*tan(f*x)^2*tan(e)^2 + 24*B*b*c*d*tan(f*x)
^2*tan(e)^2 + 12*B*a*d^2*tan(f*x)^2*tan(e)^2 + 12*A*b*d^2*tan(f*x)^2*tan(e)
^2 - 12*C*b*d^2*tan(f*x)^2*tan(e)^2 - 12*C*b*c^2*tan(f*x)*tan(e)^3 - 24*C*a
*c*d*tan(f*x)*tan(e)^3 - 24*B*b*c*d*tan(f*x)*tan(e)^3 - 12*B*a*d^2*tan(f*x)
*tan(e)^3 - 12*A*b*d^2*tan(f*x)*tan(e)^3 + 24*C*b*d^2*tan(f*x)*tan(e)^3 + 3
*C*b*d^2*tan(e)^4 + 8*C*b*c*d*tan(f*x)^3 + 4*C*a*d^2*tan(f*x)^3 + 4*B*b*d^2
*tan(f*x)^3 + 24*B*a*c^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(
f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*
tan(f*x)*tan(e) + 24*A*b*c^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*
tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) +
1))*tan(f*x)*tan(e) - 24*C*b*c^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2
- 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e
) + 1))*tan(f*x)*tan(e) + 48*A*a*c*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e
)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*t
an(e) + 1))*tan(f*x)*tan(e) - 48*C*a*c*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*t
an(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*
x)*tan(e) + 1))*tan(f*x)*tan(e) - 48*B*b*c*d*log(4*(tan(e)^2 + 1)/(tan(f*x)
^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*ta
n(f*x)*tan(e) + 1))*tan(f*x)*tan(e) - 24*B*a*d^2*log(4*(tan(e)^2 + 1)/(tan(
f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 -
2*tan(f*x)*tan(e) + 1))*tan(f*x)*tan(e) - 24*A*b*d^2*log(4*(tan(e)^2 + 1)/(
tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^
2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)*tan(e) + 24*C*b*d^2*log(4*(tan(e)^2 +
1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f
*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)*tan(e) - 36*C*a*c^2*tan(f*x)^2*tan
(e) - 36*B*b*c^2*tan(f*x)^2*tan(e) - 72*B*a*c*d*tan(f*x)^2*tan(e) - 72*A*b*
c*d*tan(f*x)^2*tan(e) + 96*C*b*c*d*tan(f*x)^2*tan(e) - 36*A*a*d^2*tan(f*x)^
2*tan(e) + 48*C*a*d^2*tan(f*x)^2*tan(e) + 48*B*b*d^2*tan(f*x)^2*tan(e) - 36
*C*a*c^2*tan(f*x)*tan(e)^2 - 36*B*b*c^2*tan(f*x)*tan(e)^2 - 72*B*a*c*d*tan(
f*x)*tan(e)^2 - 72*A*b*c*d*tan(f*x)*tan(e)^2 + 96*C*b*c*d*tan(f*x)*tan(e)^2
 - 36*A*a*d^2*tan(f*x)*tan(e)^2 + 48*C*a*d^2*tan(f*x)*tan(e)^2 + 48*B*b*d^2
*tan(f*x)*tan(e)^2 + 8*C*b*c*d*tan(e)^3 + 4*C*a*d^2*tan(e)^3 + 4*B*b*d^2*ta
n(e)^3 + 12*A*a*c^2*f*x - 12*C*a*c^2*f*x - 12*B*b*c^2*f*x - 24*B*a*c*d*f*x
- 24*A*b*c*d*f*x + 24*C*b*c*d*f*x - 12*A*a*d^2*f*x + 12*C*a*d^2*f*x + 12*B*
b*d^2*f*x + 6*C*b*c^2*tan(f*x)^2 + 12*C*a*c*d*tan(f*x)^2 + 12*B*b*c*d*tan(f
*x)^2 + 6*B*a*d^2*tan(f*x)^2 + 6*A*b*d^2*tan(f*x)^2 - 6*C*b*d^2*tan(f*x)^2
```

```
- 12*C*b*c^2*tan(f*x)*tan(e) - 24*C*a*c*d*tan(f*x)*tan(e) - 24*B*b*c*d*tan(
f*x)*tan(e) - 12*B*a*d^2*tan(f*x)*tan(e) - 12*A*b*d^2*tan(f*x)*tan(e) + 24*
C*b*d^2*tan(f*x)*tan(e) + 6*C*b*c^2*tan(e)^2 + 12*C*a*c*d*tan(e)^2 + 12*B*b
*c*d*tan(e)^2 + 6*B*a*d^2*tan(e)^2 + 6*A*b*d^2*tan(e)^2 - 6*C*b*d^2*tan(e)^
2 - 6*B*a*c^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(
e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)) - 6*A*b*c^2
*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)
^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)) + 6*C*b*c^2*log(4*(tan(e
)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 +
 tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)) - 12*A*a*c*d*log(4*(tan(e)^2 + 1)/(ta
n(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2
- 2*tan(f*x)*tan(e) + 1)) + 12*C*a*c*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan
(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)
*tan(e) + 1)) + 12*B*b*c*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*ta
n(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)
) + 6*B*a*d^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(
e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)) + 6*A*b*d^2
*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)
^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)) - 6*C*b*d^2*log(4*(tan(e
)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 +
 tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)) + 12*C*a*c^2*tan(f*x) + 12*B*b*c^2*ta
n(f*x) + 24*B*a*c*d*tan(f*x) + 24*A*b*c*d*tan(f*x) - 24*C*b*c*d*tan(f*x) +
12*A*a*d^2*tan(f*x) - 12*C*a*d^2*tan(f*x) - 12*B*b*d^2*tan(f*x) + 12*C*a*c^
2*tan(e) + 12*B*b*c^2*tan(e) + 24*B*a*c*d*tan(e) + 24*A*b*c*d*tan(e) - 24*C
*b*c*d*tan(e) + 12*A*a*d^2*tan(e) - 12*C*a*d^2*tan(e) - 12*B*b*d^2*tan(e) +
 6*C*b*c^2 + 12*C*a*c*d + 12*B*b*c*d + 6*B*a*d^2 + 6*A*b*d^2 - 9*C*b*d^2)/(
f*tan(f*x)^4*tan(e)^4 - 4*f*tan(f*x)^3*tan(e)^3 + 6*f*tan(f*x)^2*tan(e)^2 -
 4*f*tan(f*x)*tan(e) + f)
```

3.60 $\int (c+d\tan(e+fx))^2 \left(A + B\tan(e + fx) + C\tan^2(e + fx)\right) d$

Optimal. Leaf size=131

$$-\frac{\left(2cd(A-C)+B\left(c^2-d^2\right)\right)\log(\cos(e+fx))}{f}-x\left(-A\left(c^2-d^2\right)+2Bcd+c^2C-Cd^2\right)+\frac{d\tan(e+fx)(d(A-C)+Bc)}{f}+$$

```
[Out] -((c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2))*x) - ((2*c*(A - C)*d + B*(c^2 -
 d^2))*Log[Cos[e + f*x]])/f + (d*(B*c + (A - C)*d)*Tan[e + f*x])/f + (B*(c
+ d*Tan[e + f*x])^2)/(2*f) + (C*(c + d*Tan[e + f*x])^3)/(3*d*f)
```

Rubi [A] time = 0.155012, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {3630, 3528, 3525, 3475}

$$-\frac{\left(2cd(A-C)+B\left(c^2-d^2\right)\right)\log(\cos(e+fx))}{f}-x\left(-A\left(c^2-d^2\right)+2Bcd+c^2C-Cd^2\right)+\frac{d\tan(e+fx)(d(A-C)+Bc)}{f}+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] -((c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2))*x) - ((2*c*(A - C)*d + B*(c^2 -
 d^2))*Log[Cos[e + f*x]])/f + (d*(B*c + (A - C)*d)*Tan[e + f*x])/f + (B*(c
+ d*Tan[e + f*x])^2)/(2*f) + (C*(c + d*Tan[e + f*x])^3)/(3*d*f)
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx = \frac{C(c+d\tan(e+fx))^3}{3df}+\int (A-C+B\tan(e+fx$$

$$= \frac{B(c+d\tan(e+fx))^2}{2f}+\frac{C(c+d\tan(e+fx))^3}{3df}+$$

$$= -\left(c^2C+2Bcd-Cd^2-A\left(c^2-d^2\right)\right)x+\frac{d(Bc+(A}{}$$

$$= -\left(c^2C+2Bcd-Cd^2-A\left(c^2-d^2\right)\right)x-\frac{\left(2c(A-C\right.}{}$$

Mathematica [C] time = 1.12575, size = 176, normalized size = 1.34

$$\frac{3(d(C-A)+Bc)\left(-2d^2\tan(e+fx)+i\left((c+id)^2\log(-\tan(e+fx)+i)-(c-id)^2\log(\tan(e+fx)+i)\right)\right)+3B\left(6cd^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] (2*C*(c + d*Tan[e + f*x])^3 + 3*(B*c + (-A + C)*d)*(I*((c + I*d)^2*Log[I -
Tan[e + f*x]] - (c - I*d)^2*Log[I + Tan[e + f*x]]) - 2*d^2*Tan[e + f*x]) +
3*B*((I*c - d)^3*Log[I - Tan[e + f*x]] - (I*c + d)^3*Log[I + Tan[e + f*x]]
+ 6*c*d^2*Tan[e + f*x] + d^3*Tan[e + f*x]^2))/(6*d*f)
```

Maple [B] time = 0.015, size = 262, normalized size = 2.

$$\frac{Cd^2\left(\tan\left(fx+e\right)\right)^3}{3f}+\frac{B\left(\tan\left(fx+e\right)\right)^2d^2}{2f}+\frac{C\left(\tan\left(fx+e\right)\right)^2cd}{f}+\frac{Ad^2\tan\left(fx+e\right)}{f}+2\frac{Bcd\tan\left(fx+e\right)}{f}+\frac{c^2C}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x)
```

```
[Out] 1/3/f*C*d^2*tan(f*x+e)^3+1/2/f*B*tan(f*x+e)^2*d^2+1/f*C*tan(f*x+e)^2*c*d+1/
f*A*d^2*tan(f*x+e)+2/f*B*c*d*tan(f*x+e)+1/f*c^2*C*tan(f*x+e)-1/f*C*d^2*tan(
f*x+e)+1/f*ln(1+tan(f*x+e)^2)*A*c*d+1/2/f*ln(1+tan(f*x+e)^2)*B*c^2-1/2/f*ln
(1+tan(f*x+e)^2)*B*d^2-1/f*ln(1+tan(f*x+e)^2)*c*C*d+1/f*A*arctan(tan(f*x+e)
)*c^2-1/f*A*arctan(tan(f*x+e))*d^2-2/f*B*arctan(tan(f*x+e))*c*d-1/f*C*arcta
n(tan(f*x+e))*c^2+1/f*C*arctan(tan(f*x+e))*d^2
```

Maxima [A] time = 1.44305, size = 182, normalized size = 1.39

$$\frac{2\,Cd^2\tan\left(fx+e\right)^3+3\left(2\,Ccd+Bd^2\right)\tan\left(fx+e\right)^2+6\left((A-C)c^2-2\,Bcd-(A-C)d^2\right)\left(fx+e\right)+3\left(Bc^2+2\,(A-\right.}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm="
maxima")
```

```
[Out] 1/6*(2*C*d^2*tan(f*x + e)^3 + 3*(2*C*c*d + B*d^2)*tan(f*x + e)^2 + 6*((A -
C)*c^2 - 2*B*c*d - (A - C)*d^2)*(f*x + e) + 3*(B*c^2 + 2*(A - C)*c*d - B*d^
2)*log(tan(f*x + e)^2 + 1) + 6*(C*c^2 + 2*B*c*d + (A - C)*d^2)*tan(f*x + e)
)/f
```

Fricas [A] time = 1.0522, size = 308, normalized size = 2.35

$$\frac{2\,Cd^2\tan\left(fx+e\right)^3+6\left((A-C)c^2-2\,Bcd-(A-C)d^2\right)fx+3\left(2\,Ccd+Bd^2\right)\tan\left(fx+e\right)^2-3\left(Bc^2+2\,(A-C)cd-Bd\right.}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm="
fricas")
```

```
[Out] 1/6*(2*C*d^2*tan(f*x + e)^3 + 6*((A - C)*c^2 - 2*B*c*d - (A - C)*d^2)*f*x +
 3*(2*C*c*d + B*d^2)*tan(f*x + e)^2 - 3*(B*c^2 + 2*(A - C)*c*d - B*d^2)*log
(1/(tan(f*x + e)^2 + 1)) + 6*(C*c^2 + 2*B*c*d + (A - C)*d^2)*tan(f*x + e))/
f
```

Sympy [A] time = 1.33194, size = 241, normalized size = 1.84

$$\begin{cases} Ac^2x+\frac{Acd\log\left(\tan^2\left(e+fx\right)+1\right)}{f}-Ad^2x+\frac{Ad^2\tan\left(e+fx\right)}{f}+\frac{Bc^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}-2Bcdx+\frac{2Bcd\tan\left(e+fx\right)}{f}-\frac{Bd^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f} \\ x\left(c+d\tan\left(e\right)\right)^2\left(A+B\tan\left(e\right)+C\tan^2\left(e\right)\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((A*c**2*x + A*c*d*log(tan(e + f*x)**2 + 1)/f - A*d**2*x + A*d**2*
tan(e + f*x)/f + B*c**2*log(tan(e + f*x)**2 + 1)/(2*f) - 2*B*c*d*x + 2*B*c*
d*tan(e + f*x)/f - B*d**2*log(tan(e + f*x)**2 + 1)/(2*f) + B*d**2*tan(e + f
*x)**2/(2*f) - C*c**2*x + C*c**2*tan(e + f*x)/f - C*c*d*log(tan(e + f*x)**2
 + 1)/f + C*c*d*tan(e + f*x)**2/f + C*d**2*x + C*d**2*tan(e + f*x)**3/(3*f)
 - C*d**2*tan(e + f*x)/f, Ne(f, 0)), (x*(c + d*tan(e))**2*(A + B*tan(e) + C
*tan(e)**2), True))
```

Giac [B] time = 3.58531, size = 2873, normalized size = 21.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm="
giac")
```

```
[Out] 1/6*(6*A*c^2*f*x*tan(f*x)^3*tan(e)^3 - 6*C*c^2*f*x*tan(f*x)^3*tan(e)^3 - 12
*B*c*d*f*x*tan(f*x)^3*tan(e)^3 - 6*A*d^2*f*x*tan(f*x)^3*tan(e)^3 + 6*C*d^2*
f*x*tan(f*x)^3*tan(e)^3 - 3*B*c^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2
 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(
```

```
e) + 1))*tan(f*x)^3*tan(e)^3 - 6*A*c*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan
(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)
*tan(e) + 1))*tan(f*x)^3*tan(e)^3 + 6*C*c*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^
4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan
(f*x)*tan(e) + 1))*tan(f*x)^3*tan(e)^3 + 3*B*d^2*log(4*(tan(e)^2 + 1)/(tan(
f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 -
2*tan(f*x)*tan(e) + 1))*tan(f*x)^3*tan(e)^3 - 18*A*c^2*f*x*tan(f*x)^2*tan(e
)^2 + 18*C*c^2*f*x*tan(f*x)^2*tan(e)^2 + 36*B*c*d*f*x*tan(f*x)^2*tan(e)^2 +
 18*A*d^2*f*x*tan(f*x)^2*tan(e)^2 - 18*C*d^2*f*x*tan(f*x)^2*tan(e)^2 + 6*C*
c*d*tan(f*x)^3*tan(e)^3 + 3*B*d^2*tan(f*x)^3*tan(e)^3 + 9*B*c^2*log(4*(tan(
e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2
+ tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^2*tan(e)^2 + 18*A*c*d*log(4
*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan
(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^2*tan(e)^2 - 18*C*c*d
*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)
^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^2*tan(e)^2 - 9*
B*d^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan
(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^2*tan(e)^2
 - 6*C*c^2*tan(f*x)^3*tan(e)^2 - 12*B*c*d*tan(f*x)^3*tan(e)^2 - 6*A*d^2*tan
(f*x)^3*tan(e)^2 + 6*C*d^2*tan(f*x)^3*tan(e)^2 - 6*C*c^2*tan(f*x)^2*tan(e)^
3 - 12*B*c*d*tan(f*x)^2*tan(e)^3 - 6*A*d^2*tan(f*x)^2*tan(e)^3 + 6*C*d^2*ta
n(f*x)^2*tan(e)^3 + 18*A*c^2*f*x*tan(f*x)*tan(e) - 18*C*c^2*f*x*tan(f*x)*ta
n(e) - 36*B*c*d*f*x*tan(f*x)*tan(e) - 18*A*d^2*f*x*tan(f*x)*tan(e) + 18*C*d
^2*f*x*tan(f*x)*tan(e) + 6*C*c*d*tan(f*x)^3*tan(e) + 3*B*d^2*tan(f*x)^3*tan
(e) - 6*C*c*d*tan(f*x)^2*tan(e)^2 - 3*B*d^2*tan(f*x)^2*tan(e)^2 + 6*C*c*d*t
an(f*x)*tan(e)^3 + 3*B*d^2*tan(f*x)*tan(e)^3 - 2*C*d^2*tan(f*x)^3 - 9*B*c^2
*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)
^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)*tan(e) - 18*A*c
*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*
x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)*tan(e) + 18*C
*c*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(
f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)*tan(e) + 9*
B*d^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan
(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)*tan(e) + 1
2*C*c^2*tan(f*x)^2*tan(e) + 24*B*c*d*tan(f*x)^2*tan(e) + 12*A*d^2*tan(f*x)^
2*tan(e) - 18*C*d^2*tan(f*x)^2*tan(e) + 12*C*c^2*tan(f*x)*tan(e)^2 + 24*B*c
*d*tan(f*x)*tan(e)^2 + 12*A*d^2*tan(f*x)*tan(e)^2 - 18*C*d^2*tan(f*x)*tan(e
)^2 - 2*C*d^2*tan(e)^3 - 6*A*c^2*f*x + 6*C*c^2*f*x + 12*B*c*d*f*x + 6*A*d^2
*f*x - 6*C*d^2*f*x - 6*C*c*d*tan(f*x)^2 - 3*B*d^2*tan(f*x)^2 + 6*C*c*d*tan(
f*x)*tan(e) + 3*B*d^2*tan(f*x)*tan(e) - 6*C*c*d*tan(e)^2 - 3*B*d^2*tan(e)^2
 + 3*B*c^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e)
+ tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)) + 6*A*c*d*log(
4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*ta
n(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)) - 6*C*c*d*log(4*(tan(e)^2 + 1
)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*
x)^2 - 2*tan(f*x)*tan(e) + 1)) - 3*B*d^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*t
an(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*
x)*tan(e) + 1)) - 6*C*c^2*tan(f*x) - 12*B*c*d*tan(f*x) - 6*A*d^2*tan(f*x) +
 6*C*d^2*tan(f*x) - 6*C*c^2*tan(e) - 12*B*c*d*tan(e) - 6*A*d^2*tan(e) + 6*C
*d^2*tan(e) - 6*C*c*d - 3*B*d^2)/(f*tan(f*x)^3*tan(e)^3 - 3*f*tan(f*x)^2*ta
n(e)^2 + 3*f*tan(f*x)*tan(e) - f)
```

3.61 $\int \frac{(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=254

$$-\frac{\log(\cos(e+fx))\left(A\left(2acd-b\left(c^2-d^2\right)\right)+a\left(Bc^2-Bd^2-2cCd\right)+b\left(2Bcd+c^2C-Cd^2\right)\right)}{f\left(a^2+b^2\right)}-\frac{x\left(a\left(-A\left(c^2-d^2\right)+2Bcd\right.\right.}{}$$

```
[Out] -(((a*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - b*(2*c*(A - C)*d + B*(c^2
 - d^2)))*x)/(a^2 + b^2)) - ((a*(B*c^2 - 2*c*C*d - B*d^2) + b*(c^2*C + 2*B*
c*d - C*d^2) + A*(2*a*c*d - b*(c^2 - d^2)))*Log[Cos[e + f*x]])/((a^2 + b^2)
*f) + ((A*b^2 - a*(b*B - a*C))*(b*c - a*d)^2*Log[a + b*Tan[e + f*x]])/(b^3*
(a^2 + b^2)*f) + (d*(b*c*C + b*B*d - a*C*d)*Tan[e + f*x])/(b^2*f) + (C*(c +
 d*Tan[e + f*x])^2)/(2*b*f)
```

Rubi [A] time = 0.82732, antiderivative size = 252, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3647, 3637, 3626, 3617, 31, 3475}

$$-\frac{\log(\cos(e+fx))\left(2aAcd+aB\left(c^2-d^2\right)-2acCd-Ab\left(c^2-d^2\right)+b\left(2Bcd+c^2C-Cd^2\right)\right)}{f\left(a^2+b^2\right)}-\frac{x\left(a\left(-A\left(c^2-d^2\right)+2Bcd+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a + b
*Tan[e + f*x]),x]
```

```
[Out] -(((a*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - b*(2*c*(A - C)*d + B*(c^2
 - d^2)))*x)/(a^2 + b^2)) - ((2*a*A*c*d - 2*a*c*C*d - A*b*(c^2 - d^2) + a*B
*(c^2 - d^2) + b*(c^2*C + 2*B*c*d - C*d^2))*Log[Cos[e + f*x]])/((a^2 + b^2)
*f) + ((A*b^2 - a*(b*B - a*C))*(b*c - a*d)^2*Log[a + b*Tan[e + f*x]])/(b^3*
(a^2 + b^2)*f) + (d*(b*c*C + b*B*d - a*C*d)*Tan[e + f*x])/(b^2*f) + (C*(c +
 d*Tan[e + f*x])^2)/(2*b*f)
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
```

```
!LtQ[n, -1]
```

Rule 3626

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((a*A + b*B -
a*C)*x)/(a^2 + b^2), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3617

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx = \frac{C(c+d\tan(e+fx))^2}{2bf} + \frac{\int \frac{(c+d\tan(e+fx))\left(2(Abc-aCd\right.}{}}{}$$

$$= \frac{d(bcC+bBd-aCd)\tan(e+fx)}{b^2f} + \frac{C(c+d\tan(e}{2bf}$$

$$= -\frac{\left(a\left(c^2C+2Bcd-Cd^2-A\left(c^2-d^2\right)\right)-b\left(2c(A\right.\right.}{a^2+b^2}$$

$$= -\frac{\left(a\left(c^2C+2Bcd-Cd^2-A\left(c^2-d^2\right)\right)-b\left(2c(A\right.\right.}{a^2+b^2}$$

$$= -\frac{\left(a\left(c^2C+2Bcd-Cd^2-A\left(c^2-d^2\right)\right)-b\left(2c(A\right.\right.}{a^2+b^2}$$

Mathematica [C] time = 3.03524, size = 190, normalized size = 0.75

$$\frac{\frac{2(bc-ad)^2\left(a(aC-bB)+Ab^2\right)\log(a+b\tan(e+fx))}{b^2\left(a^2+b^2\right)} + \frac{b(c-id)^2(iA+B-iC)\log(\tan(e+fx)+i)}{a-ib} + \frac{b(c+id)^2(-iA+B+iC)\log(-\tan(e+fx)+i)}{a+ib} + \frac{2d\tan(e+fx)(-a}{b}}{2bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(a + b*Tan[e + f*x]),x]
```

```
[Out] ((b*((-I)*A + B + I*C)*(c + I*d)^2*Log[I - Tan[e + f*x]])/(a + I*b) + (b*(I
*A + B - I*C)*(c - I*d)^2*Log[I + Tan[e + f*x]])/(a - I*b) + (2*(A*b^2 + a*
(-(b*B) + a*C))*(b*c - a*d)^2*Log[a + b*Tan[e + f*x]])/(b^2*(a^2 + b^2)) +
(2*d*(b*c*C + b*B*d - a*C*d)*Tan[e + f*x])/b + C*(c + d*Tan[e + f*x])^2)/(2
*b*f)
```

Maple [B] time = 0.048, size = 861, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)),x)
```

```
[Out] 2/f/b/(a^2+b^2)*ln(a+b*tan(f*x+e))*a^2*B*c*d-2/f/b^2/(a^2+b^2)*ln(a+b*tan(f
*x+e))*C*a^3*c*d+1/2/f/(a^2+b^2)*ln(1+tan(f*x+e)^2)*A*b*d^2+1/2/f/(a^2+b^2)
*ln(1+tan(f*x+e)^2)*B*a*c^2-1/2/f/(a^2+b^2)*ln(1+tan(f*x+e)^2)*B*a*d^2-1/f/
(a^2+b^2)*ln(1+tan(f*x+e)^2)*C*a*c*d+2/f/(a^2+b^2)*A*arctan(tan(f*x+e))*b*c
*d-2/f/(a^2+b^2)*B*arctan(tan(f*x+e))*a*c*d-2/f/(a^2+b^2)*C*arctan(tan(f*x+
e))*b*c*d+1/f/b/(a^2+b^2)*ln(a+b*tan(f*x+e))*a^2*A*d^2-1/f/b^2/(a^2+b^2)*ln
(a+b*tan(f*x+e))*B*a^3*d^2-1/f*d^2/b^2*a*C*tan(f*x+e)+2/f*d/b*C*c*tan(f*x+e
)+1/f*b/(a^2+b^2)*ln(a+b*tan(f*x+e))*A*c^2-1/2/f/(a^2+b^2)*ln(1+tan(f*x+e)^
2)*A*b*c^2-1/f/(a^2+b^2)*ln(a+b*tan(f*x+e))*B*a*c^2-2/f/(a^2+b^2)*ln(a+b*ta
n(f*x+e))*A*a*c*d+1/f/b^3/(a^2+b^2)*ln(a+b*tan(f*x+e))*C*a^4*d^2+1/f/b/(a^2
+b^2)*ln(a+b*tan(f*x+e))*C*a^2*c^2+1/f/(a^2+b^2)*ln(1+tan(f*x+e)^2)*B*b*c*d
+1/f/(a^2+b^2)*ln(1+tan(f*x+e)^2)*A*a*c*d+1/f*d^2/b*B*tan(f*x+e)+1/2/f*d^2/
b*C*tan(f*x+e)^2+1/2/f/(a^2+b^2)*ln(1+tan(f*x+e)^2)*C*b*c^2-1/2/f/(a^2+b^2)
*ln(1+tan(f*x+e)^2)*C*b*d^2+1/f/(a^2+b^2)*A*arctan(tan(f*x+e))*a*c^2-1/f/(a
^2+b^2)*A*arctan(tan(f*x+e))*a*d^2+1/f/(a^2+b^2)*B*arctan(tan(f*x+e))*b*c^2
-1/f/(a^2+b^2)*B*arctan(tan(f*x+e))*b*d^2-1/f/(a^2+b^2)*C*arctan(tan(f*x+e)
)*a*c^2+1/f/(a^2+b^2)*C*arctan(tan(f*x+e))*a*d^2
```

Maxima [A] time = 1.52073, size = 392, normalized size = 1.54

$$\frac{\frac{2\left(((A-C)a+Bb)c^2-2\,(Ba-(A-C)b)cd-((A-C)a+Bb)d^2\right)\left(fx+e\right)}{a^2+b^2}+\frac{2\left(\left(Ca^2b^2-Bab^3+Ab^4\right)c^2-2\left(Ca^3b-Ba^2b^2+Aab^3\right)cd+\left(Ca^4-Ba^3b+Aa^2b^2\right)d^2\right)\log\left(b\tan\left(fx+e\right)\right.}{a^2b^3+b^5}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e
)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(((A - C)*a + B*b)*c^2 - 2*(B*a - (A - C)*b)*c*d - ((A - C)*a + B*b)
*d^2)*(f*x + e)/(a^2 + b^2) + 2*((C*a^2*b^2 - B*a*b^3 + A*b^4)*c^2 - 2*(C*a
^3*b - B*a^2*b^2 + A*a*b^3)*c*d + (C*a^4 - B*a^3*b + A*a^2*b^2)*d^2)*log(b*
tan(f*x + e) + a)/(a^2*b^3 + b^5) + ((B*a - (A - C)*b)*c^2 + 2*((A - C)*a +
 B*b)*c*d - (B*a - (A - C)*b)*d^2)*log(tan(f*x + e)^2 + 1)/(a^2 + b^2) + (C
*b*d^2*tan(f*x + e)^2 + 2*(2*C*b*c*d - (C*a - B*b)*d^2)*tan(f*x + e))/b^2)/
f
```

Fricas [A] time = 2.91679, size = 822, normalized size = 3.24

$$\left(Ca^2b^2+Cb^4\right)d^2\tan\left(fx+e\right)^2+2\left(\left((A-C)ab^3+Bb^4\right)c^2-2\left(Bab^3-(A-C)b^4\right)cd-\left((A-C)ab^3+Bb^4\right)d^2\right)fx+\left(\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e
)),x, algorithm="fricas")
```

```
[Out] 1/2*((C*a^2*b^2 + C*b^4)*d^2*tan(f*x + e)^2 + 2*(((A - C)*a*b^3 + B*b^4)*c^
2 - 2*(B*a*b^3 - (A - C)*b^4)*c*d - ((A - C)*a*b^3 + B*b^4)*d^2)*f*x + ((C*
a^2*b^2 - B*a*b^3 + A*b^4)*c^2 - 2*(C*a^3*b - B*a^2*b^2 + A*a*b^3)*c*d + (C
*a^4 - B*a^3*b + A*a^2*b^2)*d^2)*log((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x +
e) + a^2)/(tan(f*x + e)^2 + 1)) - ((C*a^2*b^2 + C*b^4)*c^2 - 2*(C*a^3*b - B
*a^2*b^2 + C*a*b^3 - B*b^4)*c*d + (C*a^4 - B*a^3*b + A*a^2*b^2 - B*a*b^3 +
(A - C)*b^4)*d^2)*log(1/(tan(f*x + e)^2 + 1)) + 2*(2*(C*a^2*b^2 + C*b^4)*c*
d - (C*a^3*b - B*a^2*b^2 + C*a*b^3 - B*b^4)*d^2)*tan(f*x + e))/((a^2*b^3 +
b^5)*f)
```

Sympy [A] time = 36.782, size = 4444, normalized size = 17.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x
+e)),x)
```

```
[Out] Piecewise((zoo*x*(c + d*tan(e))**2*(A + B*tan(e) + C*tan(e)**2)/tan(e), Eq(
a, 0) & Eq(b, 0) & Eq(f, 0)), ((A*c**2*x + A*c*d*log(tan(e + f*x)**2 + 1)/f
 - A*d**2*x + A*d**2*tan(e + f*x)/f + B*c**2*log(tan(e + f*x)**2 + 1)/(2*f)
 - 2*B*c*d*x + 2*B*c*d*tan(e + f*x)/f - B*d**2*log(tan(e + f*x)**2 + 1)/(2*
f) + B*d**2*tan(e + f*x)**2/(2*f) - C*c**2*x + C*c**2*tan(e + f*x)/f - C*c*
d*log(tan(e + f*x)**2 + 1)/f + C*c*d*tan(e + f*x)**2/f + C*d**2*x + C*d**2*
tan(e + f*x)**3/(3*f) - C*d**2*tan(e + f*x)/f)/a, Eq(b, 0)), (-I*A*c**2*f*x
*tan(e + f*x)/(-2*b*f*tan(e + f*x) + 2*I*b*f) - A*c**2*f*x/(-2*b*f*tan(e +
f*x) + 2*I*b*f) - I*A*c**2/(-2*b*f*tan(e + f*x) + 2*I*b*f) - 2*A*c*d*f*x*ta
n(e + f*x)/(-2*b*f*tan(e + f*x) + 2*I*b*f) + 2*I*A*c*d*f*x/(-2*b*f*tan(e +
f*x) + 2*I*b*f) + 2*A*c*d/(-2*b*f*tan(e + f*x) + 2*I*b*f) - I*A*d**2*f*x*ta
n(e + f*x)/(-2*b*f*tan(e + f*x) + 2*I*b*f) - A*d**2*f*x/(-2*b*f*tan(e + f*x
) + 2*I*b*f) - A*d**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(-2*b*f*tan(e +
 f*x) + 2*I*b*f) + I*A*d**2*log(tan(e + f*x)**2 + 1)/(-2*b*f*tan(e + f*x) +
 2*I*b*f) + I*A*d**2/(-2*b*f*tan(e + f*x) + 2*I*b*f) - B*c**2*f*x*tan(e + f
*x)/(-2*b*f*tan(e + f*x) + 2*I*b*f) + I*B*c**2*f*x/(-2*b*f*tan(e + f*x) + 2
*I*b*f) + B*c**2/(-2*b*f*tan(e + f*x) + 2*I*b*f) - 2*I*B*c*d*f*x*tan(e + f*
x)/(-2*b*f*tan(e + f*x) + 2*I*b*f) - 2*B*c*d*f*x/(-2*b*f*tan(e + f*x) + 2*I
*b*f) - 2*B*c*d*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(-2*b*f*tan(e + f*x)
+ 2*I*b*f) + 2*I*B*c*d*log(tan(e + f*x)**2 + 1)/(-2*b*f*tan(e + f*x) + 2*I*
b*f) + 2*I*B*c*d/(-2*b*f*tan(e + f*x) + 2*I*b*f) + 3*B*d**2*f*x*tan(e + f*x
)/(-2*b*f*tan(e + f*x) + 2*I*b*f) - 3*I*B*d**2*f*x/(-2*b*f*tan(e + f*x) + 2
*I*b*f) - I*B*d**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(-2*b*f*tan(e + f*
x) + 2*I*b*f) - B*d**2*log(tan(e + f*x)**2 + 1)/(-2*b*f*tan(e + f*x) + 2*I*
b*f) - 2*B*d**2*tan(e + f*x)**2/(-2*b*f*tan(e + f*x) + 2*I*b*f) - 3*B*d**2/
(-2*b*f*tan(e + f*x) + 2*I*b*f) - I*C*c**2*f*x*tan(e + f*x)/(-2*b*f*tan(e +
```

```
f*x) + 2*I*b*f) - C*c**2*f*x/(-2*b*f*tan(e + f*x) + 2*I*b*f) - C*c**2*log(
tan(e + f*x)**2 + 1)*tan(e + f*x)/(-2*b*f*tan(e + f*x) + 2*I*b*f) + I*C*c**
2*log(tan(e + f*x)**2 + 1)/(-2*b*f*tan(e + f*x) + 2*I*b*f) + I*C*c**2/(-2*b
*f*tan(e + f*x) + 2*I*b*f) + 6*C*c*d*f*x*tan(e + f*x)/(-2*b*f*tan(e + f*x)
+ 2*I*b*f) - 6*I*C*c*d*f*x/(-2*b*f*tan(e + f*x) + 2*I*b*f) - 2*I*C*c*d*log(
tan(e + f*x)**2 + 1)*tan(e + f*x)/(-2*b*f*tan(e + f*x) + 2*I*b*f) - 2*C*c*d
*log(tan(e + f*x)**2 + 1)/(-2*b*f*tan(e + f*x) + 2*I*b*f) - 4*C*c*d*tan(e +
 f*x)**2/(-2*b*f*tan(e + f*x) + 2*I*b*f) - 6*C*c*d/(-2*b*f*tan(e + f*x) + 2
*I*b*f) + 3*I*C*d**2*f*x*tan(e + f*x)/(-2*b*f*tan(e + f*x) + 2*I*b*f) + 3*C
*d**2*f*x/(-2*b*f*tan(e + f*x) + 2*I*b*f) + 2*C*d**2*log(tan(e + f*x)**2 +
1)*tan(e + f*x)/(-2*b*f*tan(e + f*x) + 2*I*b*f) - 2*I*C*d**2*log(tan(e + f*
x)**2 + 1)/(-2*b*f*tan(e + f*x) + 2*I*b*f) - C*d**2*tan(e + f*x)**3/(-2*b*f
*tan(e + f*x) + 2*I*b*f) - I*C*d**2*tan(e + f*x)**2/(-2*b*f*tan(e + f*x) +
2*I*b*f) - 3*I*C*d**2/(-2*b*f*tan(e + f*x) + 2*I*b*f), Eq(a, -I*b)), (-I*A*
c**2*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x) + 2*I*b*f) + A*c**2*f*x/(2*b*f*ta
n(e + f*x) + 2*I*b*f) - I*A*c**2/(2*b*f*tan(e + f*x) + 2*I*b*f) + 2*A*c*d*f
*x*tan(e + f*x)/(2*b*f*tan(e + f*x) + 2*I*b*f) + 2*I*A*c*d*f*x/(2*b*f*tan(e
 + f*x) + 2*I*b*f) - 2*A*c*d/(2*b*f*tan(e + f*x) + 2*I*b*f) - I*A*d**2*f*x*
tan(e + f*x)/(2*b*f*tan(e + f*x) + 2*I*b*f) + A*d**2*f*x/(2*b*f*tan(e + f*x
) + 2*I*b*f) + A*d**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*b*f*tan(e +
f*x) + 2*I*b*f) + I*A*d**2*log(tan(e + f*x)**2 + 1)/(2*b*f*tan(e + f*x) + 2
*I*b*f) + I*A*d**2/(2*b*f*tan(e + f*x) + 2*I*b*f) + B*c**2*f*x*tan(e + f*x)
/(2*b*f*tan(e + f*x) + 2*I*b*f) + I*B*c**2*f*x/(2*b*f*tan(e + f*x) + 2*I*b*
f) - B*c**2/(2*b*f*tan(e + f*x) + 2*I*b*f) - 2*I*B*c*d*f*x*tan(e + f*x)/(2*
b*f*tan(e + f*x) + 2*I*b*f) + 2*B*c*d*f*x/(2*b*f*tan(e + f*x) + 2*I*b*f) +
2*B*c*d*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*b*f*tan(e + f*x) + 2*I*b*f
) + 2*I*B*c*d*log(tan(e + f*x)**2 + 1)/(2*b*f*tan(e + f*x) + 2*I*b*f) + 2*I
*B*c*d/(2*b*f*tan(e + f*x) + 2*I*b*f) - 3*B*d**2*f*x*tan(e + f*x)/(2*b*f*ta
n(e + f*x) + 2*I*b*f) - 3*I*B*d**2*f*x/(2*b*f*tan(e + f*x) + 2*I*b*f) - I*B
*d**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*b*f*tan(e + f*x) + 2*I*b*f)
+ B*d**2*log(tan(e + f*x)**2 + 1)/(2*b*f*tan(e + f*x) + 2*I*b*f) + 2*B*d**2
*tan(e + f*x)**2/(2*b*f*tan(e + f*x) + 2*I*b*f) + 3*B*d**2/(2*b*f*tan(e + f
*x) + 2*I*b*f) - I*C*c**2*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x) + 2*I*b*f) +
 C*c**2*f*x/(2*b*f*tan(e + f*x) + 2*I*b*f) + C*c**2*log(tan(e + f*x)**2 + 1
)*tan(e + f*x)/(2*b*f*tan(e + f*x) + 2*I*b*f) + I*C*c**2*log(tan(e + f*x)**
2 + 1)/(2*b*f*tan(e + f*x) + 2*I*b*f) + I*C*c**2/(2*b*f*tan(e + f*x) + 2*I*
b*f) - 6*C*c*d*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x) + 2*I*b*f) - 6*I*C*c*d*
f*x/(2*b*f*tan(e + f*x) + 2*I*b*f) - 2*I*C*c*d*log(tan(e + f*x)**2 + 1)*tan
(e + f*x)/(2*b*f*tan(e + f*x) + 2*I*b*f) + 2*C*c*d*log(tan(e + f*x)**2 + 1)
/(2*b*f*tan(e + f*x) + 2*I*b*f) + 4*C*c*d*tan(e + f*x)**2/(2*b*f*tan(e + f*
x) + 2*I*b*f) + 6*C*c*d/(2*b*f*tan(e + f*x) + 2*I*b*f) + 3*I*C*d**2*f*x*tan
(e + f*x)/(2*b*f*tan(e + f*x) + 2*I*b*f) - 3*C*d**2*f*x/(2*b*f*tan(e + f*x)
 + 2*I*b*f) - 2*C*d**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*b*f*tan(e +
 f*x) + 2*I*b*f) - 2*I*C*d**2*log(tan(e + f*x)**2 + 1)/(2*b*f*tan(e + f*x)
+ 2*I*b*f) + C*d**2*tan(e + f*x)**3/(2*b*f*tan(e + f*x) + 2*I*b*f) - I*C*d*
*2*tan(e + f*x)**2/(2*b*f*tan(e + f*x) + 2*I*b*f) - 3*I*C*d**2/(2*b*f*tan(e
 + f*x) + 2*I*b*f), Eq(a, I*b)), (x*(c + d*tan(e))**2*(A + B*tan(e) + C*tan
(e)**2)/(a + b*tan(e)), Eq(f, 0)), (2*A*a**2*b**2*d**2*log(a/b + tan(e + f*
x))/(2*a**2*b**3*f + 2*b**5*f) + 2*A*a*b**3*c**2*f*x/(2*a**2*b**3*f + 2*b**
5*f) - 4*A*a*b**3*c*d*log(a/b + tan(e + f*x))/(2*a**2*b**3*f + 2*b**5*f) +
2*A*a*b**3*c*d*log(tan(e + f*x)**2 + 1)/(2*a**2*b**3*f + 2*b**5*f) - 2*A*a*
b**3*d**2*f*x/(2*a**2*b**3*f + 2*b**5*f) + 2*A*b**4*c**2*log(a/b + tan(e +
f*x))/(2*a**2*b**3*f + 2*b**5*f) - A*b**4*c**2*log(tan(e + f*x)**2 + 1)/(2*
a**2*b**3*f + 2*b**5*f) + 4*A*b**4*c*d*f*x/(2*a**2*b**3*f + 2*b**5*f) + A*b
**4*d**2*log(tan(e + f*x)**2 + 1)/(2*a**2*b**3*f + 2*b**5*f) - 2*B*a**3*b*d
**2*log(a/b + tan(e + f*x))/(2*a**2*b**3*f + 2*b**5*f) + 4*B*a**2*b**2*c*d*
log(a/b + tan(e + f*x))/(2*a**2*b**3*f + 2*b**5*f) + 2*B*a**2*b**2*d**2*tan
(e + f*x)/(2*a**2*b**3*f + 2*b**5*f) - 2*B*a*b**3*c**2*log(a/b + tan(e + f*
x))/(2*a**2*b**3*f + 2*b**5*f) + B*a*b**3*c**2*log(tan(e + f*x)**2 + 1)/(2*
```

```
a**2*b**3*f + 2*b**5*f) - 4*B*a*b**3*c*d*f*x/(2*a**2*b**3*f + 2*b**5*f) - B
*a*b**3*d**2*log(tan(e + f*x)**2 + 1)/(2*a**2*b**3*f + 2*b**5*f) + 2*B*b**4
*c**2*f*x/(2*a**2*b**3*f + 2*b**5*f) + 2*B*b**4*c*d*log(tan(e + f*x)**2 + 1
)/(2*a**2*b**3*f + 2*b**5*f) - 2*B*b**4*d**2*f*x/(2*a**2*b**3*f + 2*b**5*f)
 + 2*B*b**4*d**2*tan(e + f*x)/(2*a**2*b**3*f + 2*b**5*f) + 2*C*a**4*d**2*lo
g(a/b + tan(e + f*x))/(2*a**2*b**3*f + 2*b**5*f) - 4*C*a**3*b*c*d*log(a/b +
 tan(e + f*x))/(2*a**2*b**3*f + 2*b**5*f) - 2*C*a**3*b*d**2*tan(e + f*x)/(2
*a**2*b**3*f + 2*b**5*f) + 2*C*a**2*b**2*c**2*log(a/b + tan(e + f*x))/(2*a*
*2*b**3*f + 2*b**5*f) + 4*C*a**2*b**2*c*d*tan(e + f*x)/(2*a**2*b**3*f + 2*b
**5*f) + C*a**2*b**2*d**2*tan(e + f*x)**2/(2*a**2*b**3*f + 2*b**5*f) - 2*C*
a*b**3*c**2*f*x/(2*a**2*b**3*f + 2*b**5*f) - 2*C*a*b**3*c*d*log(tan(e + f*x
)**2 + 1)/(2*a**2*b**3*f + 2*b**5*f) + 2*C*a*b**3*d**2*f*x/(2*a**2*b**3*f +
 2*b**5*f) - 2*C*a*b**3*d**2*tan(e + f*x)/(2*a**2*b**3*f + 2*b**5*f) + C*b*
*4*c**2*log(tan(e + f*x)**2 + 1)/(2*a**2*b**3*f + 2*b**5*f) - 4*C*b**4*c*d*
f*x/(2*a**2*b**3*f + 2*b**5*f) + 4*C*b**4*c*d*tan(e + f*x)/(2*a**2*b**3*f +
 2*b**5*f) - C*b**4*d**2*log(tan(e + f*x)**2 + 1)/(2*a**2*b**3*f + 2*b**5*f
) + C*b**4*d**2*tan(e + f*x)**2/(2*a**2*b**3*f + 2*b**5*f), True))
```

Giac [A] time = 1.7387, size = 456, normalized size = 1.8

$$\frac{\frac{2\left(Aac^2-Cac^2+Bbc^2-2\,Bacd+2\,Abcd-2\,Cbcd-Aad^2+Cad^2-Bbd^2\right)(fx+e)}{a^2+b^2}+\frac{\left(Bac^2-Abc^2+Cbc^2+2\,Aacd-2\,Cacd+2\,Bbcd-Bad^2+Abd^2-Cbd^2\right)\log\left(\tan\left(fx+e\right.\right.}{a^2+b^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e
)),x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a*c^2 - C*a*c^2 + B*b*c^2 - 2*B*a*c*d + 2*A*b*c*d - 2*C*b*c*d - A
*a*d^2 + C*a*d^2 - B*b*d^2)*(f*x + e)/(a^2 + b^2) + (B*a*c^2 - A*b*c^2 + C*
b*c^2 + 2*A*a*c*d - 2*C*a*c*d + 2*B*b*c*d - B*a*d^2 + A*b*d^2 - C*b*d^2)*lo
g(tan(f*x + e)^2 + 1)/(a^2 + b^2) + 2*(C*a^2*b^2*c^2 - B*a*b^3*c^2 + A*b^4*
c^2 - 2*C*a^3*b*c*d + 2*B*a^2*b^2*c*d - 2*A*a*b^3*c*d + C*a^4*d^2 - B*a^3*b
*d^2 + A*a^2*b^2*d^2)*log(abs(b*tan(f*x + e) + a))/(a^2*b^3 + b^5) + (C*b*d
^2*tan(f*x + e)^2 + 4*C*b*c*d*tan(f*x + e) - 2*C*a*d^2*tan(f*x + e) + 2*B*b
*d^2*tan(f*x + e))/b^2)/f
```

3.62 $\int \frac{(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=415

$$-\frac{\log(\cos(e+fx))\left(a^2\left(2cd(A-C)+B\left(c^2-d^2\right)\right)+2ab\left(-A\left(c^2-d^2\right)+2Bcd+c^2C-Cd^2\right)-b^2\left(2cd(A-C)+B\left(c^2-d\right.\right.\right.}{f\left(a^2+b^2\right)^2}$$

```
[Out] -((((a^2*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - b^2*(c^2*C + 2*B*c*d -
C*d^2 - A*(c^2 - d^2)) - 2*a*b*(2*c*(A - C)*d + B*(c^2 - d^2)))*x)/(a^2 + b
^2)^2) - ((2*a*b*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) + a^2*(2*c*(A -
C)*d + B*(c^2 - d^2)) - b^2*(2*c*(A - C)*d + B*(c^2 - d^2)))*Log[Cos[e + f*
x]])/((a^2 + b^2)^2*f) - ((b*c - a*d)*(a^3*b*B*d - 2*a^4*C*d - b^4*(B*c + 2
*A*d) - a*b^3*(2*A*c - 2*c*C - 3*B*d) + a^2*b^2*(B*c - 4*C*d))*Log[a + b*Ta
n[e + f*x]])/(b^3*(a^2 + b^2)^2*f) + ((A*b^2 - a*b*B + 2*a^2*C + b^2*C)*d^2
*Tan[e + f*x])/(b^2*(a^2 + b^2)*f) - ((A*b^2 - a*(b*B - a*C))*(c + d*Tan[e
+ f*x])^2)/(b*(a^2 + b^2)*f*(a + b*Tan[e + f*x]))
```

Rubi [A] time = 1.05285, antiderivative size = 415, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3645, 3637, 3626, 3617, 31, 3475}

$$-\frac{\log(\cos(e+fx))\left(a^2\left(2cd(A-C)+B\left(c^2-d^2\right)\right)+2ab\left(-A\left(c^2-d^2\right)+2Bcd+c^2C-Cd^2\right)-b^2\left(2cd(A-C)+B\left(c^2-d\right.\right.\right.}{f\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a + b
*Tan[e + f*x])^2,x]
```

```
[Out] -((((a^2*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - b^2*(c^2*C + 2*B*c*d -
C*d^2 - A*(c^2 - d^2)) - 2*a*b*(2*c*(A - C)*d + B*(c^2 - d^2)))*x)/(a^2 + b
^2)^2) - ((2*a*b*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) + a^2*(2*c*(A -
C)*d + B*(c^2 - d^2)) - b^2*(2*c*(A - C)*d + B*(c^2 - d^2)))*Log[Cos[e + f*
x]])/((a^2 + b^2)^2*f) - ((b*c - a*d)*(a^3*b*B*d - 2*a^4*C*d - b^4*(B*c + 2
*A*d) - a*b^3*(2*A*c - 2*c*C - 3*B*d) + a^2*b^2*(B*c - 4*C*d))*Log[a + b*Ta
n[e + f*x]])/(b^3*(a^2 + b^2)^2*f) + ((A*b^2 - a*b*B + 2*a^2*C + b^2*C)*d^2
*Tan[e + f*x])/(b^2*(a^2 + b^2)*f) - ((A*b^2 - a*(b*B - a*C))*(c + d*Tan[e
+ f*x])^2)/(b*(a^2 + b^2)*f*(a + b*Tan[e + f*x]))
```

Rule 3645

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*d^2 + c*(c*C - B*d))*(a + b*Tan[e
+ f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3626

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((a*A + b*B -
a*C)*x)/(a^2 + b^2), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3617

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^2}{b\left(a^2+b^2\right)f(a+b\tan(e+fx))} + \frac{\int \frac{(c+}{}}{}$$

$$= \frac{\left(Ab^2-abB+2a^2C+b^2C\right)d^2\tan(e+fx)}{b^2\left(a^2+b^2\right)f} - \frac{\left(Ab\right.}{}$$

$$= -\frac{\left(a^2\left(c^2C+2Bcd-Cd^2-A\left(c^2-d^2\right)\right)-b^2\left(c^2C\right.\right.}{}$$

$$= -\frac{\left(a^2\left(c^2C+2Bcd-Cd^2-A\left(c^2-d^2\right)\right)-b^2\left(c^2C\right.\right.}{}$$

$$= -\frac{\left(a^2\left(c^2C+2Bcd-Cd^2-A\left(c^2-d^2\right)\right)-b^2\left(c^2C\right.\right.}{}$$

Mathematica [C] time = 7.78334, size = 2640, normalized size = 6.36

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(a + b*Tan[e + f*x])^2,x]
```

```
[Out] ((-I)*(-2*a^6*A*b^6*c^2 + (2*I)*a^5*A*b^7*c^2 - 2*a^4*A*b^8*c^2 + (2*I)*a^3
*A*b^9*c^2 + a^7*b^5*B*c^2 - I*a^6*b^6*B*c^2 - a^3*b^9*B*c^2 + I*a^2*b^10*B
*c^2 + 2*a^6*b^6*c^2*C - (2*I)*a^5*b^7*c^2*C + 2*a^4*b^8*c^2*C - (2*I)*a^3*
b^9*c^2*C + 2*a^7*A*b^5*c*d - (2*I)*a^6*A*b^6*c*d - 2*a^3*A*b^9*c*d + (2*I)
*a^2*A*b^10*c*d + 4*a^6*b^6*B*c*d - (4*I)*a^5*b^7*B*c*d + 4*a^4*b^8*B*c*d -
 (4*I)*a^3*b^9*B*c*d - 2*a^9*b^3*c*C*d + (2*I)*a^8*b^4*c*C*d - 8*a^7*b^5*c*
C*d + (8*I)*a^6*b^6*c*C*d - 6*a^5*b^7*c*C*d + (6*I)*a^4*b^8*c*C*d + 2*a^6*A
*b^6*d^2 - (2*I)*a^5*A*b^7*d^2 + 2*a^4*A*b^8*d^2 - (2*I)*a^3*A*b^9*d^2 - a^
9*b^3*B*d^2 + I*a^8*b^4*B*d^2 - 4*a^7*b^5*B*d^2 + (4*I)*a^6*b^6*B*d^2 - 3*a
^5*b^7*B*d^2 + (3*I)*a^4*b^8*B*d^2 + 2*a^10*b^2*C*d^2 - (2*I)*a^9*b^3*C*d^2
 + 6*a^8*b^4*C*d^2 - (6*I)*a^7*b^5*C*d^2 + 4*a^6*b^6*C*d^2 - (4*I)*a^5*b^7*
C*d^2)*(e + f*x)*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(c + d*Tan[e + f*x])^2
)/(a^2*(a - I*b)^4*(a + I*b)^3*b^5*f*(c*Cos[e + f*x] + d*Sin[e + f*x])^2*(a
 + b*Tan[e + f*x])^2) - (I*(2*a*A*b^4*c^2 - a^2*b^3*B*c^2 + b^5*B*c^2 - 2*a
*b^4*c^2*C - 2*a^2*A*b^3*c*d + 2*A*b^5*c*d - 4*a*b^4*B*c*d + 2*a^4*b*c*C*d
+ 6*a^2*b^3*c*C*d - 2*a*A*b^4*d^2 + a^4*b*B*d^2 + 3*a^2*b^3*B*d^2 - 2*a^5*C
*d^2 - 4*a^3*b^2*C*d^2)*ArcTan[Tan[e + f*x]]*(a*Cos[e + f*x] + b*Sin[e + f*
x])^2*(c + d*Tan[e + f*x])^2)/(b^3*(a^2 + b^2)^2*f*(c*Cos[e + f*x] + d*Sin[
e + f*x])^2*(a + b*Tan[e + f*x])^2) + ((-2*b*c*C*d - b*B*d^2 + 2*a*C*d^2)*L
og[Cos[e + f*x]]*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(c + d*Tan[e + f*x])^2
)/(b^3*f*(c*Cos[e + f*x] + d*Sin[e + f*x])^2*(a + b*Tan[e + f*x])^2) + ((2*
a*A*b^4*c^2 - a^2*b^3*B*c^2 + b^5*B*c^2 - 2*a*b^4*c^2*C - 2*a^2*A*b^3*c*d +
 2*A*b^5*c*d - 4*a*b^4*B*c*d + 2*a^4*b*c*C*d + 6*a^2*b^3*c*C*d - 2*a*A*b^4*
d^2 + a^4*b*B*d^2 + 3*a^2*b^3*B*d^2 - 2*a^5*C*d^2 - 4*a^3*b^2*C*d^2)*Log[(a
*Cos[e + f*x] + b*Sin[e + f*x])^2]*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(c +
 d*Tan[e + f*x])^2)/(2*b^3*(a^2 + b^2)^2*f*(c*Cos[e + f*x] + d*Sin[e + f*x]
)^2*(a + b*Tan[e + f*x])^2) + (Sec[e + f*x]*(a*Cos[e + f*x] + b*Sin[e + f*x
])*(a^5*b*C*d^2 + 2*a^3*b^3*C*d^2 + a*b^5*C*d^2 + a^4*A*b^2*c^2*(e + f*x) -
 a^2*A*b^4*c^2*(e + f*x) + 2*a^3*b^3*B*c^2*(e + f*x) - a^4*b^2*c^2*C*(e + f
*x) + a^2*b^4*c^2*C*(e + f*x) + 4*a^3*A*b^3*c*d*(e + f*x) - 2*a^4*b^2*B*c*d
*(e + f*x) + 2*a^2*b^4*B*c*d*(e + f*x) - 4*a^3*b^3*c*C*d*(e + f*x) - a^4*A*
b^2*d^2*(e + f*x) + a^2*A*b^4*d^2*(e + f*x) - 2*a^3*b^3*B*d^2*(e + f*x) + a
^4*b^2*C*d^2*(e + f*x) - a^2*b^4*C*d^2*(e + f*x) - a^5*b*C*d^2*Cos[2*(e + f
*x)] - 2*a^3*b^3*C*d^2*Cos[2*(e + f*x)] - a*b^5*C*d^2*Cos[2*(e + f*x)] + a^
4*A*b^2*c^2*(e + f*x)*Cos[2*(e + f*x)] - a^2*A*b^4*c^2*(e + f*x)*Cos[2*(e +
 f*x)] + 2*a^3*b^3*B*c^2*(e + f*x)*Cos[2*(e + f*x)] - a^4*b^2*c^2*C*(e + f*
x)*Cos[2*(e + f*x)] + a^2*b^4*c^2*C*(e + f*x)*Cos[2*(e + f*x)] + 4*a^3*A*b^
3*c*d*(e + f*x)*Cos[2*(e + f*x)] - 2*a^4*b^2*B*c*d*(e + f*x)*Cos[2*(e + f*x
)] + 2*a^2*b^4*B*c*d*(e + f*x)*Cos[2*(e + f*x)] - 4*a^3*b^3*c*C*d*(e + f*x)
*Cos[2*(e + f*x)] - a^4*A*b^2*d^2*(e + f*x)*Cos[2*(e + f*x)] + a^2*A*b^4*d^
2*(e + f*x)*Cos[2*(e + f*x)] - 2*a^3*b^3*B*d^2*(e + f*x)*Cos[2*(e + f*x)] +
 a^4*b^2*C*d^2*(e + f*x)*Cos[2*(e + f*x)] - a^2*b^4*C*d^2*(e + f*x)*Cos[2*(
e + f*x)] + a^2*A*b^4*c^2*Sin[2*(e + f*x)] + A*b^6*c^2*Sin[2*(e + f*x)] - a
^3*b^3*B*c^2*Sin[2*(e + f*x)] - a*b^5*B*c^2*Sin[2*(e + f*x)] + a^4*b^2*c^2*
C*Sin[2*(e + f*x)] + a^2*b^4*c^2*C*Sin[2*(e + f*x)] - 2*a^3*A*b^3*c*d*Sin[2
*(e + f*x)] - 2*a*A*b^5*c*d*Sin[2*(e + f*x)] + 2*a^4*b^2*B*c*d*Sin[2*(e + f
*x)] + 2*a^2*b^4*B*c*d*Sin[2*(e + f*x)] - 2*a^5*b*c*C*d*Sin[2*(e + f*x)] -
2*a^3*b^3*c*C*d*Sin[2*(e + f*x)] + a^4*A*b^2*d^2*Sin[2*(e + f*x)] + a^2*A*b
^4*d^2*Sin[2*(e + f*x)] - a^5*b*B*d^2*Sin[2*(e + f*x)] - a^3*b^3*B*d^2*Sin[
2*(e + f*x)] + 2*a^6*C*d^2*Sin[2*(e + f*x)] + 3*a^4*b^2*C*d^2*Sin[2*(e + f*
x)] + a^2*b^4*C*d^2*Sin[2*(e + f*x)] + a^3*A*b^3*c^2*(e + f*x)*Sin[2*(e + f
*x)] - a*A*b^5*c^2*(e + f*x)*Sin[2*(e + f*x)] + 2*a^2*b^4*B*c^2*(e + f*x)*S
in[2*(e + f*x)] - a^3*b^3*c^2*C*(e + f*x)*Sin[2*(e + f*x)] + a*b^5*c^2*C*(e
 + f*x)*Sin[2*(e + f*x)] + 4*a^2*A*b^4*c*d*(e + f*x)*Sin[2*(e + f*x)] - 2*a
```

```
^3*b^3*B*c*d*(e + f*x)*Sin[2*(e + f*x)] + 2*a*b^5*B*c*d*(e + f*x)*Sin[2*(e
+ f*x)] - 4*a^2*b^4*c*C*d*(e + f*x)*Sin[2*(e + f*x)] - a^3*A*b^3*d^2*(e + f
*x)*Sin[2*(e + f*x)] + a*A*b^5*d^2*(e + f*x)*Sin[2*(e + f*x)] - 2*a^2*b^4*B
*d^2*(e + f*x)*Sin[2*(e + f*x)] + a^3*b^3*C*d^2*(e + f*x)*Sin[2*(e + f*x)]
- a*b^5*C*d^2*(e + f*x)*Sin[2*(e + f*x)])*(c + d*Tan[e + f*x])^2)/(2*a*(a -
 I*b)^2*(a + I*b)^2*b^2*f*(c*Cos[e + f*x] + d*Sin[e + f*x])^2*(a + b*Tan[e
+ f*x])^2)
```

Maple [B] time = 0.061, size = 1554, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2,x
)
```

```
[Out] -4/f/(a^2+b^2)^2*C*arctan(tan(f*x+e))*a*b*c*d-2/f/b/(a^2+b^2)/(a+b*tan(f*x+
e))*B*a^2*c*d+2/f/b^2/(a^2+b^2)/(a+b*tan(f*x+e))*C*a^3*c*d-4/f*b/(a^2+b^2)^
2*ln(a+b*tan(f*x+e))*B*a*c*d+4/f/(a^2+b^2)^2*A*arctan(tan(f*x+e))*a*b*c*d+2
/f/b^2/(a^2+b^2)^2*ln(a+b*tan(f*x+e))*C*a^4*c*d+2/f/(a^2+b^2)^2*ln(1+tan(f*
x+e)^2)*B*a*b*c*d+1/2/f/(a^2+b^2)^2*ln(1+tan(f*x+e)^2)*B*b^2*d^2-1/f/(a^2+b
^2)^2*A*arctan(tan(f*x+e))*b^2*c^2+1/f/(a^2+b^2)^2*A*arctan(tan(f*x+e))*b^2
*d^2-1/f*b/(a^2+b^2)/(a+b*tan(f*x+e))*A*c^2+1/f/(a^2+b^2)/(a+b*tan(f*x+e))*
B*a*c^2+1/2/f/(a^2+b^2)^2*ln(1+tan(f*x+e)^2)*B*a^2*c^2-1/f/(a^2+b^2)^2*C*ar
ctan(tan(f*x+e))*b^2*d^2+1/f/(a^2+b^2)^2*A*arctan(tan(f*x+e))*a^2*c^2-1/f/(
a^2+b^2)^2*A*arctan(tan(f*x+e))*a^2*d^2-1/2/f/(a^2+b^2)^2*ln(1+tan(f*x+e)^2
)*B*a^2*d^2-1/2/f/(a^2+b^2)^2*ln(1+tan(f*x+e)^2)*B*b^2*c^2-1/f/(a^2+b^2)^2*
C*arctan(tan(f*x+e))*a^2*c^2-1/f/(a^2+b^2)^2*ln(a+b*tan(f*x+e))*B*a^2*c^2+3
/f/(a^2+b^2)^2*ln(a+b*tan(f*x+e))*B*a^2*d^2+1/f*b^2/(a^2+b^2)^2*ln(a+b*tan(
f*x+e))*B*c^2+1/f/(a^2+b^2)^2*C*arctan(tan(f*x+e))*a^2*d^2+1/f/(a^2+b^2)^2*
C*arctan(tan(f*x+e))*b^2*c^2+1/f*C*d^2/b^2*tan(f*x+e)+1/f/b^2/(a^2+b^2)/(a+
b*tan(f*x+e))*B*a^3*d^2-1/f/b^3/(a^2+b^2)/(a+b*tan(f*x+e))*C*a^4*d^2-1/f/b/
(a^2+b^2)/(a+b*tan(f*x+e))*C*a^2*c^2-1/f/b/(a^2+b^2)/(a+b*tan(f*x+e))*A*a^2
*d^2-2/f/(a^2+b^2)^2*B*arctan(tan(f*x+e))*a^2*c*d+1/f/b^2/(a^2+b^2)^2*ln(a+
b*tan(f*x+e))*B*a^4*d^2-2/f/b^3/(a^2+b^2)^2*ln(a+b*tan(f*x+e))*C*a^5*d^2-4/
f/b/(a^2+b^2)^2*ln(a+b*tan(f*x+e))*C*a^3*d^2-2/f*b/(a^2+b^2)^2*ln(a+b*tan(f
*x+e))*C*a*c^2+2/f/(a^2+b^2)^2*B*arctan(tan(f*x+e))*a*b*c^2-2/f/(a^2+b^2)^2
*B*arctan(tan(f*x+e))*a*b*d^2+2/f/(a^2+b^2)^2*B*arctan(tan(f*x+e))*b^2*c*d+
1/f/(a^2+b^2)^2*ln(1+tan(f*x+e)^2)*A*a^2*c*d-1/f/(a^2+b^2)^2*ln(1+tan(f*x+e
)^2)*A*a*b*c^2+1/f/(a^2+b^2)^2*ln(1+tan(f*x+e)^2)*A*a*b*d^2-1/f/(a^2+b^2)^2
*ln(1+tan(f*x+e)^2)*A*b^2*c*d+6/f/(a^2+b^2)^2*ln(a+b*tan(f*x+e))*C*a^2*c*d-
1/f/(a^2+b^2)^2*ln(1+tan(f*x+e)^2)*C*a^2*c*d+1/f/(a^2+b^2)^2*ln(1+tan(f*x+e
)^2)*C*a*b*c^2-1/f/(a^2+b^2)^2*ln(1+tan(f*x+e)^2)*C*a*b*d^2+1/f/(a^2+b^2)^2
*ln(1+tan(f*x+e)^2)*C*b^2*c*d+2/f*b/(a^2+b^2)^2*ln(a+b*tan(f*x+e))*A*a*c^2-
2/f*b/(a^2+b^2)^2*ln(a+b*tan(f*x+e))*A*a*d^2+2/f*b^2/(a^2+b^2)^2*ln(a+b*tan
(f*x+e))*A*c*d+2/f/(a^2+b^2)/(a+b*tan(f*x+e))*A*a*c*d-2/f/(a^2+b^2)^2*ln(a+
b*tan(f*x+e))*A*a^2*c*d
```

Maxima [A] time = 1.52891, size = 670, normalized size = 1.61

$$\frac{2\,Cd^2\tan\left(fx+e\right)}{b^2}+\frac{2\left(\left(\left((A-C)a^2+2\,Bab-(A-C)b^2\right)c^2-2\left(Ba^2-2\,(A-C)ab-Bb^2\right)cd-\left((A-C)a^2+2\,Bab-(A-C)b^2\right)d^2\right)\left(fx+e\right)\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\left(\left(Ba^2b^3-2\,(A-C)ab^4-Bb^5\right)c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e
))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*C*d^2*tan(f*x + e)/b^2 + 2*(((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*c^
2 - 2*(B*a^2 - 2*(A - C)*a*b - B*b^2)*c*d - ((A - C)*a^2 + 2*B*a*b - (A - C
)*b^2)*d^2)*(f*x + e)/(a^4 + 2*a^2*b^2 + b^4) - 2*((B*a^2*b^3 - 2*(A - C)*a
*b^4 - B*b^5)*c^2 - 2*(C*a^4*b - (A - 3*C)*a^2*b^3 - 2*B*a*b^4 + A*b^5)*c*d
 + (2*C*a^5 - B*a^4*b + 4*C*a^3*b^2 - 3*B*a^2*b^3 + 2*A*a*b^4)*d^2)*log(b*t
an(f*x + e) + a)/(a^4*b^3 + 2*a^2*b^5 + b^7) + ((B*a^2 - 2*(A - C)*a*b - B*
b^2)*c^2 + 2*((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*c*d - (B*a^2 - 2*(A - C)
*a*b - B*b^2)*d^2)*log(tan(f*x + e)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2*((C*
a^2*b^2 - B*a*b^3 + A*b^4)*c^2 - 2*(C*a^3*b - B*a^2*b^2 + A*a*b^3)*c*d + (C
*a^4 - B*a^3*b + A*a^2*b^2)*d^2)/(a^3*b^3 + a*b^5 + (a^2*b^4 + b^6)*tan(f*x
 + e)))/f
```

Fricas [B] time = 3.56237, size = 1987, normalized size = 4.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e
))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(C*a^4*b^2 + 2*C*a^2*b^4 + C*b^6)*d^2*tan(f*x + e)^2 - 2*(C*a^2*b^4
- B*a*b^5 + A*b^6)*c^2 + 4*(C*a^3*b^3 - B*a^2*b^4 + A*a*b^5)*c*d - 2*(C*a^4
*b^2 - B*a^3*b^3 + A*a^2*b^4)*d^2 + 2*(((A - C)*a^3*b^3 + 2*B*a^2*b^4 - (A
- C)*a*b^5)*c^2 - 2*(B*a^3*b^3 - 2*(A - C)*a^2*b^4 - B*a*b^5)*c*d - ((A - C
)*a^3*b^3 + 2*B*a^2*b^4 - (A - C)*a*b^5)*d^2)*f*x - ((B*a^3*b^3 - 2*(A - C)
*a^2*b^4 - B*a*b^5)*c^2 - 2*(C*a^5*b - (A - 3*C)*a^3*b^3 - 2*B*a^2*b^4 + A*
a*b^5)*c*d + (2*C*a^6 - B*a^5*b + 4*C*a^4*b^2 - 3*B*a^3*b^3 + 2*A*a^2*b^4)*
d^2 + ((B*a^2*b^4 - 2*(A - C)*a*b^5 - B*b^6)*c^2 - 2*(C*a^4*b^2 - (A - 3*C)
*a^2*b^4 - 2*B*a*b^5 + A*b^6)*c*d + (2*C*a^5*b - B*a^4*b^2 + 4*C*a^3*b^3 -
3*B*a^2*b^4 + 2*A*a*b^5)*d^2)*tan(f*x + e))*log((b^2*tan(f*x + e)^2 + 2*a*b
*tan(f*x + e) + a^2)/(tan(f*x + e)^2 + 1)) - (2*(C*a^5*b + 2*C*a^3*b^3 + C*
a*b^5)*c*d - (2*C*a^6 - B*a^5*b + 4*C*a^4*b^2 - 2*B*a^3*b^3 + 2*C*a^2*b^4 -
 B*a*b^5)*d^2 + (2*(C*a^4*b^2 + 2*C*a^2*b^4 + C*b^6)*c*d - (2*C*a^5*b - B*a
^4*b^2 + 4*C*a^3*b^3 - 2*B*a^2*b^4 + 2*C*a*b^5 - B*b^6)*d^2)*tan(f*x + e))*
log(1/(tan(f*x + e)^2 + 1)) + 2*((C*a^3*b^3 - B*a^2*b^4 + A*a*b^5)*c^2 - 2*
(C*a^4*b^2 - B*a^3*b^3 + A*a^2*b^4)*c*d + (2*C*a^5*b - B*a^4*b^2 + (A + 2*C
)*a^3*b^3 + C*a*b^5)*d^2 + (((A - C)*a^2*b^4 + 2*B*a*b^5 - (A - C)*b^6)*c^2
 - 2*(B*a^2*b^4 - 2*(A - C)*a*b^5 - B*b^6)*c*d - ((A - C)*a^2*b^4 + 2*B*a*b
^5 - (A - C)*b^6)*d^2)*f*x)*tan(f*x + e))/((a^4*b^4 + 2*a^2*b^6 + b^8)*f*ta
n(f*x + e) + (a^5*b^3 + 2*a^3*b^5 + a*b^7)*f)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x
+e))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.8707, size = 1231, normalized size = 2.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e
))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*C*d^2*tan(f*x + e)/b^2 + 2*(A*a^2*c^2 - C*a^2*c^2 + 2*B*a*b*c^2 - A*
b^2*c^2 + C*b^2*c^2 - 2*B*a^2*c*d + 4*A*a*b*c*d - 4*C*a*b*c*d + 2*B*b^2*c*d
 - A*a^2*d^2 + C*a^2*d^2 - 2*B*a*b*d^2 + A*b^2*d^2 - C*b^2*d^2)*(f*x + e)/(
a^4 + 2*a^2*b^2 + b^4) + (B*a^2*c^2 - 2*A*a*b*c^2 + 2*C*a*b*c^2 - B*b^2*c^2
 + 2*A*a^2*c*d - 2*C*a^2*c*d + 4*B*a*b*c*d - 2*A*b^2*c*d + 2*C*b^2*c*d - B*
a^2*d^2 + 2*A*a*b*d^2 - 2*C*a*b*d^2 + B*b^2*d^2)*log(tan(f*x + e)^2 + 1)/(a
^4 + 2*a^2*b^2 + b^4) - 2*(B*a^2*b^3*c^2 - 2*A*a*b^4*c^2 + 2*C*a*b^4*c^2 -
B*b^5*c^2 - 2*C*a^4*b*c*d + 2*A*a^2*b^3*c*d - 6*C*a^2*b^3*c*d + 4*B*a*b^4*c
*d - 2*A*b^5*c*d + 2*C*a^5*d^2 - B*a^4*b*d^2 + 4*C*a^3*b^2*d^2 - 3*B*a^2*b^
3*d^2 + 2*A*a*b^4*d^2)*log(abs(b*tan(f*x + e) + a))/(a^4*b^3 + 2*a^2*b^5 +
b^7) + 2*(B*a^2*b^4*c^2*tan(f*x + e) - 2*A*a*b^5*c^2*tan(f*x + e) + 2*C*a*b
^5*c^2*tan(f*x + e) - B*b^6*c^2*tan(f*x + e) - 2*C*a^4*b^2*c*d*tan(f*x + e)
 + 2*A*a^2*b^4*c*d*tan(f*x + e) - 6*C*a^2*b^4*c*d*tan(f*x + e) + 4*B*a*b^5*
c*d*tan(f*x + e) - 2*A*b^6*c*d*tan(f*x + e) + 2*C*a^5*b*d^2*tan(f*x + e) -
B*a^4*b^2*d^2*tan(f*x + e) + 4*C*a^3*b^3*d^2*tan(f*x + e) - 3*B*a^2*b^4*d^2
*tan(f*x + e) + 2*A*a*b^5*d^2*tan(f*x + e) - C*a^4*b^2*c^2 + 2*B*a^3*b^3*c^
2 - 3*A*a^2*b^4*c^2 + C*a^2*b^4*c^2 - A*b^6*c^2 - 2*B*a^4*b^2*c*d + 4*A*a^3
*b^3*c*d - 4*C*a^3*b^3*c*d + 2*B*a^2*b^4*c*d + C*a^6*d^2 - A*a^4*b^2*d^2 +
3*C*a^4*b^2*d^2 - 2*B*a^3*b^3*d^2 + A*a^2*b^4*d^2)/((a^4*b^3 + 2*a^2*b^5 +
b^7)*(b*tan(f*x + e) + a)))/f
```

3.63 $\int \frac{(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx$

Optimal. Leaf size=597

$$\frac{\left(-a^3b^3\left(2cd(A-C)+B\left(c^2-d^2\right)\right)-3a^2b^4\left(-A\left(c^2-d^2\right)+2Bcd+c^2C-2Cd^2\right)+3a^4b^2Cd^2+a^6Cd^2+3ab^5\left(2cd(A-C\right.\right.}{b^3f\left(a^2+b^2\right)^3}$$

```
[Out] -(((a^3*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - 3*a*b^2*(c^2*C + 2*B*c*
d - C*d^2 - A*(c^2 - d^2)) - 3*a^2*b*(2*c*(A - C)*d + B*(c^2 - d^2)) + b^3*
(2*c*(A - C)*d + B*(c^2 - d^2)))*x)/(a^2 + b^2)^3) - ((3*a^2*b*(c^2*C + 2*B
*c*d - C*d^2 - A*(c^2 - d^2)) - b^3*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2
)) + a^3*(2*c*(A - C)*d + B*(c^2 - d^2)) - 3*a*b^2*(2*c*(A - C)*d + B*(c^2
- d^2)))*Log[Cos[e + f*x]])/((a^2 + b^2)^3*f) + ((a^6*C*d^2 + 3*a^4*b^2*C*d
^2 - 3*a^2*b^4*(c^2*C + 2*B*c*d - 2*C*d^2 - A*(c^2 - d^2)) + b^6*(c*(c*C +
2*B*d) - A*(c^2 - d^2)) - a^3*b^3*(2*c*(A - C)*d + B*(c^2 - d^2)) + 3*a*b^5
*(2*c*(A - C)*d + B*(c^2 - d^2)))*Log[a + b*Tan[e + f*x]])/(b^3*(a^2 + b^2)
^3*f) - ((b*c - a*d)*(a^4*C*d + b^4*(B*c + A*d) + 2*a*b^3*(A*c - c*C - B*d)
 - a^2*b^2*(B*c + (A - 3*C)*d)))/(b^3*(a^2 + b^2)^2*f*(a + b*Tan[e + f*x]))
 - ((A*b^2 - a*(b*B - a*C))*(c + d*Tan[e + f*x])^2)/(2*b*(a^2 + b^2)*f*(a +
 b*Tan[e + f*x])^2)
```

Rubi [A] time = 1.29003, antiderivative size = 597, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3645, 3635, 3626, 3617, 31, 3475}

$$\frac{\left(-a^3b^3\left(2cd(A-C)+B\left(c^2-d^2\right)\right)-3a^2b^4\left(-A\left(c^2-d^2\right)+2Bcd+c^2C-2Cd^2\right)+3a^4b^2Cd^2+a^6Cd^2+3ab^5\left(2cd(A-C\right.\right.}{b^3f\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a + b
*Tan[e + f*x])^3,x]
```


```
[Out] -(((a^3*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - 3*a*b^2*(c^2*C + 2*B*c*
d - C*d^2 - A*(c^2 - d^2)) - 3*a^2*b*(2*c*(A - C)*d + B*(c^2 - d^2)) + b^3*
(2*c*(A - C)*d + B*(c^2 - d^2)))*x)/(a^2 + b^2)^3) - ((3*a^2*b*(c^2*C + 2*B
*c*d - C*d^2 - A*(c^2 - d^2)) - b^3*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2
)) + a^3*(2*c*(A - C)*d + B*(c^2 - d^2)) - 3*a*b^2*(2*c*(A - C)*d + B*(c^2
- d^2)))*Log[Cos[e + f*x]])/((a^2 + b^2)^3*f) + ((a^6*C*d^2 + 3*a^4*b^2*C*d
^2 - 3*a^2*b^4*(c^2*C + 2*B*c*d - 2*C*d^2 - A*(c^2 - d^2)) + b^6*(c*(c*C +
2*B*d) - A*(c^2 - d^2)) - a^3*b^3*(2*c*(A - C)*d + B*(c^2 - d^2)) + 3*a*b^5
*(2*c*(A - C)*d + B*(c^2 - d^2)))*Log[a + b*Tan[e + f*x]])/(b^3*(a^2 + b^2)
^3*f) - ((b*c - a*d)*(a^4*C*d + b^4*(B*c + A*d) + 2*a*b^3*(A*c - c*C - B*d)
 - a^2*b^2*(B*c + (A - 3*C)*d)))/(b^3*(a^2 + b^2)^2*f*(a + b*Tan[e + f*x]))
 - ((A*b^2 - a*(b*B - a*C))*(c + d*Tan[e + f*x])^2)/(2*b*(a^2 + b^2)*f*(a +
 b*Tan[e + f*x])^2)
```


Rule 3645

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*d^2 + c*(c*C - B*d))*(a + b*Tan[e
+ f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
```

```
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3635

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(c^2*C - B*c*d + A*d^2)*(c +
d*Tan[e + f*x])^(n + 1))/(d^2*f*(n + 1)*(c^2 + d^2)), x] + Dist[1/(d*(c^2 +
 d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(c^
2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*Ta
n[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -
1]
```

Rule 3626

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((a*A + b*B -
a*C)*x)/(a^2 + b^2), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3617

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^2}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2} + \int \frac{(c+d\,\mathrm{ta}}{}$$

$$= -\frac{(bc-ad)\left(a^4Cd+b^4(Bc+Ad)+2ab^3(Ac-cC-B}{b^3\left(a^2+b^2\right)^2 f(a+b\tan(e+}$$

$$= -\frac{\left(a^3\left(c^2C+2Bcd-Cd^2-A\left(c^2-d^2\right)\right)-3ab^2\left(c^2C-\right.\right.}{}$$

$$= -\frac{\left(a^3\left(c^2C+2Bcd-Cd^2-A\left(c^2-d^2\right)\right)-3ab^2\left(c^2C-\right.\right.}{}$$

$$= -\frac{\left(a^3\left(c^2C+2Bcd-Cd^2-A\left(c^2-d^2\right)\right)-3ab^2\left(c^2C-\right.\right.}{}$$

Mathematica [C] time = 7.9052, size = 2499, normalized size = 4.19

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(a + b*Tan[e + f*x])^3,x]
```

```
[Out] ((-(A*b^4*c^2) + a*b^3*B*c^2 - a^2*b^2*c^2*C + 2*a*A*b^3*c*d - 2*a^2*b^2*B*
c*d + 2*a^3*b*c*C*d - a^2*A*b^2*d^2 + a^3*b*B*d^2 - a^4*C*d^2)*Sec[e + f*x]
*(a*Cos[e + f*x] + b*Sin[e + f*x])*(c + d*Tan[e + f*x])^2)/(2*(a - I*b)^2*(
a + I*b)^2*b*f*(c*Cos[e + f*x] + d*Sin[e + f*x])^2*(a + b*Tan[e + f*x])^3)
+ ((a^3*A*c^2 - 3*a*A*b^2*c^2 + 3*a^2*b*B*c^2 - b^3*B*c^2 - a^3*c^2*C + 3*a
*b^2*c^2*C + 6*a^2*A*b*c*d - 2*A*b^3*c*d - 2*a^3*B*c*d + 6*a*b^2*B*c*d - 6*
a^2*b*c*C*d + 2*b^3*c*C*d - a^3*A*d^2 + 3*a*A*b^2*d^2 - 3*a^2*b*B*d^2 + b^3
*B*d^2 + a^3*C*d^2 - 3*a*b^2*C*d^2)*(e + f*x)*Sec[e + f*x]*(a*Cos[e + f*x]
+ b*Sin[e + f*x])^3*(c + d*Tan[e + f*x])^2)/((a - I*b)^3*(a + I*b)^3*f*(c*C
os[e + f*x] + d*Sin[e + f*x])^2*(a + b*Tan[e + f*x])^3) + (((3*I)*a^9*A*b^6
*c^2 + 3*a^8*A*b^7*c^2 + (5*I)*a^7*A*b^8*c^2 + 5*a^6*A*b^9*c^2 + I*a^5*A*b^
10*c^2 + a^4*A*b^11*c^2 - I*a^3*A*b^12*c^2 - a^2*A*b^13*c^2 - I*a^10*b^5*B*
c^2 - a^9*b^6*B*c^2 + I*a^8*b^7*B*c^2 + a^7*b^8*B*c^2 + (5*I)*a^6*b^9*B*c^2
 + 5*a^5*b^10*B*c^2 + (3*I)*a^4*b^11*B*c^2 + 3*a^3*b^12*B*c^2 - (3*I)*a^9*b
^6*c^2*C - 3*a^8*b^7*c^2*C - (5*I)*a^7*b^8*c^2*C - 5*a^6*b^9*c^2*C - I*a^5*
b^10*c^2*C - a^4*b^11*c^2*C + I*a^3*b^12*c^2*C + a^2*b^13*c^2*C - (2*I)*a^1
0*A*b^5*c*d - 2*a^9*A*b^6*c*d + (2*I)*a^8*A*b^7*c*d + 2*a^7*A*b^8*c*d + (10
*I)*a^6*A*b^9*c*d + 10*a^5*A*b^10*c*d + (6*I)*a^4*A*b^11*c*d + 6*a^3*A*b^12
*c*d - (6*I)*a^9*b^6*B*c*d - 6*a^8*b^7*B*c*d - (10*I)*a^7*b^8*B*c*d - 10*a^
6*b^9*B*c*d - (2*I)*a^5*b^10*B*c*d - 2*a^4*b^11*B*c*d + (2*I)*a^3*b^12*B*c*
d + 2*a^2*b^13*B*c*d + (2*I)*a^10*b^5*c*C*d + 2*a^9*b^6*c*C*d - (2*I)*a^8*b
^7*c*C*d - 2*a^7*b^8*c*C*d - (10*I)*a^6*b^9*c*C*d - 10*a^5*b^10*c*C*d - (6*
I)*a^4*b^11*c*C*d - 6*a^3*b^12*c*C*d - (3*I)*a^9*A*b^6*d^2 - 3*a^8*A*b^7*d^
2 - (5*I)*a^7*A*b^8*d^2 - 5*a^6*A*b^9*d^2 - I*a^5*A*b^10*d^2 - a^4*A*b^11*d
^2 + I*a^3*A*b^12*d^2 + a^2*A*b^13*d^2 + I*a^10*b^5*B*d^2 + a^9*b^6*B*d^2 -
 I*a^8*b^7*B*d^2 - a^7*b^8*B*d^2 - (5*I)*a^6*b^9*B*d^2 - 5*a^5*b^10*B*d^2 -
 (3*I)*a^4*b^11*B*d^2 - 3*a^3*b^12*B*d^2 + I*a^13*b^2*C*d^2 + a^12*b^3*C*d^
2 + (5*I)*a^11*b^4*C*d^2 + 5*a^10*b^5*C*d^2 + (13*I)*a^9*b^6*C*d^2 + 13*a^8
*b^7*C*d^2 + (15*I)*a^7*b^8*C*d^2 + 15*a^6*b^9*C*d^2 + (6*I)*a^5*b^10*C*d^2
 + 6*a^4*b^11*C*d^2)*(e + f*x)*Sec[e + f*x]*(a*Cos[e + f*x] + b*Sin[e + f*x
```

```
])^3*(c + d*Tan[e + f*x])^2)/(a^2*(a - I*b)^6*(a + I*b)^5*b^5*f*(c*Cos[e +
f*x] + d*Sin[e + f*x])^2*(a + b*Tan[e + f*x])^3) - (I*(3*a^2*A*b^4*c^2 - A*
b^6*c^2 - a^3*b^3*B*c^2 + 3*a*b^5*B*c^2 - 3*a^2*b^4*c^2*C + b^6*c^2*C - 2*a
^3*A*b^3*c*d + 6*a*A*b^5*c*d - 6*a^2*b^4*B*c*d + 2*b^6*B*c*d + 2*a^3*b^3*c*
C*d - 6*a*b^5*c*C*d - 3*a^2*A*b^4*d^2 + A*b^6*d^2 + a^3*b^3*B*d^2 - 3*a*b^5
*B*d^2 + a^6*C*d^2 + 3*a^4*b^2*C*d^2 + 6*a^2*b^4*C*d^2)*ArcTan[Tan[e + f*x]
]*Sec[e + f*x]*(a*Cos[e + f*x] + b*Sin[e + f*x])^3*(c + d*Tan[e + f*x])^2)/
(b^3*(a^2 + b^2)^3*f*(c*Cos[e + f*x] + d*Sin[e + f*x])^2*(a + b*Tan[e + f*x
])^3) - (C*d^2*Log[Cos[e + f*x]]*Sec[e + f*x]*(a*Cos[e + f*x] + b*Sin[e + f
*x])^3*(c + d*Tan[e + f*x])^2)/(b^3*f*(c*Cos[e + f*x] + d*Sin[e + f*x])^2*(
a + b*Tan[e + f*x])^3) + ((3*a^2*A*b^4*c^2 - A*b^6*c^2 - a^3*b^3*B*c^2 + 3*
a*b^5*B*c^2 - 3*a^2*b^4*c^2*C + b^6*c^2*C - 2*a^3*A*b^3*c*d + 6*a*A*b^5*c*d
 - 6*a^2*b^4*B*c*d + 2*b^6*B*c*d + 2*a^3*b^3*c*C*d - 6*a*b^5*c*C*d - 3*a^2*
A*b^4*d^2 + A*b^6*d^2 + a^3*b^3*B*d^2 - 3*a*b^5*B*d^2 + a^6*C*d^2 + 3*a^4*b
^2*C*d^2 + 6*a^2*b^4*C*d^2)*Log[(a*Cos[e + f*x] + b*Sin[e + f*x])^2]*Sec[e
+ f*x]*(a*Cos[e + f*x] + b*Sin[e + f*x])^3*(c + d*Tan[e + f*x])^2)/(2*b^3*(
a^2 + b^2)^3*f*(c*Cos[e + f*x] + d*Sin[e + f*x])^2*(a + b*Tan[e + f*x])^3)
+ (Sec[e + f*x]*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(3*a*A*b^4*c^2*Sin[e +
f*x] - 2*a^2*b^3*B*c^2*Sin[e + f*x] + b^5*B*c^2*Sin[e + f*x] + a^3*b^2*c^2*
C*Sin[e + f*x] - 2*a*b^4*c^2*C*Sin[e + f*x] - 4*a^2*A*b^3*c*d*Sin[e + f*x]
+ 2*A*b^5*c*d*Sin[e + f*x] + 2*a^3*b^2*B*c*d*Sin[e + f*x] - 4*a*b^4*B*c*d*S
in[e + f*x] + 6*a^2*b^3*c*C*d*Sin[e + f*x] + a^3*A*b^2*d^2*Sin[e + f*x] - 2
*a*A*b^4*d^2*Sin[e + f*x] + 3*a^2*b^3*B*d^2*Sin[e + f*x] - a^5*C*d^2*Sin[e
+ f*x] - 4*a^3*b^2*C*d^2*Sin[e + f*x])*(c + d*Tan[e + f*x])^2)/(a*(a - I*b)
^2*(a + I*b)^2*b^2*f*(c*Cos[e + f*x] + d*Sin[e + f*x])^2*(a + b*Tan[e + f*x
])^3)
```

Maple [B] time = 0.077, size = 2465, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^3,x
)
```

```
[Out] 1/f/b^2/(a^2+b^2)/(a+b*tan(f*x+e))^2*C*a^3*c*d-6/f/(a^2+b^2)^3*C*arctan(tan
(f*x+e))*a^2*b*c*d-1/f/b/(a^2+b^2)/(a+b*tan(f*x+e))^2*B*a^2*c*d+6/f/(a^2+b^
2)^3*B*arctan(tan(f*x+e))*a*b^2*c*d-3/f/(a^2+b^2)^3*ln(1+tan(f*x+e)^2)*A*a*
b^2*c*d-6/f/(a^2+b^2)^3*b^2*ln(a+b*tan(f*x+e))*C*a*c*d-6/f/(a^2+b^2)^3*b*ln
(a+b*tan(f*x+e))*B*a^2*c*d+3/f/(a^2+b^2)^3*ln(1+tan(f*x+e)^2)*C*a*b^2*c*d+6
/f/(a^2+b^2)^3*A*arctan(tan(f*x+e))*a^2*b*c*d+3/f/(a^2+b^2)^3*ln(1+tan(f*x+
e)^2)*B*a^2*b*c*d-2/f/b^2/(a^2+b^2)^2/(a+b*tan(f*x+e))*C*a^4*c*d+4/f*b/(a^2
+b^2)^2/(a+b*tan(f*x+e))*B*a*c*d+6/f/(a^2+b^2)^3*b^2*ln(a+b*tan(f*x+e))*A*a
*c*d-1/2/f/b^3/(a^2+b^2)/(a+b*tan(f*x+e))^2*C*d^2*a^4-1/2/f/b/(a^2+b^2)/(a+
b*tan(f*x+e))^2*C*a^2*c^2-2/f/(a^2+b^2)^3*B*arctan(tan(f*x+e))*a^3*c*d+3/f/
(a^2+b^2)^3*B*arctan(tan(f*x+e))*a^2*b*c^2-3/f/(a^2+b^2)^3*B*arctan(tan(f*x
+e))*a^2*b*d^2+3/f/(a^2+b^2)^3*C*arctan(tan(f*x+e))*a*b^2*c^2-3/f/(a^2+b^2)
^3*C*arctan(tan(f*x+e))*a*b^2*d^2+2/f/(a^2+b^2)^3*C*arctan(tan(f*x+e))*b^3*
c*d+1/f/(a^2+b^2)^3*ln(1+tan(f*x+e)^2)*A*a^3*c*d+3/f/(a^2+b^2)^3*b^2*ln(a+b
*tan(f*x+e))*a*B*c^2+2/f/(a^2+b^2)^2/(a+b*tan(f*x+e))*A*a^2*c*d-6/f/(a^2+b^
2)^2/(a+b*tan(f*x+e))*C*a^2*c*d+1/2/f/(a^2+b^2)^3*ln(1+tan(f*x+e)^2)*B*a^3*
c^2-1/2/f/(a^2+b^2)^3*ln(1+tan(f*x+e)^2)*B*a^3*d^2-1/2/f/(a^2+b^2)^3*ln(1+t
an(f*x+e)^2)*C*b^3*c^2+1/2/f/(a^2+b^2)^3*ln(1+tan(f*x+e)^2)*C*b^3*d^2+1/f/(
a^2+b^2)^3*A*arctan(tan(f*x+e))*a^3*c^2-1/f/(a^2+b^2)^3*A*arctan(tan(f*x+e)
)*a^3*d^2-1/f/(a^2+b^2)^3*B*arctan(tan(f*x+e))*b^3*c^2+1/f/(a^2+b^2)^3*B*ar
ctan(tan(f*x+e))*b^3*d^2-1/f/(a^2+b^2)^3*C*arctan(tan(f*x+e))*a^3*c^2+1/f/(
```

```
a^2+b^2)^3*C*arctan(tan(f*x+e))*a^3*d^2-1/2/f*b/(a^2+b^2)/(a+b*tan(f*x+e))^
2*A*c^2-1/f*b^2/(a^2+b^2)^2/(a+b*tan(f*x+e))*B*c^2-1/f/(a^2+b^2)^3*b^3*ln(a
+b*tan(f*x+e))*A*c^2+1/f/(a^2+b^2)^3*b^3*ln(a+b*tan(f*x+e))*A*d^2+1/f/(a^2+
b^2)^3*b^3*ln(a+b*tan(f*x+e))*C*c^2-1/f/(a^2+b^2)^3*ln(a+b*tan(f*x+e))*B*a^
3*c^2+3/f/(a^2+b^2)^3/b*ln(a+b*tan(f*x+e))*a^4*C*d^2+1/f/(a^2+b^2)^3/b^3*ln
(a+b*tan(f*x+e))*a^6*C*d^2-2/f/(a^2+b^2)^3*ln(a+b*tan(f*x+e))*A*a^3*c*d+2/f
/(a^2+b^2)^3*ln(a+b*tan(f*x+e))*C*a^3*c*d+1/f/(a^2+b^2)/(a+b*tan(f*x+e))^2*
A*a*c*d+3/f/(a^2+b^2)^3*A*arctan(tan(f*x+e))*a*b^2*d^2+1/f/(a^2+b^2)^3*ln(a
+b*tan(f*x+e))*B*a^3*d^2+1/2/f/(a^2+b^2)/(a+b*tan(f*x+e))^2*B*a*c^2+1/f/(a^
2+b^2)^2/(a+b*tan(f*x+e))*B*a^2*c^2-3/f/(a^2+b^2)^2/(a+b*tan(f*x+e))*B*a^2*
d^2+1/2/f/(a^2+b^2)^3*ln(1+tan(f*x+e)^2)*A*b^3*c^2-1/2/f/(a^2+b^2)^3*ln(1+t
an(f*x+e)^2)*A*b^3*d^2+3/f/(a^2+b^2)^3*b*ln(a+b*tan(f*x+e))*A*a^2*c^2+2/f*b
/(a^2+b^2)^2/(a+b*tan(f*x+e))*A*a*d^2-3/f/(a^2+b^2)^3*b^2*ln(a+b*tan(f*x+e)
)*B*a*d^2-3/f/(a^2+b^2)^3*b*ln(a+b*tan(f*x+e))*C*a^2*c^2+6/f/(a^2+b^2)^3*b*
ln(a+b*tan(f*x+e))*C*a^2*d^2-3/2/f/(a^2+b^2)^3*ln(1+tan(f*x+e)^2)*A*a^2*b*c
^2+3/2/f/(a^2+b^2)^3*ln(1+tan(f*x+e)^2)*A*a^2*b*d^2-3/2/f/(a^2+b^2)^3*ln(1+
tan(f*x+e)^2)*B*a*b^2*c^2+3/2/f/(a^2+b^2)^3*ln(1+tan(f*x+e)^2)*B*a*b^2*d^2-
3/f/(a^2+b^2)^3*b*ln(a+b*tan(f*x+e))*A*a^2*d^2+4/f/b/(a^2+b^2)^2/(a+b*tan(f
*x+e))*C*a^3*d^2+2/f*b/(a^2+b^2)^2/(a+b*tan(f*x+e))*C*a*c^2-1/f/(a^2+b^2)^3
*ln(1+tan(f*x+e)^2)*B*b^3*c*d-1/f/(a^2+b^2)^3*ln(1+tan(f*x+e)^2)*C*a^3*c*d-
1/2/f/b/(a^2+b^2)/(a+b*tan(f*x+e))^2*A*a^2*d^2+3/2/f/(a^2+b^2)^3*ln(1+tan(f
*x+e)^2)*C*a^2*b*c^2+1/2/f/b^2/(a^2+b^2)/(a+b*tan(f*x+e))^2*B*a^3*d^2-2/f*b
^2/(a^2+b^2)^2/(a+b*tan(f*x+e))*A*c*d-3/2/f/(a^2+b^2)^3*ln(1+tan(f*x+e)^2)*
C*a^2*b*d^2-3/f/(a^2+b^2)^3*A*arctan(tan(f*x+e))*a*b^2*c^2-2/f*b/(a^2+b^2)^
2/(a+b*tan(f*x+e))*A*a*c^2-1/f/b^2/(a^2+b^2)^2/(a+b*tan(f*x+e))*B*a^4*d^2+2
/f/b^3/(a^2+b^2)^2/(a+b*tan(f*x+e))*C*a^5*d^2-2/f/(a^2+b^2)^3*A*arctan(tan(
f*x+e))*b^3*c*d+2/f/(a^2+b^2)^3*b^3*ln(a+b*tan(f*x+e))*B*c*d
```

Maxima [A] time = 1.59634, size = 1133, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e
))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(((A - C)*a^3 + 3*B*a^2*b - 3*(A - C)*a*b^2 - B*b^3)*c^2 - 2*(B*a^3
- 3*(A - C)*a^2*b - 3*B*a*b^2 + (A - C)*b^3)*c*d - ((A - C)*a^3 + 3*B*a^2*b
 - 3*(A - C)*a*b^2 - B*b^3)*d^2)*(f*x + e)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6) - 2*((B*a^3*b^3 - 3*(A - C)*a^2*b^4 - 3*B*a*b^5 + (A - C)*b^6)*c^2 + 2*
((A - C)*a^3*b^3 + 3*B*a^2*b^4 - 3*(A - C)*a*b^5 - B*b^6)*c*d - (C*a^6 + 3*
C*a^4*b^2 + B*a^3*b^3 - 3*(A - 2*C)*a^2*b^4 - 3*B*a*b^5 + A*b^6)*d^2)*log(b
*tan(f*x + e) + a)/(a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9) + ((B*a^3 - 3*(A
 - C)*a^2*b - 3*B*a*b^2 + (A - C)*b^3)*c^2 + 2*((A - C)*a^3 + 3*B*a^2*b - 3
*(A - C)*a*b^2 - B*b^3)*c*d - (B*a^3 - 3*(A - C)*a^2*b - 3*B*a*b^2 + (A - C
)*b^3)*d^2)*log(tan(f*x + e)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (
(C*a^4*b^2 - 3*B*a^3*b^3 + (5*A - 3*C)*a^2*b^4 + B*a*b^5 + A*b^6)*c^2 + 2*(
C*a^5*b + B*a^4*b^2 - (3*A - 5*C)*a^3*b^3 - 3*B*a^2*b^4 + A*a*b^5)*c*d - (3
*C*a^6 - B*a^5*b - (A - 7*C)*a^4*b^2 - 5*B*a^3*b^3 + 3*A*a^2*b^4)*d^2 - 2*(
(B*a^2*b^4 - 2*(A - C)*a*b^5 - B*b^6)*c^2 - 2*(C*a^4*b^2 - (A - 3*C)*a^2*b^
4 - 2*B*a*b^5 + A*b^6)*c*d + (2*C*a^5*b - B*a^4*b^2 + 4*C*a^3*b^3 - 3*B*a^2
*b^4 + 2*A*a*b^5)*d^2)*tan(f*x + e))/(a^6*b^3 + 2*a^4*b^5 + a^2*b^7 + (a^4*
b^5 + 2*a^2*b^7 + b^9)*tan(f*x + e)^2 + 2*(a^5*b^4 + 2*a^3*b^6 + a*b^8)*tan
(f*x + e)))/f
```

Fricas [B] time = 4.65581, size = 3513, normalized size = 5.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e
))^3,x, algorithm="fricas")
```

```
[Out] -1/2*((3*C*a^4*b^4 - 5*B*a^3*b^5 + (7*A - 3*C)*a^2*b^6 + B*a*b^7 + A*b^8)*c
^2 - 2*(C*a^5*b^3 - 3*B*a^4*b^4 + 5*(A - C)*a^3*b^5 + 3*B*a^2*b^6 - A*a*b^7
)*c*d - (C*a^6*b^2 + B*a^5*b^3 - (3*A - 7*C)*a^4*b^4 - 5*B*a^3*b^5 + 3*A*a^
2*b^6)*d^2 - 2*(((A - C)*a^5*b^3 + 3*B*a^4*b^4 - 3*(A - C)*a^3*b^5 - B*a^2*
b^6)*c^2 - 2*(B*a^5*b^3 - 3*(A - C)*a^4*b^4 - 3*B*a^3*b^5 + (A - C)*a^2*b^6
)*c*d - ((A - C)*a^5*b^3 + 3*B*a^4*b^4 - 3*(A - C)*a^3*b^5 - B*a^2*b^6)*d^2
)*f*x - ((C*a^4*b^4 - 3*B*a^3*b^5 + 5*(A - C)*a^2*b^6 + 3*B*a*b^7 - A*b^8)*
c^2 + 2*(C*a^5*b^3 + B*a^4*b^4 - (3*A - 7*C)*a^3*b^5 - 5*B*a^2*b^6 + 3*A*a*
b^7)*c*d - (3*C*a^6*b^2 - B*a^5*b^3 - (A - 9*C)*a^4*b^4 - 7*B*a^3*b^5 + 5*A
*a^2*b^6)*d^2 + 2*(((A - C)*a^3*b^5 + 3*B*a^2*b^6 - 3*(A - C)*a*b^7 - B*b^8
)*c^2 - 2*(B*a^3*b^5 - 3*(A - C)*a^2*b^6 - 3*B*a*b^7 + (A - C)*b^8)*c*d - (
(A - C)*a^3*b^5 + 3*B*a^2*b^6 - 3*(A - C)*a*b^7 - B*b^8)*d^2)*f*x)*tan(f*x
+ e)^2 + ((B*a^5*b^3 - 3*(A - C)*a^4*b^4 - 3*B*a^3*b^5 + (A - C)*a^2*b^6)*c
^2 + 2*((A - C)*a^5*b^3 + 3*B*a^4*b^4 - 3*(A - C)*a^3*b^5 - B*a^2*b^6)*c*d
- (C*a^8 + 3*C*a^6*b^2 + B*a^5*b^3 - 3*(A - 2*C)*a^4*b^4 - 3*B*a^3*b^5 + A*
a^2*b^6)*d^2 + ((B*a^3*b^5 - 3*(A - C)*a^2*b^6 - 3*B*a*b^7 + (A - C)*b^8)*c
^2 + 2*((A - C)*a^3*b^5 + 3*B*a^2*b^6 - 3*(A - C)*a*b^7 - B*b^8)*c*d - (C*a
^6*b^2 + 3*C*a^4*b^4 + B*a^3*b^5 - 3*(A - 2*C)*a^2*b^6 - 3*B*a*b^7 + A*b^8)
*d^2)*tan(f*x + e)^2 + 2*((B*a^4*b^4 - 3*(A - C)*a^3*b^5 - 3*B*a^2*b^6 + (A
 - C)*a*b^7)*c^2 + 2*((A - C)*a^4*b^4 + 3*B*a^3*b^5 - 3*(A - C)*a^2*b^6 - B
*a*b^7)*c*d - (C*a^7*b + 3*C*a^5*b^3 + B*a^4*b^4 - 3*(A - 2*C)*a^3*b^5 - 3*
B*a^2*b^6 + A*a*b^7)*d^2)*tan(f*x + e))*log((b^2*tan(f*x + e)^2 + 2*a*b*tan
(f*x + e) + a^2)/(tan(f*x + e)^2 + 1)) + ((C*a^6*b^2 + 3*C*a^4*b^4 + 3*C*a^
2*b^6 + C*b^8)*d^2*tan(f*x + e)^2 + 2*(C*a^7*b + 3*C*a^5*b^3 + 3*C*a^3*b^5
+ C*a*b^7)*d^2*tan(f*x + e) + (C*a^8 + 3*C*a^6*b^2 + 3*C*a^4*b^4 + C*a^2*b^
6)*d^2)*log(1/(tan(f*x + e)^2 + 1)) - 2*((C*a^5*b^3 - 2*B*a^4*b^4 + 3*(A -
C)*a^3*b^5 + 3*B*a^2*b^6 - (3*A - 2*C)*a*b^7 - B*b^8)*c^2 + 2*(B*a^5*b^3 -
(2*A - 3*C)*a^4*b^4 - 3*B*a^3*b^5 + 3*(A - C)*a^2*b^6 + 2*B*a*b^7 - A*b^8)*
c*d - (C*a^7*b - (A - 3*C)*a^5*b^3 - 3*B*a^4*b^4 + (3*A - 4*C)*a^3*b^5 + 3*
B*a^2*b^6 - 2*A*a*b^7)*d^2 + 2*(((A - C)*a^4*b^4 + 3*B*a^3*b^5 - 3*(A - C)*
a^2*b^6 - B*a*b^7)*c^2 - 2*(B*a^4*b^4 - 3*(A - C)*a^3*b^5 - 3*B*a^2*b^6 + (
A - C)*a*b^7)*c*d - ((A - C)*a^4*b^4 + 3*B*a^3*b^5 - 3*(A - C)*a^2*b^6 - B*
a*b^7)*d^2)*f*x)*tan(f*x + e))/((a^6*b^5 + 3*a^4*b^7 + 3*a^2*b^9 + b^11)*f*
tan(f*x + e)^2 + 2*(a^7*b^4 + 3*a^5*b^6 + 3*a^3*b^8 + a*b^10)*f*tan(f*x + e
) + (a^8*b^3 + 3*a^6*b^5 + 3*a^4*b^7 + a^2*b^9)*f)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x
+e))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.87047, size = 2314, normalized size = 3.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e
))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a^3*c^2 - C*a^3*c^2 + 3*B*a^2*b*c^2 - 3*A*a*b^2*c^2 + 3*C*a*b^2*c
^2 - B*b^3*c^2 - 2*B*a^3*c*d + 6*A*a^2*b*c*d - 6*C*a^2*b*c*d + 6*B*a*b^2*c*
d - 2*A*b^3*c*d + 2*C*b^3*c*d - A*a^3*d^2 + C*a^3*d^2 - 3*B*a^2*b*d^2 + 3*A
*a*b^2*d^2 - 3*C*a*b^2*d^2 + B*b^3*d^2)*(f*x + e)/(a^6 + 3*a^4*b^2 + 3*a^2*
b^4 + b^6) + (B*a^3*c^2 - 3*A*a^2*b*c^2 + 3*C*a^2*b*c^2 - 3*B*a*b^2*c^2 + A
*b^3*c^2 - C*b^3*c^2 + 2*A*a^3*c*d - 2*C*a^3*c*d + 6*B*a^2*b*c*d - 6*A*a*b^
2*c*d + 6*C*a*b^2*c*d - 2*B*b^3*c*d - B*a^3*d^2 + 3*A*a^2*b*d^2 - 3*C*a^2*b
*d^2 + 3*B*a*b^2*d^2 - A*b^3*d^2 + C*b^3*d^2)*log(tan(f*x + e)^2 + 1)/(a^6
+ 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 2*(B*a^3*b^3*c^2 - 3*A*a^2*b^4*c^2 + 3*C*a
^2*b^4*c^2 - 3*B*a*b^5*c^2 + A*b^6*c^2 - C*b^6*c^2 + 2*A*a^3*b^3*c*d - 2*C*
a^3*b^3*c*d + 6*B*a^2*b^4*c*d - 6*A*a*b^5*c*d + 6*C*a*b^5*c*d - 2*B*b^6*c*d
 - C*a^6*d^2 - 3*C*a^4*b^2*d^2 - B*a^3*b^3*d^2 + 3*A*a^2*b^4*d^2 - 6*C*a^2*
b^4*d^2 + 3*B*a*b^5*d^2 - A*b^6*d^2)*log(abs(b*tan(f*x + e) + a))/(a^6*b^3
+ 3*a^4*b^5 + 3*a^2*b^7 + b^9) + (3*B*a^3*b^4*c^2*tan(f*x + e)^2 - 9*A*a^2*
b^5*c^2*tan(f*x + e)^2 + 9*C*a^2*b^5*c^2*tan(f*x + e)^2 - 9*B*a*b^6*c^2*tan
(f*x + e)^2 + 3*A*b^7*c^2*tan(f*x + e)^2 - 3*C*b^7*c^2*tan(f*x + e)^2 + 6*A
*a^3*b^4*c*d*tan(f*x + e)^2 - 6*C*a^3*b^4*c*d*tan(f*x + e)^2 + 18*B*a^2*b^5
*c*d*tan(f*x + e)^2 - 18*A*a*b^6*c*d*tan(f*x + e)^2 + 18*C*a*b^6*c*d*tan(f*
x + e)^2 - 6*B*b^7*c*d*tan(f*x + e)^2 - 3*C*a^6*b*d^2*tan(f*x + e)^2 - 9*C*
a^4*b^3*d^2*tan(f*x + e)^2 - 3*B*a^3*b^4*d^2*tan(f*x + e)^2 + 9*A*a^2*b^5*d
^2*tan(f*x + e)^2 - 18*C*a^2*b^5*d^2*tan(f*x + e)^2 + 9*B*a*b^6*d^2*tan(f*x
 + e)^2 - 3*A*b^7*d^2*tan(f*x + e)^2 + 8*B*a^4*b^3*c^2*tan(f*x + e) - 22*A*
a^3*b^4*c^2*tan(f*x + e) + 22*C*a^3*b^4*c^2*tan(f*x + e) - 18*B*a^2*b^5*c^2
*tan(f*x + e) + 2*A*a*b^6*c^2*tan(f*x + e) - 2*C*a*b^6*c^2*tan(f*x + e) - 2
*B*b^7*c^2*tan(f*x + e) - 4*C*a^6*b*c*d*tan(f*x + e) + 16*A*a^4*b^3*c*d*tan
(f*x + e) - 28*C*a^4*b^3*c*d*tan(f*x + e) + 44*B*a^3*b^4*c*d*tan(f*x + e) -
 36*A*a^2*b^5*c*d*tan(f*x + e) + 24*C*a^2*b^5*c*d*tan(f*x + e) - 4*B*a*b^6*
c*d*tan(f*x + e) - 4*A*b^7*c*d*tan(f*x + e) - 2*C*a^7*d^2*tan(f*x + e) - 2*
B*a^6*b*d^2*tan(f*x + e) - 6*C*a^5*b^2*d^2*tan(f*x + e) - 14*B*a^4*b^3*d^2*
tan(f*x + e) + 22*A*a^3*b^4*d^2*tan(f*x + e) - 28*C*a^3*b^4*d^2*tan(f*x + e
) + 12*B*a^2*b^5*d^2*tan(f*x + e) - 2*A*a*b^6*d^2*tan(f*x + e) - C*a^6*b*c^
2 + 6*B*a^5*b^2*c^2 - 14*A*a^4*b^3*c^2 + 11*C*a^4*b^3*c^2 - 7*B*a^3*b^4*c^2
 - 3*A*a^2*b^5*c^2 - B*a*b^6*c^2 - A*b^7*c^2 - 2*C*a^7*c*d - 2*B*a^6*b*c*d
+ 12*A*a^5*b^2*c*d - 18*C*a^5*b^2*c*d + 22*B*a^4*b^3*c*d - 14*A*a^3*b^4*c*d
 + 8*C*a^3*b^4*c*d - 2*A*a*b^6*c*d - B*a^7*d^2 - A*a^6*b*d^2 + C*a^6*b*d^2
- 9*B*a^5*b^2*d^2 + 11*A*a^4*b^3*d^2 - 11*C*a^4*b^3*d^2 + 4*B*a^3*b^4*d^2)/
((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*(b*tan(f*x + e) + a)^2))/f
```

3.64 $\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^3 \left(A + B\tan(e + fx) \right.$

Optimal. Leaf size=603

$$\frac{(c+d\tan(e+fx))^4\left(5a^2Cd^2-6abd(cC-5Bd)+b^2\left(15d^2(A-C)-3Bcd+c^2C\right)\right)}{60d^3f}-\frac{d\tan(e+fx)\left(a^2\left(-\left(2cd(A-C\right.\right.\right.}{}$$

```
[Out] (a^2*(A*c^3 - c^3*C - 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 + B*d^3) + b^2*(c^3
*C + 3*B*c^2*d - 3*c*C*d^2 - B*d^3 - A*(c^3 - 3*c*d^2)) - 2*a*b*((A - C)*d*
(3*c^2 - d^2) + B*(c^3 - 3*c*d^2)))*x + ((2*a*b*(c^3*C + 3*B*c^2*d - 3*c*C*
d^2 - B*d^3 - A*(c^3 - 3*c*d^2)) - a^2*((A - C)*d*(3*c^2 - d^2) + B*(c^3 -
3*c*d^2)) + b^2*((A - C)*d*(3*c^2 - d^2) + B*(c^3 - 3*c*d^2)))*Log[Cos[e +
f*x]])/f - (d*(2*a*b*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - a^2*(2*c*(
A - C)*d + B*(c^2 - d^2)) + b^2*(2*c*(A - C)*d + B*(c^2 - d^2)))*Tan[e + f*
x])/f + ((2*a*b*(A*c - c*C - B*d) + a^2*(B*c + (A - C)*d) - b^2*(B*c + (A -
 C)*d))*(c + d*Tan[e + f*x])^2)/(2*f) + ((a^2*B - b^2*B + 2*a*b*(A - C))*(c
 + d*Tan[e + f*x])^3)/(3*f) + ((5*a^2*C*d^2 - 6*a*b*d*(c*C - 5*B*d) + b^2*(
c^2*C - 3*B*c*d + 15*(A - C)*d^2))*(c + d*Tan[e + f*x])^4)/(60*d^3*f) - (b*
(b*c*C - 3*b*B*d - a*C*d)*Tan[e + f*x]*(c + d*Tan[e + f*x])^4)/(15*d^2*f) +
 (C*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^4)/(6*d*f)
```

Rubi [A] time = 1.53279, antiderivative size = 603, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3647, 3637, 3630, 3528, 3525, 3475}

$$\frac{(c+d\tan(e+fx))^4\left(5a^2Cd^2-6abd(cC-5Bd)+b^2\left(15d^2(A-C)-3Bcd+c^2C\right)\right)}{60d^3f}-\frac{d\tan(e+fx)\left(a^2\left(-\left(2cd(A-C\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*T
an[e + f*x]^2),x]
```


```
[Out] (a^2*(A*c^3 - c^3*C - 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 + B*d^3) + b^2*(c^3
*C + 3*B*c^2*d - 3*c*C*d^2 - B*d^3 - A*(c^3 - 3*c*d^2)) - 2*a*b*((A - C)*d*
(3*c^2 - d^2) + B*(c^3 - 3*c*d^2)))*x + ((2*a*b*(c^3*C + 3*B*c^2*d - 3*c*C*
d^2 - B*d^3 - A*(c^3 - 3*c*d^2)) - a^2*((A - C)*d*(3*c^2 - d^2) + B*(c^3 -
3*c*d^2)) + b^2*((A - C)*d*(3*c^2 - d^2) + B*(c^3 - 3*c*d^2)))*Log[Cos[e +
f*x]])/f - (d*(2*a*b*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - a^2*(2*c*(
A - C)*d + B*(c^2 - d^2)) + b^2*(2*c*(A - C)*d + B*(c^2 - d^2)))*Tan[e + f*
x])/f + ((2*a*b*(A*c - c*C - B*d) + a^2*(B*c + (A - C)*d) - b^2*(B*c + (A -
 C)*d))*(c + d*Tan[e + f*x])^2)/(2*f) + ((a^2*B - b^2*B + 2*a*b*(A - C))*(c
 + d*Tan[e + f*x])^3)/(3*f) + ((5*a^2*C*d^2 - 6*a*b*d*(c*C - 5*B*d) + b^2*(
c^2*C - 3*B*c*d + 15*(A - C)*d^2))*(c + d*Tan[e + f*x])^4)/(60*d^3*f) - (b*
(b*c*C - 3*b*B*d - a*C*d)*Tan[e + f*x]*(c + d*Tan[e + f*x])^4)/(15*d^2*f) +
 (C*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^4)/(6*d*f)
```


Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
```

```
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx = \frac{C(a+b\tan(e+fx))^2(c+d\,\mathrm{ta}}{6df}$$

$$= -\frac{b(bcC-3bBd-aCd)\tan(e}{15d^2}$$

$$= \frac{\left(5a^2Cd^2-6abd(cC-5Bd)+\right.}{}$$

$$= \frac{\left(a^2B-b^2B+2ab(A-C)\right)(c}{3f}$$

$$= \frac{\left(2ab(Ac-cC-Bd)+a^2(Bc+\right.}{}$$

$$= \left(a^2\left(Ac^3-c^3C-3Bc^2d-3Ac\right.\right.$$

$$= \left(a^2\left(Ac^3-c^3C-3Bc^2d-3Ac\right.\right.$$

Mathematica [C] time = 6.58338, size = 419, normalized size = 0.69

$$\frac{C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^4}{6df} + \frac{-\frac{2b\tan(e+fx)(-aCd-3bBd+bcC)(c+d\tan(e+fx))^4}{5df} - \frac{-\frac{(c+d\tan(e+fx))^4\left(5a^2Cd^2-6abd(cC-5Bd)+b^2\right.}{2df}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x]
 + C*Tan[e + f*x]^2),x]
```

```
[Out] (C*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^4)/(6*d*f) + ((-2*b*(b*c*C -
 3*b*B*d - a*C*d)*Tan[e + f*x]*(c + d*Tan[e + f*x])^4)/(5*d*f) - (-((5*a^2*
C*d^2 - 6*a*b*d*(c*C - 5*B*d) + b^2*(c^2*C - 3*B*c*d + 15*(A - C)*d^2))*(c
+ d*Tan[e + f*x])^4)/(2*d*f) + (5*(3*d*(2*a*b*(A*c - c*C + B*d) + a^2*(B*c
- (A - C)*d) - b^2*(B*c - (A - C)*d))*((I*c - d)^3*Log[I - Tan[e + f*x]] -
(I*c + d)^3*Log[I + Tan[e + f*x]] + 6*c*d^2*Tan[e + f*x] + d^3*Tan[e + f*x]
^2) + (a^2*B - b^2*B + 2*a*b*(A - C))*d*((3*I)*(c + I*d)^4*Log[I - Tan[e +
f*x]] - (3*I)*(c - I*d)^4*Log[I + Tan[e + f*x]] - 6*d^2*(6*c^2 - d^2)*Tan[e
 + f*x] - 12*c*d^3*Tan[e + f*x]^2 - 2*d^4*Tan[e + f*x]^3)))/f)/(5*d))/(6*d)
```

Maple [B] time = 0.021, size = 1807, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x
)
```

```
[Out] 3/f*B*tan(f*x+e)^2*a*b*c^2*d-3/f*C*tan(f*x+e)^2*a*b*c*d^2+2/f*C*tan(f*x+e)^
3*a*b*c^2*d+3/2/f*C*tan(f*x+e)^4*a*b*c*d^2+6/f*A*a*b*c^2*d*tan(f*x+e)-3/f*l
n(1+tan(f*x+e)^2)*A*a*b*c*d^2+3/4/f*C*tan(f*x+e)^4*b^2*c^2*d-3/2/f*ln(1+tan
(f*x+e)^2)*B*a^2*c*d^2+1/f*ln(1+tan(f*x+e)^2)*B*a*b*d^3+3/2/f*ln(1+tan(f*x+
e)^2)*B*b^2*c*d^2-3/2/f*ln(1+tan(f*x+e)^2)*C*a^2*c^2*d+1/f*C*tan(f*x+e)^2*a
```

```
*b*c^3+1/f*C*tan(f*x+e)^3*a^2*c*d^2-1/f*C*tan(f*x+e)^3*b^2*c*d^2+2/f*A*arct
an(tan(f*x+e))*a*b*d^3+3/f*A*arctan(tan(f*x+e))*b^2*c*d^2+2/f*B*a*b*c^3*tan
(f*x+e)+3/2/f*A*tan(f*x+e)^2*b^2*c^2*d+3/2/f*B*tan(f*x+e)^2*a^2*c*d^2+3/2/f
*C*tan(f*x+e)^2*a^2*c^2*d-1/f*ln(1+tan(f*x+e)^2)*C*a*b*c^3+3/2/f*ln(1+tan(f
*x+e)^2)*C*b^2*c^2*d-3/f*A*arctan(tan(f*x+e))*a^2*c*d^2+3/f*A*tan(f*x+e)^2*
a*b*c*d^2-6/f*B*a*b*c*d^2*tan(f*x+e)-3/f*ln(1+tan(f*x+e)^2)*B*a*b*c^2*d+3/f
*ln(1+tan(f*x+e)^2)*C*a*b*c*d^2-6/f*A*arctan(tan(f*x+e))*a*b*c^2*d+6/f*B*ar
ctan(tan(f*x+e))*a*b*c*d^2+6/f*C*arctan(tan(f*x+e))*a*b*c^2*d-6/f*C*a*b*c^2
*d*tan(f*x+e)+2/f*B*tan(f*x+e)^3*a*b*c*d^2-1/3/f*B*tan(f*x+e)^3*b^2*d^3+1/3
/f*C*tan(f*x+e)^3*b^2*c^3+1/2/f*C*tan(f*x+e)^2*b^2*d^3+1/5/f*B*tan(f*x+e)^5
*b^2*d^3+1/f*C*a^2*c^3*tan(f*x+e)+1/f*B*b^2*d^3*tan(f*x+e)-1/2/f*C*tan(f*x+
e)^2*a^2*d^3+1/4/f*A*tan(f*x+e)^4*b^2*d^3+1/4/f*C*tan(f*x+e)^4*a^2*d^3-1/f*
C*b^2*c^3*tan(f*x+e)+1/6/f*C*b^2*d^3*tan(f*x+e)^6+1/2/f*ln(1+tan(f*x+e)^2)*
B*a^2*c^3-1/2/f*ln(1+tan(f*x+e)^2)*B*b^2*c^3-2/f*C*arctan(tan(f*x+e))*a*b*d
^3-3/f*C*arctan(tan(f*x+e))*b^2*c*d^2-3/2/f*ln(1+tan(f*x+e)^2)*A*b^2*c^2*d+
3/2/f*ln(1+tan(f*x+e)^2)*A*a^2*c^2*d+1/f*ln(1+tan(f*x+e)^2)*A*a*b*c^3+1/f*A
*tan(f*x+e)^3*b^2*c*d^2+1/f*B*tan(f*x+e)^3*b^2*c^2*d-3/f*A*b^2*c*d^2*tan(f*
x+e)+3/f*B*a^2*c^2*d*tan(f*x+e)+1/2/f*ln(1+tan(f*x+e)^2)*C*a^2*d^3+1/2/f*A*
tan(f*x+e)^2*a^2*d^3-1/4/f*C*tan(f*x+e)^4*b^2*d^3-1/2/f*A*tan(f*x+e)^2*b^2*
d^3+1/2/f*B*tan(f*x+e)^2*b^2*c^3+1/f*C*arctan(tan(f*x+e))*b^2*c^3+1/3/f*B*t
an(f*x+e)^3*a^2*d^3+1/f*A*b^2*c^3*tan(f*x+e)-1/f*B*a^2*d^3*tan(f*x+e)-1/f*A
*arctan(tan(f*x+e))*b^2*c^3+1/f*B*arctan(tan(f*x+e))*a^2*d^3-1/f*B*arctan(t
an(f*x+e))*b^2*d^3-1/f*C*arctan(tan(f*x+e))*a^2*c^3+1/f*A*arctan(tan(f*x+e)
)*a^2*c^3-1/2/f*ln(1+tan(f*x+e)^2)*C*b^2*d^3-1/2/f*ln(1+tan(f*x+e)^2)*A*a^2
*d^3+1/2/f*ln(1+tan(f*x+e)^2)*A*b^2*d^3+3/f*C*b^2*c*d^2*tan(f*x+e)-3/f*C*a^
2*c*d^2*tan(f*x+e)+2/f*C*a*b*d^3*tan(f*x+e)-3/f*B*b^2*c^2*d*tan(f*x+e)+2/3/
f*A*tan(f*x+e)^3*a*b*d^3-2/3/f*C*tan(f*x+e)^3*a*b*d^3+2/5/f*C*tan(f*x+e)^5*
a*b*d^3+3/5/f*C*tan(f*x+e)^5*b^2*c*d^2-3/2/f*B*tan(f*x+e)^2*b^2*c*d^2-3/f*B
*arctan(tan(f*x+e))*a^2*c^2*d-2/f*B*arctan(tan(f*x+e))*a*b*c^3+3/f*B*arctan
(tan(f*x+e))*b^2*c^2*d+3/f*C*arctan(tan(f*x+e))*a^2*c*d^2+1/2/f*B*tan(f*x+e
)^4*a*b*d^3+3/4/f*B*tan(f*x+e)^4*b^2*c*d^2+3/f*A*a^2*c*d^2*tan(f*x+e)-2/f*A
*a*b*d^3*tan(f*x+e)-3/2/f*C*tan(f*x+e)^2*b^2*c^2*d-1/f*B*tan(f*x+e)^2*a*b*d
^3
```

Maxima [A] time = 1.49778, size = 918, normalized size = 1.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e
)^2),x, algorithm="maxima")
```

```
[Out] 1/60*(10*C*b^2*d^3*tan(f*x + e)^6 + 12*(3*C*b^2*c*d^2 + (2*C*a*b + B*b^2)*d
^3)*tan(f*x + e)^5 + 15*(3*C*b^2*c^2*d + 3*(2*C*a*b + B*b^2)*c*d^2 + (C*a^2
 + 2*B*a*b + (A - C)*b^2)*d^3)*tan(f*x + e)^4 + 20*(C*b^2*c^3 + 3*(2*C*a*b
+ B*b^2)*c^2*d + 3*(C*a^2 + 2*B*a*b + (A - C)*b^2)*c*d^2 + (B*a^2 + 2*(A -
C)*a*b - B*b^2)*d^3)*tan(f*x + e)^3 + 30*((2*C*a*b + B*b^2)*c^3 + 3*(C*a^2
+ 2*B*a*b + (A - C)*b^2)*c^2*d + 3*(B*a^2 + 2*(A - C)*a*b - B*b^2)*c*d^2 +
((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*d^3)*tan(f*x + e)^2 + 60*(((A - C)*a^
2 - 2*B*a*b - (A - C)*b^2)*c^3 - 3*(B*a^2 + 2*(A - C)*a*b - B*b^2)*c^2*d -
3*((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c*d^2 + (B*a^2 + 2*(A - C)*a*b - B*
b^2)*d^3)*(f*x + e) + 30*((B*a^2 + 2*(A - C)*a*b - B*b^2)*c^3 + 3*((A - C)*
a^2 - 2*B*a*b - (A - C)*b^2)*c^2*d - 3*(B*a^2 + 2*(A - C)*a*b - B*b^2)*c*d^
2 - ((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*d^3)*log(tan(f*x + e)^2 + 1) + 60
*((C*a^2 + 2*B*a*b + (A - C)*b^2)*c^3 + 3*(B*a^2 + 2*(A - C)*a*b - B*b^2)*c
^2*d + 3*((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c*d^2 - (B*a^2 + 2*(A - C)*a
```

```
*b - B*b^2)*d^3)*tan(f*x + e))/f
```

Fricas [A] time = 1.31179, size = 1461, normalized size = 2.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e
)^2),x, algorithm="fricas")
```

```
[Out] 1/60*(10*C*b^2*d^3*tan(f*x + e)^6 + 12*(3*C*b^2*c*d^2 + (2*C*a*b + B*b^2)*d
^3)*tan(f*x + e)^5 + 15*(3*C*b^2*c^2*d + 3*(2*C*a*b + B*b^2)*c*d^2 + (C*a^2
 + 2*B*a*b + (A - C)*b^2)*d^3)*tan(f*x + e)^4 + 20*(C*b^2*c^3 + 3*(2*C*a*b
+ B*b^2)*c^2*d + 3*(C*a^2 + 2*B*a*b + (A - C)*b^2)*c*d^2 + (B*a^2 + 2*(A -
C)*a*b - B*b^2)*d^3)*tan(f*x + e)^3 + 60*(((A - C)*a^2 - 2*B*a*b - (A - C)*
b^2)*c^3 - 3*(B*a^2 + 2*(A - C)*a*b - B*b^2)*c^2*d - 3*((A - C)*a^2 - 2*B*a
*b - (A - C)*b^2)*c*d^2 + (B*a^2 + 2*(A - C)*a*b - B*b^2)*d^3)*f*x + 30*((2
*C*a*b + B*b^2)*c^3 + 3*(C*a^2 + 2*B*a*b + (A - C)*b^2)*c^2*d + 3*(B*a^2 +
2*(A - C)*a*b - B*b^2)*c*d^2 + ((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*d^3)*t
an(f*x + e)^2 - 30*((B*a^2 + 2*(A - C)*a*b - B*b^2)*c^3 + 3*((A - C)*a^2 -
2*B*a*b - (A - C)*b^2)*c^2*d - 3*(B*a^2 + 2*(A - C)*a*b - B*b^2)*c*d^2 - ((
A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*d^3)*log(1/(tan(f*x + e)^2 + 1)) + 60*(
(C*a^2 + 2*B*a*b + (A - C)*b^2)*c^3 + 3*(B*a^2 + 2*(A - C)*a*b - B*b^2)*c^2
*d + 3*((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c*d^2 - (B*a^2 + 2*(A - C)*a*b
 - B*b^2)*d^3)*tan(f*x + e))/f
```

Sympy [A] time = 7.86021, size = 1819, normalized size = 3.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2*(c+d*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*x
+e)**2),x)
```

```
[Out] Piecewise((A*a**2*c**3*x + 3*A*a**2*c**2*d*log(tan(e + f*x)**2 + 1)/(2*f) -
 3*A*a**2*c*d**2*x + 3*A*a**2*c*d**2*tan(e + f*x)/f - A*a**2*d**3*log(tan(e
 + f*x)**2 + 1)/(2*f) + A*a**2*d**3*tan(e + f*x)**2/(2*f) + A*a*b*c**3*log(
tan(e + f*x)**2 + 1)/f - 6*A*a*b*c**2*d*x + 6*A*a*b*c**2*d*tan(e + f*x)/f -
 3*A*a*b*c*d**2*log(tan(e + f*x)**2 + 1)/f + 3*A*a*b*c*d**2*tan(e + f*x)**2
/f + 2*A*a*b*d**3*x + 2*A*a*b*d**3*tan(e + f*x)**3/(3*f) - 2*A*a*b*d**3*tan
(e + f*x)/f - A*b**2*c**3*x + A*b**2*c**3*tan(e + f*x)/f - 3*A*b**2*c**2*d*
log(tan(e + f*x)**2 + 1)/(2*f) + 3*A*b**2*c**2*d*tan(e + f*x)**2/(2*f) + 3*
A*b**2*c*d**2*x + A*b**2*c*d**2*tan(e + f*x)**3/f - 3*A*b**2*c*d**2*tan(e +
 f*x)/f + A*b**2*d**3*log(tan(e + f*x)**2 + 1)/(2*f) + A*b**2*d**3*tan(e +
f*x)**4/(4*f) - A*b**2*d**3*tan(e + f*x)**2/(2*f) + B*a**2*c**3*log(tan(e +
 f*x)**2 + 1)/(2*f) - 3*B*a**2*c**2*d*x + 3*B*a**2*c**2*d*tan(e + f*x)/f -
3*B*a**2*c*d**2*log(tan(e + f*x)**2 + 1)/(2*f) + 3*B*a**2*c*d**2*tan(e + f*
x)**2/(2*f) + B*a**2*d**3*x + B*a**2*d**3*tan(e + f*x)**3/(3*f) - B*a**2*d*
*3*tan(e + f*x)/f - 2*B*a*b*c**3*x + 2*B*a*b*c**3*tan(e + f*x)/f - 3*B*a*b*
c**2*d*log(tan(e + f*x)**2 + 1)/f + 3*B*a*b*c**2*d*tan(e + f*x)**2/f + 6*B*
a*b*c*d**2*x + 2*B*a*b*c*d**2*tan(e + f*x)**3/f - 6*B*a*b*c*d**2*tan(e + f*
x)/f + B*a*b*d**3*log(tan(e + f*x)**2 + 1)/f + B*a*b*d**3*tan(e + f*x)**4/(
```

```
2*f) - B*a*b*d**3*tan(e + f*x)**2/f - B*b**2*c**3*log(tan(e + f*x)**2 + 1)/
(2*f) + B*b**2*c**3*tan(e + f*x)**2/(2*f) + 3*B*b**2*c**2*d*x + B*b**2*c**2
*d*tan(e + f*x)**3/f - 3*B*b**2*c**2*d*tan(e + f*x)/f + 3*B*b**2*c*d**2*log
(tan(e + f*x)**2 + 1)/(2*f) + 3*B*b**2*c*d**2*tan(e + f*x)**4/(4*f) - 3*B*b
**2*c*d**2*tan(e + f*x)**2/(2*f) - B*b**2*d**3*x + B*b**2*d**3*tan(e + f*x)
**5/(5*f) - B*b**2*d**3*tan(e + f*x)**3/(3*f) + B*b**2*d**3*tan(e + f*x)/f
- C*a**2*c**3*x + C*a**2*c**3*tan(e + f*x)/f - 3*C*a**2*c**2*d*log(tan(e +
f*x)**2 + 1)/(2*f) + 3*C*a**2*c**2*d*tan(e + f*x)**2/(2*f) + 3*C*a**2*c*d**
2*x + C*a**2*c*d**2*tan(e + f*x)**3/f - 3*C*a**2*c*d**2*tan(e + f*x)/f + C*
a**2*d**3*log(tan(e + f*x)**2 + 1)/(2*f) + C*a**2*d**3*tan(e + f*x)**4/(4*f
) - C*a**2*d**3*tan(e + f*x)**2/(2*f) - C*a*b*c**3*log(tan(e + f*x)**2 + 1)
/f + C*a*b*c**3*tan(e + f*x)**2/f + 6*C*a*b*c**2*d*x + 2*C*a*b*c**2*d*tan(e
 + f*x)**3/f - 6*C*a*b*c**2*d*tan(e + f*x)/f + 3*C*a*b*c*d**2*log(tan(e + f
*x)**2 + 1)/f + 3*C*a*b*c*d**2*tan(e + f*x)**4/(2*f) - 3*C*a*b*c*d**2*tan(e
 + f*x)**2/f - 2*C*a*b*d**3*x + 2*C*a*b*d**3*tan(e + f*x)**5/(5*f) - 2*C*a*
b*d**3*tan(e + f*x)**3/(3*f) + 2*C*a*b*d**3*tan(e + f*x)/f + C*b**2*c**3*x
+ C*b**2*c**3*tan(e + f*x)**3/(3*f) - C*b**2*c**3*tan(e + f*x)/f + 3*C*b**2
*c**2*d*log(tan(e + f*x)**2 + 1)/(2*f) + 3*C*b**2*c**2*d*tan(e + f*x)**4/(4
*f) - 3*C*b**2*c**2*d*tan(e + f*x)**2/(2*f) - 3*C*b**2*c*d**2*x + 3*C*b**2*
c*d**2*tan(e + f*x)**5/(5*f) - C*b**2*c*d**2*tan(e + f*x)**3/f + 3*C*b**2*c
*d**2*tan(e + f*x)/f - C*b**2*d**3*log(tan(e + f*x)**2 + 1)/(2*f) + C*b**2*
d**3*tan(e + f*x)**6/(6*f) - C*b**2*d**3*tan(e + f*x)**4/(4*f) + C*b**2*d**
3*tan(e + f*x)**2/(2*f), Ne(f, 0)), (x*(a + b*tan(e))**2*(c + d*tan(e))**3*
(A + B*tan(e) + C*tan(e)**2), True))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e
)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.65 $\int (a+b\tan(e+fx))(c+d\tan(e+fx))^3 \left(A + B\tan(e+fx) +\right.$

Optimal. Leaf size=389

$$\frac{d\tan(e+fx)\left(A\left(2acd+b\left(c^2-d^2\right)\right)+a\left(Bc^2-Bd^2-2cCd\right)-b\left(2Bcd+c^2C-Cd^2\right)\right)}{f}-\frac{\log(\cos(e+fx))\left(A\left(3ac^2d\right.\right.}{}$$

```
[Out] (a*(A*c^3 - c^3*C - 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 + B*d^3) - b*((A - C)
*d*(3*c^2 - d^2) + B*(c^3 - 3*c*d^2)))*x - ((A*(b*c^3 + 3*a*c^2*d - 3*b*c*d
^2 - a*d^3) - b*(c^3*C + 3*B*c^2*d - 3*c*C*d^2 - B*d^3) + a*(B*c^3 - 3*c^2*
C*d - 3*B*c*d^2 + C*d^3))*Log[Cos[e + f*x]])/f + (d*(a*(B*c^2 - 2*c*C*d - B
*d^2) - b*(c^2*C + 2*B*c*d - C*d^2) + A*(2*a*c*d + b*(c^2 - d^2)))*Tan[e +
f*x])/f + ((A*b*c + a*B*c - b*c*C + a*A*d - b*B*d - a*C*d)*(c + d*Tan[e + f
*x])^2)/(2*f) + ((A*b + a*B - b*C)*(c + d*Tan[e + f*x])^3)/(3*f) - ((b*c*C
- 5*b*B*d - 5*a*C*d)*(c + d*Tan[e + f*x])^4)/(20*d^2*f) + (b*C*Tan[e + f*x]
*(c + d*Tan[e + f*x])^4)/(5*d*f)
```

Rubi [A] time = 0.705025, antiderivative size = 387, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {3637, 3630, 3528, 3525, 3475}

$$\frac{d\tan(e+fx)\left(2aAcd+aB\left(c^2-d^2\right)-2acCd+Ab\left(c^2-d^2\right)-b\left(2Bcd+c^2C-Cd^2\right)\right)}{f}-\frac{\log(\cos(e+fx))\left(A\left(3ac^2d-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan
[e + f*x]^2),x]
```

```
[Out] -((b*(A - C)*d*(3*c^2 - d^2) + b*B*(c^3 - 3*c*d^2) - a*(A*c^3 - c^3*C - 3*B
*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 + B*d^3))*x) - ((A*(b*c^3 + 3*a*c^2*d - 3*b*
c*d^2 - a*d^3) - b*(c^3*C + 3*B*c^2*d - 3*c*C*d^2 - B*d^3) + a*(B*c^3 - 3*c
^2*C*d - 3*B*c*d^2 + C*d^3))*Log[Cos[e + f*x]])/f + (d*(2*a*A*c*d - 2*a*c*C
*d + A*b*(c^2 - d^2) + a*B*(c^2 - d^2) - b*(c^2*C + 2*B*c*d - C*d^2))*Tan[e
 + f*x])/f + ((A*b*c + a*B*c - b*c*C + a*A*d - b*B*d - a*C*d)*(c + d*Tan[e
+ f*x])^2)/(2*f) + ((A*b + a*B - b*C)*(c + d*Tan[e + f*x])^3)/(3*f) - ((b*c
*C - 5*b*B*d - 5*a*C*d)*(c + d*Tan[e + f*x])^4)/(20*d^2*f) + (b*C*Tan[e + f
*x]*(c + d*Tan[e + f*x])^4)/(5*d*f)
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx=\frac{bC\tan(e+fx)(c+d\tan(e+fx))}{5df}$$

$$=-\frac{(bcC-5bBd-5aCd)(c+d\tan(}{20d^2f}$$

$$=\frac{(Ab+aB-bC)(c+d\tan(e+fx)}{3f}$$

$$=\frac{(Abc+aBc-bcC+aAd-bBd-}{2f}$$

$$=-\left(b(A-C)d\left(3c^2-d^2\right)+bB\left(c^3-\right.\right.$$

$$=-\left(b(A-C)d\left(3c^2-d^2\right)+bB\left(c^3-\right.\right.$$

Mathematica [C] time = 6.34188, size = 297, normalized size = 0.76

$$\frac{bC\tan(e+fx)(c+d\tan(e+fx))^4}{5df}-\frac{\frac{(-5aCd-5bBd+bcC)(c+d\tan(e+fx))^4}{4df}+\frac{5\left((aB+Ab-bC)\left(-6d^2\left(6c^2-d^2\right)\tan(e+fx)-12cd^3\tan^2(e+fx)-3i(c-\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x] +
 C*Tan[e + f*x]^2),x]
```

```
[Out] (b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^4)/(5*d*f) - (((b*c*C - 5*b*B*d - 5*
a*C*d)*(c + d*Tan[e + f*x])^4)/(4*d*f) + (5*(3*(A*b*c + a*B*c - b*c*C - a*A
*d + b*B*d + a*C*d)*((I*c - d)^3*Log[I - Tan[e + f*x]] - (I*c + d)^3*Log[I
+ Tan[e + f*x]] + 6*c*d^2*Tan[e + f*x] + d^3*Tan[e + f*x]^2) + (A*b + a*B -
 b*C)*((3*I)*(c + I*d)^4*Log[I - Tan[e + f*x]] - (3*I)*(c - I*d)^4*Log[I +
Tan[e + f*x]] - 6*d^2*(6*c^2 - d^2)*Tan[e + f*x] - 12*c*d^3*Tan[e + f*x]^2
```

```
- 2*d^4*Tan[e + f*x]^3)))/(6*f))/(5*d)
```

Maple [B] time = 0.017, size = 994, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x)
```

```
[Out] 1/2/f*ln(1+tan(f*x+e)^2)*C*a*d^3+1/f*B*arctan(tan(f*x+e))*a*d^3-1/2/f*ln(1+
tan(f*x+e)^2)*C*b*c^3-1/2/f*B*tan(f*x+e)^2*b*d^3+1/f*C*a*c^3*tan(f*x+e)+1/2
/f*ln(1+tan(f*x+e)^2)*B*a*c^3+1/4/f*B*tan(f*x+e)^4*b*d^3-1/3/f*C*tan(f*x+e)
^3*b*d^3-1/2/f*C*tan(f*x+e)^2*a*d^3+1/5/f*C*b*d^3*tan(f*x+e)^5-1/f*A*b*d^3*
tan(f*x+e)-1/f*B*a*d^3*tan(f*x+e)+1/3/f*B*tan(f*x+e)^3*a*d^3+1/2/f*C*tan(f*
x+e)^2*b*c^3+1/f*A*arctan(tan(f*x+e))*a*c^3+1/2/f*A*tan(f*x+e)^2*a*d^3+1/4/
f*C*tan(f*x+e)^4*a*d^3+1/3/f*A*tan(f*x+e)^3*b*d^3+1/2/f*ln(1+tan(f*x+e)^2)*
B*b*d^3+1/f*B*b*c^3*tan(f*x+e)+1/f*C*b*d^3*tan(f*x+e)+1/f*A*arctan(tan(f*x+
e))*b*d^3-1/2/f*ln(1+tan(f*x+e)^2)*A*a*d^3+1/2/f*ln(1+tan(f*x+e)^2)*A*b*c^3
-1/f*B*arctan(tan(f*x+e))*b*c^3-1/f*C*arctan(tan(f*x+e))*a*c^3-1/f*C*arctan
(tan(f*x+e))*b*d^3-3/f*A*arctan(tan(f*x+e))*b*c^2*d-3/f*B*arctan(tan(f*x+e)
)*a*c^2*d+3/f*B*arctan(tan(f*x+e))*b*c*d^2+3/f*C*arctan(tan(f*x+e))*a*c*d^2
-3/2/f*ln(1+tan(f*x+e)^2)*B*b*c^2*d-3/2/f*ln(1+tan(f*x+e)^2)*C*a*c^2*d+3/2/
f*ln(1+tan(f*x+e)^2)*C*b*c*d^2-3/f*A*arctan(tan(f*x+e))*a*c*d^2-3/2/f*ln(1+
tan(f*x+e)^2)*A*b*c*d^2-3/2/f*ln(1+tan(f*x+e)^2)*B*a*c*d^2+1/f*C*tan(f*x+e)
^3*b*c^2*d+3/2/f*B*tan(f*x+e)^2*a*c*d^2-3/f*C*a*c*d^2*tan(f*x+e)-3/f*C*b*c^
2*d*tan(f*x+e)+3/f*A*a*c*d^2*tan(f*x+e)+3/f*A*b*c^2*d*tan(f*x+e)+3/f*B*a*c^
2*d*tan(f*x+e)+3/2/f*A*tan(f*x+e)^2*b*c*d^2-3/2/f*C*tan(f*x+e)^2*b*c*d^2-3/
f*B*b*c*d^2*tan(f*x+e)+1/f*B*tan(f*x+e)^3*b*c*d^2+3/f*C*arctan(tan(f*x+e))*
b*c^2*d+3/4/f*C*tan(f*x+e)^4*b*c*d^2+3/2/f*B*tan(f*x+e)^2*b*c^2*d+1/f*C*tan
(f*x+e)^3*a*c*d^2+3/2/f*C*tan(f*x+e)^2*a*c^2*d+3/2/f*ln(1+tan(f*x+e)^2)*A*a
*c^2*d
```

Maxima [A] time = 1.55974, size = 522, normalized size = 1.34

$$\frac{12\,Cbd^3\tan\left(fx+e\right)^5+15\left(3\,Cbcd^2+(Ca+Bb)d^3\right)\tan\left(fx+e\right)^4+20\left(3\,Cbc^2d+3\,(Ca+Bb)cd^2+(Ba+(A-C)b)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x, algorithm="maxima")
```

```
[Out] 1/60*(12*C*b*d^3*tan(f*x + e)^5 + 15*(3*C*b*c*d^2 + (C*a + B*b)*d^3)*tan(f*
x + e)^4 + 20*(3*C*b*c^2*d + 3*(C*a + B*b)*c*d^2 + (B*a + (A - C)*b)*d^3)*t
an(f*x + e)^3 + 30*(C*b*c^3 + 3*(C*a + B*b)*c^2*d + 3*(B*a + (A - C)*b)*c*d
^2 + ((A - C)*a - B*b)*d^3)*tan(f*x + e)^2 + 60*(((A - C)*a - B*b)*c^3 - 3*
(B*a + (A - C)*b)*c^2*d - 3*((A - C)*a - B*b)*c*d^2 + (B*a + (A - C)*b)*d^3
)*(f*x + e) + 30*((B*a + (A - C)*b)*c^3 + 3*((A - C)*a - B*b)*c^2*d - 3*(B*
a + (A - C)*b)*c*d^2 - ((A - C)*a - B*b)*d^3)*log(tan(f*x + e)^2 + 1) + 60*
((C*a + B*b)*c^3 + 3*(B*a + (A - C)*b)*c^2*d + 3*((A - C)*a - B*b)*c*d^2 -
(B*a + (A - C)*b)*d^3)*tan(f*x + e))/f
```

Fricas [A] time = 1.2365, size = 861, normalized size = 2.21

$$\frac{12\,Cbd^{3}\tan\left(fx+e\right)^{5}+15\left(3\,Cbcd^{2}+(Ca+Bb)d^{3}\right)\tan\left(fx+e\right)^{4}+20\left(3\,Cbc^{2}d+3\,(Ca+Bb)cd^{2}+(Ba+(A-C)b)d^{3}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x, algorithm="fricas")
```

```
[Out] 1/60*(12*C*b*d^3*tan(f*x + e)^5 + 15*(3*C*b*c*d^2 + (C*a + B*b)*d^3)*tan(f*
x + e)^4 + 20*(3*C*b*c^2*d + 3*(C*a + B*b)*c*d^2 + (B*a + (A - C)*b)*d^3)*t
an(f*x + e)^3 + 60*(((A - C)*a - B*b)*c^3 - 3*(B*a + (A - C)*b)*c^2*d - 3*(
(A - C)*a - B*b)*c*d^2 + (B*a + (A - C)*b)*d^3)*f*x + 30*(C*b*c^3 + 3*(C*a
+ B*b)*c^2*d + 3*(B*a + (A - C)*b)*c*d^2 + ((A - C)*a - B*b)*d^3)*tan(f*x +
 e)^2 - 30*((B*a + (A - C)*b)*c^3 + 3*((A - C)*a - B*b)*c^2*d - 3*(B*a + (A
 - C)*b)*c*d^2 - ((A - C)*a - B*b)*d^3)*log(1/(tan(f*x + e)^2 + 1)) + 60*((
C*a + B*b)*c^3 + 3*(B*a + (A - C)*b)*c^2*d + 3*((A - C)*a - B*b)*c*d^2 - (B
*a + (A - C)*b)*d^3)*tan(f*x + e))/f
```

Sympy [A] time = 5.76826, size = 1001, normalized size = 2.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*x+e)
**2),x)
```

```
[Out] Piecewise((A*a*c**3*x + 3*A*a*c**2*d*log(tan(e + f*x)**2 + 1)/(2*f) - 3*A*a
*c*d**2*x + 3*A*a*c*d**2*tan(e + f*x)/f - A*a*d**3*log(tan(e + f*x)**2 + 1)
/(2*f) + A*a*d**3*tan(e + f*x)**2/(2*f) + A*b*c**3*log(tan(e + f*x)**2 + 1)
/(2*f) - 3*A*b*c**2*d*x + 3*A*b*c**2*d*tan(e + f*x)/f - 3*A*b*c*d**2*log(ta
n(e + f*x)**2 + 1)/(2*f) + 3*A*b*c*d**2*tan(e + f*x)**2/(2*f) + A*b*d**3*x
+ A*b*d**3*tan(e + f*x)**3/(3*f) - A*b*d**3*tan(e + f*x)/f + B*a*c**3*log(t
an(e + f*x)**2 + 1)/(2*f) - 3*B*a*c**2*d*x + 3*B*a*c**2*d*tan(e + f*x)/f -
3*B*a*c*d**2*log(tan(e + f*x)**2 + 1)/(2*f) + 3*B*a*c*d**2*tan(e + f*x)**2/
(2*f) + B*a*d**3*x + B*a*d**3*tan(e + f*x)**3/(3*f) - B*a*d**3*tan(e + f*x)
/f - B*b*c**3*x + B*b*c**3*tan(e + f*x)/f - 3*B*b*c**2*d*log(tan(e + f*x)**
2 + 1)/(2*f) + 3*B*b*c**2*d*tan(e + f*x)**2/(2*f) + 3*B*b*c*d**2*x + B*b*c*
d**2*tan(e + f*x)**3/f - 3*B*b*c*d**2*tan(e + f*x)/f + B*b*d**3*log(tan(e +
 f*x)**2 + 1)/(2*f) + B*b*d**3*tan(e + f*x)**4/(4*f) - B*b*d**3*tan(e + f*x
)**2/(2*f) - C*a*c**3*x + C*a*c**3*tan(e + f*x)/f - 3*C*a*c**2*d*log(tan(e
+ f*x)**2 + 1)/(2*f) + 3*C*a*c**2*d*tan(e + f*x)**2/(2*f) + 3*C*a*c*d**2*x
+ C*a*c*d**2*tan(e + f*x)**3/f - 3*C*a*c*d**2*tan(e + f*x)/f + C*a*d**3*log
(tan(e + f*x)**2 + 1)/(2*f) + C*a*d**3*tan(e + f*x)**4/(4*f) - C*a*d**3*tan
(e + f*x)**2/(2*f) - C*b*c**3*log(tan(e + f*x)**2 + 1)/(2*f) + C*b*c**3*tan
(e + f*x)**2/(2*f) + 3*C*b*c**2*d*x + C*b*c**2*d*tan(e + f*x)**3/f - 3*C*b*
c**2*d*tan(e + f*x)/f + 3*C*b*c*d**2*log(tan(e + f*x)**2 + 1)/(2*f) + 3*C*b
*c*d**2*tan(e + f*x)**4/(4*f) - 3*C*b*c*d**2*tan(e + f*x)**2/(2*f) - C*b*d*
*3*x + C*b*d**3*tan(e + f*x)**5/(5*f) - C*b*d**3*tan(e + f*x)**3/(3*f) + C*
b*d**3*tan(e + f*x)/f, Ne(f, 0)), (x*(a + b*tan(e))*(c + d*tan(e))**3*(A +
B*tan(e) + C*tan(e)**2), True))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.66 $\int (c+d\tan(e+fx))^3 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) d$

Optimal. Leaf size=191

$$\frac{d\tan(e+fx)\left(2cd(A-C)+B\left(c^2-d^2\right)\right)}{f}-\frac{\left(d(A-C)\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)\log(\cos(e+fx))}{f}-x\left(-A\left(c^3-3cd^2\right)+\right.$$

```
[Out] -((c^3*C + 3*B*c^2*d - 3*c*C*d^2 - B*d^3 - A*(c^3 - 3*c*d^2))*x) - (((A - C
)*d*(3*c^2 - d^2) + B*(c^3 - 3*c*d^2))*Log[Cos[e + f*x]])/f + (d*(2*c*(A -
C)*d + B*(c^2 - d^2))*Tan[e + f*x])/f + ((B*c + (A - C)*d)*(c + d*Tan[e + f
*x])^2)/(2*f) + (B*(c + d*Tan[e + f*x])^3)/(3*f) + (C*(c + d*Tan[e + f*x])^
4)/(4*d*f)
```

Rubi [A] time = 0.243865, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {3630, 3528, 3525, 3475}

$$\frac{d\tan(e+fx)\left(2cd(A-C)+B\left(c^2-d^2\right)\right)}{f}-\frac{\left(d(A-C)\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)\log(\cos(e+fx))}{f}-x\left(-A\left(c^3-3cd^2\right)+\right.$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] -((c^3*C + 3*B*c^2*d - 3*c*C*d^2 - B*d^3 - A*(c^3 - 3*c*d^2))*x) - (((A - C
)*d*(3*c^2 - d^2) + B*(c^3 - 3*c*d^2))*Log[Cos[e + f*x]])/f + (d*(2*c*(A -
C)*d + B*(c^2 - d^2))*Tan[e + f*x])/f + ((B*c + (A - C)*d)*(c + d*Tan[e + f
*x])^2)/(2*f) + (B*(c + d*Tan[e + f*x])^3)/(3*f) + (C*(c + d*Tan[e + f*x])^
4)/(4*d*f)
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx = \frac{C(c+d\tan(e+fx))^4}{4df}+\int (A-C+B\tan(e+fx$$
$$= \frac{B(c+d\tan(e+fx))^3}{3f}+\frac{C(c+d\tan(e+fx))^4}{4df}+$$
$$= \frac{(Bc+(A-C)d)(c+d\tan(e+fx))^2}{2f}+\frac{B(c+d\tan}{3}$$
$$= -\left(c^3C+3Bc^2d-3cCd^2-Bd^3-A\left(c^3-3cd^2\right)\right)x$$
$$= -\left(c^3C+3Bc^2d-3cCd^2-Bd^3-A\left(c^3-3cd^2\right)\right)x$$

Mathematica [C] time = 2.41027, size = 212, normalized size = 1.11

$$\frac{-6(d(C-A)+Bc)\left(6cd^2\tan(e+fx)+(-d+ic)^3\log(-\tan(e+fx)+i)-(d+ic)^3\log(\tan(e+fx)+i)+d^3\tan^2(e$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] (3*C*(c + d*Tan[e + f*x])^4 - 6*(B*c + (-A + C)*d)*((I*c - d)^3*Log[I - Tan
[e + f*x]] - (I*c + d)^3*Log[I + Tan[e + f*x]] + 6*c*d^2*Tan[e + f*x] + d^3
*Tan[e + f*x]^2) + 2*B*((-3*I)*(c + I*d)^4*Log[I - Tan[e + f*x]] + (3*I)*(c
 - I*d)^4*Log[I + Tan[e + f*x]] - 6*d^2*(-6*c^2 + d^2)*Tan[e + f*x] + 12*c*
d^3*Tan[e + f*x]^2 + 2*d^4*Tan[e + f*x]^3))/(12*d*f)
```

Maple [B] time = 0.014, size = 420, normalized size = 2.2

$$\frac{Cd^3\left(\tan\left(fx+e\right)\right)^4}{4f}+\frac{B\left(\tan\left(fx+e\right)\right)^3d^3}{3f}+\frac{C\left(\tan\left(fx+e\right)\right)^3cd^2}{f}+\frac{A\left(\tan\left(fx+e\right)\right)^2d^3}{2f}+\frac{3B\left(\tan\left(fx+e\right)\right)^2cd}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x)
```

```
[Out] 1/4/f*C*d^3*tan(f*x+e)^4+1/3/f*B*tan(f*x+e)^3*d^3+1/f*C*tan(f*x+e)^3*c*d^2+
1/2/f*A*tan(f*x+e)^2*d^3+3/2/f*B*tan(f*x+e)^2*c*d^2+3/2/f*C*tan(f*x+e)^2*c^
2*d-1/2/f*C*tan(f*x+e)^2*d^3+3/f*A*c*d^2*tan(f*x+e)+3/f*B*c^2*d*tan(f*x+e)-
1/f*B*d^3*tan(f*x+e)+1/f*c^3*C*tan(f*x+e)-3/f*c*C*d^2*tan(f*x+e)+3/2/f*ln(1
+tan(f*x+e)^2)*A*c^2*d-1/2/f*ln(1+tan(f*x+e)^2)*A*d^3+1/2/f*ln(1+tan(f*x+e)
^2)*B*c^3-3/2/f*ln(1+tan(f*x+e)^2)*B*c*d^2-3/2/f*ln(1+tan(f*x+e)^2)*C*c^2*d
+1/2/f*ln(1+tan(f*x+e)^2)*C*d^3+1/f*A*arctan(tan(f*x+e))*c^3-3/f*A*arctan(t
an(f*x+e))*c*d^2-3/f*B*arctan(tan(f*x+e))*c^2*d+1/f*B*arctan(tan(f*x+e))*d^
3-1/f*C*arctan(tan(f*x+e))*c^3+3/f*C*arctan(tan(f*x+e))*c*d^2
```

Maxima [A] time = 1.49359, size = 273, normalized size = 1.43

$$3\,Cd^3\tan\left(fx+e\right)^4+4\left(3\,Ccd^2+Bd^3\right)\tan\left(fx+e\right)^3+6\left(3\,Cc^2d+3\,Bcd^2+(A-C)d^3\right)\tan\left(fx+e\right)^2+12\left((A-C)c^3-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm="
maxima")
```

```
[Out] 1/12*(3*C*d^3*tan(f*x + e)^4 + 4*(3*C*c*d^2 + B*d^3)*tan(f*x + e)^3 + 6*(3*
C*c^2*d + 3*B*c*d^2 + (A - C)*d^3)*tan(f*x + e)^2 + 12*((A - C)*c^3 - 3*B*c
^2*d - 3*(A - C)*c*d^2 + B*d^3)*(f*x + e) + 6*(B*c^3 + 3*(A - C)*c^2*d - 3*
B*c*d^2 - (A - C)*d^3)*log(tan(f*x + e)^2 + 1) + 12*(C*c^3 + 3*B*c^2*d + 3*
(A - C)*c*d^2 - B*d^3)*tan(f*x + e))/f
```

Fricas [A] time = 1.13902, size = 456, normalized size = 2.39

$$3\,Cd^3\tan\left(fx+e\right)^4+4\left(3\,Ccd^2+Bd^3\right)\tan\left(fx+e\right)^3+12\left((A-C)c^3-3\,Bc^2d-3\,(A-C)cd^2+Bd^3\right)fx+6\left(3\,Cc^2d+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm="
fricas")
```

```
[Out] 1/12*(3*C*d^3*tan(f*x + e)^4 + 4*(3*C*c*d^2 + B*d^3)*tan(f*x + e)^3 + 12*((
A - C)*c^3 - 3*B*c^2*d - 3*(A - C)*c*d^2 + B*d^3)*f*x + 6*(3*C*c^2*d + 3*B*
c*d^2 + (A - C)*d^3)*tan(f*x + e)^2 - 6*(B*c^3 + 3*(A - C)*c^2*d - 3*B*c*d^
2 - (A - C)*d^3)*log(1/(tan(f*x + e)^2 + 1)) + 12*(C*c^3 + 3*B*c^2*d + 3*(A
 - C)*c*d^2 - B*d^3)*tan(f*x + e))/f
```

Sympy [A] time = 2.49255, size = 410, normalized size = 2.15

$$\begin{cases} Ac^3x+\frac{3Ac^2d\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}-3Acd^2x+\frac{3Acd^2\tan\left(e+fx\right)}{f}-\frac{Ad^3\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{Ad^3\tan^2\left(e+fx\right)}{2f}+\frac{Bc^3\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}- \\ x\left(c+d\tan\left(e\right)\right)^3\left(A+B\tan\left(e\right)+C\tan^2\left(e\right)\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((A*c**3*x + 3*A*c**2*d*log(tan(e + f*x)**2 + 1)/(2*f) - 3*A*c*d**
2*x + 3*A*c*d**2*tan(e + f*x)/f - A*d**3*log(tan(e + f*x)**2 + 1)/(2*f) + A
*d**3*tan(e + f*x)**2/(2*f) + B*c**3*log(tan(e + f*x)**2 + 1)/(2*f) - 3*B*c
**2*d*x + 3*B*c**2*d*tan(e + f*x)/f - 3*B*c*d**2*log(tan(e + f*x)**2 + 1)/(
2*f) + 3*B*c*d**2*tan(e + f*x)**2/(2*f) + B*d**3*x + B*d**3*tan(e + f*x)**3
/(3*f) - B*d**3*tan(e + f*x)/f - C*c**3*x + C*c**3*tan(e + f*x)/f - 3*C*c**
2*d*log(tan(e + f*x)**2 + 1)/(2*f) + 3*C*c**2*d*tan(e + f*x)**2/(2*f) + 3*C
*c*d**2*x + C*c*d**2*tan(e + f*x)**3/f - 3*C*c*d**2*tan(e + f*x)/f + C*d**3
*log(tan(e + f*x)**2 + 1)/(2*f) + C*d**3*tan(e + f*x)**4/(4*f) - C*d**3*tan
(e + f*x)**2/(2*f), Ne(f, 0)), (x*(c + d*tan(e))**3*(A + B*tan(e) + C*tan(e
```

```
)**2), True))
```

Giac [B] time = 7.63522, size = 5805, normalized size = 30.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm="
giac")
```

```
[Out] 1/12*(12*A*c^3*f*x*tan(f*x)^4*tan(e)^4 - 12*C*c^3*f*x*tan(f*x)^4*tan(e)^4 -
 36*B*c^2*d*f*x*tan(f*x)^4*tan(e)^4 - 36*A*c*d^2*f*x*tan(f*x)^4*tan(e)^4 +
36*C*c*d^2*f*x*tan(f*x)^4*tan(e)^4 + 12*B*d^3*f*x*tan(f*x)^4*tan(e)^4 - 6*B
*c^3*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(
f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^4*tan(e)^4
- 18*A*c^2*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e
) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^4*t
an(e)^4 + 18*C*c^2*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)
^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(
f*x)^4*tan(e)^4 + 18*B*c*d^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*
tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) +
1))*tan(f*x)^4*tan(e)^4 + 6*A*d^3*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2
 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(
e) + 1))*tan(f*x)^4*tan(e)^4 - 6*C*d^3*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan
(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)
*tan(e) + 1))*tan(f*x)^4*tan(e)^4 - 48*A*c^3*f*x*tan(f*x)^3*tan(e)^3 + 48*C
*c^3*f*x*tan(f*x)^3*tan(e)^3 + 144*B*c^2*d*f*x*tan(f*x)^3*tan(e)^3 + 144*A*
c*d^2*f*x*tan(f*x)^3*tan(e)^3 - 144*C*c*d^2*f*x*tan(f*x)^3*tan(e)^3 - 48*B*
d^3*f*x*tan(f*x)^3*tan(e)^3 + 18*C*c^2*d*tan(f*x)^4*tan(e)^4 + 18*B*c*d^2*t
an(f*x)^4*tan(e)^4 + 6*A*d^3*tan(f*x)^4*tan(e)^4 - 9*C*d^3*tan(f*x)^4*tan(e
)^4 + 24*B*c^3*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan
(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^3
*tan(e)^3 + 72*A*c^2*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*
x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*ta
n(f*x)^3*tan(e)^3 - 72*C*c^2*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 -
2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e)
+ 1))*tan(f*x)^3*tan(e)^3 - 72*B*c*d^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan
(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)
*tan(e) + 1))*tan(f*x)^3*tan(e)^3 - 24*A*d^3*log(4*(tan(e)^2 + 1)/(tan(f*x)
^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*ta
n(f*x)*tan(e) + 1))*tan(f*x)^3*tan(e)^3 + 24*C*d^3*log(4*(tan(e)^2 + 1)/(ta
n(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2
- 2*tan(f*x)*tan(e) + 1))*tan(f*x)^3*tan(e)^3 - 12*C*c^3*tan(f*x)^4*tan(e)^
3 - 36*B*c^2*d*tan(f*x)^4*tan(e)^3 - 36*A*c*d^2*tan(f*x)^4*tan(e)^3 + 36*C*
c*d^2*tan(f*x)^4*tan(e)^3 + 12*B*d^3*tan(f*x)^4*tan(e)^3 - 12*C*c^3*tan(f*x
)^3*tan(e)^4 - 36*B*c^2*d*tan(f*x)^3*tan(e)^4 - 36*A*c*d^2*tan(f*x)^3*tan(e
)^4 + 36*C*c*d^2*tan(f*x)^3*tan(e)^4 + 12*B*d^3*tan(f*x)^3*tan(e)^4 + 72*A*
c^3*f*x*tan(f*x)^2*tan(e)^2 - 72*C*c^3*f*x*tan(f*x)^2*tan(e)^2 - 216*B*c^2*
d*f*x*tan(f*x)^2*tan(e)^2 - 216*A*c*d^2*f*x*tan(f*x)^2*tan(e)^2 + 216*C*c*d
^2*f*x*tan(f*x)^2*tan(e)^2 + 72*B*d^3*f*x*tan(f*x)^2*tan(e)^2 + 18*C*c^2*d*
tan(f*x)^4*tan(e)^2 + 18*B*c*d^2*tan(f*x)^4*tan(e)^2 + 6*A*d^3*tan(f*x)^4*t
an(e)^2 - 6*C*d^3*tan(f*x)^4*tan(e)^2 - 36*C*c^2*d*tan(f*x)^3*tan(e)^3 - 36
*B*c*d^2*tan(f*x)^3*tan(e)^3 - 12*A*d^3*tan(f*x)^3*tan(e)^3 + 24*C*d^3*tan(
f*x)^3*tan(e)^3 + 18*C*c^2*d*tan(f*x)^2*tan(e)^4 + 18*B*c*d^2*tan(f*x)^2*ta
n(e)^4 + 6*A*d^3*tan(f*x)^2*tan(e)^4 - 6*C*d^3*tan(f*x)^2*tan(e)^4 - 12*C*c
*d^2*tan(f*x)^4*tan(e) - 4*B*d^3*tan(f*x)^4*tan(e) - 36*B*c^3*log(4*(tan(e)
```

```
^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 +
tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^2*tan(e)^2 - 108*A*c^2*d*log(
4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*ta
n(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^2*tan(e)^2 + 108*C*c
^2*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(
f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^2*tan(e)^2
+ 108*B*c*d^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(
e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)^2*
tan(e)^2 + 36*A*d^3*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^
3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f
*x)^2*tan(e)^2 - 36*C*d^3*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan
(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))
*tan(f*x)^2*tan(e)^2 + 36*C*c^3*tan(f*x)^3*tan(e)^2 + 108*B*c^2*d*tan(f*x)^
3*tan(e)^2 + 108*A*c*d^2*tan(f*x)^3*tan(e)^2 - 144*C*c*d^2*tan(f*x)^3*tan(e
)^2 - 48*B*d^3*tan(f*x)^3*tan(e)^2 + 36*C*c^3*tan(f*x)^2*tan(e)^3 + 108*B*c
^2*d*tan(f*x)^2*tan(e)^3 + 108*A*c*d^2*tan(f*x)^2*tan(e)^3 - 144*C*c*d^2*ta
n(f*x)^2*tan(e)^3 - 48*B*d^3*tan(f*x)^2*tan(e)^3 - 12*C*c*d^2*tan(f*x)*tan(
e)^4 - 4*B*d^3*tan(f*x)*tan(e)^4 + 3*C*d^3*tan(f*x)^4 - 48*A*c^3*f*x*tan(f*
x)*tan(e) + 48*C*c^3*f*x*tan(f*x)*tan(e) + 144*B*c^2*d*f*x*tan(f*x)*tan(e)
+ 144*A*c*d^2*f*x*tan(f*x)*tan(e) - 144*C*c*d^2*f*x*tan(f*x)*tan(e) - 48*B*
d^3*f*x*tan(f*x)*tan(e) - 36*C*c^2*d*tan(f*x)^3*tan(e) - 36*B*c*d^2*tan(f*x
)^3*tan(e) - 12*A*d^3*tan(f*x)^3*tan(e) + 24*C*d^3*tan(f*x)^3*tan(e) + 36*C
*c^2*d*tan(f*x)^2*tan(e)^2 + 36*B*c*d^2*tan(f*x)^2*tan(e)^2 + 12*A*d^3*tan(
f*x)^2*tan(e)^2 - 12*C*d^3*tan(f*x)^2*tan(e)^2 - 36*C*c^2*d*tan(f*x)*tan(e)
^3 - 36*B*c*d^2*tan(f*x)*tan(e)^3 - 12*A*d^3*tan(f*x)*tan(e)^3 + 24*C*d^3*t
an(f*x)*tan(e)^3 + 3*C*d^3*tan(e)^4 + 12*C*c*d^2*tan(f*x)^3 + 4*B*d^3*tan(f
*x)^3 + 24*B*c^3*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*t
an(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(f*x)
*tan(e) + 72*A*c^2*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)
^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*tan(
f*x)*tan(e) - 72*C*c^2*d*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(
f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1))*
tan(f*x)*tan(e) - 72*B*c*d^2*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*
tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) +
1))*tan(f*x)*tan(e) - 24*A*d^3*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 -
2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e)
+ 1))*tan(f*x)*tan(e) + 24*C*d^3*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2
- 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e
) + 1))*tan(f*x)*tan(e) - 36*C*c^3*tan(f*x)^2*tan(e) - 108*B*c^2*d*tan(f*x)
^2*tan(e) - 108*A*c*d^2*tan(f*x)^2*tan(e) + 144*C*c*d^2*tan(f*x)^2*tan(e) +
 48*B*d^3*tan(f*x)^2*tan(e) - 36*C*c^3*tan(f*x)*tan(e)^2 - 108*B*c^2*d*tan(
f*x)*tan(e)^2 - 108*A*c*d^2*tan(f*x)*tan(e)^2 + 144*C*c*d^2*tan(f*x)*tan(e)
^2 + 48*B*d^3*tan(f*x)*tan(e)^2 + 12*C*c*d^2*tan(e)^3 + 4*B*d^3*tan(e)^3 +
12*A*c^3*f*x - 12*C*c^3*f*x - 36*B*c^2*d*f*x - 36*A*c*d^2*f*x + 36*C*c*d^2*
f*x + 12*B*d^3*f*x + 18*C*c^2*d*tan(f*x)^2 + 18*B*c*d^2*tan(f*x)^2 + 6*A*d^
3*tan(f*x)^2 - 6*C*d^3*tan(f*x)^2 - 36*C*c^2*d*tan(f*x)*tan(e) - 36*B*c*d^2
*tan(f*x)*tan(e) - 12*A*d^3*tan(f*x)*tan(e) + 24*C*d^3*tan(f*x)*tan(e) + 18
*C*c^2*d*tan(e)^2 + 18*B*c*d^2*tan(e)^2 + 6*A*d^3*tan(e)^2 - 6*C*d^3*tan(e)
^2 - 6*B*c^3*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e
) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)) - 18*A*c^2*d
*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)
^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)) + 18*C*c^2*d*log(4*(tan(
e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2
+ tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)) + 18*B*c*d^2*log(4*(tan(e)^2 + 1)/(t
an(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2
 - 2*tan(f*x)*tan(e) + 1)) + 6*A*d^3*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e
)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*t
an(e) + 1)) - 6*C*d^3*log(4*(tan(e)^2 + 1)/(tan(f*x)^4*tan(e)^2 - 2*tan(f*x
)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)) + 1
```

```
2*C*c^3*tan(f*x) + 36*B*c^2*d*tan(f*x) + 36*A*c*d^2*tan(f*x) - 36*C*c*d^2*t
an(f*x) - 12*B*d^3*tan(f*x) + 12*C*c^3*tan(e) + 36*B*c^2*d*tan(e) + 36*A*c*
d^2*tan(e) - 36*C*c*d^2*tan(e) - 12*B*d^3*tan(e) + 18*C*c^2*d + 18*B*c*d^2
+ 6*A*d^3 - 9*C*d^3)/(f*tan(f*x)^4*tan(e)^4 - 4*f*tan(f*x)^3*tan(e)^3 + 6*f
*tan(f*x)^2*tan(e)^2 - 4*f*tan(f*x)*tan(e) + f)
```

3.67 $\int \frac{(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=363

$$-\frac{\log(\cos(e+fx))\left(A\left(ad\left(3c^2-d^2\right)-b\left(c^3-3cd^2\right)\right)+a\left(Bc^3-3Bcd^2-3c^2Cd+Cd^3\right)+b\left(3Bc^2d-Bd^3+c^3C-3cCd^2\right)\right)}{f\left(a^2+b^2\right)}$$

```
[Out] -(((a*(c^3*C + 3*B*c^2*d - 3*c*C*d^2 - B*d^3 - A*(c^3 - 3*c*d^2)) - b*((A -
 C)*d*(3*c^2 - d^2) + B*(c^3 - 3*c*d^2)))*x)/(a^2 + b^2)) - ((b*(c^3*C + 3*
B*c^2*d - 3*c*C*d^2 - B*d^3) + a*(B*c^3 - 3*c^2*C*d - 3*B*c*d^2 + C*d^3) +
A*(a*d*(3*c^2 - d^2) - b*(c^3 - 3*c*d^2)))*Log[Cos[e + f*x]])/((a^2 + b^2)*
f) + ((A*b^2 - a*(b*B - a*C))*(b*c - a*d)^3*Log[a + b*Tan[e + f*x]])/(b^4*(
a^2 + b^2)*f) + (d*(b^2*d*(B*c + (A - C)*d) + (b*c - a*d)*(b*c*C + b*B*d -
a*C*d))*Tan[e + f*x])/(b^3*f) + ((b*c*C + b*B*d - a*C*d)*(c + d*Tan[e + f*x
])^2)/(2*b^2*f) + (C*(c + d*Tan[e + f*x])^3)/(3*b*f)
```

Rubi [A] time = 1.51166, antiderivative size = 363, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3647, 3637, 3626, 3617, 31, 3475}

$$-\frac{\log(\cos(e+fx))\left(A\left(ad\left(3c^2-d^2\right)-b\left(c^3-3cd^2\right)\right)+a\left(Bc^3-3Bcd^2-3c^2Cd+Cd^3\right)+b\left(3Bc^2d-Bd^3+c^3C-3cCd^2\right)\right)}{f\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a + b
*Tan[e + f*x]),x]
```


```
[Out] -(((a*(c^3*C + 3*B*c^2*d - 3*c*C*d^2 - B*d^3 - A*(c^3 - 3*c*d^2)) - b*((A -
 C)*d*(3*c^2 - d^2) + B*(c^3 - 3*c*d^2)))*x)/(a^2 + b^2)) - ((b*(c^3*C + 3*
B*c^2*d - 3*c*C*d^2 - B*d^3) + a*(B*c^3 - 3*c^2*C*d - 3*B*c*d^2 + C*d^3) +
A*(a*d*(3*c^2 - d^2) - b*(c^3 - 3*c*d^2)))*Log[Cos[e + f*x]])/((a^2 + b^2)*
f) + ((A*b^2 - a*(b*B - a*C))*(b*c - a*d)^3*Log[a + b*Tan[e + f*x]])/(b^4*(
a^2 + b^2)*f) + (d*(b^2*d*(B*c + (A - C)*d) + (b*c - a*d)*(b*c*C + b*B*d -
a*C*d))*Tan[e + f*x])/(b^3*f) + ((b*c*C + b*B*d - a*C*d)*(c + d*Tan[e + f*x
])^2)/(2*b^2*f) + (C*(c + d*Tan[e + f*x])^3)/(3*b*f)
```


Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
```

```
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3626

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((a*A + b*B -
a*C)*x)/(a^2 + b^2), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3617

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx = \frac{C(c+d\tan(e+fx))^3}{3bf} + \frac{\int \frac{(c+d\tan(e+fx))^2(3(Abc-aC\cdots}{\cdots}}{\cdots}$$

$$= \frac{(bcC+bBd-aCd)(c+d\tan(e+fx))^2}{2b^2f} + \frac{C(c+\cdots}{\cdots}$$

$$= \frac{d\left(b^2d(Bc+(A-C)d)+(bc-ad)(bcC+bBd-a\cdots\right.}{b^3f}$$

$$= -\frac{\left(a\left(c^3C+3Bc^2d-3cCd^2-Bd^3-A\left(c^3-3cd^2\right)\cdots\right.\right.}{a^2+\cdots}$$

$$= -\frac{\left(a\left(c^3C+3Bc^2d-3cCd^2-Bd^3-A\left(c^3-3cd^2\right)\cdots\right.\right.}{a^2+\cdots}$$

$$= -\frac{\left(a\left(c^3C+3Bc^2d-3cCd^2-Bd^3-A\left(c^3-3cd^2\right)\cdots\right.\right.}{a^2+\cdots}$$

Mathematica [C] time = 4.73759, size = 255, normalized size = 0.7

$$\frac{\frac{6(bc-ad)^3\left(a(aC-bB)+Ab^2\right)\log(a+b\tan(e+fx))}{b^2\left(a^2+b^2\right)}+\frac{3b^2(c+id)^3(-iA+B+iC)\log(-\tan(e+fx)+i)}{a+ib}-\frac{3b^2(d+ic)^3(A-iB-C)\log(\tan(e+fx)+i)}{a-ib}+3(-aCd+\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(a + b*Tan[e + f*x]),x]
```

```
[Out] ((3*b^2*((-I)*A + B + I*C)*(c + I*d)^3*Log[I - Tan[e + f*x]])/(a + I*b) - (
3*b^2*(A - I*B - C)*(I*c + d)^3*Log[I + Tan[e + f*x]])/(a - I*b) + (6*(A*b^
2 + a*(-(b*B) + a*C))*(b*c - a*d)^3*Log[a + b*Tan[e + f*x]])/(b^2*(a^2 + b^
2)) + 6*b*d^2*(B*c + (A - C)*d)*Tan[e + f*x] + (6*d*(b*c - a*d)*(b*c*C + b*
B*d - a*C*d)*Tan[e + f*x])/b + 3*(b*c*C + b*B*d - a*C*d)*(c + d*Tan[e + f*x
])^2 + 2*b*C*(c + d*Tan[e + f*x])^3)/(6*b^2*f)
```

Maple [B] time = 0.051, size = 1304, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)),x)
```

```
[Out] 3/f/(a^2+b^2)*C*arctan(tan(f*x+e))*a*c*d^2-3/f/(a^2+b^2)*C*arctan(tan(f*x+e
))*b*c^2*d-3/2/f/(a^2+b^2)*ln(1+tan(f*x+e)^2)*C*a*c^2*d-1/f/b^4/(a^2+b^2)*l
n(a+b*tan(f*x+e))*C*a^5*d^3+1/f/b/(a^2+b^2)*ln(a+b*tan(f*x+e))*C*a^2*c^3-1/
f/b^2/(a^2+b^2)*ln(a+b*tan(f*x+e))*A*a^3*d^3+1/f/b^3/(a^2+b^2)*ln(a+b*tan(f
*x+e))*B*a^4*d^3-3/f/(a^2+b^2)*A*arctan(tan(f*x+e))*a*c*d^2-3/f*d^2/b^2*C*a
*c*tan(f*x+e)+3/2/f/(a^2+b^2)*ln(1+tan(f*x+e)^2)*A*a*c^2*d+3/2/f/(a^2+b^2)*
ln(1+tan(f*x+e)^2)*A*b*c*d^2+3/f/b^3/(a^2+b^2)*ln(a+b*tan(f*x+e))*C*a^4*c*d
^2+3/f/b/(a^2+b^2)*ln(a+b*tan(f*x+e))*B*a^2*c^2*d+3/f/b/(a^2+b^2)*ln(a+b*ta
n(f*x+e))*A*a^2*c*d^2-3/f/b^2/(a^2+b^2)*ln(a+b*tan(f*x+e))*C*a^3*c^2*d-3/f/
b^2/(a^2+b^2)*ln(a+b*tan(f*x+e))*B*a^3*c*d^2-3/2/f/(a^2+b^2)*ln(1+tan(f*x+e
)^2)*C*b*c*d^2+1/3/f*d^3/b*C*tan(f*x+e)^3-3/f/(a^2+b^2)*B*arctan(tan(f*x+e)
)*a*c^2*d+3/f/(a^2+b^2)*A*arctan(tan(f*x+e))*b*c^2*d-3/f/(a^2+b^2)*ln(a+b*t
an(f*x+e))*A*a*c^2*d-3/2/f/(a^2+b^2)*ln(1+tan(f*x+e)^2)*B*a*c*d^2+3/2/f/(a^
2+b^2)*ln(1+tan(f*x+e)^2)*B*b*c^2*d-3/f/(a^2+b^2)*B*arctan(tan(f*x+e))*b*c*
d^2+3/f*d/b*C*c^2*tan(f*x+e)-1/f/(a^2+b^2)*ln(a+b*tan(f*x+e))*B*a*c^3-1/2/f
/(a^2+b^2)*ln(1+tan(f*x+e)^2)*A*a*d^3-1/2/f/(a^2+b^2)*ln(1+tan(f*x+e)^2)*A*
b*c^3+1/2/f/(a^2+b^2)*ln(1+tan(f*x+e)^2)*B*a*c^3-1/2/f/(a^2+b^2)*ln(1+tan(f
*x+e)^2)*B*b*d^3+1/2/f/(a^2+b^2)*ln(1+tan(f*x+e)^2)*C*a*d^3+1/2/f/(a^2+b^2)
*ln(1+tan(f*x+e)^2)*C*b*c^3+1/f/(a^2+b^2)*A*arctan(tan(f*x+e))*a*c^3-1/f/(a
^2+b^2)*A*arctan(tan(f*x+e))*b*d^3+1/f/(a^2+b^2)*B*arctan(tan(f*x+e))*a*d^3
+1/f/(a^2+b^2)*B*arctan(tan(f*x+e))*b*c^3-1/f/(a^2+b^2)*C*arctan(tan(f*x+e)
)*a*c^3+1/f/(a^2+b^2)*C*arctan(tan(f*x+e))*b*d^3+1/f*b/(a^2+b^2)*ln(a+b*tan
(f*x+e))*A*c^3-1/2/f*d^3/b^2*C*tan(f*x+e)^2*a+3/2/f*d^2/b*C*tan(f*x+e)^2*c-
1/f*d^3/b^2*B*a*tan(f*x+e)+3/f*d^2/b*B*c*tan(f*x+e)+1/f*d^3/b^3*a^2*C*tan(f
*x+e)+1/2/f*d^3/b*B*tan(f*x+e)^2+1/f*d^3/b*A*tan(f*x+e)-1/f*d^3/b*C*tan(f*x
+e)
```

Maxima [A] time = 1.53438, size = 589, normalized size = 1.62

$$\frac{6\left(((A-C)a+Bb)c^3-3\,(Ba-(A-C)b)c^2d-3\,((A-C)a+Bb)cd^2+(Ba-(A-C)b)d^3\right)\left(fx+e\right)}{a^2+b^2}+\frac{6\left(\left(Ca^2b^3-Bab^4+Ab^5\right)c^3-3\left(Ca^3b^2-Ba^2b^3+Aab^4\right)c^2d+3\left(Ca^4b-Ba^3b^2+\cdots\right.\right.}{a^2b^4+b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e
)),x, algorithm="maxima")
```

```
[Out] 1/6*(6*(((A - C)*a + B*b)*c^3 - 3*(B*a - (A - C)*b)*c^2*d - 3*((A - C)*a +
B*b)*c*d^2 + (B*a - (A - C)*b)*d^3)*(f*x + e)/(a^2 + b^2) + 6*((C*a^2*b^3 -
 B*a*b^4 + A*b^5)*c^3 - 3*(C*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*c^2*d + 3*(C*a^
4*b - B*a^3*b^2 + A*a^2*b^3)*c*d^2 - (C*a^5 - B*a^4*b + A*a^3*b^2)*d^3)*log
(b*tan(f*x + e) + a)/(a^2*b^4 + b^6) + 3*((B*a - (A - C)*b)*c^3 + 3*((A - C
)*a + B*b)*c^2*d - 3*(B*a - (A - C)*b)*c*d^2 - ((A - C)*a + B*b)*d^3)*log(t
an(f*x + e)^2 + 1)/(a^2 + b^2) + (2*C*b^2*d^3*tan(f*x + e)^3 + 3*(3*C*b^2*c
*d^2 - (C*a*b - B*b^2)*d^3)*tan(f*x + e)^2 + 6*(3*C*b^2*c^2*d - 3*(C*a*b -
B*b^2)*c*d^2 + (C*a^2 - B*a*b + (A - C)*b^2)*d^3)*tan(f*x + e))/b^3)/f
```

Fricas [A] time = 5.57819, size = 1269, normalized size = 3.5

$$2\left(Ca^2b^3 + Cb^5\right)d^3\tan\left(fx+e\right)^3 + 6\left(\left((A-C)ab^4 + Bb^5\right)c^3 - 3\left(Bab^4 - (A-C)b^5\right)c^2d - 3\left((A-C)ab^4 + Bb^5\right)cd^2 + \left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e
)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*(C*a^2*b^3 + C*b^5)*d^3*tan(f*x + e)^3 + 6*(((A - C)*a*b^4 + B*b^5)*
c^3 - 3*(B*a*b^4 - (A - C)*b^5)*c^2*d - 3*((A - C)*a*b^4 + B*b^5)*c*d^2 + (
B*a*b^4 - (A - C)*b^5)*d^3)*f*x + 3*(3*(C*a^2*b^3 + C*b^5)*c*d^2 - (C*a^3*b
^2 - B*a^2*b^3 + C*a*b^4 - B*b^5)*d^3)*tan(f*x + e)^2 + 3*((C*a^2*b^3 - B*a
*b^4 + A*b^5)*c^3 - 3*(C*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*c^2*d + 3*(C*a^4*b
- B*a^3*b^2 + A*a^2*b^3)*c*d^2 - (C*a^5 - B*a^4*b + A*a^3*b^2)*d^3)*log((b^
2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)/(tan(f*x + e)^2 + 1)) - 3*((C*
a^2*b^3 + C*b^5)*c^3 - 3*(C*a^3*b^2 - B*a^2*b^3 + C*a*b^4 - B*b^5)*c^2*d +
3*(C*a^4*b - B*a^3*b^2 + A*a^2*b^3 - B*a*b^4 + (A - C)*b^5)*c*d^2 - (C*a^5
- B*a^4*b + A*a^3*b^2 + (A - C)*a*b^4 + B*b^5)*d^3)*log(1/(tan(f*x + e)^2 +
 1)) + 6*(3*(C*a^2*b^3 + C*b^5)*c^2*d - 3*(C*a^3*b^2 - B*a^2*b^3 + C*a*b^4
- B*b^5)*c*d^2 + (C*a^4*b - B*a^3*b^2 + A*a^2*b^3 - B*a*b^4 + (A - C)*b^5)*
d^3)*tan(f*x + e))/((a^2*b^4 + b^6)*f)
```

Sympy [A] time = 46.9782, size = 7096, normalized size = 19.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x
+e)),x)
```

```
[Out] Piecewise((zoo*x*(c + d*tan(e))**3*(A + B*tan(e) + C*tan(e)**2)/tan(e), Eq(
a, 0) & Eq(b, 0) & Eq(f, 0)), (-3*I*A*c**3*f*x*tan(e + f*x)/(-6*b*f*tan(e +
 f*x) + 6*I*b*f) - 3*A*c**3*f*x/(-6*b*f*tan(e + f*x) + 6*I*b*f) - 3*I*A*c**
3/(-6*b*f*tan(e + f*x) + 6*I*b*f) - 9*A*c**2*d*f*x*tan(e + f*x)/(-6*b*f*tan
(e + f*x) + 6*I*b*f) + 9*I*A*c**2*d*f*x/(-6*b*f*tan(e + f*x) + 6*I*b*f) + 9
*A*c**2*d/(-6*b*f*tan(e + f*x) + 6*I*b*f) - 9*I*A*c*d**2*f*x*tan(e + f*x)/(
```

```
-6*b*f*tan(e + f*x) + 6*I*b*f) - 9*A*c*d**2*f*x/(-6*b*f*tan(e + f*x) + 6*I*
b*f) - 9*A*c*d**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(-6*b*f*tan(e + f*x
) + 6*I*b*f) + 9*I*A*c*d**2*log(tan(e + f*x)**2 + 1)/(-6*b*f*tan(e + f*x) +
 6*I*b*f) + 9*I*A*c*d**2/(-6*b*f*tan(e + f*x) + 6*I*b*f) + 9*A*d**3*f*x*tan
(e + f*x)/(-6*b*f*tan(e + f*x) + 6*I*b*f) - 9*I*A*d**3*f*x/(-6*b*f*tan(e +
f*x) + 6*I*b*f) - 3*I*A*d**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(-6*b*f*
tan(e + f*x) + 6*I*b*f) - 3*A*d**3*log(tan(e + f*x)**2 + 1)/(-6*b*f*tan(e +
 f*x) + 6*I*b*f) - 6*A*d**3*tan(e + f*x)**2/(-6*b*f*tan(e + f*x) + 6*I*b*f)
 - 9*A*d**3/(-6*b*f*tan(e + f*x) + 6*I*b*f) - 3*B*c**3*f*x*tan(e + f*x)/(-6
*b*f*tan(e + f*x) + 6*I*b*f) + 3*I*B*c**3*f*x/(-6*b*f*tan(e + f*x) + 6*I*b*
f) + 3*B*c**3/(-6*b*f*tan(e + f*x) + 6*I*b*f) - 9*I*B*c**2*d*f*x*tan(e + f*
x)/(-6*b*f*tan(e + f*x) + 6*I*b*f) - 9*B*c**2*d*f*x/(-6*b*f*tan(e + f*x) +
6*I*b*f) - 9*B*c**2*d*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(-6*b*f*tan(e +
 f*x) + 6*I*b*f) + 9*I*B*c**2*d*log(tan(e + f*x)**2 + 1)/(-6*b*f*tan(e + f*
x) + 6*I*b*f) + 9*I*B*c**2*d/(-6*b*f*tan(e + f*x) + 6*I*b*f) + 27*B*c*d**2*
f*x*tan(e + f*x)/(-6*b*f*tan(e + f*x) + 6*I*b*f) - 27*I*B*c*d**2*f*x/(-6*b*
f*tan(e + f*x) + 6*I*b*f) - 9*I*B*c*d**2*log(tan(e + f*x)**2 + 1)*tan(e + f
*x)/(-6*b*f*tan(e + f*x) + 6*I*b*f) - 9*B*c*d**2*log(tan(e + f*x)**2 + 1)/(
-6*b*f*tan(e + f*x) + 6*I*b*f) - 18*B*c*d**2*tan(e + f*x)**2/(-6*b*f*tan(e
+ f*x) + 6*I*b*f) - 27*B*c*d**2/(-6*b*f*tan(e + f*x) + 6*I*b*f) + 9*I*B*d**
3*f*x*tan(e + f*x)/(-6*b*f*tan(e + f*x) + 6*I*b*f) + 9*B*d**3*f*x/(-6*b*f*t
an(e + f*x) + 6*I*b*f) + 6*B*d**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(-6
*b*f*tan(e + f*x) + 6*I*b*f) - 6*I*B*d**3*log(tan(e + f*x)**2 + 1)/(-6*b*f*
tan(e + f*x) + 6*I*b*f) - 3*B*d**3*tan(e + f*x)**3/(-6*b*f*tan(e + f*x) + 6
*I*b*f) - 3*I*B*d**3*tan(e + f*x)**2/(-6*b*f*tan(e + f*x) + 6*I*b*f) - 9*I*
B*d**3/(-6*b*f*tan(e + f*x) + 6*I*b*f) - 3*I*C*c**3*f*x*tan(e + f*x)/(-6*b*
f*tan(e + f*x) + 6*I*b*f) - 3*C*c**3*f*x/(-6*b*f*tan(e + f*x) + 6*I*b*f) -
3*C*c**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(-6*b*f*tan(e + f*x) + 6*I*b
*f) + 3*I*C*c**3*log(tan(e + f*x)**2 + 1)/(-6*b*f*tan(e + f*x) + 6*I*b*f) +
 3*I*C*c**3/(-6*b*f*tan(e + f*x) + 6*I*b*f) + 27*C*c**2*d*f*x*tan(e + f*x)/
(-6*b*f*tan(e + f*x) + 6*I*b*f) - 27*I*C*c**2*d*f*x/(-6*b*f*tan(e + f*x) +
6*I*b*f) - 9*I*C*c**2*d*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(-6*b*f*tan(e
 + f*x) + 6*I*b*f) - 9*C*c**2*d*log(tan(e + f*x)**2 + 1)/(-6*b*f*tan(e + f*
x) + 6*I*b*f) - 18*C*c**2*d*tan(e + f*x)**2/(-6*b*f*tan(e + f*x) + 6*I*b*f)
 - 27*C*c**2*d/(-6*b*f*tan(e + f*x) + 6*I*b*f) + 27*I*C*c*d**2*f*x*tan(e +
f*x)/(-6*b*f*tan(e + f*x) + 6*I*b*f) + 27*C*c*d**2*f*x/(-6*b*f*tan(e + f*x)
 + 6*I*b*f) + 18*C*c*d**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(-6*b*f*tan
(e + f*x) + 6*I*b*f) - 18*I*C*c*d**2*log(tan(e + f*x)**2 + 1)/(-6*b*f*tan(e
 + f*x) + 6*I*b*f) - 9*C*c*d**2*tan(e + f*x)**3/(-6*b*f*tan(e + f*x) + 6*I*
b*f) - 9*I*C*c*d**2*tan(e + f*x)**2/(-6*b*f*tan(e + f*x) + 6*I*b*f) - 27*I*
C*c*d**2/(-6*b*f*tan(e + f*x) + 6*I*b*f) - 15*C*d**3*f*x*tan(e + f*x)/(-6*b
*f*tan(e + f*x) + 6*I*b*f) + 15*I*C*d**3*f*x/(-6*b*f*tan(e + f*x) + 6*I*b*f
) + 6*I*C*d**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(-6*b*f*tan(e + f*x) +
 6*I*b*f) + 6*C*d**3*log(tan(e + f*x)**2 + 1)/(-6*b*f*tan(e + f*x) + 6*I*b*
f) - 2*C*d**3*tan(e + f*x)**4/(-6*b*f*tan(e + f*x) + 6*I*b*f) - I*C*d**3*ta
n(e + f*x)**3/(-6*b*f*tan(e + f*x) + 6*I*b*f) + 9*C*d**3*tan(e + f*x)**2/(-
6*b*f*tan(e + f*x) + 6*I*b*f) + 15*C*d**3/(-6*b*f*tan(e + f*x) + 6*I*b*f),
Eq(a, -I*b)), (-3*I*A*c**3*f*x*tan(e + f*x)/(6*b*f*tan(e + f*x) + 6*I*b*f)
+ 3*A*c**3*f*x/(6*b*f*tan(e + f*x) + 6*I*b*f) - 3*I*A*c**3/(6*b*f*tan(e + f
*x) + 6*I*b*f) + 9*A*c**2*d*f*x*tan(e + f*x)/(6*b*f*tan(e + f*x) + 6*I*b*f)
 + 9*I*A*c**2*d*f*x/(6*b*f*tan(e + f*x) + 6*I*b*f) - 9*A*c**2*d/(6*b*f*tan(
e + f*x) + 6*I*b*f) - 9*I*A*c*d**2*f*x*tan(e + f*x)/(6*b*f*tan(e + f*x) + 6
*I*b*f) + 9*A*c*d**2*f*x/(6*b*f*tan(e + f*x) + 6*I*b*f) + 9*A*c*d**2*log(ta
n(e + f*x)**2 + 1)*tan(e + f*x)/(6*b*f*tan(e + f*x) + 6*I*b*f) + 9*I*A*c*d*
*2*log(tan(e + f*x)**2 + 1)/(6*b*f*tan(e + f*x) + 6*I*b*f) + 9*I*A*c*d**2/(
6*b*f*tan(e + f*x) + 6*I*b*f) - 9*A*d**3*f*x*tan(e + f*x)/(6*b*f*tan(e + f*
x) + 6*I*b*f) - 9*I*A*d**3*f*x/(6*b*f*tan(e + f*x) + 6*I*b*f) - 3*I*A*d**3*
log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(6*b*f*tan(e + f*x) + 6*I*b*f) + 3*A*
d**3*log(tan(e + f*x)**2 + 1)/(6*b*f*tan(e + f*x) + 6*I*b*f) + 6*A*d**3*tan
```

```
(e + f*x)**2/(6*b*f*tan(e + f*x) + 6*I*b*f) + 9*A*d**3/(6*b*f*tan(e + f*x)
+ 6*I*b*f) + 3*B*c**3*f*x*tan(e + f*x)/(6*b*f*tan(e + f*x) + 6*I*b*f) + 3*I
*B*c**3*f*x/(6*b*f*tan(e + f*x) + 6*I*b*f) - 3*B*c**3/(6*b*f*tan(e + f*x) +
 6*I*b*f) - 9*I*B*c**2*d*f*x*tan(e + f*x)/(6*b*f*tan(e + f*x) + 6*I*b*f) +
9*B*c**2*d*f*x/(6*b*f*tan(e + f*x) + 6*I*b*f) + 9*B*c**2*d*log(tan(e + f*x)
**2 + 1)*tan(e + f*x)/(6*b*f*tan(e + f*x) + 6*I*b*f) + 9*I*B*c**2*d*log(tan
(e + f*x)**2 + 1)/(6*b*f*tan(e + f*x) + 6*I*b*f) + 9*I*B*c**2*d/(6*b*f*tan(
e + f*x) + 6*I*b*f) - 27*B*c*d**2*f*x*tan(e + f*x)/(6*b*f*tan(e + f*x) + 6*
I*b*f) - 27*I*B*c*d**2*f*x/(6*b*f*tan(e + f*x) + 6*I*b*f) - 9*I*B*c*d**2*lo
g(tan(e + f*x)**2 + 1)*tan(e + f*x)/(6*b*f*tan(e + f*x) + 6*I*b*f) + 9*B*c*
d**2*log(tan(e + f*x)**2 + 1)/(6*b*f*tan(e + f*x) + 6*I*b*f) + 18*B*c*d**2*
tan(e + f*x)**2/(6*b*f*tan(e + f*x) + 6*I*b*f) + 27*B*c*d**2/(6*b*f*tan(e +
 f*x) + 6*I*b*f) + 9*I*B*d**3*f*x*tan(e + f*x)/(6*b*f*tan(e + f*x) + 6*I*b*
f) - 9*B*d**3*f*x/(6*b*f*tan(e + f*x) + 6*I*b*f) - 6*B*d**3*log(tan(e + f*x
)**2 + 1)*tan(e + f*x)/(6*b*f*tan(e + f*x) + 6*I*b*f) - 6*I*B*d**3*log(tan(
e + f*x)**2 + 1)/(6*b*f*tan(e + f*x) + 6*I*b*f) + 3*B*d**3*tan(e + f*x)**3/
(6*b*f*tan(e + f*x) + 6*I*b*f) - 3*I*B*d**3*tan(e + f*x)**2/(6*b*f*tan(e +
f*x) + 6*I*b*f) - 9*I*B*d**3/(6*b*f*tan(e + f*x) + 6*I*b*f) - 3*I*C*c**3*f*
x*tan(e + f*x)/(6*b*f*tan(e + f*x) + 6*I*b*f) + 3*C*c**3*f*x/(6*b*f*tan(e +
 f*x) + 6*I*b*f) + 3*C*c**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(6*b*f*ta
n(e + f*x) + 6*I*b*f) + 3*I*C*c**3*log(tan(e + f*x)**2 + 1)/(6*b*f*tan(e +
f*x) + 6*I*b*f) + 3*I*C*c**3/(6*b*f*tan(e + f*x) + 6*I*b*f) - 27*C*c**2*d*f
*x*tan(e + f*x)/(6*b*f*tan(e + f*x) + 6*I*b*f) - 27*I*C*c**2*d*f*x/(6*b*f*t
an(e + f*x) + 6*I*b*f) - 9*I*C*c**2*d*log(tan(e + f*x)**2 + 1)*tan(e + f*x)
/(6*b*f*tan(e + f*x) + 6*I*b*f) + 9*C*c**2*d*log(tan(e + f*x)**2 + 1)/(6*b*
f*tan(e + f*x) + 6*I*b*f) + 18*C*c**2*d*tan(e + f*x)**2/(6*b*f*tan(e + f*x)
 + 6*I*b*f) + 27*C*c**2*d/(6*b*f*tan(e + f*x) + 6*I*b*f) + 27*I*C*c*d**2*f*
x*tan(e + f*x)/(6*b*f*tan(e + f*x) + 6*I*b*f) - 27*C*c*d**2*f*x/(6*b*f*tan(
e + f*x) + 6*I*b*f) - 18*C*c*d**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(6*
b*f*tan(e + f*x) + 6*I*b*f) - 18*I*C*c*d**2*log(tan(e + f*x)**2 + 1)/(6*b*f
*tan(e + f*x) + 6*I*b*f) + 9*C*c*d**2*tan(e + f*x)**3/(6*b*f*tan(e + f*x) +
 6*I*b*f) - 9*I*C*c*d**2*tan(e + f*x)**2/(6*b*f*tan(e + f*x) + 6*I*b*f) - 2
7*I*C*c*d**2/(6*b*f*tan(e + f*x) + 6*I*b*f) + 15*C*d**3*f*x*tan(e + f*x)/(6
*b*f*tan(e + f*x) + 6*I*b*f) + 15*I*C*d**3*f*x/(6*b*f*tan(e + f*x) + 6*I*b*
f) + 6*I*C*d**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(6*b*f*tan(e + f*x) +
 6*I*b*f) - 6*C*d**3*log(tan(e + f*x)**2 + 1)/(6*b*f*tan(e + f*x) + 6*I*b*f
) + 2*C*d**3*tan(e + f*x)**4/(6*b*f*tan(e + f*x) + 6*I*b*f) - I*C*d**3*tan(
e + f*x)**3/(6*b*f*tan(e + f*x) + 6*I*b*f) - 9*C*d**3*tan(e + f*x)**2/(6*b*
f*tan(e + f*x) + 6*I*b*f) - 15*C*d**3/(6*b*f*tan(e + f*x) + 6*I*b*f), Eq(a,
 I*b)), ((A*c**3*x + 3*A*c**2*d*log(tan(e + f*x)**2 + 1)/(2*f) - 3*A*c*d**2
*x + 3*A*c*d**2*tan(e + f*x)/f - A*d**3*log(tan(e + f*x)**2 + 1)/(2*f) + A*
d**3*tan(e + f*x)**2/(2*f) + B*c**3*log(tan(e + f*x)**2 + 1)/(2*f) - 3*B*c*
*2*d*x + 3*B*c**2*d*tan(e + f*x)/f - 3*B*c*d**2*log(tan(e + f*x)**2 + 1)/(2
*f) + 3*B*c*d**2*tan(e + f*x)**2/(2*f) + B*d**3*x + B*d**3*tan(e + f*x)**3/
(3*f) - B*d**3*tan(e + f*x)/f - C*c**3*x + C*c**3*tan(e + f*x)/f - 3*C*c**2
*d*log(tan(e + f*x)**2 + 1)/(2*f) + 3*C*c**2*d*tan(e + f*x)**2/(2*f) + 3*C*
c*d**2*x + C*c*d**2*tan(e + f*x)**3/f - 3*C*c*d**2*tan(e + f*x)/f + C*d**3*
log(tan(e + f*x)**2 + 1)/(2*f) + C*d**3*tan(e + f*x)**4/(4*f) - C*d**3*tan(
e + f*x)**2/(2*f))/a, Eq(b, 0)), (x*(c + d*tan(e))**3*(A + B*tan(e) + C*tan
(e)**2)/(a + b*tan(e)), Eq(f, 0)), (-6*A*a**3*b**2*d**3*log(a/b + tan(e + f
*x))/(6*a**2*b**4*f + 6*b**6*f) + 18*A*a**2*b**3*c*d**2*log(a/b + tan(e + f
*x))/(6*a**2*b**4*f + 6*b**6*f) + 6*A*a**2*b**3*d**3*tan(e + f*x)/(6*a**2*b
**4*f + 6*b**6*f) + 6*A*a*b**4*c**3*f*x/(6*a**2*b**4*f + 6*b**6*f) - 18*A*a
*b**4*c**2*d*log(a/b + tan(e + f*x))/(6*a**2*b**4*f + 6*b**6*f) + 9*A*a*b**
4*c**2*d*log(tan(e + f*x)**2 + 1)/(6*a**2*b**4*f + 6*b**6*f) - 18*A*a*b**4*
c*d**2*f*x/(6*a**2*b**4*f + 6*b**6*f) - 3*A*a*b**4*d**3*log(tan(e + f*x)**2
 + 1)/(6*a**2*b**4*f + 6*b**6*f) + 6*A*b**5*c**3*log(a/b + tan(e + f*x))/(6
*a**2*b**4*f + 6*b**6*f) - 3*A*b**5*c**3*log(tan(e + f*x)**2 + 1)/(6*a**2*b
**4*f + 6*b**6*f) + 18*A*b**5*c**2*d*f*x/(6*a**2*b**4*f + 6*b**6*f) + 9*A*b
```

```
**5*c*d**2*log(tan(e + f*x)**2 + 1)/(6*a**2*b**4*f + 6*b**6*f) - 6*A*b**5*d
**3*f*x/(6*a**2*b**4*f + 6*b**6*f) + 6*A*b**5*d**3*tan(e + f*x)/(6*a**2*b**
4*f + 6*b**6*f) + 6*B*a**4*b*d**3*log(a/b + tan(e + f*x))/(6*a**2*b**4*f +
6*b**6*f) - 18*B*a**3*b**2*c*d**2*log(a/b + tan(e + f*x))/(6*a**2*b**4*f +
6*b**6*f) - 6*B*a**3*b**2*d**3*tan(e + f*x)/(6*a**2*b**4*f + 6*b**6*f) + 18
*B*a**2*b**3*c**2*d*log(a/b + tan(e + f*x))/(6*a**2*b**4*f + 6*b**6*f) + 18
*B*a**2*b**3*c*d**2*tan(e + f*x)/(6*a**2*b**4*f + 6*b**6*f) + 3*B*a**2*b**3
*d**3*tan(e + f*x)**2/(6*a**2*b**4*f + 6*b**6*f) - 6*B*a*b**4*c**3*log(a/b
+ tan(e + f*x))/(6*a**2*b**4*f + 6*b**6*f) + 3*B*a*b**4*c**3*log(tan(e + f*
x)**2 + 1)/(6*a**2*b**4*f + 6*b**6*f) - 18*B*a*b**4*c**2*d*f*x/(6*a**2*b**4
*f + 6*b**6*f) - 9*B*a*b**4*c*d**2*log(tan(e + f*x)**2 + 1)/(6*a**2*b**4*f
+ 6*b**6*f) + 6*B*a*b**4*d**3*f*x/(6*a**2*b**4*f + 6*b**6*f) - 6*B*a*b**4*d
**3*tan(e + f*x)/(6*a**2*b**4*f + 6*b**6*f) + 6*B*b**5*c**3*f*x/(6*a**2*b**
4*f + 6*b**6*f) + 9*B*b**5*c**2*d*log(tan(e + f*x)**2 + 1)/(6*a**2*b**4*f +
 6*b**6*f) - 18*B*b**5*c*d**2*f*x/(6*a**2*b**4*f + 6*b**6*f) + 18*B*b**5*c*
d**2*tan(e + f*x)/(6*a**2*b**4*f + 6*b**6*f) - 3*B*b**5*d**3*log(tan(e + f*
x)**2 + 1)/(6*a**2*b**4*f + 6*b**6*f) + 3*B*b**5*d**3*tan(e + f*x)**2/(6*a*
*2*b**4*f + 6*b**6*f) - 6*C*a**5*d**3*log(a/b + tan(e + f*x))/(6*a**2*b**4*
f + 6*b**6*f) + 18*C*a**4*b*c*d**2*log(a/b + tan(e + f*x))/(6*a**2*b**4*f +
 6*b**6*f) + 6*C*a**4*b*d**3*tan(e + f*x)/(6*a**2*b**4*f + 6*b**6*f) - 18*C
*a**3*b**2*c**2*d*log(a/b + tan(e + f*x))/(6*a**2*b**4*f + 6*b**6*f) - 18*C
*a**3*b**2*c*d**2*tan(e + f*x)/(6*a**2*b**4*f + 6*b**6*f) - 3*C*a**3*b**2*d
**3*tan(e + f*x)**2/(6*a**2*b**4*f + 6*b**6*f) + 6*C*a**2*b**3*c**3*log(a/b
 + tan(e + f*x))/(6*a**2*b**4*f + 6*b**6*f) + 18*C*a**2*b**3*c**2*d*tan(e +
 f*x)/(6*a**2*b**4*f + 6*b**6*f) + 9*C*a**2*b**3*c*d**2*tan(e + f*x)**2/(6*
a**2*b**4*f + 6*b**6*f) + 2*C*a**2*b**3*d**3*tan(e + f*x)**3/(6*a**2*b**4*f
 + 6*b**6*f) - 6*C*a*b**4*c**3*f*x/(6*a**2*b**4*f + 6*b**6*f) - 9*C*a*b**4*
c**2*d*log(tan(e + f*x)**2 + 1)/(6*a**2*b**4*f + 6*b**6*f) + 18*C*a*b**4*c*
d**2*f*x/(6*a**2*b**4*f + 6*b**6*f) - 18*C*a*b**4*c*d**2*tan(e + f*x)/(6*a*
*2*b**4*f + 6*b**6*f) + 3*C*a*b**4*d**3*log(tan(e + f*x)**2 + 1)/(6*a**2*b*
*4*f + 6*b**6*f) - 3*C*a*b**4*d**3*tan(e + f*x)**2/(6*a**2*b**4*f + 6*b**6*
f) + 3*C*b**5*c**3*log(tan(e + f*x)**2 + 1)/(6*a**2*b**4*f + 6*b**6*f) - 18
*C*b**5*c**2*d*f*x/(6*a**2*b**4*f + 6*b**6*f) + 18*C*b**5*c**2*d*tan(e + f*
x)/(6*a**2*b**4*f + 6*b**6*f) - 9*C*b**5*c*d**2*log(tan(e + f*x)**2 + 1)/(6
*a**2*b**4*f + 6*b**6*f) + 9*C*b**5*c*d**2*tan(e + f*x)**2/(6*a**2*b**4*f +
 6*b**6*f) + 6*C*b**5*d**3*f*x/(6*a**2*b**4*f + 6*b**6*f) + 2*C*b**5*d**3*t
an(e + f*x)**3/(6*a**2*b**4*f + 6*b**6*f) - 6*C*b**5*d**3*tan(e + f*x)/(6*a
**2*b**4*f + 6*b**6*f), True))
```

Giac [A] time = 2.25161, size = 774, normalized size = 2.13

$$\frac{\frac{6\left(Aac^3-Cac^3+Bbc^3-3\,Bac^2d+3\,Abc^2d-3\,Cbc^2d-3\,Aacd^2+3\,Cacd^2-3\,Bbcd^2+Bad^3-Abd^3+Cbd^3\right)\left(fx+e\right)}{a^2+b^2}+\frac{3\left(Bac^3-Abc^3+Cbc^3+3\,Aac^2d-3\,Cac^2d+3\,Bbc^2d-3\,B\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e
)),x, algorithm="giac")
```

```
[Out] 1/6*(6*(A*a*c^3 - C*a*c^3 + B*b*c^3 - 3*B*a*c^2*d + 3*A*b*c^2*d - 3*C*b*c^2
*d - 3*A*a*c*d^2 + 3*C*a*c*d^2 - 3*B*b*c*d^2 + B*a*d^3 - A*b*d^3 + C*b*d^3)
*(f*x + e)/(a^2 + b^2) + 3*(B*a*c^3 - A*b*c^3 + C*b*c^3 + 3*A*a*c^2*d - 3*C
*a*c^2*d + 3*B*b*c^2*d - 3*B*a*c*d^2 + 3*A*b*c*d^2 - 3*C*b*c*d^2 - A*a*d^3
+ C*a*d^3 - B*b*d^3)*log(tan(f*x + e)^2 + 1)/(a^2 + b^2) + 6*(C*a^2*b^3*c^3
 - B*a*b^4*c^3 + A*b^5*c^3 - 3*C*a^3*b^2*c^2*d + 3*B*a^2*b^3*c^2*d - 3*A*a*
b^4*c^2*d + 3*C*a^4*b*c*d^2 - 3*B*a^3*b^2*c*d^2 + 3*A*a^2*b^3*c*d^2 - C*a^5
```

```
*d^3 + B*a^4*b*d^3 - A*a^3*b^2*d^3)*log(abs(b*tan(f*x + e) + a))/(a^2*b^4 +
 b^6) + (2*C*b^2*d^3*tan(f*x + e)^3 + 9*C*b^2*c*d^2*tan(f*x + e)^2 - 3*C*a*
b*d^3*tan(f*x + e)^2 + 3*B*b^2*d^3*tan(f*x + e)^2 + 18*C*b^2*c^2*d*tan(f*x
+ e) - 18*C*a*b*c*d^2*tan(f*x + e) + 18*B*b^2*c*d^2*tan(f*x + e) + 6*C*a^2*
d^3*tan(f*x + e) - 6*B*a*b*d^3*tan(f*x + e) + 6*A*b^2*d^3*tan(f*x + e) - 6*
C*b^2*d^3*tan(f*x + e))/b^3)/f
```

3.68 $\int \frac{(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=574

$$\frac{\log(\cos(e+fx))\left(a^2\left(-\left(d(A-C)\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)\right)+2ab\left(Ac^3-3Acd^2-3Bc^2d+Bd^3-c^3C+3cCd^2\right)+b^2\left(\right.}{f\left(a^2+b^2\right)^2}$$

```
[Out] -(((b^2*(A*c^3 - c^3*C - 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 + B*d^3) + a^2*(
c^3*C + 3*B*c^2*d - 3*c*C*d^2 - B*d^3 - A*(c^3 - 3*c*d^2)) - 2*a*b*((A - C)
*d*(3*c^2 - d^2) + B*(c^3 - 3*c*d^2)))*x)/(a^2 + b^2)^2) + ((2*a*b*(A*c^3 -
 c^3*C - 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 + B*d^3) - a^2*((A - C)*d*(3*c^2
 - d^2) + B*(c^3 - 3*c*d^2)) + b^2*((A - C)*d*(3*c^2 - d^2) + B*(c^3 - 3*c*
d^2)))*Log[Cos[e + f*x]])/((a^2 + b^2)^2*f) - ((b*c - a*d)^2*(2*a^3*b*B*d -
 3*a^4*C*d - b^4*(B*c + 3*A*d) - 2*a*b^3*(A*c - c*C - 2*B*d) + a^2*b^2*(B*c
 - (A + 5*C)*d))*Log[a + b*Tan[e + f*x]])/(b^4*(a^2 + b^2)^2*f) - (d^2*(3*a
^3*C*d - A*b^2*(b*c - a*d) - b^3*(2*c*C + B*d) - a^2*b*(3*c*C + 2*B*d) + a*
b^2*(B*c + 2*C*d))*Tan[e + f*x])/(b^3*(a^2 + b^2)*f) + ((2*A*b^2 - 2*a*b*B
+ 3*a^2*C + b^2*C)*d*(c + d*Tan[e + f*x])^2)/(2*b^2*(a^2 + b^2)*f) - ((A*b^
2 - a*(b*B - a*C))*(c + d*Tan[e + f*x])^3)/(b*(a^2 + b^2)*f*(a + b*Tan[e +
f*x]))
```

Rubi [A] time = 2.32142, antiderivative size = 574, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3645, 3647, 3637, 3626, 3617, 31, 3475}

$$\frac{\log(\cos(e+fx))\left(a^2\left(-\left(d(A-C)\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)\right)+2ab\left(Ac^3-3Acd^2-3Bc^2d+Bd^3-c^3C+3cCd^2\right)+b^2\left(\right.}{f\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a + b
*Tan[e + f*x])^2,x]
```

```
[Out] -(((b^2*(A*c^3 - c^3*C - 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 + B*d^3) + a^2*(
c^3*C + 3*B*c^2*d - 3*c*C*d^2 - B*d^3 - A*(c^3 - 3*c*d^2)) - 2*a*b*((A - C)
*d*(3*c^2 - d^2) + B*(c^3 - 3*c*d^2)))*x)/(a^2 + b^2)^2) + ((2*a*b*(A*c^3 -
 c^3*C - 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 + B*d^3) - a^2*((A - C)*d*(3*c^2
 - d^2) + B*(c^3 - 3*c*d^2)) + b^2*((A - C)*d*(3*c^2 - d^2) + B*(c^3 - 3*c*
d^2)))*Log[Cos[e + f*x]])/((a^2 + b^2)^2*f) - ((b*c - a*d)^2*(2*a^3*b*B*d -
 3*a^4*C*d - b^4*(B*c + 3*A*d) - 2*a*b^3*(A*c - c*C - 2*B*d) + a^2*b^2*(B*c
 - (A + 5*C)*d))*Log[a + b*Tan[e + f*x]])/(b^4*(a^2 + b^2)^2*f) - (d^2*(3*a
^3*C*d - A*b^2*(b*c - a*d) - b^3*(2*c*C + B*d) - a^2*b*(3*c*C + 2*B*d) + a*
b^2*(B*c + 2*C*d))*Tan[e + f*x])/(b^3*(a^2 + b^2)*f) + ((2*A*b^2 - 2*a*b*B
+ 3*a^2*C + b^2*C)*d*(c + d*Tan[e + f*x])^2)/(2*b^2*(a^2 + b^2)*f) - ((A*b^
2 - a*(b*B - a*C))*(c + d*Tan[e + f*x])^3)/(b*(a^2 + b^2)*f*(a + b*Tan[e +
f*x]))
```

Rule 3645

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*d^2 + c*(c*C - B*d))*(a + b*Tan[e
+ f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
```

```
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3626

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((a*A + b*B -
a*C)*x)/(a^2 + b^2), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3617

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^3}{b\left(a^2+b^2\right)f(a+b\tan(e+fx))} + \int \frac{(c+d\,\mathrm{ta}}{}$$

$$= \frac{\left(2Ab^2-2abB+3a^2C+b^2C\right)d(c+d\tan(e+fx))^2}{2b^2\left(a^2+b^2\right)f}$$

$$= -\frac{d^2\left(3a^3Cd-Ab^2(bc-ad)-b^3(2cC+Bd)-a^2b(3c\right.}{b^3\left(a^2+b^2\right)}$$

$$= -\frac{\left(b^2\left(Ac^3-c^3C-3Bc^2d-3Acd^2+3cCd^2+Bd^3\right)+\right.}{}$$

$$= -\frac{\left(b^2\left(Ac^3-c^3C-3Bc^2d-3Acd^2+3cCd^2+Bd^3\right)+\right.}{}$$

$$= -\frac{\left(b^2\left(Ac^3-c^3C-3Bc^2d-3Acd^2+3cCd^2+Bd^3\right)+\right.}{}$$

Mathematica [C] time = 8.36127, size = 2467, normalized size = 4.3

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(a + b*Tan[e + f*x])^2,x]
```

```
[Out] ((a^2*A*c^3 - A*b^2*c^3 + 2*a*b*B*c^3 - a^2*c^3*C + b^2*c^3*C + 6*a*A*b*c^2
*d - 3*a^2*B*c^2*d + 3*b^2*B*c^2*d - 6*a*b*c^2*C*d - 3*a^2*A*c*d^2 + 3*A*b^
2*c*d^2 - 6*a*b*B*c*d^2 + 3*a^2*c*C*d^2 - 3*b^2*c*C*d^2 - 2*a*A*b*d^3 + a^2
*B*d^3 - b^2*B*d^3 + 2*a*b*C*d^3)*(e + f*x)*Cos[e + f*x]*(a*Cos[e + f*x] +
b*Sin[e + f*x])^2*(c + d*Tan[e + f*x])^3)/((a - I*b)^2*(a + I*b)^2*f*(c*Cos
[e + f*x] + d*Sin[e + f*x])^3*(a + b*Tan[e + f*x])^2) - (I*(-2*a^6*A*b^8*c^
3 + (2*I)*a^5*A*b^9*c^3 - 2*a^4*A*b^10*c^3 + (2*I)*a^3*A*b^11*c^3 + a^7*b^7
*B*c^3 - I*a^6*b^8*B*c^3 - a^3*b^11*B*c^3 + I*a^2*b^12*B*c^3 + 2*a^6*b^8*c^
3*C - (2*I)*a^5*b^9*c^3*C + 2*a^4*b^10*c^3*C - (2*I)*a^3*b^11*c^3*C + 3*a^7
*A*b^7*c^2*d - (3*I)*a^6*A*b^8*c^2*d - 3*a^3*A*b^11*c^2*d + (3*I)*a^2*A*b^1
2*c^2*d + 6*a^6*b^8*B*c^2*d - (6*I)*a^5*b^9*B*c^2*d + 6*a^4*b^10*B*c^2*d -
(6*I)*a^3*b^11*B*c^2*d - 3*a^9*b^5*c^2*C*d + (3*I)*a^8*b^6*c^2*C*d - 12*a^7
*b^7*c^2*C*d + (12*I)*a^6*b^8*c^2*C*d - 9*a^5*b^9*c^2*C*d + (9*I)*a^4*b^10*
c^2*C*d + 6*a^6*A*b^8*c*d^2 - (6*I)*a^5*A*b^9*c*d^2 + 6*a^4*A*b^10*c*d^2 -
(6*I)*a^3*A*b^11*c*d^2 - 3*a^9*b^5*B*c*d^2 + (3*I)*a^8*b^6*B*c*d^2 - 12*a^7
*b^7*B*c*d^2 + (12*I)*a^6*b^8*B*c*d^2 - 9*a^5*b^9*B*c*d^2 + (9*I)*a^4*b^10*
B*c*d^2 + 6*a^10*b^4*c*C*d^2 - (6*I)*a^9*b^5*c*C*d^2 + 18*a^8*b^6*c*C*d^2 -
 (18*I)*a^7*b^7*c*C*d^2 + 12*a^6*b^8*c*C*d^2 - (12*I)*a^5*b^9*c*C*d^2 - a^9
*A*b^5*d^3 + I*a^8*A*b^6*d^3 - 4*a^7*A*b^7*d^3 + (4*I)*a^6*A*b^8*d^3 - 3*a^
5*A*b^9*d^3 + (3*I)*a^4*A*b^10*d^3 + 2*a^10*b^4*B*d^3 - (2*I)*a^9*b^5*B*d^3
 + 6*a^8*b^6*B*d^3 - (6*I)*a^7*b^7*B*d^3 + 4*a^6*b^8*B*d^3 - (4*I)*a^5*b^9*
B*d^3 - 3*a^11*b^3*C*d^3 + (3*I)*a^10*b^4*C*d^3 - 8*a^9*b^5*C*d^3 + (8*I)*a
^8*b^6*C*d^3 - 5*a^7*b^7*C*d^3 + (5*I)*a^6*b^8*C*d^3)*(e + f*x)*Cos[e + f*x
]*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(c + d*Tan[e + f*x])^3)/(a^2*(a - I*b
)^4*(a + I*b)^3*b^7*f*(c*Cos[e + f*x] + d*Sin[e + f*x])^3*(a + b*Tan[e + f*
x])^2) - (I*(2*a*A*b^5*c^3 - a^2*b^4*B*c^3 + b^6*B*c^3 - 2*a*b^5*c^3*C - 3*
a^2*A*b^4*c^2*d + 3*A*b^6*c^2*d - 6*a*b^5*B*c^2*d + 3*a^4*b^2*c^2*C*d + 9*a
```

```
^2*b^4*c^2*C*d - 6*a*A*b^5*c*d^2 + 3*a^4*b^2*B*c*d^2 + 9*a^2*b^4*B*c*d^2 -
6*a^5*b*c*C*d^2 - 12*a^3*b^3*c*C*d^2 + a^4*A*b^2*d^3 + 3*a^2*A*b^4*d^3 - 2*
a^5*b*B*d^3 - 4*a^3*b^3*B*d^3 + 3*a^6*C*d^3 + 5*a^4*b^2*C*d^3)*ArcTan[Tan[e
 + f*x]]*Cos[e + f*x]*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(c + d*Tan[e + f*
x])^3)/(b^4*(a^2 + b^2)^2*f*(c*Cos[e + f*x] + d*Sin[e + f*x])^3*(a + b*Tan[
e + f*x])^2) + ((-3*b^2*c^2*C*d - 3*b^2*B*c*d^2 + 6*a*b*c*C*d^2 - A*b^2*d^3
 + 2*a*b*B*d^3 - 3*a^2*C*d^3 + b^2*C*d^3)*Cos[e + f*x]*Log[Cos[e + f*x]]*(a
*Cos[e + f*x] + b*Sin[e + f*x])^2*(c + d*Tan[e + f*x])^3)/(b^4*f*(c*Cos[e +
 f*x] + d*Sin[e + f*x])^3*(a + b*Tan[e + f*x])^2) + ((2*a*A*b^5*c^3 - a^2*b
^4*B*c^3 + b^6*B*c^3 - 2*a*b^5*c^3*C - 3*a^2*A*b^4*c^2*d + 3*A*b^6*c^2*d -
6*a*b^5*B*c^2*d + 3*a^4*b^2*c^2*C*d + 9*a^2*b^4*c^2*C*d - 6*a*A*b^5*c*d^2 +
 3*a^4*b^2*B*c*d^2 + 9*a^2*b^4*B*c*d^2 - 6*a^5*b*c*C*d^2 - 12*a^3*b^3*c*C*d
^2 + a^4*A*b^2*d^3 + 3*a^2*A*b^4*d^3 - 2*a^5*b*B*d^3 - 4*a^3*b^3*B*d^3 + 3*
a^6*C*d^3 + 5*a^4*b^2*C*d^3)*Cos[e + f*x]*Log[(a*Cos[e + f*x] + b*Sin[e + f
*x])^2]*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(c + d*Tan[e + f*x])^3)/(2*b^4*
(a^2 + b^2)^2*f*(c*Cos[e + f*x] + d*Sin[e + f*x])^3*(a + b*Tan[e + f*x])^2)
 + (C*d^3*Sec[e + f*x]*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(c + d*Tan[e + f
*x])^3)/(2*b^2*f*(c*Cos[e + f*x] + d*Sin[e + f*x])^3*(a + b*Tan[e + f*x])^2
) + ((a*Cos[e + f*x] + b*Sin[e + f*x])^2*(3*b*c*C*d^2*Sin[e + f*x] + b*B*d^
3*Sin[e + f*x] - 2*a*C*d^3*Sin[e + f*x])*(c + d*Tan[e + f*x])^3)/(b^3*f*(c*
Cos[e + f*x] + d*Sin[e + f*x])^3*(a + b*Tan[e + f*x])^2) + (Cos[e + f*x]*(a
*Cos[e + f*x] + b*Sin[e + f*x])*(A*b^5*c^3*Sin[e + f*x] - a*b^4*B*c^3*Sin[e
 + f*x] + a^2*b^3*c^3*C*Sin[e + f*x] - 3*a*A*b^4*c^2*d*Sin[e + f*x] + 3*a^2
*b^3*B*c^2*d*Sin[e + f*x] - 3*a^3*b^2*c^2*C*d*Sin[e + f*x] + 3*a^2*A*b^3*c*
d^2*Sin[e + f*x] - 3*a^3*b^2*B*c*d^2*Sin[e + f*x] + 3*a^4*b*c*C*d^2*Sin[e +
 f*x] - a^3*A*b^2*d^3*Sin[e + f*x] + a^4*b*B*d^3*Sin[e + f*x] - a^5*C*d^3*S
in[e + f*x])*(c + d*Tan[e + f*x])^3)/(a*(a - I*b)*(a + I*b)*b^3*f*(c*Cos[e
+ f*x] + d*Sin[e + f*x])^3*(a + b*Tan[e + f*x])^2)
```

Maple [B] time = 0.07, size = 2250, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2,x
)
```

```
[Out] -6/f/(a^2+b^2)^2*C*arctan(tan(f*x+e))*a*b*c^2*d+6/f/(a^2+b^2)^2*A*arctan(ta
n(f*x+e))*a*b*c^2*d-6/f/b^3/(a^2+b^2)^2*ln(a+b*tan(f*x+e))*C*a^5*c*d^2-6/f*
b/(a^2+b^2)^2*ln(a+b*tan(f*x+e))*B*a*c^2*d-6/f/(a^2+b^2)^2*B*arctan(tan(f*x
+e))*a*b*c*d^2+3/f/(a^2+b^2)^2*ln(1+tan(f*x+e)^2)*A*a*b*c*d^2-3/f/b/(a^2+b^
2)/(a+b*tan(f*x+e))*A*a^2*c*d^2-3/f/(a^2+b^2)^2*ln(1+tan(f*x+e)^2)*C*a*b*c*
d^2+3/f/(a^2+b^2)^2*ln(1+tan(f*x+e)^2)*B*a*b*c^2*d+3/f/b^2/(a^2+b^2)^2*ln(a
+b*tan(f*x+e))*C*a^4*c^2*d-12/f/b/(a^2+b^2)^2*ln(a+b*tan(f*x+e))*C*a^3*c*d^
2-3/f/b^3/(a^2+b^2)/(a+b*tan(f*x+e))*C*a^4*c*d^2-3/f/b/(a^2+b^2)/(a+b*tan(f
*x+e))*B*a^2*c^2*d+3/f/b^2/(a^2+b^2)^2*ln(a+b*tan(f*x+e))*B*a^4*c*d^2+3/f/b
^2/(a^2+b^2)/(a+b*tan(f*x+e))*B*a^3*c*d^2+3/f/b^2/(a^2+b^2)/(a+b*tan(f*x+e)
)*C*a^3*c^2*d-6/f*b/(a^2+b^2)^2*ln(a+b*tan(f*x+e))*A*a*c*d^2-2/f/(a^2+b^2)^
2*A*arctan(tan(f*x+e))*a*b*d^3+3/f/(a^2+b^2)^2*A*arctan(tan(f*x+e))*b^2*c*d
^2+3/f/b^4/(a^2+b^2)^2*ln(a+b*tan(f*x+e))*C*a^6*d^3+5/f/b^2/(a^2+b^2)^2*ln(
a+b*tan(f*x+e))*C*a^4*d^3-2/f*b/(a^2+b^2)^2*ln(a+b*tan(f*x+e))*C*a*c^3+3/f/
(a^2+b^2)^2*B*arctan(tan(f*x+e))*b^2*c^2*d-3/f/(a^2+b^2)^2*B*arctan(tan(f*x
+e))*a^2*c^2*d+2/f/(a^2+b^2)^2*B*arctan(tan(f*x+e))*a*b*c^3+1/f/b^2/(a^2+b^
2)^2*ln(a+b*tan(f*x+e))*A*a^4*d^3+3/f/(a^2+b^2)^2*C*arctan(tan(f*x+e))*a^2*
c*d^2+2/f/(a^2+b^2)^2*C*arctan(tan(f*x+e))*a*b*d^3-2/f/b^3/(a^2+b^2)^2*ln(a
+b*tan(f*x+e))*B*a^5*d^3-4/f/b/(a^2+b^2)^2*ln(a+b*tan(f*x+e))*B*a^3*d^3+3/f
```

```
/(a^2+b^2)^2*ln(a+b*tan(f*x+e))*A*a^2*d^3-1/f/(a^2+b^2)^2*ln(a+b*tan(f*x+e)
)*B*a^2*c^3+1/f/(a^2+b^2)/(a+b*tan(f*x+e))*B*a*c^3-1/f*b/(a^2+b^2)/(a+b*tan
(f*x+e))*A*c^3+1/f*b^2/(a^2+b^2)^2*ln(a+b*tan(f*x+e))*B*c^3+1/2/f/(a^2+b^2)
^2*ln(1+tan(f*x+e)^2)*C*a^2*d^3-1/2/f/(a^2+b^2)^2*ln(1+tan(f*x+e)^2)*C*b^2*
d^3+1/f/(a^2+b^2)^2*A*arctan(tan(f*x+e))*a^2*c^3-1/f/(a^2+b^2)^2*A*arctan(t
an(f*x+e))*b^2*c^3+1/f/(a^2+b^2)^2*B*arctan(tan(f*x+e))*a^2*d^3-1/f/(a^2+b^
2)^2*B*arctan(tan(f*x+e))*b^2*d^3-1/f/(a^2+b^2)^2*C*arctan(tan(f*x+e))*a^2*
c^3+1/f/(a^2+b^2)^2*C*arctan(tan(f*x+e))*b^2*c^3-1/2/f/(a^2+b^2)^2*ln(1+tan
(f*x+e)^2)*A*a^2*d^3+1/2/f/(a^2+b^2)^2*ln(1+tan(f*x+e)^2)*A*b^2*d^3+1/2/f/(
a^2+b^2)^2*ln(1+tan(f*x+e)^2)*B*a^2*c^3-1/2/f/(a^2+b^2)^2*ln(1+tan(f*x+e)^2
)*B*b^2*c^3-2/f*d^3/b^3*a*C*tan(f*x+e)+3/f*d^2/b^2*C*c*tan(f*x+e)+1/f*d^3/b
^2*B*tan(f*x+e)+1/f/b^2/(a^2+b^2)/(a+b*tan(f*x+e))*A*a^3*d^3-1/f/b^3/(a^2+b
^2)/(a+b*tan(f*x+e))*B*a^4*d^3+1/f/b^4/(a^2+b^2)/(a+b*tan(f*x+e))*C*a^5*d^3
-1/f/b/(a^2+b^2)/(a+b*tan(f*x+e))*C*a^2*c^3+3/2/f/(a^2+b^2)^2*ln(1+tan(f*x+
e)^2)*C*b^2*c^2*d-3/f/(a^2+b^2)^2*A*arctan(tan(f*x+e))*a^2*c*d^2-3/f/(a^2+b
^2)^2*C*arctan(tan(f*x+e))*b^2*c*d^2+3/2/f/(a^2+b^2)^2*ln(1+tan(f*x+e)^2)*A
*a^2*c^2*d-1/f/(a^2+b^2)^2*ln(1+tan(f*x+e)^2)*A*a*b*c^3-3/2/f/(a^2+b^2)^2*l
n(1+tan(f*x+e)^2)*A*b^2*c^2*d-3/2/f/(a^2+b^2)^2*ln(1+tan(f*x+e)^2)*B*a^2*c*
d^2-1/f/(a^2+b^2)^2*ln(1+tan(f*x+e)^2)*B*a*b*d^3+3/2/f/(a^2+b^2)^2*ln(1+tan
(f*x+e)^2)*B*b^2*c*d^2-3/2/f/(a^2+b^2)^2*ln(1+tan(f*x+e)^2)*C*a^2*c^2*d+1/f
/(a^2+b^2)^2*ln(1+tan(f*x+e)^2)*C*a*b*c^3-3/f/(a^2+b^2)^2*ln(a+b*tan(f*x+e)
)*A*a^2*c^2*d+9/f/(a^2+b^2)^2*ln(a+b*tan(f*x+e))*B*a^2*c*d^2+9/f/(a^2+b^2)^
2*ln(a+b*tan(f*x+e))*C*a^2*c^2*d+3/f/(a^2+b^2)/(a+b*tan(f*x+e))*A*a*c^2*d+2
/f*b/(a^2+b^2)^2*ln(a+b*tan(f*x+e))*A*a*c^3+3/f*b^2/(a^2+b^2)^2*ln(a+b*tan(
f*x+e))*A*c^2*d+1/2/f*d^3/b^2*C*tan(f*x+e)^2
```

Maxima [A] time = 1.64338, size = 925, normalized size = 1.61

$$\frac{\frac{2\left(\left(\left(A-C\right)a^2+2\,Bab-\left(A-C\right)b^2\right)c^3-3\left(Ba^2-2\left(A-C\right)ab-Bb^2\right)c^2d-3\left(\left(A-C\right)a^2+2\,Bab-\left(A-C\right)b^2\right)cd^2+\left(Ba^2-2\left(A-C\right)ab-Bb^2\right)d^3\right)\left(fx+e\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\left(\left(Ba^2b^4-2\left(A-C\right)ab^5-B\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e
))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*c^3 - 3*(B*a^2 - 2*(A - C)*a*
b - B*b^2)*c^2*d - 3*((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*c*d^2 + (B*a^2 -
 2*(A - C)*a*b - B*b^2)*d^3)*(f*x + e)/(a^4 + 2*a^2*b^2 + b^4) - 2*((B*a^2*
b^4 - 2*(A - C)*a*b^5 - B*b^6)*c^3 - 3*(C*a^4*b^2 - (A - 3*C)*a^2*b^4 - 2*B
*a*b^5 + A*b^6)*c^2*d + 3*(2*C*a^5*b - B*a^4*b^2 + 4*C*a^3*b^3 - 3*B*a^2*b^
4 + 2*A*a*b^5)*c*d^2 - (3*C*a^6 - 2*B*a^5*b + (A + 5*C)*a^4*b^2 - 4*B*a^3*b
^3 + 3*A*a^2*b^4)*d^3)*log(b*tan(f*x + e) + a)/(a^4*b^4 + 2*a^2*b^6 + b^8)
+ ((B*a^2 - 2*(A - C)*a*b - B*b^2)*c^3 + 3*((A - C)*a^2 + 2*B*a*b - (A - C)
*b^2)*c^2*d - 3*(B*a^2 - 2*(A - C)*a*b - B*b^2)*c*d^2 - ((A - C)*a^2 + 2*B*
a*b - (A - C)*b^2)*d^3)*log(tan(f*x + e)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2
*((C*a^2*b^3 - B*a*b^4 + A*b^5)*c^3 - 3*(C*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*c
^2*d + 3*(C*a^4*b - B*a^3*b^2 + A*a^2*b^3)*c*d^2 - (C*a^5 - B*a^4*b + A*a^3
*b^2)*d^3)/(a^3*b^4 + a*b^6 + (a^2*b^5 + b^7)*tan(f*x + e)) + (C*b*d^3*tan(
f*x + e)^2 + 2*(3*C*b*c*d^2 - (2*C*a - B*b)*d^3)*tan(f*x + e))/b^3)/f
```

Fricas [B] time = 8.35889, size = 3131, normalized size = 5.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e
))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((C*a^4*b^3 + 2*C*a^2*b^5 + C*b^7)*d^3*tan(f*x + e)^3 - 2*(C*a^2*b^5 -
B*a*b^6 + A*b^7)*c^3 + 6*(C*a^3*b^4 - B*a^2*b^5 + A*a*b^6)*c^2*d - 6*(C*a^4
*b^3 - B*a^3*b^4 + A*a^2*b^5)*c*d^2 + (3*C*a^5*b^2 - 2*B*a^4*b^3 + 2*(A + C
)*a^3*b^4 + C*a*b^6)*d^3 + 2*(((A - C)*a^3*b^4 + 2*B*a^2*b^5 - (A - C)*a*b^
6)*c^3 - 3*(B*a^3*b^4 - 2*(A - C)*a^2*b^5 - B*a*b^6)*c^2*d - 3*((A - C)*a^3
*b^4 + 2*B*a^2*b^5 - (A - C)*a*b^6)*c*d^2 + (B*a^3*b^4 - 2*(A - C)*a^2*b^5
- B*a*b^6)*d^3)*f*x + (6*(C*a^4*b^3 + 2*C*a^2*b^5 + C*b^7)*c*d^2 - (3*C*a^5
*b^2 - 2*B*a^4*b^3 + 6*C*a^3*b^4 - 4*B*a^2*b^5 + 3*C*a*b^6 - 2*B*b^7)*d^3)*
tan(f*x + e)^2 - ((B*a^3*b^4 - 2*(A - C)*a^2*b^5 - B*a*b^6)*c^3 - 3*(C*a^5*
b^2 - (A - 3*C)*a^3*b^4 - 2*B*a^2*b^5 + A*a*b^6)*c^2*d + 3*(2*C*a^6*b - B*a
^5*b^2 + 4*C*a^4*b^3 - 3*B*a^3*b^4 + 2*A*a^2*b^5)*c*d^2 - (3*C*a^7 - 2*B*a^
6*b + (A + 5*C)*a^5*b^2 - 4*B*a^4*b^3 + 3*A*a^3*b^4)*d^3 + ((B*a^2*b^5 - 2*
(A - C)*a*b^6 - B*b^7)*c^3 - 3*(C*a^4*b^3 - (A - 3*C)*a^2*b^5 - 2*B*a*b^6 +
 A*b^7)*c^2*d + 3*(2*C*a^5*b^2 - B*a^4*b^3 + 4*C*a^3*b^4 - 3*B*a^2*b^5 + 2*
A*a*b^6)*c*d^2 - (3*C*a^6*b - 2*B*a^5*b^2 + (A + 5*C)*a^4*b^3 - 4*B*a^3*b^4
 + 3*A*a^2*b^5)*d^3)*tan(f*x + e))*log((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x
+ e) + a^2)/(tan(f*x + e)^2 + 1)) - (3*(C*a^5*b^2 + 2*C*a^3*b^4 + C*a*b^6)*
c^2*d - 3*(2*C*a^6*b - B*a^5*b^2 + 4*C*a^4*b^3 - 2*B*a^3*b^4 + 2*C*a^2*b^5
- B*a*b^6)*c*d^2 + (3*C*a^7 - 2*B*a^6*b + (A + 5*C)*a^5*b^2 - 4*B*a^4*b^3 +
 (2*A + C)*a^3*b^4 - 2*B*a^2*b^5 + (A - C)*a*b^6)*d^3 + (3*(C*a^4*b^3 + 2*C
*a^2*b^5 + C*b^7)*c^2*d - 3*(2*C*a^5*b^2 - B*a^4*b^3 + 4*C*a^3*b^4 - 2*B*a^
2*b^5 + 2*C*a*b^6 - B*b^7)*c*d^2 + (3*C*a^6*b - 2*B*a^5*b^2 + (A + 5*C)*a^4
*b^3 - 4*B*a^3*b^4 + (2*A + C)*a^2*b^5 - 2*B*a*b^6 + (A - C)*b^7)*d^3)*tan(
f*x + e))*log(1/(tan(f*x + e)^2 + 1)) + (2*(C*a^3*b^4 - B*a^2*b^5 + A*a*b^6
)*c^3 - 6*(C*a^4*b^3 - B*a^3*b^4 + A*a^2*b^5)*c^2*d + 6*(2*C*a^5*b^2 - B*a^
4*b^3 + (A + 2*C)*a^3*b^4 + C*a*b^6)*c*d^2 - (6*C*a^6*b - 4*B*a^5*b^2 + (2*
A + 7*C)*a^4*b^3 - 4*B*a^3*b^4 + 2*C*a^2*b^5 - 2*B*a*b^6 - C*b^7)*d^3 + 2*(
((A - C)*a^2*b^5 + 2*B*a*b^6 - (A - C)*b^7)*c^3 - 3*(B*a^2*b^5 - 2*(A - C)*
a*b^6 - B*b^7)*c^2*d - 3*((A - C)*a^2*b^5 + 2*B*a*b^6 - (A - C)*b^7)*c*d^2
+ (B*a^2*b^5 - 2*(A - C)*a*b^6 - B*b^7)*d^3)*f*x)*tan(f*x + e))/((a^4*b^5 +
 2*a^2*b^7 + b^9)*f*tan(f*x + e) + (a^5*b^4 + 2*a^3*b^6 + a*b^8)*f)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x
+e))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 2.42252, size = 1832, normalized size = 3.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e
))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a^2*c^3 - C*a^2*c^3 + 2*B*a*b*c^3 - A*b^2*c^3 + C*b^2*c^3 - 3*B*a
^2*c^2*d + 6*A*a*b*c^2*d - 6*C*a*b*c^2*d + 3*B*b^2*c^2*d - 3*A*a^2*c*d^2 +
3*C*a^2*c*d^2 - 6*B*a*b*c*d^2 + 3*A*b^2*c*d^2 - 3*C*b^2*c*d^2 + B*a^2*d^3 -
 2*A*a*b*d^3 + 2*C*a*b*d^3 - B*b^2*d^3)*(f*x + e)/(a^4 + 2*a^2*b^2 + b^4) +
 (B*a^2*c^3 - 2*A*a*b*c^3 + 2*C*a*b*c^3 - B*b^2*c^3 + 3*A*a^2*c^2*d - 3*C*a
^2*c^2*d + 6*B*a*b*c^2*d - 3*A*b^2*c^2*d + 3*C*b^2*c^2*d - 3*B*a^2*c*d^2 +
6*A*a*b*c*d^2 - 6*C*a*b*c*d^2 + 3*B*b^2*c*d^2 - A*a^2*d^3 + C*a^2*d^3 - 2*B
*a*b*d^3 + A*b^2*d^3 - C*b^2*d^3)*log(tan(f*x + e)^2 + 1)/(a^4 + 2*a^2*b^2
+ b^4) - 2*(B*a^2*b^4*c^3 - 2*A*a*b^5*c^3 + 2*C*a*b^5*c^3 - B*b^6*c^3 - 3*C
*a^4*b^2*c^2*d + 3*A*a^2*b^4*c^2*d - 9*C*a^2*b^4*c^2*d + 6*B*a*b^5*c^2*d -
3*A*b^6*c^2*d + 6*C*a^5*b*c*d^2 - 3*B*a^4*b^2*c*d^2 + 12*C*a^3*b^3*c*d^2 -
9*B*a^2*b^4*c*d^2 + 6*A*a*b^5*c*d^2 - 3*C*a^6*d^3 + 2*B*a^5*b*d^3 - A*a^4*b
^2*d^3 - 5*C*a^4*b^2*d^3 + 4*B*a^3*b^3*d^3 - 3*A*a^2*b^4*d^3)*log(abs(b*tan
(f*x + e) + a))/(a^4*b^4 + 2*a^2*b^6 + b^8) + 2*(B*a^2*b^5*c^3*tan(f*x + e)
 - 2*A*a*b^6*c^3*tan(f*x + e) + 2*C*a*b^6*c^3*tan(f*x + e) - B*b^7*c^3*tan(
f*x + e) - 3*C*a^4*b^3*c^2*d*tan(f*x + e) + 3*A*a^2*b^5*c^2*d*tan(f*x + e)
- 9*C*a^2*b^5*c^2*d*tan(f*x + e) + 6*B*a*b^6*c^2*d*tan(f*x + e) - 3*A*b^7*c
^2*d*tan(f*x + e) + 6*C*a^5*b^2*c*d^2*tan(f*x + e) - 3*B*a^4*b^3*c*d^2*tan(
f*x + e) + 12*C*a^3*b^4*c*d^2*tan(f*x + e) - 9*B*a^2*b^5*c*d^2*tan(f*x + e)
 + 6*A*a*b^6*c*d^2*tan(f*x + e) - 3*C*a^6*b*d^3*tan(f*x + e) + 2*B*a^5*b^2*
d^3*tan(f*x + e) - A*a^4*b^3*d^3*tan(f*x + e) - 5*C*a^4*b^3*d^3*tan(f*x + e
) + 4*B*a^3*b^4*d^3*tan(f*x + e) - 3*A*a^2*b^5*d^3*tan(f*x + e) - C*a^4*b^3
*c^3 + 2*B*a^3*b^4*c^3 - 3*A*a^2*b^5*c^3 + C*a^2*b^5*c^3 - A*b^7*c^3 - 3*B*
a^4*b^3*c^2*d + 6*A*a^3*b^4*c^2*d - 6*C*a^3*b^4*c^2*d + 3*B*a^2*b^5*c^2*d +
 3*C*a^6*b*c*d^2 - 3*A*a^4*b^3*c*d^2 + 9*C*a^4*b^3*c*d^2 - 6*B*a^3*b^4*c*d^
2 + 3*A*a^2*b^5*c*d^2 - 2*C*a^7*d^3 + B*a^6*b*d^3 - 4*C*a^5*b^2*d^3 + 3*B*a
^4*b^3*d^3 - 2*A*a^3*b^4*d^3)/((a^4*b^4 + 2*a^2*b^6 + b^8)*(b*tan(f*x + e)
+ a)) + (C*b^2*d^3*tan(f*x + e)^2 + 6*C*b^2*c*d^2*tan(f*x + e) - 4*C*a*b*d^
3*tan(f*x + e) + 2*B*b^2*d^3*tan(f*x + e))/b^4)/f
```

3.69 $\int \frac{(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx$

Optimal. Leaf size=798

$$-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^3}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}+\frac{\left(-3Cda^4+bBda^3+b^2(2Bc+(A-7C)d)a^2-b^3(4Ac-4Cc-5Bd)a-b\right.}{2b^2\left(a^2+b^2\right)^2f(a+b\tan(e+fx))}$$

```
[Out] -((((3*a*b^2*(A*c^3 - c^3*C - 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 + B*d^3) + a
^3*(c^3*C + 3*B*c^2*d - 3*c*C*d^2 - B*d^3 - A*(c^3 - 3*c*d^2)) - 3*a^2*b*((
A - C)*d*(3*c^2 - d^2) + B*(c^3 - 3*c*d^2)) + b^3*((A - C)*d*(3*c^2 - d^2)
+ B*(c^3 - 3*c*d^2)))*x)/(a^2 + b^2)^3) - ((b^3*(A*c^3 - c^3*C - 3*B*c^2*d
- 3*A*c*d^2 + 3*c*C*d^2 + B*d^3) + 3*a^2*b*(c^3*C + 3*B*c^2*d - 3*c*C*d^2 -
 B*d^3 - A*(c^3 - 3*c*d^2)) + a^3*((A - C)*d*(3*c^2 - d^2) + B*(c^3 - 3*c*d
^2)) - 3*a*b^2*((A - C)*d*(3*c^2 - d^2) + B*(c^3 - 3*c*d^2)))*Log[Cos[e + f
*x]])/((a^2 + b^2)^3*f) - ((b*c - a*d)*(a^5*b*B*d^2 - 3*a^6*C*d^2 + a^4*b^2
*d*(B*c - 9*C*d) + a^3*b^3*B*(c^2 + 3*d^2) - b^6*(c*(c*C + 3*B*d) - A*(c^2
- 3*d^2)) - a*b^5*(8*c*(A - C)*d + 3*B*(c^2 - 2*d^2)) + a^2*b^4*(3*c^2*C +
6*B*c*d - 10*C*d^2 - A*(3*c^2 - d^2)))*Log[a + b*Tan[e + f*x]])/(b^4*(a^2 +
 b^2)^3*f) - (d^2*(a^3*b*B*d - 3*a^4*C*d - a*b^3*(2*A*c - 2*c*C - 3*B*d) +
a^2*b^2*(B*c - 6*C*d) - b^4*(B*c + (2*A + C)*d))*Tan[e + f*x])/(b^3*(a^2 +
b^2)^2*f) + ((a^3*b*B*d - 3*a^4*C*d - b^4*(2*B*c + 3*A*d) - a*b^3*(4*A*c -
4*c*C - 5*B*d) + a^2*b^2*(2*B*c + (A - 7*C)*d))*(c + d*Tan[e + f*x])^2)/(2*
b^2*(a^2 + b^2)^2*f*(a + b*Tan[e + f*x])) - ((A*b^2 - a*(b*B - a*C))*(c + d
*Tan[e + f*x])^3)/(2*b*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^2)
```

Rubi [A] time = 2.83884, antiderivative size = 798, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3645, 3637, 3626, 3617, 31, 3475}

$$-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^3}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}+\frac{\left(-3Cda^4+bBda^3+b^2(2Bc+(A-7C)d)a^2-b^3(4Ac-4Cc-5Bd)a-b\right.}{2b^2\left(a^2+b^2\right)^2f(a+b\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a + b
*Tan[e + f*x])^3,x]
```

```
[Out] -((((3*a*b^2*(A*c^3 - c^3*C - 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 + B*d^3) + a
^3*(c^3*C + 3*B*c^2*d - 3*c*C*d^2 - B*d^3 - A*(c^3 - 3*c*d^2)) - 3*a^2*b*((
A - C)*d*(3*c^2 - d^2) + B*(c^3 - 3*c*d^2)) + b^3*((A - C)*d*(3*c^2 - d^2)
+ B*(c^3 - 3*c*d^2)))*x)/(a^2 + b^2)^3) - ((b^3*(A*c^3 - c^3*C - 3*B*c^2*d
- 3*A*c*d^2 + 3*c*C*d^2 + B*d^3) + 3*a^2*b*(c^3*C + 3*B*c^2*d - 3*c*C*d^2 -
 B*d^3 - A*(c^3 - 3*c*d^2)) + a^3*((A - C)*d*(3*c^2 - d^2) + B*(c^3 - 3*c*d
^2)) - 3*a*b^2*((A - C)*d*(3*c^2 - d^2) + B*(c^3 - 3*c*d^2)))*Log[Cos[e + f
*x]])/((a^2 + b^2)^3*f) - ((b*c - a*d)*(a^5*b*B*d^2 - 3*a^6*C*d^2 + a^4*b^2
*d*(B*c - 9*C*d) + a^3*b^3*B*(c^2 + 3*d^2) - b^6*(c*(c*C + 3*B*d) - A*(c^2
- 3*d^2)) - a*b^5*(8*c*(A - C)*d + 3*B*(c^2 - 2*d^2)) + a^2*b^4*(3*c^2*C +
6*B*c*d - 10*C*d^2 - A*(3*c^2 - d^2)))*Log[a + b*Tan[e + f*x]])/(b^4*(a^2 +
 b^2)^3*f) - (d^2*(a^3*b*B*d - 3*a^4*C*d - a*b^3*(2*A*c - 2*c*C - 3*B*d) +
a^2*b^2*(B*c - 6*C*d) - b^4*(B*c + (2*A + C)*d))*Tan[e + f*x])/(b^3*(a^2 +
b^2)^2*f) + ((a^3*b*B*d - 3*a^4*C*d - b^4*(2*B*c + 3*A*d) - a*b^3*(4*A*c -
4*c*C - 5*B*d) + a^2*b^2*(2*B*c + (A - 7*C)*d))*(c + d*Tan[e + f*x])^2)/(2*
b^2*(a^2 + b^2)^2*f*(a + b*Tan[e + f*x])) - ((A*b^2 - a*(b*B - a*C))*(c + d
*Tan[e + f*x])^3)/(2*b*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^2)
```

Rule 3645

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*d^2 + c*(c*C - B*d))*(a + b*Tan[e
+ f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3626

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((a*A + b*B -
a*C)*x)/(a^2 + b^2), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3617

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}dx = -\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^3}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2} + \int \frac{(c+}{}$$

$$= \frac{\left(a^3bBd-3a^4Cd-b^4(2Bc+3Ad)-ab^3(4Ac-4c\right.}{2b^2\left(a^2+b^2\right)^2}$$

$$= -\frac{d^2\left(a^3bBd-3a^4Cd-ab^3(2Ac-2cC-3Bd)+a\right.}{b^3\left(a^2\right.}$$

$$= -\frac{\left(3ab^2\left(Ac^3-c^3C-3Bc^2d-3Acd^2+3cCd^2+B\right.\right.}{}$$

$$= -\frac{\left(3ab^2\left(Ac^3-c^3C-3Bc^2d-3Acd^2+3cCd^2+B\right.\right.}{}$$

$$= -\frac{\left(3ab^2\left(Ac^3-c^3C-3Bc^2d-3Acd^2+3cCd^2+B\right.\right.}{}$$

<u>Mathematica</u> [A] time = 15.0509, size = 1451, normalized size = 1.82

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(a + b*Tan[e + f*x])^3,x]
```

```
[Out] ((3*a*b^2*(-(A*c^3) + c^3*C + 3*B*c^2*d + 3*A*c*d^2 - 3*c*C*d^2 - B*d^3) +
a^3*(-(c^3*C) - 3*B*c^2*d + 3*c*C*d^2 + B*d^3 + A*(c^3 - 3*c*d^2)) + b^3*((
A - C)*d*(-3*c^2 + d^2) - B*(c^3 - 3*c*d^2)) + 3*a^2*b*(-((A - C)*d*(-3*c^2
 + d^2)) + B*(c^3 - 3*c*d^2)))*(e + f*x)*(a*Cos[e + f*x] + b*Sin[e + f*x])^
3*(c + d*Tan[e + f*x])^3)/((a^2 + b^2)^3*f*(c*Cos[e + f*x] + d*Sin[e + f*x]
)^3*(a + b*Tan[e + f*x])^3) - (d^2*(3*b*c*C + b*B*d - 3*a*C*d)*Log[1 - Tan[
(e + f*x)/2]^2]*(a*Cos[e + f*x] + b*Sin[e + f*x])^3*(c + d*Tan[e + f*x])^3)
/(b^4*f*(c*Cos[e + f*x] + d*Sin[e + f*x])^3*(a + b*Tan[e + f*x])^3) + ((3*a
^2*b*(-(A*c^3) + c^3*C + 3*B*c^2*d + 3*A*c*d^2 - 3*c*C*d^2 - B*d^3) + b^3*(
-(c^3*C) - 3*B*c^2*d + 3*c*C*d^2 + B*d^3 + A*(c^3 - 3*c*d^2)) + a^3*(-((A -
 C)*d*(-3*c^2 + d^2)) + B*(c^3 - 3*c*d^2)) - 3*a*b^2*(-((A - C)*d*(-3*c^2 +
 d^2)) + B*(c^3 - 3*c*d^2)))*Log[1 + Tan[(e + f*x)/2]^2]*(a*Cos[e + f*x] +
b*Sin[e + f*x])^3*(c + d*Tan[e + f*x])^3)/((a^2 + b^2)^3*f*(c*Cos[e + f*x]
+ d*Sin[e + f*x])^3*(a + b*Tan[e + f*x])^3) - ((b*c - a*d)*(a^5*b*B*d^2 - 3
*a^6*C*d^2 + a^4*b^2*d*(B*c - 9*C*d) + a^3*b^3*B*(c^2 + 3*d^2) + b^6*(-(c*(
c*C + 3*B*d)) + A*(c^2 - 3*d^2)) + a*b^5*(8*c*(-A + C)*d - 3*B*(c^2 - 2*d^2
)) + a^2*b^4*(3*c^2*C + 6*B*c*d - 10*C*d^2 + A*(-3*c^2 + d^2)))*Log[-2*b*Ta
n[(e + f*x)/2] + a*(-1 + Tan[(e + f*x)/2]^2)]*(a*Cos[e + f*x] + b*Sin[e + f
*x])^3*(c + d*Tan[e + f*x])^3)/(b^4*(a^2 + b^2)^3*f*(c*Cos[e + f*x] + d*Sin
[e + f*x])^3*(a + b*Tan[e + f*x])^3) - (2*C*d^3*(a*Cos[e + f*x] + b*Sin[e +
 f*x])^3*Tan[(e + f*x)/2]*(c + d*Tan[e + f*x])^3)/(b^3*f*(c*Cos[e + f*x] +
d*Sin[e + f*x])^3*(-1 + Tan[(e + f*x)/2]^2)*(a + b*Tan[e + f*x])^3) + (2*(A
*b^2 + a*(-(b*B) + a*C))*(-(b*c) + a*d)^3*(a*Cos[e + f*x] + b*Sin[e + f*x])
^3*(a + 2*b*Tan[(e + f*x)/2])*(c + d*Tan[e + f*x])^3)/(a^3*b^2*(a^2 + b^2)*
f*(c*Cos[e + f*x] + d*Sin[e + f*x])^3*(a + 2*b*Tan[(e + f*x)/2] - a*Tan[(e
+ f*x)/2]^2)^2*(a + b*Tan[e + f*x])^3) - (2*(b*c - a*d)^2*(a*Cos[e + f*x] +
```

```
b*Sin[e + f*x])^3*(A*b^6*c + 2*a^6*C*d*Tan[(e + f*x)/2] - a*b^5*(B*c + A*(
d - c*Tan[(e + f*x)/2])) - a^5*b*(B*d*Tan[(e + f*x)/2] + C*(d - c*Tan[(e +
f*x)/2])) + a^4*b^2*(c*(C - 2*B*Tan[(e + f*x)/2]) + d*(B + 4*C*Tan[(e + f*x
)/2])) + a^2*b^4*(c*C + B*d + A*(c + 2*d*Tan[(e + f*x)/2])) - a^3*b^3*(A*d
+ C*d - 3*A*c*Tan[(e + f*x)/2] + c*C*Tan[(e + f*x)/2] + B*(c + 3*d*Tan[(e +
 f*x)/2])))*(c + d*Tan[e + f*x])^3)/(a^3*b^3*(a^2 + b^2)^2*f*(c*Cos[e + f*x
] + d*Sin[e + f*x])^3*(-2*b*Tan[(e + f*x)/2] + a*(-1 + Tan[(e + f*x)/2]^2))
*(a + b*Tan[e + f*x])^3)
```

Maple [B] time = 0.079, size = 3522, normalized size = 4.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^3,x
)
```

```
[Out] -9/f/(a^2+b^2)^3*B*arctan(tan(f*x+e))*a^2*b*c*d^2+9/f/(a^2+b^2)^3*B*arctan(
tan(f*x+e))*a*b^2*c^2*d-9/f/(a^2+b^2)^3*C*arctan(tan(f*x+e))*a^2*b*c^2*d+9/
2/f/(a^2+b^2)^3*ln(1+tan(f*x+e)^2)*B*a*b^2*c*d^2+6/f/b^3/(a^2+b^2)^2/(a+b*t
an(f*x+e))*C*a^5*c*d^2+9/f/(a^2+b^2)^3*A*arctan(tan(f*x+e))*a*b^2*c*d^2+3/f
/b^3/(a^2+b^2)^3*ln(a+b*tan(f*x+e))*C*a^6*c*d^2-9/2/f/(a^2+b^2)^3*ln(1+tan(
f*x+e)^2)*C*a^2*b*c*d^2+9/2/f/(a^2+b^2)^3*ln(1+tan(f*x+e)^2)*C*a*b^2*c^2*d+
9/f/(a^2+b^2)^3*A*arctan(tan(f*x+e))*a^2*b*c^2*d-3/2/f/b/(a^2+b^2)/(a+b*tan
(f*x+e))^2*B*a^2*c^2*d-3/2/f/b^3/(a^2+b^2)/(a+b*tan(f*x+e))^2*C*a^4*c*d^2+3
/2/f/b^2/(a^2+b^2)/(a+b*tan(f*x+e))^2*C*a^3*c^2*d-9/f/(a^2+b^2)^3*C*arctan(
tan(f*x+e))*a*b^2*c*d^2-9/2/f/(a^2+b^2)^3*ln(1+tan(f*x+e)^2)*A*a*b^2*c^2*d+
6/f*b/(a^2+b^2)^2/(a+b*tan(f*x+e))*B*a*c^2*d+9/2/f/(a^2+b^2)^3*ln(1+tan(f*x
+e)^2)*B*a^2*b*c^2*d+12/f/b/(a^2+b^2)^2/(a+b*tan(f*x+e))*C*a^3*c*d^2-9/f*b/
(a^2+b^2)^3*ln(a+b*tan(f*x+e))*B*a^2*c^2*d-9/f*b^2/(a^2+b^2)^3*ln(a+b*tan(f
*x+e))*B*a*c*d^2+9/f*b^2/(a^2+b^2)^3*ln(a+b*tan(f*x+e))*A*a*c^2*d+9/2/f/(a^
2+b^2)^3*ln(1+tan(f*x+e)^2)*A*a^2*b*c*d^2-9/f*b/(a^2+b^2)^3*ln(a+b*tan(f*x+
e))*A*a^2*c*d^2+9/f/b/(a^2+b^2)^3*ln(a+b*tan(f*x+e))*C*a^4*c*d^2+18/f*b/(a^
2+b^2)^3*ln(a+b*tan(f*x+e))*C*a^2*c*d^2-9/f*b^2/(a^2+b^2)^3*ln(a+b*tan(f*x+
e))*C*a*c^2*d-3/2/f/b/(a^2+b^2)/(a+b*tan(f*x+e))^2*A*a^2*c*d^2+3/2/f/b^2/(a
^2+b^2)/(a+b*tan(f*x+e))^2*B*a^3*c*d^2+6/f*b/(a^2+b^2)^2/(a+b*tan(f*x+e))*A
*a*c*d^2-3/f/b^2/(a^2+b^2)^2/(a+b*tan(f*x+e))*B*a^4*c*d^2-3/2/f/(a^2+b^2)^3
*ln(1+tan(f*x+e)^2)*B*a^3*c*d^2-3/2/f/(a^2+b^2)^3*ln(1+tan(f*x+e)^2)*B*a^2*
b*d^3-3/2/f/(a^2+b^2)^3*ln(1+tan(f*x+e)^2)*B*a*b^2*c^3-3/2/f/(a^2+b^2)^3*ln
(1+tan(f*x+e)^2)*B*b^3*c^2*d-3/2/f/(a^2+b^2)^3*ln(1+tan(f*x+e)^2)*C*a^3*c^2
*d+3/2/f/(a^2+b^2)^3*ln(1+tan(f*x+e)^2)*C*a^2*b*c^3-3/2/f/(a^2+b^2)^3*ln(1+
tan(f*x+e)^2)*C*a*b^2*d^3+3/2/f/(a^2+b^2)^3*ln(1+tan(f*x+e)^2)*C*b^3*c*d^2-
3/f/b^2/(a^2+b^2)^2/(a+b*tan(f*x+e))*C*a^4*c^2*d-3/f/(a^2+b^2)^3*A*arctan(t
an(f*x+e))*a^3*c*d^2-3/f/(a^2+b^2)^3*A*arctan(tan(f*x+e))*a^2*b*d^3-3/f/(a^
2+b^2)^3*A*arctan(tan(f*x+e))*a*b^2*c^3-3/f/(a^2+b^2)^3*A*arctan(tan(f*x+e)
)*b^3*c^2*d-3/f/(a^2+b^2)^3*B*arctan(tan(f*x+e))*a^3*c^2*d+3/f/(a^2+b^2)^3*
B*arctan(tan(f*x+e))*a^2*b*c^3+3/f/(a^2+b^2)^3*ln(a+b*tan(f*x+e))*B*a^3*c*d
^2+3/f/(a^2+b^2)^3*ln(a+b*tan(f*x+e))*C*a^3*c^2*d+3/2/f/(a^2+b^2)/(a+b*tan(
f*x+e))^2*A*a*c^2*d+3/f/(a^2+b^2)^2/(a+b*tan(f*x+e))*A*a^2*c^2*d-9/f/(a^2+b
^2)^2/(a+b*tan(f*x+e))*B*a^2*c*d^2+3/f/b/(a^2+b^2)^3*ln(a+b*tan(f*x+e))*B*a
^4*d^3+6/f*b/(a^2+b^2)^3*ln(a+b*tan(f*x+e))*B*a^2*d^3+3/f*b^2/(a^2+b^2)^3*l
n(a+b*tan(f*x+e))*B*a*c^3-3/f/(a^2+b^2)^3*B*arctan(tan(f*x+e))*a*b^2*d^3+3/
f/(a^2+b^2)^3*B*arctan(tan(f*x+e))*b^3*c*d^2+3/f/(a^2+b^2)^3*C*arctan(tan(f
*x+e))*a^3*c*d^2+3/f/(a^2+b^2)^3*C*arctan(tan(f*x+e))*a^2*b*d^3+3/f/(a^2+b^
2)^3*C*arctan(tan(f*x+e))*a*b^2*c^3+3/f/(a^2+b^2)^3*C*arctan(tan(f*x+e))*b^
3*c^2*d+1/2/f/b^2/(a^2+b^2)/(a+b*tan(f*x+e))^2*A*a^3*d^3-1/2/f/b^3/(a^2+b^2
```

```
)/(a+b*tan(f*x+e))^2*B*a^4*d^3+1/2/f/b^4/(a^2+b^2)/(a+b*tan(f*x+e))^2*C*a^5
*d^3+1/f/b^3/(a^2+b^2)^3*ln(a+b*tan(f*x+e))*B*a^6*d^3+3/f*b^3/(a^2+b^2)^3*l
n(a+b*tan(f*x+e))*B*c^2*d-3/f/b^4/(a^2+b^2)^3*ln(a+b*tan(f*x+e))*C*a^7*d^3-
9/f/b^2/(a^2+b^2)^3*ln(a+b*tan(f*x+e))*C*a^5*d^3-3/f*b/(a^2+b^2)^3*ln(a+b*t
an(f*x+e))*C*a^2*c^3-1/2/f/b/(a^2+b^2)/(a+b*tan(f*x+e))^2*C*a^2*c^3-9/f/(a^
2+b^2)^2/(a+b*tan(f*x+e))*C*a^2*c^2*d-3/f/(a^2+b^2)^3*ln(a+b*tan(f*x+e))*A*
a^3*c^2*d-3/f*b^2/(a^2+b^2)^3*ln(a+b*tan(f*x+e))*A*a*d^3+3/f*b^3/(a^2+b^2)^
3*ln(a+b*tan(f*x+e))*A*c*d^2-3/f/b^4/(a^2+b^2)^2/(a+b*tan(f*x+e))*C*a^6*d^3
-5/f/b^2/(a^2+b^2)^2/(a+b*tan(f*x+e))*C*a^4*d^3+2/f*b/(a^2+b^2)^2/(a+b*tan(
f*x+e))*C*a*c^3+3/f*b/(a^2+b^2)^3*ln(a+b*tan(f*x+e))*A*a^2*c^3+3/2/f/(a^2+b
^2)^3*ln(1+tan(f*x+e)^2)*A*a^3*c^2*d-3/2/f/(a^2+b^2)^3*ln(1+tan(f*x+e)^2)*A
*a^2*b*c^3+3/2/f/(a^2+b^2)^3*ln(1+tan(f*x+e)^2)*A*a*b^2*d^3-3/2/f/(a^2+b^2)
^3*ln(1+tan(f*x+e)^2)*A*b^3*c*d^2+1/f*C*d^3/b^3*tan(f*x+e)-1/f/b^2/(a^2+b^2
)^2/(a+b*tan(f*x+e))*A*a^4*d^3-2/f*b/(a^2+b^2)^2/(a+b*tan(f*x+e))*A*a*c^3-3
/f*b^2/(a^2+b^2)^2/(a+b*tan(f*x+e))*A*c^2*d+2/f/b^3/(a^2+b^2)^2/(a+b*tan(f*
x+e))*B*a^5*d^3+4/f/b/(a^2+b^2)^2/(a+b*tan(f*x+e))*B*a^3*d^3-1/f*b^3/(a^2+b
^2)^3*ln(a+b*tan(f*x+e))*A*c^3+1/f*b^3/(a^2+b^2)^3*ln(a+b*tan(f*x+e))*C*c^3
-1/f/(a^2+b^2)^3*ln(a+b*tan(f*x+e))*B*a^3*c^3-10/f/(a^2+b^2)^3*ln(a+b*tan(f
*x+e))*C*a^3*d^3-3/f/(a^2+b^2)^2/(a+b*tan(f*x+e))*A*a^2*d^3+1/f/(a^2+b^2)^2
/(a+b*tan(f*x+e))*B*a^2*c^3+1/f/(a^2+b^2)^3*ln(a+b*tan(f*x+e))*A*a^3*d^3+1/
2/f/(a^2+b^2)/(a+b*tan(f*x+e))^2*B*a*c^3-1/f/(a^2+b^2)^3*C*arctan(tan(f*x+e
))*b^3*d^3-1/2/f/(a^2+b^2)^3*ln(1+tan(f*x+e)^2)*A*a^3*d^3+1/2/f/(a^2+b^2)^3
*ln(1+tan(f*x+e)^2)*A*b^3*c^3+1/2/f/(a^2+b^2)^3*ln(1+tan(f*x+e)^2)*B*a^3*c^
3+1/2/f/(a^2+b^2)^3*ln(1+tan(f*x+e)^2)*B*b^3*d^3+1/2/f/(a^2+b^2)^3*ln(1+tan
(f*x+e)^2)*a^3*C*d^3-1/2/f/(a^2+b^2)^3*ln(1+tan(f*x+e)^2)*C*b^3*c^3+1/f/(a^
2+b^2)^3*A*arctan(tan(f*x+e))*a^3*c^3+1/f/(a^2+b^2)^3*A*arctan(tan(f*x+e))*
b^3*d^3+1/f/(a^2+b^2)^3*B*arctan(tan(f*x+e))*a^3*d^3-1/f/(a^2+b^2)^3*B*arct
an(tan(f*x+e))*b^3*c^3-1/f/(a^2+b^2)^3*C*arctan(tan(f*x+e))*a^3*c^3-1/2/f*b
/(a^2+b^2)/(a+b*tan(f*x+e))^2*A*c^3-1/f*b^2/(a^2+b^2)^2/(a+b*tan(f*x+e))*B*
c^3
```

Maxima [A] time = 1.77136, size = 1511, normalized size = 1.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e
))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*C*d^3*tan(f*x + e)/b^3 + 2*(((A - C)*a^3 + 3*B*a^2*b - 3*(A - C)*a*b
^2 - B*b^3)*c^3 - 3*(B*a^3 - 3*(A - C)*a^2*b - 3*B*a*b^2 + (A - C)*b^3)*c^2
*d - 3*((A - C)*a^3 + 3*B*a^2*b - 3*(A - C)*a*b^2 - B*b^3)*c*d^2 + (B*a^3 -
 3*(A - C)*a^2*b - 3*B*a*b^2 + (A - C)*b^3)*d^3)*(f*x + e)/(a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6) - 2*((B*a^3*b^4 - 3*(A - C)*a^2*b^5 - 3*B*a*b^6 + (A -
C)*b^7)*c^3 + 3*((A - C)*a^3*b^4 + 3*B*a^2*b^5 - 3*(A - C)*a*b^6 - B*b^7)*c
^2*d - 3*(C*a^6*b + 3*C*a^4*b^3 + B*a^3*b^4 - 3*(A - 2*C)*a^2*b^5 - 3*B*a*b
^6 + A*b^7)*c*d^2 + (3*C*a^7 - B*a^6*b + 9*C*a^5*b^2 - 3*B*a^4*b^3 - (A - 1
0*C)*a^3*b^4 - 6*B*a^2*b^5 + 3*A*a*b^6)*d^3)*log(b*tan(f*x + e) + a)/(a^6*b
^4 + 3*a^4*b^6 + 3*a^2*b^8 + b^10) + ((B*a^3 - 3*(A - C)*a^2*b - 3*B*a*b^2
+ (A - C)*b^3)*c^3 + 3*((A - C)*a^3 + 3*B*a^2*b - 3*(A - C)*a*b^2 - B*b^3)*
c^2*d - 3*(B*a^3 - 3*(A - C)*a^2*b - 3*B*a*b^2 + (A - C)*b^3)*c*d^2 - ((A -
 C)*a^3 + 3*B*a^2*b - 3*(A - C)*a*b^2 - B*b^3)*d^3)*log(tan(f*x + e)^2 + 1)
/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - ((C*a^4*b^3 - 3*B*a^3*b^4 + (5*A - 3
*C)*a^2*b^5 + B*a*b^6 + A*b^7)*c^3 + 3*(C*a^5*b^2 + B*a^4*b^3 - (3*A - 5*C)
*a^3*b^4 - 3*B*a^2*b^5 + A*a*b^6)*c^2*d - 3*(3*C*a^6*b - B*a^5*b^2 - (A - 7
*C)*a^4*b^3 - 5*B*a^3*b^4 + 3*A*a^2*b^5)*c*d^2 + (5*C*a^7 - 3*B*a^6*b + (A
```

```
+ 9*C)*a^5*b^2 - 7*B*a^4*b^3 + 5*A*a^3*b^4)*d^3 - 2*((B*a^2*b^5 - 2*(A - C)
*a*b^6 - B*b^7)*c^3 - 3*(C*a^4*b^3 - (A - 3*C)*a^2*b^5 - 2*B*a*b^6 + A*b^7)
*c^2*d + 3*(2*C*a^5*b^2 - B*a^4*b^3 + 4*C*a^3*b^4 - 3*B*a^2*b^5 + 2*A*a*b^6
)*c*d^2 - (3*C*a^6*b - 2*B*a^5*b^2 + (A + 5*C)*a^4*b^3 - 4*B*a^3*b^4 + 3*A*
a^2*b^5)*d^3)*tan(f*x + e))/(a^6*b^4 + 2*a^4*b^6 + a^2*b^8 + (a^4*b^6 + 2*a
^2*b^8 + b^10)*tan(f*x + e)^2 + 2*(a^5*b^5 + 2*a^3*b^7 + a*b^9)*tan(f*x + e
)))/f
```

Fricas [B] time = 10.9242, size = 5261, normalized size = 6.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e
))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(C*a^6*b^3 + 3*C*a^4*b^5 + 3*C*a^2*b^7 + C*b^9)*d^3*tan(f*x + e)^3 -
 (3*C*a^4*b^5 - 5*B*a^3*b^6 + (7*A - 3*C)*a^2*b^7 + B*a*b^8 + A*b^9)*c^3 +
3*(C*a^5*b^4 - 3*B*a^4*b^5 + 5*(A - C)*a^3*b^6 + 3*B*a^2*b^7 - A*a*b^8)*c^2
*d + 3*(C*a^6*b^3 + B*a^5*b^4 - (3*A - 7*C)*a^4*b^5 - 5*B*a^3*b^6 + 3*A*a^2
*b^7)*c*d^2 - (3*C*a^7*b^2 - B*a^6*b^3 - (A - 9*C)*a^5*b^4 - 7*B*a^4*b^5 +
5*A*a^3*b^6)*d^3 + 2*(((A - C)*a^5*b^4 + 3*B*a^4*b^5 - 3*(A - C)*a^3*b^6 -
B*a^2*b^7)*c^3 - 3*(B*a^5*b^4 - 3*(A - C)*a^4*b^5 - 3*B*a^3*b^6 + (A - C)*a
^2*b^7)*c^2*d - 3*((A - C)*a^5*b^4 + 3*B*a^4*b^5 - 3*(A - C)*a^3*b^6 - B*a^
2*b^7)*c*d^2 + (B*a^5*b^4 - 3*(A - C)*a^4*b^5 - 3*B*a^3*b^6 + (A - C)*a^2*b
^7)*d^3)*f*x + ((C*a^4*b^5 - 3*B*a^3*b^6 + 5*(A - C)*a^2*b^7 + 3*B*a*b^8 -
A*b^9)*c^3 + 3*(C*a^5*b^4 + B*a^4*b^5 - (3*A - 7*C)*a^3*b^6 - 5*B*a^2*b^7 +
 3*A*a*b^8)*c^2*d - 3*(3*C*a^6*b^3 - B*a^5*b^4 - (A - 9*C)*a^4*b^5 - 7*B*a^
3*b^6 + 5*A*a^2*b^7)*c*d^2 + (9*C*a^7*b^2 - 3*B*a^6*b^3 + (A + 23*C)*a^5*b^
4 - 9*B*a^4*b^5 + (7*A + 12*C)*a^3*b^6 + 4*C*a*b^8)*d^3 + 2*(((A - C)*a^3*b
^6 + 3*B*a^2*b^7 - 3*(A - C)*a*b^8 - B*b^9)*c^3 - 3*(B*a^3*b^6 - 3*(A - C)*
a^2*b^7 - 3*B*a*b^8 + (A - C)*b^9)*c^2*d - 3*((A - C)*a^3*b^6 + 3*B*a^2*b^7
 - 3*(A - C)*a*b^8 - B*b^9)*c*d^2 + (B*a^3*b^6 - 3*(A - C)*a^2*b^7 - 3*B*a*
b^8 + (A - C)*b^9)*d^3)*f*x)*tan(f*x + e)^2 - ((B*a^5*b^4 - 3*(A - C)*a^4*b
^5 - 3*B*a^3*b^6 + (A - C)*a^2*b^7)*c^3 + 3*((A - C)*a^5*b^4 + 3*B*a^4*b^5
- 3*(A - C)*a^3*b^6 - B*a^2*b^7)*c^2*d - 3*(C*a^8*b + 3*C*a^6*b^3 + B*a^5*b
^4 - 3*(A - 2*C)*a^4*b^5 - 3*B*a^3*b^6 + A*a^2*b^7)*c*d^2 + (3*C*a^9 - B*a^
8*b + 9*C*a^7*b^2 - 3*B*a^6*b^3 - (A - 10*C)*a^5*b^4 - 6*B*a^4*b^5 + 3*A*a^
3*b^6)*d^3 + ((B*a^3*b^6 - 3*(A - C)*a^2*b^7 - 3*B*a*b^8 + (A - C)*b^9)*c^3
 + 3*((A - C)*a^3*b^6 + 3*B*a^2*b^7 - 3*(A - C)*a*b^8 - B*b^9)*c^2*d - 3*(C
*a^6*b^3 + 3*C*a^4*b^5 + B*a^3*b^6 - 3*(A - 2*C)*a^2*b^7 - 3*B*a*b^8 + A*b^
9)*c*d^2 + (3*C*a^7*b^2 - B*a^6*b^3 + 9*C*a^5*b^4 - 3*B*a^4*b^5 - (A - 10*C
)*a^3*b^6 - 6*B*a^2*b^7 + 3*A*a*b^8)*d^3)*tan(f*x + e)^2 + 2*((B*a^4*b^5 -
3*(A - C)*a^3*b^6 - 3*B*a^2*b^7 + (A - C)*a*b^8)*c^3 + 3*((A - C)*a^4*b^5 +
 3*B*a^3*b^6 - 3*(A - C)*a^2*b^7 - B*a*b^8)*c^2*d - 3*(C*a^7*b^2 + 3*C*a^5*
b^4 + B*a^4*b^5 - 3*(A - 2*C)*a^3*b^6 - 3*B*a^2*b^7 + A*a*b^8)*c*d^2 + (3*C
*a^8*b - B*a^7*b^2 + 9*C*a^6*b^3 - 3*B*a^5*b^4 - (A - 10*C)*a^4*b^5 - 6*B*a
^3*b^6 + 3*A*a^2*b^7)*d^3)*tan(f*x + e))*log((b^2*tan(f*x + e)^2 + 2*a*b*ta
n(f*x + e) + a^2)/(tan(f*x + e)^2 + 1)) - (3*(C*a^8*b + 3*C*a^6*b^3 + 3*C*a
^4*b^5 + C*a^2*b^7)*c*d^2 - (3*C*a^9 - B*a^8*b + 9*C*a^7*b^2 - 3*B*a^6*b^3
+ 9*C*a^5*b^4 - 3*B*a^4*b^5 + 3*C*a^3*b^6 - B*a^2*b^7)*d^3 + (3*(C*a^6*b^3
+ 3*C*a^4*b^5 + 3*C*a^2*b^7 + C*b^9)*c*d^2 - (3*C*a^7*b^2 - B*a^6*b^3 + 9*C
*a^5*b^4 - 3*B*a^4*b^5 + 9*C*a^3*b^6 - 3*B*a^2*b^7 + 3*C*a*b^8 - B*b^9)*d^3
)*tan(f*x + e)^2 + 2*(3*(C*a^7*b^2 + 3*C*a^5*b^4 + 3*C*a^3*b^6 + C*a*b^8)*c
*d^2 - (3*C*a^8*b - B*a^7*b^2 + 9*C*a^6*b^3 - 3*B*a^5*b^4 + 9*C*a^4*b^5 - 3
*B*a^3*b^6 + 3*C*a^2*b^7 - B*a*b^8)*d^3)*tan(f*x + e))*log(1/(tan(f*x + e)^
```

```
2 + 1)) + 2*((C*a^5*b^4 - 2*B*a^4*b^5 + 3*(A - C)*a^3*b^6 + 3*B*a^2*b^7 - (
3*A - 2*C)*a*b^8 - B*b^9)*c^3 + 3*(B*a^5*b^4 - (2*A - 3*C)*a^4*b^5 - 3*B*a^
3*b^6 + 3*(A - C)*a^2*b^7 + 2*B*a*b^8 - A*b^9)*c^2*d - 3*(C*a^7*b^2 - (A -
3*C)*a^5*b^4 - 3*B*a^4*b^5 + (3*A - 4*C)*a^3*b^6 + 3*B*a^2*b^7 - 2*A*a*b^8)
*c*d^2 + (3*C*a^8*b - B*a^7*b^2 + 6*C*a^6*b^3 - 3*B*a^5*b^4 + (3*A - 2*C)*a
^4*b^5 + 4*B*a^3*b^6 - (3*A - C)*a^2*b^7)*d^3 + 2*(((A - C)*a^4*b^5 + 3*B*a
^3*b^6 - 3*(A - C)*a^2*b^7 - B*a*b^8)*c^3 - 3*(B*a^4*b^5 - 3*(A - C)*a^3*b^
6 - 3*B*a^2*b^7 + (A - C)*a*b^8)*c^2*d - 3*((A - C)*a^4*b^5 + 3*B*a^3*b^6 -
 3*(A - C)*a^2*b^7 - B*a*b^8)*c*d^2 + (B*a^4*b^5 - 3*(A - C)*a^3*b^6 - 3*B*
a^2*b^7 + (A - C)*a*b^8)*d^3)*f*x)*tan(f*x + e))/((a^6*b^6 + 3*a^4*b^8 + 3*
a^2*b^10 + b^12)*f*tan(f*x + e)^2 + 2*(a^7*b^5 + 3*a^5*b^7 + 3*a^3*b^9 + a*
b^11)*f*tan(f*x + e) + (a^8*b^4 + 3*a^6*b^6 + 3*a^4*b^8 + a^2*b^10)*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x
+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.48784, size = 3382, normalized size = 4.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e
))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*C*d^3*tan(f*x + e)/b^3 + 2*(A*a^3*c^3 - C*a^3*c^3 + 3*B*a^2*b*c^3 -
3*A*a*b^2*c^3 + 3*C*a*b^2*c^3 - B*b^3*c^3 - 3*B*a^3*c^2*d + 9*A*a^2*b*c^2*d
 - 9*C*a^2*b*c^2*d + 9*B*a*b^2*c^2*d - 3*A*b^3*c^2*d + 3*C*b^3*c^2*d - 3*A*
a^3*c*d^2 + 3*C*a^3*c*d^2 - 9*B*a^2*b*c*d^2 + 9*A*a*b^2*c*d^2 - 9*C*a*b^2*c
*d^2 + 3*B*b^3*c*d^2 + B*a^3*d^3 - 3*A*a^2*b*d^3 + 3*C*a^2*b*d^3 - 3*B*a*b^
2*d^3 + A*b^3*d^3 - C*b^3*d^3)*(f*x + e)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
) + (B*a^3*c^3 - 3*A*a^2*b*c^3 + 3*C*a^2*b*c^3 - 3*B*a*b^2*c^3 + A*b^3*c^3
- C*b^3*c^3 + 3*A*a^3*c^2*d - 3*C*a^3*c^2*d + 9*B*a^2*b*c^2*d - 9*A*a*b^2*c
^2*d + 9*C*a*b^2*c^2*d - 3*B*b^3*c^2*d - 3*B*a^3*c*d^2 + 9*A*a^2*b*c*d^2 -
9*C*a^2*b*c*d^2 + 9*B*a*b^2*c*d^2 - 3*A*b^3*c*d^2 + 3*C*b^3*c*d^2 - A*a^3*d
^3 + C*a^3*d^3 - 3*B*a^2*b*d^3 + 3*A*a*b^2*d^3 - 3*C*a*b^2*d^3 + B*b^3*d^3)
*log(tan(f*x + e)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 2*(B*a^3*b^4
*c^3 - 3*A*a^2*b^5*c^3 + 3*C*a^2*b^5*c^3 - 3*B*a*b^6*c^3 + A*b^7*c^3 - C*b^
7*c^3 + 3*A*a^3*b^4*c^2*d - 3*C*a^3*b^4*c^2*d + 9*B*a^2*b^5*c^2*d - 9*A*a*b
^6*c^2*d + 9*C*a*b^6*c^2*d - 3*B*b^7*c^2*d - 3*C*a^6*b*c*d^2 - 9*C*a^4*b^3*
c*d^2 - 3*B*a^3*b^4*c*d^2 + 9*A*a^2*b^5*c*d^2 - 18*C*a^2*b^5*c*d^2 + 9*B*a*
b^6*c*d^2 - 3*A*b^7*c*d^2 + 3*C*a^7*d^3 - B*a^6*b*d^3 + 9*C*a^5*b^2*d^3 - 3
*B*a^4*b^3*d^3 - A*a^3*b^4*d^3 + 10*C*a^3*b^4*d^3 - 6*B*a^2*b^5*d^3 + 3*A*a
*b^6*d^3)*log(abs(b*tan(f*x + e) + a))/(a^6*b^4 + 3*a^4*b^6 + 3*a^2*b^8 + b
^10) + (3*B*a^3*b^6*c^3*tan(f*x + e)^2 - 9*A*a^2*b^7*c^3*tan(f*x + e)^2 + 9
*C*a^2*b^7*c^3*tan(f*x + e)^2 - 9*B*a*b^8*c^3*tan(f*x + e)^2 + 3*A*b^9*c^3*
```

```
tan(f*x + e)^2 - 3*C*b^9*c^3*tan(f*x + e)^2 + 9*A*a^3*b^6*c^2*d*tan(f*x + e
)^2 - 9*C*a^3*b^6*c^2*d*tan(f*x + e)^2 + 27*B*a^2*b^7*c^2*d*tan(f*x + e)^2
- 27*A*a*b^8*c^2*d*tan(f*x + e)^2 + 27*C*a*b^8*c^2*d*tan(f*x + e)^2 - 9*B*b
^9*c^2*d*tan(f*x + e)^2 - 9*C*a^6*b^3*c*d^2*tan(f*x + e)^2 - 27*C*a^4*b^5*c
*d^2*tan(f*x + e)^2 - 9*B*a^3*b^6*c*d^2*tan(f*x + e)^2 + 27*A*a^2*b^7*c*d^2
*tan(f*x + e)^2 - 54*C*a^2*b^7*c*d^2*tan(f*x + e)^2 + 27*B*a*b^8*c*d^2*tan(
f*x + e)^2 - 9*A*b^9*c*d^2*tan(f*x + e)^2 + 9*C*a^7*b^2*d^3*tan(f*x + e)^2
- 3*B*a^6*b^3*d^3*tan(f*x + e)^2 + 27*C*a^5*b^4*d^3*tan(f*x + e)^2 - 9*B*a^
4*b^5*d^3*tan(f*x + e)^2 - 3*A*a^3*b^6*d^3*tan(f*x + e)^2 + 30*C*a^3*b^6*d^
3*tan(f*x + e)^2 - 18*B*a^2*b^7*d^3*tan(f*x + e)^2 + 9*A*a*b^8*d^3*tan(f*x
+ e)^2 + 8*B*a^4*b^5*c^3*tan(f*x + e) - 22*A*a^3*b^6*c^3*tan(f*x + e) + 22*
C*a^3*b^6*c^3*tan(f*x + e) - 18*B*a^2*b^7*c^3*tan(f*x + e) + 2*A*a*b^8*c^3*
tan(f*x + e) - 2*C*a*b^8*c^3*tan(f*x + e) - 2*B*b^9*c^3*tan(f*x + e) - 6*C*
a^6*b^3*c^2*d*tan(f*x + e) + 24*A*a^4*b^5*c^2*d*tan(f*x + e) - 42*C*a^4*b^5
*c^2*d*tan(f*x + e) + 66*B*a^3*b^6*c^2*d*tan(f*x + e) - 54*A*a^2*b^7*c^2*d*
tan(f*x + e) + 36*C*a^2*b^7*c^2*d*tan(f*x + e) - 6*B*a*b^8*c^2*d*tan(f*x +
e) - 6*A*b^9*c^2*d*tan(f*x + e) - 6*C*a^7*b^2*c*d^2*tan(f*x + e) - 6*B*a^6*
b^3*c*d^2*tan(f*x + e) - 18*C*a^5*b^4*c*d^2*tan(f*x + e) - 42*B*a^4*b^5*c*d
^2*tan(f*x + e) + 66*A*a^3*b^6*c*d^2*tan(f*x + e) - 84*C*a^3*b^6*c*d^2*tan(
f*x + e) + 36*B*a^2*b^7*c*d^2*tan(f*x + e) - 6*A*a*b^8*c*d^2*tan(f*x + e) +
 12*C*a^8*b*d^3*tan(f*x + e) - 2*B*a^7*b^2*d^3*tan(f*x + e) - 2*A*a^6*b^3*d
^3*tan(f*x + e) + 38*C*a^6*b^3*d^3*tan(f*x + e) - 6*B*a^5*b^4*d^3*tan(f*x +
 e) - 14*A*a^4*b^5*d^3*tan(f*x + e) + 50*C*a^4*b^5*d^3*tan(f*x + e) - 28*B*
a^3*b^6*d^3*tan(f*x + e) + 12*A*a^2*b^7*d^3*tan(f*x + e) - C*a^6*b^3*c^3 +
6*B*a^5*b^4*c^3 - 14*A*a^4*b^5*c^3 + 11*C*a^4*b^5*c^3 - 7*B*a^3*b^6*c^3 - 3
*A*a^2*b^7*c^3 - B*a*b^8*c^3 - A*b^9*c^3 - 3*C*a^7*b^2*c^2*d - 3*B*a^6*b^3*
c^2*d + 18*A*a^5*b^4*c^2*d - 27*C*a^5*b^4*c^2*d + 33*B*a^4*b^5*c^2*d - 21*A
*a^3*b^6*c^2*d + 12*C*a^3*b^6*c^2*d - 3*A*a*b^8*c^2*d - 3*B*a^7*b^2*c*d^2 -
 3*A*a^6*b^3*c*d^2 + 3*C*a^6*b^3*c*d^2 - 27*B*a^5*b^4*c*d^2 + 33*A*a^4*b^5*
c*d^2 - 33*C*a^4*b^5*c*d^2 + 12*B*a^3*b^6*c*d^2 + 4*C*a^9*d^3 - A*a^7*b^2*d
^3 + 13*C*a^7*b^2*d^3 + B*a^6*b^3*d^3 - 9*A*a^5*b^4*d^3 + 21*C*a^5*b^4*d^3
- 11*B*a^4*b^5*d^3 + 4*A*a^3*b^6*d^3)/((a^6*b^4 + 3*a^4*b^6 + 3*a^2*b^8 + b
^10)*(b*tan(f*x + e) + a)^2))/f
```

3.70 $\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=337

$$-\frac{\log(\cos(e+fx))\left(3a^2b(Ac+Bd-cC)+a^3(Bc-d(A-C))-3ab^2(Bc-d(A-C))-b^3(Ac+Bd-cC)\right)}{f\left(c^2+d^2\right)}+\frac{x\left(-3a^2b(\right.}{}$$

```
[Out] ((a^3*(A*c - c*C + B*d) - 3*a*b^2*(A*c - c*C + B*d) - 3*a^2*b*(B*c - (A - C
)*d) + b^3*(B*c - (A - C)*d))*x)/(c^2 + d^2) - ((3*a^2*b*(A*c - c*C + B*d)
- b^3*(A*c - c*C + B*d) + a^3*(B*c - (A - C)*d) - 3*a*b^2*(B*c - (A - C)*d)
)*Log[Cos[e + f*x]])/((c^2 + d^2)*f) - ((b*c - a*d)^3*(c^2*C - B*c*d + A*d^
2)*Log[c + d*Tan[e + f*x]])/(d^4*(c^2 + d^2)*f) + (b*(b*(A*b + a*B - b*C)*d
^2 + (b*c - a*d)*(b*c*C - b*B*d - a*C*d))*Tan[e + f*x])/(d^3*f) - ((b*c*C -
 b*B*d - a*C*d)*(a + b*Tan[e + f*x])^2)/(2*d^2*f) + (C*(a + b*Tan[e + f*x])
^3)/(3*d*f)
```

Rubi [A] time = 1.58769, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3647, 3637, 3626, 3617, 31, 3475}

$$-\frac{\log(\cos(e+fx))\left(3a^2b(Ac+Bd-cC)+a^3(Bc-d(A-C))-3ab^2(Bc-d(A-C))-b^3(Ac+Bd-cC)\right)}{f\left(c^2+d^2\right)}+\frac{x\left(-3a^2b(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d
*Tan[e + f*x]),x]
```


```
[Out] ((a^3*(A*c - c*C + B*d) - 3*a*b^2*(A*c - c*C + B*d) - 3*a^2*b*(B*c - (A - C
)*d) + b^3*(B*c - (A - C)*d))*x)/(c^2 + d^2) - ((3*a^2*b*(A*c - c*C + B*d)
- b^3*(A*c - c*C + B*d) + a^3*(B*c - (A - C)*d) - 3*a*b^2*(B*c - (A - C)*d)
)*Log[Cos[e + f*x]])/((c^2 + d^2)*f) - ((b*c - a*d)^3*(c^2*C - B*c*d + A*d^
2)*Log[c + d*Tan[e + f*x]])/(d^4*(c^2 + d^2)*f) + (b*(b*(A*b + a*B - b*C)*d
^2 + (b*c - a*d)*(b*c*C - b*B*d - a*C*d))*Tan[e + f*x])/(d^3*f) - ((b*c*C -
 b*B*d - a*C*d)*(a + b*Tan[e + f*x])^2)/(2*d^2*f) + (C*(a + b*Tan[e + f*x])
^3)/(3*d*f)
```


Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
```

```
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3626

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((a*A + b*B -
a*C)*x)/(a^2 + b^2), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3617

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{c+d\tan(e+fx)}\,dx = \frac{C(a+b\tan(e+fx))^3}{3df} + \frac{\int \frac{(a+b\tan(e+fx))^2\left(-3(bcC-aAd)\right.}{}}{}$$

$$= -\frac{(bcC-bBd-aCd)(a+b\tan(e+fx))^2}{2d^2f} + \frac{C(a+b\,t}{}$$

$$= \frac{b\left(b(Ab+aB-bC)d^2+(bc-ad)(bcC-bBd-aCd)\right.}{d^3f}$$

$$= \frac{\left(a^3(Ac-cC+Bd)-3ab^2(Ac-cC+Bd)-3a^2b(Bc\right.}{c^2+d^2}$$

$$= \frac{\left(a^3(Ac-cC+Bd)-3ab^2(Ac-cC+Bd)-3a^2b(Bc\right.}{c^2+d^2}$$

$$= \frac{\left(a^3(Ac-cC+Bd)-3ab^2(Ac-cC+Bd)-3a^2b(Bc\right.}{c^2+d^2}$$

Mathematica [C] time = 4.2852, size = 258, normalized size = 0.77

$$\frac{6b^2d\tan(e+fx)(aB+Ab-bC)+\frac{6(ad-bc)^3\left(Ad^2-Bcd+c^2C\right)\log(c+d\tan(e+fx))}{d^2\left(c^2+d^2\right)}+\frac{3d^2(a-ib)^3(iA+B-iC)\log(\tan(e+fx)+i)}{c-id}+\frac{3d^2(a+ib)^3(-iA+}{}}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(c + d*Tan[e + f*x]),x]
```

```
[Out] ((3*(a + I*b)^3*((-I)*A + B + I*C)*d^2*Log[I - Tan[e + f*x]])/(c + I*d) + (
3*(a - I*b)^3*(I*A + B - I*C)*d^2*Log[I + Tan[e + f*x]])/(c - I*d) + (6*(-(
b*c) + a*d)^3*(c^2*C - B*c*d + A*d^2)*Log[c + d*Tan[e + f*x]])/(d^2*(c^2 +
d^2)) + 6*b^2*(A*b + a*B - b*C)*d*Tan[e + f*x] - (6*b*(b*c - a*d)*(-(b*c*C)
 + b*B*d + a*C*d)*Tan[e + f*x])/d - 3*(b*c*C - b*B*d - a*C*d)*(a + b*Tan[e
+ f*x])^2 + 2*C*d*(a + b*Tan[e + f*x])^3)/(6*d^2*f)
```

Maple [B] time = 0.054, size = 1304, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)),x)
```

```
[Out] 1/f/(c^2+d^2)*B*arctan(tan(f*x+e))*a^3*d+1/f/(c^2+d^2)*B*arctan(tan(f*x+e))
*b^3*c-1/f/(c^2+d^2)*C*arctan(tan(f*x+e))*a^3*c+1/f/(c^2+d^2)*C*arctan(tan(
f*x+e))*b^3*d+3/2/f*b^2/d*C*tan(f*x+e)^2*a-1/2/f*b^3/d^2*C*tan(f*x+e)^2*c+3
/f*b^2/d*B*a*tan(f*x+e)-1/f*b^3/d^2*B*c*tan(f*x+e)+3/f*b/d*a^2*C*tan(f*x+e)
+1/f*b^3/d^3*C*c^2*tan(f*x+e)+1/3/f*b^3/d*C*tan(f*x+e)^3+3/2/f/(c^2+d^2)*ln
(1+tan(f*x+e)^2)*B*a^2*b*d-3/2/f/(c^2+d^2)*ln(1+tan(f*x+e)^2)*B*a*b^2*c-3/2
/f/(c^2+d^2)*ln(1+tan(f*x+e)^2)*C*a^2*b*c-3/2/f/(c^2+d^2)*ln(1+tan(f*x+e)^2
)*C*a*b^2*d+3/f/(c^2+d^2)*A*arctan(tan(f*x+e))*a^2*b*d-3/f/(c^2+d^2)*A*arct
an(tan(f*x+e))*a*b^2*c-3/f/(c^2+d^2)*B*arctan(tan(f*x+e))*a^2*b*c-3/f/(c^2+
d^2)*B*arctan(tan(f*x+e))*a*b^2*d+1/f/d^3/(c^2+d^2)*ln(c+d*tan(f*x+e))*B*c^
4*b^3+1/f/d/(c^2+d^2)*ln(c+d*tan(f*x+e))*C*c^2*a^3-3/f/(c^2+d^2)*ln(c+d*tan
(f*x+e))*A*a^2*c*b-1/f/d^4/(c^2+d^2)*ln(c+d*tan(f*x+e))*C*c^5*b^3+3/f/d/(c^
2+d^2)*ln(c+d*tan(f*x+e))*A*c^2*a*b^2-3/f/d^2/(c^2+d^2)*ln(c+d*tan(f*x+e))*
B*c^3*a*b^2-3/f/d^2/(c^2+d^2)*ln(c+d*tan(f*x+e))*C*c^3*a^2*b+3/f/d^3/(c^2+d
^2)*ln(c+d*tan(f*x+e))*C*c^4*a*b^2+3/f/d/(c^2+d^2)*ln(c+d*tan(f*x+e))*B*c^2
*a^2*b+3/f/(c^2+d^2)*C*arctan(tan(f*x+e))*a*b^2*c-1/f/d^2/(c^2+d^2)*ln(c+d*
tan(f*x+e))*A*c^3*b^3+3/2/f/(c^2+d^2)*ln(1+tan(f*x+e)^2)*A*a*b^2*d+1/2/f*b^
3/d*B*tan(f*x+e)^2+1/f*b^3/d*A*tan(f*x+e)-1/f*b^3/d*C*tan(f*x+e)-3/f*b^2/d^
2*C*a*c*tan(f*x+e)+3/2/f/(c^2+d^2)*ln(1+tan(f*x+e)^2)*A*a^2*b*c-3/f/(c^2+d^
2)*C*arctan(tan(f*x+e))*a^2*b*d+1/f/(c^2+d^2)*A*arctan(tan(f*x+e))*a^3*c-1/
f/(c^2+d^2)*A*arctan(tan(f*x+e))*b^3*d+1/f*d/(c^2+d^2)*ln(c+d*tan(f*x+e))*A
*a^3-1/f/(c^2+d^2)*ln(c+d*tan(f*x+e))*B*a^3*c-1/2/f/(c^2+d^2)*ln(1+tan(f*x+
e)^2)*A*a^3*d-1/2/f/(c^2+d^2)*ln(1+tan(f*x+e)^2)*A*b^3*c+1/2/f/(c^2+d^2)*ln
(1+tan(f*x+e)^2)*B*a^3*c-1/2/f/(c^2+d^2)*ln(1+tan(f*x+e)^2)*B*b^3*d+1/2/f/(
c^2+d^2)*ln(1+tan(f*x+e)^2)*a^3*C*d+1/2/f/(c^2+d^2)*ln(1+tan(f*x+e)^2)*C*b^
3*c
```

Maxima [A] time = 1.54641, size = 601, normalized size = 1.78

$$\frac{\frac{6\left(\left(\left(A-C\right)a^3-3\,Ba^2b-3\,\left(A-C\right)ab^2+Bb^3\right)c+\left(Ba^3+3\,\left(A-C\right)a^2b-3\,Bab^2-\left(A-C\right)b^3\right)d\right)\left(fx+e\right)}{c^2+d^2}-\frac{6\left(Cb^3c^5-Aa^3d^5-\left(3\,Cab^2+Bb^3\right)c^4d+\left(3\,Ca^2b+3\,Bab^2+Ab^3\right)c^3d^2}{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
)),x, algorithm="maxima")
```

```
[Out] 1/6*(6*(((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*c + (B*a^3 + 3*
(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*d)*(f*x + e)/(c^2 + d^2) - 6*(C*b^
3*c^5 - A*a^3*d^5 - (3*C*a*b^2 + B*b^3)*c^4*d + (3*C*a^2*b + 3*B*a*b^2 + A*
b^3)*c^3*d^2 - (C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c^2*d^3 + (B*a^3 + 3*A*a^2*b
)*c*d^4)*log(d*tan(f*x + e) + c)/(c^2*d^4 + d^6) + 3*((B*a^3 + 3*(A - C)*a^
2*b - 3*B*a*b^2 - (A - C)*b^3)*c - ((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b
^2 + B*b^3)*d)*log(tan(f*x + e)^2 + 1)/(c^2 + d^2) + (2*C*b^3*d^2*tan(f*x +
 e)^3 - 3*(C*b^3*c*d - (3*C*a*b^2 + B*b^3)*d^2)*tan(f*x + e)^2 + 6*(C*b^3*c
^2 - (3*C*a*b^2 + B*b^3)*c*d + (3*C*a^2*b + 3*B*a*b^2 + (A - C)*b^3)*d^2)*t
an(f*x + e))/d^3)/f
```

Fricas [A] time = 5.58118, size = 1315, normalized size = 3.9

$$\frac{2\left(Cb^3c^2d^3 + Cb^3d^5\right)\tan\left(fx+e\right)^3 + 6\left(\left((A-C)a^3 - 3\,Ba^2b - 3\,(A-C)ab^2 + Bb^3\right)cd^4 + \left(Ba^3 + 3\,(A-C)a^2b - 3\,Bab^2 -\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*(C*b^3*c^2*d^3 + C*b^3*d^5)*tan(f*x + e)^3 + 6*(((A - C)*a^3 - 3*B*a
^2*b - 3*(A - C)*a*b^2 + B*b^3)*c*d^4 + (B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^
2 - (A - C)*b^3)*d^5)*f*x - 3*(C*b^3*c^3*d^2 + C*b^3*c*d^4 - (3*C*a*b^2 + B
*b^3)*c^2*d^3 - (3*C*a*b^2 + B*b^3)*d^5)*tan(f*x + e)^2 - 3*(C*b^3*c^5 - A*
a^3*d^5 - (3*C*a*b^2 + B*b^3)*c^4*d + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^3*d
^2 - (C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c^2*d^3 + (B*a^3 + 3*A*a^2*b)*c*d^4)*l
og((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) +
3*(C*b^3*c^5 - (3*C*a*b^2 + B*b^3)*c^4*d + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*
c^3*d^2 - (C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c^2*d^3 + (3*C*a^2*b + 3*B*a*b^2
+ (A - C)*b^3)*c*d^4 - (C*a^3 + 3*B*a^2*b + 3*(A - C)*a*b^2 - B*b^3)*d^5)*l
og(1/(tan(f*x + e)^2 + 1)) + 6*(C*b^3*c^4*d - (3*C*a*b^2 + B*b^3)*c^3*d^2 +
 (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^2*d^3 - (3*C*a*b^2 + B*b^3)*c*d^4 + (3*C
*a^2*b + 3*B*a*b^2 + (A - C)*b^3)*d^5)*tan(f*x + e))/((c^2*d^4 + d^6)*f)
```

Sympy [A] time = 46.6404, size = 7096, normalized size = 21.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x
+e)),x)
```

```
[Out] Piecewise((zoo*x*(a + b*tan(e))**3*(A + B*tan(e) + C*tan(e)**2)/tan(e), Eq(
c, 0) & Eq(d, 0) & Eq(f, 0)), (-3*I*A*a**3*f*x*tan(e + f*x)/(-6*d*f*tan(e +
 f*x) + 6*I*d*f) - 3*A*a**3*f*x/(-6*d*f*tan(e + f*x) + 6*I*d*f) - 3*I*A*a**
3/(-6*d*f*tan(e + f*x) + 6*I*d*f) - 9*A*a**2*b*f*x*tan(e + f*x)/(-6*d*f*tan
(e + f*x) + 6*I*d*f) + 9*I*A*a**2*b*f*x/(-6*d*f*tan(e + f*x) + 6*I*d*f) + 9
```

```
*A*a**2*b/(-6*d*f*tan(e + f*x) + 6*I*d*f) - 9*I*A*a*b**2*f*x*tan(e + f*x)/(
-6*d*f*tan(e + f*x) + 6*I*d*f) - 9*A*a*b**2*f*x/(-6*d*f*tan(e + f*x) + 6*I*
d*f) - 9*A*a*b**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(-6*d*f*tan(e + f*x
) + 6*I*d*f) + 9*I*A*a*b**2*log(tan(e + f*x)**2 + 1)/(-6*d*f*tan(e + f*x) +
 6*I*d*f) + 9*I*A*a*b**2/(-6*d*f*tan(e + f*x) + 6*I*d*f) + 9*A*b**3*f*x*tan
(e + f*x)/(-6*d*f*tan(e + f*x) + 6*I*d*f) - 9*I*A*b**3*f*x/(-6*d*f*tan(e +
f*x) + 6*I*d*f) - 3*I*A*b**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(-6*d*f*
tan(e + f*x) + 6*I*d*f) - 3*A*b**3*log(tan(e + f*x)**2 + 1)/(-6*d*f*tan(e +
 f*x) + 6*I*d*f) - 6*A*b**3*tan(e + f*x)**2/(-6*d*f*tan(e + f*x) + 6*I*d*f)
 - 9*A*b**3/(-6*d*f*tan(e + f*x) + 6*I*d*f) - 3*B*a**3*f*x*tan(e + f*x)/(-6
*d*f*tan(e + f*x) + 6*I*d*f) + 3*I*B*a**3*f*x/(-6*d*f*tan(e + f*x) + 6*I*d*
f) + 3*B*a**3/(-6*d*f*tan(e + f*x) + 6*I*d*f) - 9*I*B*a**2*b*f*x*tan(e + f*
x)/(-6*d*f*tan(e + f*x) + 6*I*d*f) - 9*B*a**2*b*f*x/(-6*d*f*tan(e + f*x) +
6*I*d*f) - 9*B*a**2*b*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(-6*d*f*tan(e +
 f*x) + 6*I*d*f) + 9*I*B*a**2*b*log(tan(e + f*x)**2 + 1)/(-6*d*f*tan(e + f*
x) + 6*I*d*f) + 9*I*B*a**2*b/(-6*d*f*tan(e + f*x) + 6*I*d*f) + 27*B*a*b**2*
f*x*tan(e + f*x)/(-6*d*f*tan(e + f*x) + 6*I*d*f) - 27*I*B*a*b**2*f*x/(-6*d*
f*tan(e + f*x) + 6*I*d*f) - 9*I*B*a*b**2*log(tan(e + f*x)**2 + 1)*tan(e + f
*x)/(-6*d*f*tan(e + f*x) + 6*I*d*f) - 9*B*a*b**2*log(tan(e + f*x)**2 + 1)/(
-6*d*f*tan(e + f*x) + 6*I*d*f) - 18*B*a*b**2*tan(e + f*x)**2/(-6*d*f*tan(e
+ f*x) + 6*I*d*f) - 27*B*a*b**2/(-6*d*f*tan(e + f*x) + 6*I*d*f) + 9*I*B*b**
3*f*x*tan(e + f*x)/(-6*d*f*tan(e + f*x) + 6*I*d*f) + 9*B*b**3*f*x/(-6*d*f*t
an(e + f*x) + 6*I*d*f) + 6*B*b**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(-6
*d*f*tan(e + f*x) + 6*I*d*f) - 6*I*B*b**3*log(tan(e + f*x)**2 + 1)/(-6*d*f*
tan(e + f*x) + 6*I*d*f) - 3*B*b**3*tan(e + f*x)**3/(-6*d*f*tan(e + f*x) + 6
*I*d*f) - 3*I*B*b**3*tan(e + f*x)**2/(-6*d*f*tan(e + f*x) + 6*I*d*f) - 9*I*
B*b**3/(-6*d*f*tan(e + f*x) + 6*I*d*f) - 3*I*C*a**3*f*x*tan(e + f*x)/(-6*d*
f*tan(e + f*x) + 6*I*d*f) - 3*C*a**3*f*x/(-6*d*f*tan(e + f*x) + 6*I*d*f) -
3*C*a**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(-6*d*f*tan(e + f*x) + 6*I*d
*f) + 3*I*C*a**3*log(tan(e + f*x)**2 + 1)/(-6*d*f*tan(e + f*x) + 6*I*d*f) +
 3*I*C*a**3/(-6*d*f*tan(e + f*x) + 6*I*d*f) + 27*C*a**2*b*f*x*tan(e + f*x)/
(-6*d*f*tan(e + f*x) + 6*I*d*f) - 27*I*C*a**2*b*f*x/(-6*d*f*tan(e + f*x) +
6*I*d*f) - 9*I*C*a**2*b*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(-6*d*f*tan(e
 + f*x) + 6*I*d*f) - 9*C*a**2*b*log(tan(e + f*x)**2 + 1)/(-6*d*f*tan(e + f*
x) + 6*I*d*f) - 18*C*a**2*b*tan(e + f*x)**2/(-6*d*f*tan(e + f*x) + 6*I*d*f)
 - 27*C*a**2*b/(-6*d*f*tan(e + f*x) + 6*I*d*f) + 27*I*C*a*b**2*f*x*tan(e +
f*x)/(-6*d*f*tan(e + f*x) + 6*I*d*f) + 27*C*a*b**2*f*x/(-6*d*f*tan(e + f*x)
 + 6*I*d*f) + 18*C*a*b**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(-6*d*f*tan
(e + f*x) + 6*I*d*f) - 18*I*C*a*b**2*log(tan(e + f*x)**2 + 1)/(-6*d*f*tan(e
 + f*x) + 6*I*d*f) - 9*C*a*b**2*tan(e + f*x)**3/(-6*d*f*tan(e + f*x) + 6*I*
d*f) - 9*I*C*a*b**2*tan(e + f*x)**2/(-6*d*f*tan(e + f*x) + 6*I*d*f) - 27*I*
C*a*b**2/(-6*d*f*tan(e + f*x) + 6*I*d*f) - 15*C*b**3*f*x*tan(e + f*x)/(-6*d
*f*tan(e + f*x) + 6*I*d*f) + 15*I*C*b**3*f*x/(-6*d*f*tan(e + f*x) + 6*I*d*f
) + 6*I*C*b**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(-6*d*f*tan(e + f*x) +
 6*I*d*f) + 6*C*b**3*log(tan(e + f*x)**2 + 1)/(-6*d*f*tan(e + f*x) + 6*I*d*
f) - 2*C*b**3*tan(e + f*x)**4/(-6*d*f*tan(e + f*x) + 6*I*d*f) - I*C*b**3*ta
n(e + f*x)**3/(-6*d*f*tan(e + f*x) + 6*I*d*f) + 9*C*b**3*tan(e + f*x)**2/(-
6*d*f*tan(e + f*x) + 6*I*d*f) + 15*C*b**3/(-6*d*f*tan(e + f*x) + 6*I*d*f),
Eq(c, -I*d)), (-3*I*A*a**3*f*x*tan(e + f*x)/(6*d*f*tan(e + f*x) + 6*I*d*f)
+ 3*A*a**3*f*x/(6*d*f*tan(e + f*x) + 6*I*d*f) - 3*I*A*a**3/(6*d*f*tan(e + f
*x) + 6*I*d*f) + 9*A*a**2*b*f*x*tan(e + f*x)/(6*d*f*tan(e + f*x) + 6*I*d*f)
 + 9*I*A*a**2*b*f*x/(6*d*f*tan(e + f*x) + 6*I*d*f) - 9*A*a**2*b/(6*d*f*tan(
e + f*x) + 6*I*d*f) - 9*I*A*a*b**2*f*x*tan(e + f*x)/(6*d*f*tan(e + f*x) + 6
*I*d*f) + 9*A*a*b**2*f*x/(6*d*f*tan(e + f*x) + 6*I*d*f) + 9*A*a*b**2*log(ta
n(e + f*x)**2 + 1)*tan(e + f*x)/(6*d*f*tan(e + f*x) + 6*I*d*f) + 9*I*A*a*b*
*2*log(tan(e + f*x)**2 + 1)/(6*d*f*tan(e + f*x) + 6*I*d*f) + 9*I*A*a*b**2/(
6*d*f*tan(e + f*x) + 6*I*d*f) - 9*A*b**3*f*x*tan(e + f*x)/(6*d*f*tan(e + f*
x) + 6*I*d*f) - 9*I*A*b**3*f*x/(6*d*f*tan(e + f*x) + 6*I*d*f) - 3*I*A*b**3*
log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(6*d*f*tan(e + f*x) + 6*I*d*f) + 3*A*
```

```
b**3*log(tan(e + f*x)**2 + 1)/(6*d*f*tan(e + f*x) + 6*I*d*f) + 6*A*b**3*tan
(e + f*x)**2/(6*d*f*tan(e + f*x) + 6*I*d*f) + 9*A*b**3/(6*d*f*tan(e + f*x)
+ 6*I*d*f) + 3*B*a**3*f*x*tan(e + f*x)/(6*d*f*tan(e + f*x) + 6*I*d*f) + 3*I
*B*a**3*f*x/(6*d*f*tan(e + f*x) + 6*I*d*f) - 3*B*a**3/(6*d*f*tan(e + f*x) +
 6*I*d*f) - 9*I*B*a**2*b*f*x*tan(e + f*x)/(6*d*f*tan(e + f*x) + 6*I*d*f) +
9*B*a**2*b*f*x/(6*d*f*tan(e + f*x) + 6*I*d*f) + 9*B*a**2*b*log(tan(e + f*x)
**2 + 1)*tan(e + f*x)/(6*d*f*tan(e + f*x) + 6*I*d*f) + 9*I*B*a**2*b*log(tan
(e + f*x)**2 + 1)/(6*d*f*tan(e + f*x) + 6*I*d*f) + 9*I*B*a**2*b/(6*d*f*tan(
e + f*x) + 6*I*d*f) - 27*B*a*b**2*f*x*tan(e + f*x)/(6*d*f*tan(e + f*x) + 6*
I*d*f) - 27*I*B*a*b**2*f*x/(6*d*f*tan(e + f*x) + 6*I*d*f) - 9*I*B*a*b**2*lo
g(tan(e + f*x)**2 + 1)*tan(e + f*x)/(6*d*f*tan(e + f*x) + 6*I*d*f) + 9*B*a*
b**2*log(tan(e + f*x)**2 + 1)/(6*d*f*tan(e + f*x) + 6*I*d*f) + 18*B*a*b**2*
tan(e + f*x)**2/(6*d*f*tan(e + f*x) + 6*I*d*f) + 27*B*a*b**2/(6*d*f*tan(e +
 f*x) + 6*I*d*f) + 9*I*B*b**3*f*x*tan(e + f*x)/(6*d*f*tan(e + f*x) + 6*I*d*
f) - 9*B*b**3*f*x/(6*d*f*tan(e + f*x) + 6*I*d*f) - 6*B*b**3*log(tan(e + f*x
)**2 + 1)*tan(e + f*x)/(6*d*f*tan(e + f*x) + 6*I*d*f) - 6*I*B*b**3*log(tan(
e + f*x)**2 + 1)/(6*d*f*tan(e + f*x) + 6*I*d*f) + 3*B*b**3*tan(e + f*x)**3/
(6*d*f*tan(e + f*x) + 6*I*d*f) - 3*I*B*b**3*tan(e + f*x)**2/(6*d*f*tan(e +
f*x) + 6*I*d*f) - 9*I*B*b**3/(6*d*f*tan(e + f*x) + 6*I*d*f) - 3*I*C*a**3*f*
x*tan(e + f*x)/(6*d*f*tan(e + f*x) + 6*I*d*f) + 3*C*a**3*f*x/(6*d*f*tan(e +
 f*x) + 6*I*d*f) + 3*C*a**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(6*d*f*ta
n(e + f*x) + 6*I*d*f) + 3*I*C*a**3*log(tan(e + f*x)**2 + 1)/(6*d*f*tan(e +
f*x) + 6*I*d*f) + 3*I*C*a**3/(6*d*f*tan(e + f*x) + 6*I*d*f) - 27*C*a**2*b*f
*x*tan(e + f*x)/(6*d*f*tan(e + f*x) + 6*I*d*f) - 27*I*C*a**2*b*f*x/(6*d*f*t
an(e + f*x) + 6*I*d*f) - 9*I*C*a**2*b*log(tan(e + f*x)**2 + 1)*tan(e + f*x)
/(6*d*f*tan(e + f*x) + 6*I*d*f) + 9*C*a**2*b*log(tan(e + f*x)**2 + 1)/(6*d*
f*tan(e + f*x) + 6*I*d*f) + 18*C*a**2*b*tan(e + f*x)**2/(6*d*f*tan(e + f*x)
 + 6*I*d*f) + 27*C*a**2*b/(6*d*f*tan(e + f*x) + 6*I*d*f) + 27*I*C*a*b**2*f*
x*tan(e + f*x)/(6*d*f*tan(e + f*x) + 6*I*d*f) - 27*C*a*b**2*f*x/(6*d*f*tan(
e + f*x) + 6*I*d*f) - 18*C*a*b**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(6*
d*f*tan(e + f*x) + 6*I*d*f) - 18*I*C*a*b**2*log(tan(e + f*x)**2 + 1)/(6*d*f
*tan(e + f*x) + 6*I*d*f) + 9*C*a*b**2*tan(e + f*x)**3/(6*d*f*tan(e + f*x) +
 6*I*d*f) - 9*I*C*a*b**2*tan(e + f*x)**2/(6*d*f*tan(e + f*x) + 6*I*d*f) - 2
7*I*C*a*b**2/(6*d*f*tan(e + f*x) + 6*I*d*f) + 15*C*b**3*f*x*tan(e + f*x)/(6
*d*f*tan(e + f*x) + 6*I*d*f) + 15*I*C*b**3*f*x/(6*d*f*tan(e + f*x) + 6*I*d*
f) + 6*I*C*b**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(6*d*f*tan(e + f*x) +
 6*I*d*f) - 6*C*b**3*log(tan(e + f*x)**2 + 1)/(6*d*f*tan(e + f*x) + 6*I*d*f
) + 2*C*b**3*tan(e + f*x)**4/(6*d*f*tan(e + f*x) + 6*I*d*f) - I*C*b**3*tan(
e + f*x)**3/(6*d*f*tan(e + f*x) + 6*I*d*f) - 9*C*b**3*tan(e + f*x)**2/(6*d*
f*tan(e + f*x) + 6*I*d*f) - 15*C*b**3/(6*d*f*tan(e + f*x) + 6*I*d*f), Eq(c,
 I*d)), ((A*a**3*x + 3*A*a**2*b*log(tan(e + f*x)**2 + 1)/(2*f) - 3*A*a*b**2
*x + 3*A*a*b**2*tan(e + f*x)/f - A*b**3*log(tan(e + f*x)**2 + 1)/(2*f) + A*
b**3*tan(e + f*x)**2/(2*f) + B*a**3*log(tan(e + f*x)**2 + 1)/(2*f) - 3*B*a*
*2*b*x + 3*B*a**2*b*tan(e + f*x)/f - 3*B*a*b**2*log(tan(e + f*x)**2 + 1)/(2
*f) + 3*B*a*b**2*tan(e + f*x)**2/(2*f) + B*b**3*x + B*b**3*tan(e + f*x)**3/
(3*f) - B*b**3*tan(e + f*x)/f - C*a**3*x + C*a**3*tan(e + f*x)/f - 3*C*a**2
*b*log(tan(e + f*x)**2 + 1)/(2*f) + 3*C*a**2*b*tan(e + f*x)**2/(2*f) + 3*C*
a*b**2*x + C*a*b**2*tan(e + f*x)**3/f - 3*C*a*b**2*tan(e + f*x)/f + C*b**3*
log(tan(e + f*x)**2 + 1)/(2*f) + C*b**3*tan(e + f*x)**4/(4*f) - C*b**3*tan(
e + f*x)**2/(2*f))/c, Eq(d, 0)), (x*(a + b*tan(e))**3*(A + B*tan(e) + C*tan
(e)**2)/(c + d*tan(e)), Eq(f, 0)), (6*A*a**3*c*d**4*f*x/(6*c**2*d**4*f + 6*
d**6*f) + 6*A*a**3*d**5*log(c/d + tan(e + f*x))/(6*c**2*d**4*f + 6*d**6*f)
- 3*A*a**3*d**5*log(tan(e + f*x)**2 + 1)/(6*c**2*d**4*f + 6*d**6*f) - 18*A*
a**2*b*c*d**4*log(c/d + tan(e + f*x))/(6*c**2*d**4*f + 6*d**6*f) + 9*A*a**2
*b*c*d**4*log(tan(e + f*x)**2 + 1)/(6*c**2*d**4*f + 6*d**6*f) + 18*A*a**2*b
*d**5*f*x/(6*c**2*d**4*f + 6*d**6*f) + 18*A*a*b**2*c**2*d**3*log(c/d + tan(
e + f*x))/(6*c**2*d**4*f + 6*d**6*f) - 18*A*a*b**2*c*d**4*f*x/(6*c**2*d**4*
f + 6*d**6*f) + 9*A*a*b**2*d**5*log(tan(e + f*x)**2 + 1)/(6*c**2*d**4*f + 6
*d**6*f) - 6*A*b**3*c**3*d**2*log(c/d + tan(e + f*x))/(6*c**2*d**4*f + 6*d*
```

```
*6*f) + 6*A*b**3*c**2*d**3*tan(e + f*x)/(6*c**2*d**4*f + 6*d**6*f) - 3*A*b*
*3*c*d**4*log(tan(e + f*x)**2 + 1)/(6*c**2*d**4*f + 6*d**6*f) - 6*A*b**3*d*
*5*f*x/(6*c**2*d**4*f + 6*d**6*f) + 6*A*b**3*d**5*tan(e + f*x)/(6*c**2*d**4
*f + 6*d**6*f) - 6*B*a**3*c*d**4*log(c/d + tan(e + f*x))/(6*c**2*d**4*f + 6
*d**6*f) + 3*B*a**3*c*d**4*log(tan(e + f*x)**2 + 1)/(6*c**2*d**4*f + 6*d**6
*f) + 6*B*a**3*d**5*f*x/(6*c**2*d**4*f + 6*d**6*f) + 18*B*a**2*b*c**2*d**3*
log(c/d + tan(e + f*x))/(6*c**2*d**4*f + 6*d**6*f) - 18*B*a**2*b*c*d**4*f*x
/(6*c**2*d**4*f + 6*d**6*f) + 9*B*a**2*b*d**5*log(tan(e + f*x)**2 + 1)/(6*c
**2*d**4*f + 6*d**6*f) - 18*B*a*b**2*c**3*d**2*log(c/d + tan(e + f*x))/(6*c
**2*d**4*f + 6*d**6*f) + 18*B*a*b**2*c**2*d**3*tan(e + f*x)/(6*c**2*d**4*f
+ 6*d**6*f) - 9*B*a*b**2*c*d**4*log(tan(e + f*x)**2 + 1)/(6*c**2*d**4*f + 6
*d**6*f) - 18*B*a*b**2*d**5*f*x/(6*c**2*d**4*f + 6*d**6*f) + 18*B*a*b**2*d*
*5*tan(e + f*x)/(6*c**2*d**4*f + 6*d**6*f) + 6*B*b**3*c**4*d*log(c/d + tan(
e + f*x))/(6*c**2*d**4*f + 6*d**6*f) - 6*B*b**3*c**3*d**2*tan(e + f*x)/(6*c
**2*d**4*f + 6*d**6*f) + 3*B*b**3*c**2*d**3*tan(e + f*x)**2/(6*c**2*d**4*f
+ 6*d**6*f) + 6*B*b**3*c*d**4*f*x/(6*c**2*d**4*f + 6*d**6*f) - 6*B*b**3*c*d
**4*tan(e + f*x)/(6*c**2*d**4*f + 6*d**6*f) - 3*B*b**3*d**5*log(tan(e + f*x
)**2 + 1)/(6*c**2*d**4*f + 6*d**6*f) + 3*B*b**3*d**5*tan(e + f*x)**2/(6*c**
2*d**4*f + 6*d**6*f) + 6*C*a**3*c**2*d**3*log(c/d + tan(e + f*x))/(6*c**2*d
**4*f + 6*d**6*f) - 6*C*a**3*c*d**4*f*x/(6*c**2*d**4*f + 6*d**6*f) + 3*C*a*
*3*d**5*log(tan(e + f*x)**2 + 1)/(6*c**2*d**4*f + 6*d**6*f) - 18*C*a**2*b*c
**3*d**2*log(c/d + tan(e + f*x))/(6*c**2*d**4*f + 6*d**6*f) + 18*C*a**2*b*c
**2*d**3*tan(e + f*x)/(6*c**2*d**4*f + 6*d**6*f) - 9*C*a**2*b*c*d**4*log(ta
n(e + f*x)**2 + 1)/(6*c**2*d**4*f + 6*d**6*f) - 18*C*a**2*b*d**5*f*x/(6*c**
2*d**4*f + 6*d**6*f) + 18*C*a**2*b*d**5*tan(e + f*x)/(6*c**2*d**4*f + 6*d**
6*f) + 18*C*a*b**2*c**4*d*log(c/d + tan(e + f*x))/(6*c**2*d**4*f + 6*d**6*f
) - 18*C*a*b**2*c**3*d**2*tan(e + f*x)/(6*c**2*d**4*f + 6*d**6*f) + 9*C*a*b
**2*c**2*d**3*tan(e + f*x)**2/(6*c**2*d**4*f + 6*d**6*f) + 18*C*a*b**2*c*d*
*4*f*x/(6*c**2*d**4*f + 6*d**6*f) - 18*C*a*b**2*c*d**4*tan(e + f*x)/(6*c**2
*d**4*f + 6*d**6*f) - 9*C*a*b**2*d**5*log(tan(e + f*x)**2 + 1)/(6*c**2*d**4
*f + 6*d**6*f) + 9*C*a*b**2*d**5*tan(e + f*x)**2/(6*c**2*d**4*f + 6*d**6*f)
 - 6*C*b**3*c**5*log(c/d + tan(e + f*x))/(6*c**2*d**4*f + 6*d**6*f) + 6*C*b
**3*c**4*d*tan(e + f*x)/(6*c**2*d**4*f + 6*d**6*f) - 3*C*b**3*c**3*d**2*tan
(e + f*x)**2/(6*c**2*d**4*f + 6*d**6*f) + 2*C*b**3*c**2*d**3*tan(e + f*x)**
3/(6*c**2*d**4*f + 6*d**6*f) + 3*C*b**3*c*d**4*log(tan(e + f*x)**2 + 1)/(6*
c**2*d**4*f + 6*d**6*f) - 3*C*b**3*c*d**4*tan(e + f*x)**2/(6*c**2*d**4*f +
6*d**6*f) + 6*C*b**3*d**5*f*x/(6*c**2*d**4*f + 6*d**6*f) + 2*C*b**3*d**5*ta
n(e + f*x)**3/(6*c**2*d**4*f + 6*d**6*f) - 6*C*b**3*d**5*tan(e + f*x)/(6*c*
*2*d**4*f + 6*d**6*f), True))
```

Giac [A] time = 2.42818, size = 774, normalized size = 2.3

$$\frac{\frac{6\left(Aa^3c-Ca^3c-3\,Ba^2bc-3\,Aab^2c+3\,Cab^2c+Bb^3c+Ba^3d+3\,Aa^2bd-3\,Ca^2bd-3\,Bab^2d-Ab^3d+Cb^3d\right)(fx+e)}{c^2+d^2}+\frac{3\left(Ba^3c+3\,Aa^2bc-3\,Ca^2bc-3\,Bab^2c-Ab^3c+Cb^3c\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
)),x, algorithm="giac")
```

```
[Out] 1/6*(6*(A*a^3*c - C*a^3*c - 3*B*a^2*b*c - 3*A*a*b^2*c + 3*C*a*b^2*c + B*b^3
*c + B*a^3*d + 3*A*a^2*b*d - 3*C*a^2*b*d - 3*B*a*b^2*d - A*b^3*d + C*b^3*d)
*(f*x + e)/(c^2 + d^2) + 3*(B*a^3*c + 3*A*a^2*b*c - 3*C*a^2*b*c - 3*B*a*b^2
*c - A*b^3*c + C*b^3*c - A*a^3*d + C*a^3*d + 3*B*a^2*b*d + 3*A*a*b^2*d - 3*
C*a*b^2*d - B*b^3*d)*log(tan(f*x + e)^2 + 1)/(c^2 + d^2) - 6*(C*b^3*c^5 - 3
*C*a*b^2*c^4*d - B*b^3*c^4*d + 3*C*a^2*b*c^3*d^2 + 3*B*a*b^2*c^3*d^2 + A*b^
```

```
3*c^3*d^2 - C*a^3*c^2*d^3 - 3*B*a^2*b*c^2*d^3 - 3*A*a*b^2*c^2*d^3 + B*a^3*c
*d^4 + 3*A*a^2*b*c*d^4 - A*a^3*d^5)*log(abs(d*tan(f*x + e) + c))/(c^2*d^4 +
 d^6) + (2*C*b^3*d^2*tan(f*x + e)^3 - 3*C*b^3*c*d*tan(f*x + e)^2 + 9*C*a*b^
2*d^2*tan(f*x + e)^2 + 3*B*b^3*d^2*tan(f*x + e)^2 + 6*C*b^3*c^2*tan(f*x + e
) - 18*C*a*b^2*c*d*tan(f*x + e) - 6*B*b^3*c*d*tan(f*x + e) + 18*C*a^2*b*d^2
*tan(f*x + e) + 18*B*a*b^2*d^2*tan(f*x + e) + 6*A*b^3*d^2*tan(f*x + e) - 6*
C*b^3*d^2*tan(f*x + e))/d^3)/f
```

3.71 $\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=236

$$-\frac{\log(\cos(e+fx))\left(a^2(Bc-d(A-C))+2ab(Ac+Bd-cC)-b^2(Bc-d(A-C))\right)}{f\left(c^2+d^2\right)}+\frac{x\left(a^2(Ac+Bd-cC)-2ab(Bc-\right.}{c^2+}$$

```
[Out] ((a^2*(A*c - c*C + B*d) - b^2*(A*c - c*C + B*d) - 2*a*b*(B*c - (A - C)*d))*
x)/(c^2 + d^2) - ((2*a*b*(A*c - c*C + B*d) + a^2*(B*c - (A - C)*d) - b^2*(B
*c - (A - C)*d))*Log[Cos[e + f*x]])/((c^2 + d^2)*f) + ((b*c - a*d)^2*(c^2*C
 - B*c*d + A*d^2)*Log[c + d*Tan[e + f*x]])/(d^3*(c^2 + d^2)*f) - (b*(b*c*C
- b*B*d - a*C*d)*Tan[e + f*x])/(d^2*f) + (C*(a + b*Tan[e + f*x])^2)/(2*d*f)
```

Rubi [A] time = 0.804035, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3647, 3637, 3626, 3617, 31, 3475}

$$-\frac{\log(\cos(e+fx))\left(a^2(Bc-d(A-C))+2ab(Ac+Bd-cC)-b^2(Bc-d(A-C))\right)}{f\left(c^2+d^2\right)}+\frac{x\left(a^2(Ac+Bd-cC)-2ab(Bc-\right.}{c^2+}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d
*Tan[e + f*x]),x]
```

```
[Out] ((a^2*(A*c - c*C + B*d) - b^2*(A*c - c*C + B*d) - 2*a*b*(B*c - (A - C)*d))*
x)/(c^2 + d^2) - ((2*a*b*(A*c - c*C + B*d) + a^2*(B*c - (A - C)*d) - b^2*(B
*c - (A - C)*d))*Log[Cos[e + f*x]])/((c^2 + d^2)*f) + ((b*c - a*d)^2*(c^2*C
 - B*c*d + A*d^2)*Log[c + d*Tan[e + f*x]])/(d^3*(c^2 + d^2)*f) - (b*(b*c*C
- b*B*d - a*C*d)*Tan[e + f*x])/(d^2*f) + (C*(a + b*Tan[e + f*x])^2)/(2*d*f)
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3626

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((a*A + b*B -
a*C)*x)/(a^2 + b^2), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3617

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{c+d\tan(e+fx)}\,dx = \frac{C(a+b\tan(e+fx))^2}{2df} + \frac{\int \frac{(a+b\tan(e+fx))\left(-2(bcC-aAd)+\right.}{}}{}$$

$$= -\frac{b(bcC-bBd-aCd)\tan(e+fx)}{d^2f} + \frac{C(a+b\tan(e+}{2df}$$

$$= \frac{\left(a^2(Ac-cC+Bd)-b^2(Ac-cC+Bd)-2ab(Bc-(\right.}{c^2+d^2}$$

$$= \frac{\left(a^2(Ac-cC+Bd)-b^2(Ac-cC+Bd)-2ab(Bc-(\right.}{c^2+d^2}$$

$$= \frac{\left(a^2(Ac-cC+Bd)-b^2(Ac-cC+Bd)-2ab(Bc-(\right.}{c^2+d^2}$$

Mathematica [C] time = 2.89784, size = 190, normalized size = 0.81

$$\frac{\frac{2(bc-ad)^2\left(Ad^2-Bcd+c^2C\right)\log(c+d\tan(e+fx))}{d^2\left(c^2+d^2\right)}+\frac{d(a-ib)^2(iA+B-iC)\log(\tan(e+fx)+i)}{c-id}+\frac{d(a+ib)^2(-iA+B+iC)\log(-\tan(e+fx)+i)}{c+id}+\frac{2b\tan(e+fx)(aCd+b}{d}}{2df}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(c + d*Tan[e + f*x]),x]
```

```
[Out] (((a + I*b)^2*((-I)*A + B + I*C)*d*Log[I - Tan[e + f*x]])/(c + I*d) + ((a -
 I*b)^2*(I*A + B - I*C)*d*Log[I + Tan[e + f*x]])/(c - I*d) + (2*(b*c - a*d)
```

```
^2*(c^2*C - B*c*d + A*d^2)*Log[c + d*Tan[e + f*x]])/(d^2*(c^2 + d^2)) + (2*
b*(-(b*c*C) + b*B*d + a*C*d)*Tan[e + f*x])/d + C*(a + b*Tan[e + f*x])^2)/(2
*d*f)
```

Maple [B] time = 0.046, size = 861, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)),x)
```

```
[Out] 2/f*b/d*a*C*tan(f*x+e)-1/f*b^2/d^2*C*c*tan(f*x+e)+2/f/d/(c^2+d^2)*ln(c+d*ta
n(f*x+e))*B*c^2*a*b-2/f/d^2/(c^2+d^2)*ln(c+d*tan(f*x+e))*C*c^3*a*b-1/f/d^2/
(c^2+d^2)*ln(c+d*tan(f*x+e))*B*c^3*b^2+1/f*b^2/d*B*tan(f*x+e)+1/2/f*b^2/d*C
*tan(f*x+e)^2+1/f*d/(c^2+d^2)*ln(c+d*tan(f*x+e))*A*a^2-1/f/(c^2+d^2)*B*arct
an(tan(f*x+e))*b^2*d-1/f/(c^2+d^2)*C*arctan(tan(f*x+e))*a^2*c+1/f/(c^2+d^2)
*C*arctan(tan(f*x+e))*b^2*c-1/2/f/(c^2+d^2)*ln(1+tan(f*x+e)^2)*A*a^2*d+1/2/
f/(c^2+d^2)*ln(1+tan(f*x+e)^2)*A*b^2*d+1/2/f/(c^2+d^2)*ln(1+tan(f*x+e)^2)*B
*a^2*c-1/2/f/(c^2+d^2)*ln(1+tan(f*x+e)^2)*B*b^2*c+1/2/f/(c^2+d^2)*ln(1+tan(
f*x+e)^2)*C*a^2*d-1/2/f/(c^2+d^2)*ln(1+tan(f*x+e)^2)*C*b^2*d+1/f/(c^2+d^2)*
A*arctan(tan(f*x+e))*a^2*c-1/f/(c^2+d^2)*A*arctan(tan(f*x+e))*b^2*c+1/f/(c^
2+d^2)*B*arctan(tan(f*x+e))*a^2*d-1/f/(c^2+d^2)*ln(c+d*tan(f*x+e))*B*a^2*c+
1/f/d/(c^2+d^2)*ln(c+d*tan(f*x+e))*C*c^2*a^2+1/f/d^3/(c^2+d^2)*ln(c+d*tan(f
*x+e))*C*c^4*b^2+1/f/(c^2+d^2)*ln(1+tan(f*x+e)^2)*B*a*b*d-1/f/(c^2+d^2)*ln(
1+tan(f*x+e)^2)*C*a*b*c+2/f/(c^2+d^2)*A*arctan(tan(f*x+e))*a*b*d-2/f/(c^2+d
^2)*B*arctan(tan(f*x+e))*a*b*c-2/f/(c^2+d^2)*C*arctan(tan(f*x+e))*a*b*d-2/f
/(c^2+d^2)*ln(c+d*tan(f*x+e))*A*a*c*b+1/f/(c^2+d^2)*ln(1+tan(f*x+e)^2)*A*a*
b*c+1/f/d/(c^2+d^2)*ln(c+d*tan(f*x+e))*A*c^2*b^2
```

Maxima [A] time = 1.46339, size = 397, normalized size = 1.68

$$\frac{\frac{2\left(\left(\left((A-C)a^2-2\,Bab-(A-C)b^2\right)c+\left(Ba^2+2\,(A-C)ab-Bb^2\right)d\right)\left(fx+e\right)}{c^2+d^2}+\frac{2\left(Cb^2c^4+Aa^2d^4-\left(2\,Cab+Bb^2\right)c^3d+\left(Ca^2+2\,Bab+Ab^2\right)c^2d^2-\left(Ba^2+2\,Aab\right)cd^3\right)\log\left(d\,\mathrm{ta}\right.}{c^2d^3+d^5}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c + (B*a^2 + 2*(A - C)*a*b -
B*b^2)*d)*(f*x + e)/(c^2 + d^2) + 2*(C*b^2*c^4 + A*a^2*d^4 - (2*C*a*b + B*b
^2)*c^3*d + (C*a^2 + 2*B*a*b + A*b^2)*c^2*d^2 - (B*a^2 + 2*A*a*b)*c*d^3)*lo
g(d*tan(f*x + e) + c)/(c^2*d^3 + d^5) + ((B*a^2 + 2*(A - C)*a*b - B*b^2)*c
- ((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*d)*log(tan(f*x + e)^2 + 1)/(c^2 + d
^2) + (C*b^2*d*tan(f*x + e)^2 - 2*(C*b^2*c - (2*C*a*b + B*b^2)*d)*tan(f*x +
 e))/d^2)/f
```

Fricas [A] time = 2.7326, size = 830, normalized size = 3.52

$$\frac{2\left(\left(\left((A-C)a^2-2\,Bab-(A-C)b^2\right)cd^3+\left(Ba^2+2\,(A-C)ab-Bb^2\right)d^4\right)fx+\left(Cb^2c^2d^2+Cb^2d^4\right)\tan\left(fx+e\right)^2+\left(Cb^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c*d^3 + (B*a^2 + 2*(A - C)*a*
b - B*b^2)*d^4)*f*x + (C*b^2*c^2*d^2 + C*b^2*d^4)*tan(f*x + e)^2 + (C*b^2*c
^4 + A*a^2*d^4 - (2*C*a*b + B*b^2)*c^3*d + (C*a^2 + 2*B*a*b + A*b^2)*c^2*d^
2 - (B*a^2 + 2*A*a*b)*c*d^3)*log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) +
 c^2)/(tan(f*x + e)^2 + 1)) - (C*b^2*c^4 - (2*C*a*b + B*b^2)*c^3*d + (C*a^2
 + 2*B*a*b + A*b^2)*c^2*d^2 - (2*C*a*b + B*b^2)*c*d^3 + (C*a^2 + 2*B*a*b +
(A - C)*b^2)*d^4)*log(1/(tan(f*x + e)^2 + 1)) - 2*(C*b^2*c^3*d + C*b^2*c*d^
3 - (2*C*a*b + B*b^2)*c^2*d^2 - (2*C*a*b + B*b^2)*d^4)*tan(f*x + e))/((c^2*
d^3 + d^5)*f)
```

Sympy [A] time = 40.7374, size = 4444, normalized size = 18.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x
+e)),x)
```

```
[Out] Piecewise((zoo*x*(a + b*tan(e))**2*(A + B*tan(e) + C*tan(e)**2)/tan(e), Eq(
c, 0) & Eq(d, 0) & Eq(f, 0)), (-I*A*a**2*f*x*tan(e + f*x)/(-2*d*f*tan(e + f
*x) + 2*I*d*f) - A*a**2*f*x/(-2*d*f*tan(e + f*x) + 2*I*d*f) - I*A*a**2/(-2*
d*f*tan(e + f*x) + 2*I*d*f) - 2*A*a*b*f*x*tan(e + f*x)/(-2*d*f*tan(e + f*x)
 + 2*I*d*f) + 2*I*A*a*b*f*x/(-2*d*f*tan(e + f*x) + 2*I*d*f) + 2*A*a*b/(-2*d
*f*tan(e + f*x) + 2*I*d*f) - I*A*b**2*f*x*tan(e + f*x)/(-2*d*f*tan(e + f*x)
 + 2*I*d*f) - A*b**2*f*x/(-2*d*f*tan(e + f*x) + 2*I*d*f) - A*b**2*log(tan(e
 + f*x)**2 + 1)*tan(e + f*x)/(-2*d*f*tan(e + f*x) + 2*I*d*f) + I*A*b**2*log
(tan(e + f*x)**2 + 1)/(-2*d*f*tan(e + f*x) + 2*I*d*f) + I*A*b**2/(-2*d*f*ta
n(e + f*x) + 2*I*d*f) - B*a**2*f*x*tan(e + f*x)/(-2*d*f*tan(e + f*x) + 2*I*
d*f) + I*B*a**2*f*x/(-2*d*f*tan(e + f*x) + 2*I*d*f) + B*a**2/(-2*d*f*tan(e
+ f*x) + 2*I*d*f) - 2*I*B*a*b*f*x*tan(e + f*x)/(-2*d*f*tan(e + f*x) + 2*I*d
*f) - 2*B*a*b*f*x/(-2*d*f*tan(e + f*x) + 2*I*d*f) - 2*B*a*b*log(tan(e + f*x
)**2 + 1)*tan(e + f*x)/(-2*d*f*tan(e + f*x) + 2*I*d*f) + 2*I*B*a*b*log(tan(
e + f*x)**2 + 1)/(-2*d*f*tan(e + f*x) + 2*I*d*f) + 2*I*B*a*b/(-2*d*f*tan(e
+ f*x) + 2*I*d*f) + 3*B*b**2*f*x*tan(e + f*x)/(-2*d*f*tan(e + f*x) + 2*I*d*
f) - 3*I*B*b**2*f*x/(-2*d*f*tan(e + f*x) + 2*I*d*f) - I*B*b**2*log(tan(e +
f*x)**2 + 1)*tan(e + f*x)/(-2*d*f*tan(e + f*x) + 2*I*d*f) - B*b**2*log(tan(
e + f*x)**2 + 1)/(-2*d*f*tan(e + f*x) + 2*I*d*f) - 2*B*b**2*tan(e + f*x)**2
/(-2*d*f*tan(e + f*x) + 2*I*d*f) - 3*B*b**2/(-2*d*f*tan(e + f*x) + 2*I*d*f)
 - I*C*a**2*f*x*tan(e + f*x)/(-2*d*f*tan(e + f*x) + 2*I*d*f) - C*a**2*f*x/(
-2*d*f*tan(e + f*x) + 2*I*d*f) - C*a**2*log(tan(e + f*x)**2 + 1)*tan(e + f*
x)/(-2*d*f*tan(e + f*x) + 2*I*d*f) + I*C*a**2*log(tan(e + f*x)**2 + 1)/(-2*
d*f*tan(e + f*x) + 2*I*d*f) + I*C*a**2/(-2*d*f*tan(e + f*x) + 2*I*d*f) + 6*
C*a*b*f*x*tan(e + f*x)/(-2*d*f*tan(e + f*x) + 2*I*d*f) - 6*I*C*a*b*f*x/(-2*
d*f*tan(e + f*x) + 2*I*d*f) - 2*I*C*a*b*log(tan(e + f*x)**2 + 1)*tan(e + f*
x)/(-2*d*f*tan(e + f*x) + 2*I*d*f) - 2*C*a*b*log(tan(e + f*x)**2 + 1)/(-2*d
*f*tan(e + f*x) + 2*I*d*f) - 4*C*a*b*tan(e + f*x)**2/(-2*d*f*tan(e + f*x) +
 2*I*d*f) - 6*C*a*b/(-2*d*f*tan(e + f*x) + 2*I*d*f) + 3*I*C*b**2*f*x*tan(e
+ f*x)/(-2*d*f*tan(e + f*x) + 2*I*d*f) + 3*C*b**2*f*x/(-2*d*f*tan(e + f*x)
+ 2*I*d*f) + 2*C*b**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(-2*d*f*tan(e +
 f*x) + 2*I*d*f) - 2*I*C*b**2*log(tan(e + f*x)**2 + 1)/(-2*d*f*tan(e + f*x)
 + 2*I*d*f) - C*b**2*tan(e + f*x)**3/(-2*d*f*tan(e + f*x) + 2*I*d*f) - I*C*
```

```
b**2*tan(e + f*x)**2/(-2*d*f*tan(e + f*x) + 2*I*d*f) - 3*I*C*b**2/(-2*d*f*t
an(e + f*x) + 2*I*d*f), Eq(c, -I*d)), (-I*A*a**2*f*x*tan(e + f*x)/(2*d*f*ta
n(e + f*x) + 2*I*d*f) + A*a**2*f*x/(2*d*f*tan(e + f*x) + 2*I*d*f) - I*A*a**
2/(2*d*f*tan(e + f*x) + 2*I*d*f) + 2*A*a*b*f*x*tan(e + f*x)/(2*d*f*tan(e +
f*x) + 2*I*d*f) + 2*I*A*a*b*f*x/(2*d*f*tan(e + f*x) + 2*I*d*f) - 2*A*a*b/(2
*d*f*tan(e + f*x) + 2*I*d*f) - I*A*b**2*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x
) + 2*I*d*f) + A*b**2*f*x/(2*d*f*tan(e + f*x) + 2*I*d*f) + A*b**2*log(tan(e
 + f*x)**2 + 1)*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) + I*A*b**2*log(
tan(e + f*x)**2 + 1)/(2*d*f*tan(e + f*x) + 2*I*d*f) + I*A*b**2/(2*d*f*tan(e
 + f*x) + 2*I*d*f) + B*a**2*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f)
 + I*B*a**2*f*x/(2*d*f*tan(e + f*x) + 2*I*d*f) - B*a**2/(2*d*f*tan(e + f*x)
 + 2*I*d*f) - 2*I*B*a*b*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) + 2
*B*a*b*f*x/(2*d*f*tan(e + f*x) + 2*I*d*f) + 2*B*a*b*log(tan(e + f*x)**2 + 1
)*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) + 2*I*B*a*b*log(tan(e + f*x)*
*2 + 1)/(2*d*f*tan(e + f*x) + 2*I*d*f) + 2*I*B*a*b/(2*d*f*tan(e + f*x) + 2*
I*d*f) - 3*B*b**2*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) - 3*I*B*b
**2*f*x/(2*d*f*tan(e + f*x) + 2*I*d*f) - I*B*b**2*log(tan(e + f*x)**2 + 1)*
tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) + B*b**2*log(tan(e + f*x)**2 +
1)/(2*d*f*tan(e + f*x) + 2*I*d*f) + 2*B*b**2*tan(e + f*x)**2/(2*d*f*tan(e +
 f*x) + 2*I*d*f) + 3*B*b**2/(2*d*f*tan(e + f*x) + 2*I*d*f) - I*C*a**2*f*x*t
an(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) + C*a**2*f*x/(2*d*f*tan(e + f*x)
 + 2*I*d*f) + C*a**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*d*f*tan(e + f
*x) + 2*I*d*f) + I*C*a**2*log(tan(e + f*x)**2 + 1)/(2*d*f*tan(e + f*x) + 2*
I*d*f) + I*C*a**2/(2*d*f*tan(e + f*x) + 2*I*d*f) - 6*C*a*b*f*x*tan(e + f*x)
/(2*d*f*tan(e + f*x) + 2*I*d*f) - 6*I*C*a*b*f*x/(2*d*f*tan(e + f*x) + 2*I*d
*f) - 2*I*C*a*b*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*d*f*tan(e + f*x) +
 2*I*d*f) + 2*C*a*b*log(tan(e + f*x)**2 + 1)/(2*d*f*tan(e + f*x) + 2*I*d*f)
 + 4*C*a*b*tan(e + f*x)**2/(2*d*f*tan(e + f*x) + 2*I*d*f) + 6*C*a*b/(2*d*f*
tan(e + f*x) + 2*I*d*f) + 3*I*C*b**2*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) +
 2*I*d*f) - 3*C*b**2*f*x/(2*d*f*tan(e + f*x) + 2*I*d*f) - 2*C*b**2*log(tan(
e + f*x)**2 + 1)*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) - 2*I*C*b**2*l
og(tan(e + f*x)**2 + 1)/(2*d*f*tan(e + f*x) + 2*I*d*f) + C*b**2*tan(e + f*x
)**3/(2*d*f*tan(e + f*x) + 2*I*d*f) - I*C*b**2*tan(e + f*x)**2/(2*d*f*tan(e
 + f*x) + 2*I*d*f) - 3*I*C*b**2/(2*d*f*tan(e + f*x) + 2*I*d*f), Eq(c, I*d))
, ((A*a**2*x + A*a*b*log(tan(e + f*x)**2 + 1)/f - A*b**2*x + A*b**2*tan(e +
 f*x)/f + B*a**2*log(tan(e + f*x)**2 + 1)/(2*f) - 2*B*a*b*x + 2*B*a*b*tan(e
 + f*x)/f - B*b**2*log(tan(e + f*x)**2 + 1)/(2*f) + B*b**2*tan(e + f*x)**2/
(2*f) - C*a**2*x + C*a**2*tan(e + f*x)/f - C*a*b*log(tan(e + f*x)**2 + 1)/f
 + C*a*b*tan(e + f*x)**2/f + C*b**2*x + C*b**2*tan(e + f*x)**3/(3*f) - C*b*
*2*tan(e + f*x)/f)/c, Eq(d, 0)), (x*(a + b*tan(e))**2*(A + B*tan(e) + C*tan
(e)**2)/(c + d*tan(e)), Eq(f, 0)), (2*A*a**2*c*d**3*f*x/(2*c**2*d**3*f + 2*
d**5*f) + 2*A*a**2*d**4*log(c/d + tan(e + f*x))/(2*c**2*d**3*f + 2*d**5*f)
- A*a**2*d**4*log(tan(e + f*x)**2 + 1)/(2*c**2*d**3*f + 2*d**5*f) - 4*A*a*b
*c*d**3*log(c/d + tan(e + f*x))/(2*c**2*d**3*f + 2*d**5*f) + 2*A*a*b*c*d**3
*log(tan(e + f*x)**2 + 1)/(2*c**2*d**3*f + 2*d**5*f) + 4*A*a*b*d**4*f*x/(2*
c**2*d**3*f + 2*d**5*f) + 2*A*b**2*c**2*d**2*log(c/d + tan(e + f*x))/(2*c**
2*d**3*f + 2*d**5*f) - 2*A*b**2*c*d**3*f*x/(2*c**2*d**3*f + 2*d**5*f) + A*b
**2*d**4*log(tan(e + f*x)**2 + 1)/(2*c**2*d**3*f + 2*d**5*f) - 2*B*a**2*c*d
**3*log(c/d + tan(e + f*x))/(2*c**2*d**3*f + 2*d**5*f) + B*a**2*c*d**3*log(
tan(e + f*x)**2 + 1)/(2*c**2*d**3*f + 2*d**5*f) + 2*B*a**2*d**4*f*x/(2*c**2
*d**3*f + 2*d**5*f) + 4*B*a*b*c**2*d**2*log(c/d + tan(e + f*x))/(2*c**2*d**
3*f + 2*d**5*f) - 4*B*a*b*c*d**3*f*x/(2*c**2*d**3*f + 2*d**5*f) + 2*B*a*b*d
**4*log(tan(e + f*x)**2 + 1)/(2*c**2*d**3*f + 2*d**5*f) - 2*B*b**2*c**3*d*l
og(c/d + tan(e + f*x))/(2*c**2*d**3*f + 2*d**5*f) + 2*B*b**2*c**2*d**2*tan(
e + f*x)/(2*c**2*d**3*f + 2*d**5*f) - B*b**2*c*d**3*log(tan(e + f*x)**2 + 1
)/(2*c**2*d**3*f + 2*d**5*f) - 2*B*b**2*d**4*f*x/(2*c**2*d**3*f + 2*d**5*f)
 + 2*B*b**2*d**4*tan(e + f*x)/(2*c**2*d**3*f + 2*d**5*f) + 2*C*a**2*c**2*d*
*2*log(c/d + tan(e + f*x))/(2*c**2*d**3*f + 2*d**5*f) - 2*C*a**2*c*d**3*f*x
/(2*c**2*d**3*f + 2*d**5*f) + C*a**2*d**4*log(tan(e + f*x)**2 + 1)/(2*c**2*
```

```
d**3*f + 2*d**5*f) - 4*C*a*b*c**3*d*log(c/d + tan(e + f*x))/(2*c**2*d**3*f
+ 2*d**5*f) + 4*C*a*b*c**2*d**2*tan(e + f*x)/(2*c**2*d**3*f + 2*d**5*f) - 2
*C*a*b*c*d**3*log(tan(e + f*x)**2 + 1)/(2*c**2*d**3*f + 2*d**5*f) - 4*C*a*b
*d**4*f*x/(2*c**2*d**3*f + 2*d**5*f) + 4*C*a*b*d**4*tan(e + f*x)/(2*c**2*d*
*3*f + 2*d**5*f) + 2*C*b**2*c**4*log(c/d + tan(e + f*x))/(2*c**2*d**3*f + 2
*d**5*f) - 2*C*b**2*c**3*d*tan(e + f*x)/(2*c**2*d**3*f + 2*d**5*f) + C*b**2
*c**2*d**2*tan(e + f*x)**2/(2*c**2*d**3*f + 2*d**5*f) + 2*C*b**2*c*d**3*f*x
/(2*c**2*d**3*f + 2*d**5*f) - 2*C*b**2*c*d**3*tan(e + f*x)/(2*c**2*d**3*f +
 2*d**5*f) - C*b**2*d**4*log(tan(e + f*x)**2 + 1)/(2*c**2*d**3*f + 2*d**5*f
) + C*b**2*d**4*tan(e + f*x)**2/(2*c**2*d**3*f + 2*d**5*f), True))
```

Giac [A] time = 2.06508, size = 456, normalized size = 1.93

$$\frac{\frac{2\left(Aa^2c-Ca^2c-2\,Babc-Ab^2c+Cb^2c+Ba^2d+2\,Aabd-2\,Cabd-Bb^2d\right)\left(fx+e\right)}{c^2+d^2}+\frac{\left(Ba^2c+2\,Aabc-2\,Cabc-Bb^2c-Aa^2d+Ca^2d+2\,Babd+Ab^2d-Cb^2d\right)\log\left(\tan\left(fx+e\right)^2+\right.}{c^2+d^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
)),x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a^2*c - C*a^2*c - 2*B*a*b*c - A*b^2*c + C*b^2*c + B*a^2*d + 2*A*a
*b*d - 2*C*a*b*d - B*b^2*d)*(f*x + e)/(c^2 + d^2) + (B*a^2*c + 2*A*a*b*c -
2*C*a*b*c - B*b^2*c - A*a^2*d + C*a^2*d + 2*B*a*b*d + A*b^2*d - C*b^2*d)*lo
g(tan(f*x + e)^2 + 1)/(c^2 + d^2) + 2*(C*b^2*c^4 - 2*C*a*b*c^3*d - B*b^2*c^
3*d + C*a^2*c^2*d^2 + 2*B*a*b*c^2*d^2 + A*b^2*c^2*d^2 - B*a^2*c*d^3 - 2*A*a
*b*c*d^3 + A*a^2*d^4)*log(abs(d*tan(f*x + e) + c))/(c^2*d^3 + d^5) + (C*b^2
*d*tan(f*x + e)^2 - 2*C*b^2*c*tan(f*x + e) + 4*C*a*b*d*tan(f*x + e) + 2*B*b
^2*d*tan(f*x + e))/d^2)/f
```

3.72 $\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=156

$$-\frac{(bc-ad)\left(Ad^2-Bcd+c^2C\right)\log(c+d\tan(e+fx))}{d^2f\left(c^2+d^2\right)}-\frac{\log(\cos(e+fx))(-aAd+aBc+aCd+Abc+bBd-bcC)}{f\left(c^2+d^2\right)}+$$

```
[Out] ((a*(A*c - c*C + B*d) - b*(B*c - (A - C)*d))*x)/(c^2 + d^2) - ((A*b*c + a*B
*c - b*c*C - a*A*d + b*B*d + a*C*d)*Log[Cos[e + f*x]])/((c^2 + d^2)*f) - ((
b*c - a*d)*(c^2*C - B*c*d + A*d^2)*Log[c + d*Tan[e + f*x]])/(d^2*(c^2 + d^2
)*f) + (b*C*Tan[e + f*x])/(d*f)
```

Rubi [A] time = 0.341521, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {3637, 3626, 3617, 31, 3475}

$$-\frac{(bc-ad)\left(Ad^2-Bcd+c^2C\right)\log(c+d\tan(e+fx))}{d^2f\left(c^2+d^2\right)}-\frac{\log(\cos(e+fx))(-aAd+aBc+aCd+Abc+bBd-bcC)}{f\left(c^2+d^2\right)}+$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d*T
an[e + f*x]),x]
```

```
[Out] ((a*(A*c - c*C + B*d) - b*(B*c - (A - C)*d))*x)/(c^2 + d^2) - ((A*b*c + a*B
*c - b*c*C - a*A*d + b*B*d + a*C*d)*Log[Cos[e + f*x]])/((c^2 + d^2)*f) - ((
b*c - a*d)*(c^2*C - B*c*d + A*d^2)*Log[c + d*Tan[e + f*x]])/(d^2*(c^2 + d^2
)*f) + (b*C*Tan[e + f*x])/(d*f)
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3626

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((a*A + b*B -
a*C)*x)/(a^2 + b^2), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3617

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
```

```
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{c+d\tan(e+fx)}\,dx = \frac{bC\tan(e+fx)}{df} - \frac{\int \frac{bcC-aAd-(Ab+aB-bC)d\tan(e+fx)+(bcC-}{c+d\tan(e+fx)}}{d}$$

$$= \frac{(a(Ac-cC+Bd)-b(Bc-(A-C)d))x}{c^2+d^2} + \frac{bC\tan(e+}{df}$$

$$= \frac{(a(Ac-cC+Bd)-b(Bc-(A-C)d))x}{c^2+d^2} - \frac{(Abc+aBc}{}$$

$$= \frac{(a(Ac-cC+Bd)-b(Bc-(A-C)d))x}{c^2+d^2} - \frac{(Abc+aBc}{}$$

Mathematica [C] time = 1.05483, size = 148, normalized size = 0.95

$$\frac{\frac{2(ad-bc)\left(Ad^2-Bcd+c^2C\right)\log(c+d\tan(e+fx))}{d^2\left(c^2+d^2\right)} + \frac{(b-ia)(A+iB-C)\log(-\tan(e+fx)+i)}{c+id} + \frac{(b+ia)(A-iB-C)\log(\tan(e+fx)+i)}{c-id} + \frac{2bC\tan(e+fx)}{d}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c
 + d*Tan[e + f*x]),x]
```

```
[Out] ((((-I)*a + b)*(A + I*B - C)*Log[I - Tan[e + f*x]])/(c + I*d) + ((I*a + b)*
(A - I*B - C)*Log[I + Tan[e + f*x]])/(c - I*d) + (2*(-(b*c) + a*d)*(c^2*C -
 B*c*d + A*d^2)*Log[c + d*Tan[e + f*x]])/(d^2*(c^2 + d^2)) + (2*b*C*Tan[e +
 f*x])/d)/(2*f)
```

Maple [B] time = 0.05, size = 506, normalized size = 3.2

$$\frac{Cb\tan\left(fx+e\right)}{fd} - \frac{\ln\left(1+\left(\tan\left(fx+e\right)\right)^2\right)Aad}{2f\left(c^2+d^2\right)} + \frac{\ln\left(1+\left(\tan\left(fx+e\right)\right)^2\right)Abc}{2f\left(c^2+d^2\right)} + \frac{\ln\left(1+\left(\tan\left(fx+e\right)\right)^2\right)Bac}{2f\left(c^2+d^2\right)} + \frac{\ln\left(1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)),x)
```

```
[Out] b*C*tan(f*x+e)/f/d-1/2/f/(c^2+d^2)*ln(1+tan(f*x+e)^2)*A*a*d+1/2/f/(c^2+d^2)
*ln(1+tan(f*x+e)^2)*A*b*c+1/2/f/(c^2+d^2)*ln(1+tan(f*x+e)^2)*B*a*c+1/2/f/(c
^2+d^2)*ln(1+tan(f*x+e)^2)*B*b*d+1/2/f/(c^2+d^2)*ln(1+tan(f*x+e)^2)*a*C*d-1
/2/f/(c^2+d^2)*ln(1+tan(f*x+e)^2)*C*b*c+1/f/(c^2+d^2)*A*arctan(tan(f*x+e))*
a*c+1/f/(c^2+d^2)*A*arctan(tan(f*x+e))*b*d+1/f/(c^2+d^2)*B*arctan(tan(f*x+e
))*a*d-1/f/(c^2+d^2)*B*arctan(tan(f*x+e))*b*c-1/f/(c^2+d^2)*C*arctan(tan(f*
x+e))*a*c-1/f/(c^2+d^2)*C*arctan(tan(f*x+e))*b*d+1/f*d/(c^2+d^2)*ln(c+d*tan
(f*x+e))*A*a-1/f/(c^2+d^2)*ln(c+d*tan(f*x+e))*A*b*c-1/f/(c^2+d^2)*ln(c+d*ta
n(f*x+e))*B*a*c+1/f/d/(c^2+d^2)*ln(c+d*tan(f*x+e))*B*c^2*b+1/f/d/(c^2+d^2)*
ln(c+d*tan(f*x+e))*C*c^2*a-1/f/d^2/(c^2+d^2)*ln(c+d*tan(f*x+e))*C*c^3*b
```

Maxima [A] time = 1.48502, size = 240, normalized size = 1.54

$$\frac{\frac{2\,Cb\tan\left(fx+e\right)}{d} + \frac{2\,(((A-C)a-Bb)c+(Ba+(A-C)b)d)\left(fx+e\right)}{c^2+d^2} - \frac{2\left(Cbc^3-Aad^3-(Ca+Bb)c^2d+(Ba+Ab)cd^2\right)\log\left(d\tan\left(fx+e\right)+c\right)}{c^2d^2+d^4} + \frac{((Ba+(A-C)b)c-((A-C}{}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
,x, algorithm="maxima")
```

```
[Out] 1/2*(2*C*b*tan(f*x + e)/d + 2*(((A - C)*a - B*b)*c + (B*a + (A - C)*b)*d)*(
f*x + e)/(c^2 + d^2) - 2*(C*b*c^3 - A*a*d^3 - (C*a + B*b)*c^2*d + (B*a + A*
b)*c*d^2)*log(d*tan(f*x + e) + c)/(c^2*d^2 + d^4) + ((B*a + (A - C)*b)*c -
((A - C)*a - B*b)*d)*log(tan(f*x + e)^2 + 1)/(c^2 + d^2))/f
```

Fricas [A] time = 1.59051, size = 470, normalized size = 3.01

$$\frac{2\left(((A-C)a-Bb)cd^2+(Ba+(A-C)b)d^3\right)fx-\left(Cbc^3-Aad^3-(Ca+Bb)c^2d+(Ba+Ab)cd^2\right)\log\left(\frac{d^2\tan\left(fx+e\right)^2+2\,cd}{\tan\left(fx+e\right.}\right.}{2\left(c^2d^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(((A - C)*a - B*b)*c*d^2 + (B*a + (A - C)*b)*d^3)*f*x - (C*b*c^3 - A
*a*d^3 - (C*a + B*b)*c^2*d + (B*a + A*b)*c*d^2)*log((d^2*tan(f*x + e)^2 + 2
*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) + (C*b*c^3 + C*b*c*d^2 - (C*
a + B*b)*c^2*d - (C*a + B*b)*d^3)*log(1/(tan(f*x + e)^2 + 1)) + 2*(C*b*c^2*
d + C*b*d^3)*tan(f*x + e))/((c^2*d^2 + d^4)*f)
```

Sympy [A] time = 23.7434, size = 2387, normalized size = 15.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e)
),x)
```

```
[Out] Piecewise((zoo*x*(a + b*tan(e))*(A + B*tan(e) + C*tan(e)**2)/tan(e), Eq(c,
0) & Eq(d, 0) & Eq(f, 0)), (-I*A*a*f*x*tan(e + f*x)/(-2*d*f*tan(e + f*x) +
2*I*d*f) - A*a*f*x/(-2*d*f*tan(e + f*x) + 2*I*d*f) - I*A*a/(-2*d*f*tan(e +
f*x) + 2*I*d*f) - A*b*f*x*tan(e + f*x)/(-2*d*f*tan(e + f*x) + 2*I*d*f) + I*
A*b*f*x/(-2*d*f*tan(e + f*x) + 2*I*d*f) + A*b/(-2*d*f*tan(e + f*x) + 2*I*d*
f) - B*a*f*x*tan(e + f*x)/(-2*d*f*tan(e + f*x) + 2*I*d*f) + I*B*a*f*x/(-2*d
*f*tan(e + f*x) + 2*I*d*f) + B*a/(-2*d*f*tan(e + f*x) + 2*I*d*f) - I*B*b*f*
x*tan(e + f*x)/(-2*d*f*tan(e + f*x) + 2*I*d*f) - B*b*f*x/(-2*d*f*tan(e + f*
x) + 2*I*d*f) - B*b*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(-2*d*f*tan(e + f
*x) + 2*I*d*f) + I*B*b*log(tan(e + f*x)**2 + 1)/(-2*d*f*tan(e + f*x) + 2*I*
d*f) + I*B*b/(-2*d*f*tan(e + f*x) + 2*I*d*f) - I*C*a*f*x*tan(e + f*x)/(-2*d
*f*tan(e + f*x) + 2*I*d*f) - C*a*f*x/(-2*d*f*tan(e + f*x) + 2*I*d*f) - C*a*
log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(-2*d*f*tan(e + f*x) + 2*I*d*f) + I*C
*a*log(tan(e + f*x)**2 + 1)/(-2*d*f*tan(e + f*x) + 2*I*d*f) + I*C*a/(-2*d*f
*tan(e + f*x) + 2*I*d*f) + 3*C*b*f*x*tan(e + f*x)/(-2*d*f*tan(e + f*x) + 2*
I*d*f) - 3*I*C*b*f*x/(-2*d*f*tan(e + f*x) + 2*I*d*f) - I*C*b*log(tan(e + f*
x)**2 + 1)*tan(e + f*x)/(-2*d*f*tan(e + f*x) + 2*I*d*f) - C*b*log(tan(e + f
*x)**2 + 1)/(-2*d*f*tan(e + f*x) + 2*I*d*f) - 2*C*b*tan(e + f*x)**2/(-2*d*f
*tan(e + f*x) + 2*I*d*f) - 3*C*b/(-2*d*f*tan(e + f*x) + 2*I*d*f), Eq(c, -I*
d)), (-I*A*a*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) + A*a*f*x/(2*d
*f*tan(e + f*x) + 2*I*d*f) - I*A*a/(2*d*f*tan(e + f*x) + 2*I*d*f) + A*b*f*x
*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) + I*A*b*f*x/(2*d*f*tan(e + f*x
) + 2*I*d*f) - A*b/(2*d*f*tan(e + f*x) + 2*I*d*f) + B*a*f*x*tan(e + f*x)/(2
*d*f*tan(e + f*x) + 2*I*d*f) + I*B*a*f*x/(2*d*f*tan(e + f*x) + 2*I*d*f) - B
*a/(2*d*f*tan(e + f*x) + 2*I*d*f) - I*B*b*f*x*tan(e + f*x)/(2*d*f*tan(e + f
*x) + 2*I*d*f) + B*b*f*x/(2*d*f*tan(e + f*x) + 2*I*d*f) + B*b*log(tan(e + f
*x)**2 + 1)*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) + I*B*b*log(tan(e +
 f*x)**2 + 1)/(2*d*f*tan(e + f*x) + 2*I*d*f) + I*B*b/(2*d*f*tan(e + f*x) +
2*I*d*f) - I*C*a*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) + C*a*f*x/
(2*d*f*tan(e + f*x) + 2*I*d*f) + C*a*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/
(2*d*f*tan(e + f*x) + 2*I*d*f) + I*C*a*log(tan(e + f*x)**2 + 1)/(2*d*f*tan(
e + f*x) + 2*I*d*f) + I*C*a/(2*d*f*tan(e + f*x) + 2*I*d*f) - 3*C*b*f*x*tan(
e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) - 3*I*C*b*f*x/(2*d*f*tan(e + f*x) +
 2*I*d*f) - I*C*b*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*d*f*tan(e + f*x)
 + 2*I*d*f) + C*b*log(tan(e + f*x)**2 + 1)/(2*d*f*tan(e + f*x) + 2*I*d*f) +
 2*C*b*tan(e + f*x)**2/(2*d*f*tan(e + f*x) + 2*I*d*f) + 3*C*b/(2*d*f*tan(e
+ f*x) + 2*I*d*f), Eq(c, I*d)), ((A*a*x + A*b*log(tan(e + f*x)**2 + 1)/(2*f
) + B*a*log(tan(e + f*x)**2 + 1)/(2*f) - B*b*x + B*b*tan(e + f*x)/f - C*a*x
 + C*a*tan(e + f*x)/f - C*b*log(tan(e + f*x)**2 + 1)/(2*f) + C*b*tan(e + f*
x)**2/(2*f))/c, Eq(d, 0)), (x*(a + b*tan(e))*(A + B*tan(e) + C*tan(e)**2)/(
c + d*tan(e)), Eq(f, 0)), (2*A*a*c*d**2*f*x/(2*c**2*d**2*f + 2*d**4*f) + 2*
A*a*d**3*log(c/d + tan(e + f*x))/(2*c**2*d**2*f + 2*d**4*f) - A*a*d**3*log(
tan(e + f*x)**2 + 1)/(2*c**2*d**2*f + 2*d**4*f) - 2*A*b*c*d**2*log(c/d + ta
n(e + f*x))/(2*c**2*d**2*f + 2*d**4*f) + A*b*c*d**2*log(tan(e + f*x)**2 + 1
)/(2*c**2*d**2*f + 2*d**4*f) + 2*A*b*d**3*f*x/(2*c**2*d**2*f + 2*d**4*f) -
2*B*a*c*d**2*log(c/d + tan(e + f*x))/(2*c**2*d**2*f + 2*d**4*f) + B*a*c*d**
2*log(tan(e + f*x)**2 + 1)/(2*c**2*d**2*f + 2*d**4*f) + 2*B*a*d**3*f*x/(2*c
**2*d**2*f + 2*d**4*f) + 2*B*b*c**2*d*log(c/d + tan(e + f*x))/(2*c**2*d**2*
f + 2*d**4*f) - 2*B*b*c*d**2*f*x/(2*c**2*d**2*f + 2*d**4*f) + B*b*d**3*log(
tan(e + f*x)**2 + 1)/(2*c**2*d**2*f + 2*d**4*f) + 2*C*a*c**2*d*log(c/d + ta
n(e + f*x))/(2*c**2*d**2*f + 2*d**4*f) - 2*C*a*c*d**2*f*x/(2*c**2*d**2*f +
2*d**4*f) + C*a*d**3*log(tan(e + f*x)**2 + 1)/(2*c**2*d**2*f + 2*d**4*f) -
2*C*b*c**3*log(c/d + tan(e + f*x))/(2*c**2*d**2*f + 2*d**4*f) + 2*C*b*c**2*
d*tan(e + f*x)/(2*c**2*d**2*f + 2*d**4*f) - C*b*c*d**2*log(tan(e + f*x)**2
+ 1)/(2*c**2*d**2*f + 2*d**4*f) - 2*C*b*d**3*f*x/(2*c**2*d**2*f + 2*d**4*f)
 + 2*C*b*d**3*tan(e + f*x)/(2*c**2*d**2*f + 2*d**4*f), True))
```

Giac [A] time = 1.68285, size = 251, normalized size = 1.61

$$\frac{\frac{2\,Cb\tan\left(fx+e\right)}{d}+\frac{2\,(Aac-Cac-Bbc+Bad+Abd-Cbd)\left(fx+e\right)}{c^2+d^2}+\frac{(Bac+Abc-Cbc-Aad+Cad+Bbd)\log\left(\tan\left(fx+e\right)^2+1\right)}{c^2+d^2}-\frac{2\left(Cbc^3-Cac^2d-Bbc^2d+Bacd^2+\right.}{c^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
,x, algorithm="giac")
```

```
[Out] 1/2*(2*C*b*tan(f*x + e)/d + 2*(A*a*c - C*a*c - B*b*c + B*a*d + A*b*d - C*b*
d)*(f*x + e)/(c^2 + d^2) + (B*a*c + A*b*c - C*b*c - A*a*d + C*a*d + B*b*d)*
log(tan(f*x + e)^2 + 1)/(c^2 + d^2) - 2*(C*b*c^3 - C*a*c^2*d - B*b*c^2*d +
B*a*c*d^2 + A*b*c*d^2 - A*a*d^3)*log(abs(d*tan(f*x + e) + c))/(c^2*d^2 + d^
4))/f
```

3.73 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=99

$$\frac{(Ad^2-Bcd+c^2C)\log(c+d\tan(e+fx))}{df(c^2+d^2)}-\frac{(Bc-d(A-C))\log(\cos(e+fx))}{f(c^2+d^2)}+\frac{x(Ac+Bd-cC)}{c^2+d^2}$$

```
[Out] ((A*c - c*C + B*d)*x)/(c^2 + d^2) - ((B*c - (A - C)*d)*Log[Cos[e + f*x]])/(
(c^2 + d^2)*f) + ((c^2*C - B*c*d + A*d^2)*Log[c + d*Tan[e + f*x]])/(d*(c^2
+ d^2)*f)
```

Rubi [A] time = 0.097683, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {3626, 3617, 31, 3475}

$$\frac{(Ad^2-Bcd+c^2C)\log(c+d\tan(e+fx))}{df(c^2+d^2)}-\frac{(Bc-d(A-C))\log(\cos(e+fx))}{f(c^2+d^2)}+\frac{x(Ac+Bd-cC)}{c^2+d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(c + d*Tan[e + f*x]),x]
```

```
[Out] ((A*c - c*C + B*d)*x)/(c^2 + d^2) - ((B*c - (A - C)*d)*Log[Cos[e + f*x]])/(
(c^2 + d^2)*f) + ((c^2*C - B*c*d + A*d^2)*Log[c + d*Tan[e + f*x]])/(d*(c^2
+ d^2)*f)
```

Rule 3626

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((a*A + b*B -
a*C)*x)/(a^2 + b^2), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3617

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{c + d\tan(e + fx)}\,dx = \frac{(Ac - cC + Bd)x}{c^2 + d^2} - \frac{(-Bc + Ad - Cd)\int \tan(e + fx)\,dx}{c^2 + d^2} + \frac{\left(c^2C - Bcd + \right.}{}$$

$$= \frac{(Ac - cC + Bd)x}{c^2 + d^2} - \frac{(Bc - (A - C)d)\log(\cos(e + fx))}{\left(c^2 + d^2\right)f} + \frac{\left(c^2C - Bcd + A\right.}{}$$

$$= \frac{(Ac - cC + Bd)x}{c^2 + d^2} - \frac{(Bc - (A - C)d)\log(\cos(e + fx))}{\left(c^2 + d^2\right)f} + \frac{\left(c^2C - Bcd + A\right.}{d}$$

Mathematica [C] time = 0.213477, size = 117, normalized size = 1.18

$$\frac{\frac{2\left(Ad^2-Bcd+c^2C\right)\log(c+d\tan(e+fx))}{d\left(c^2+d^2\right)}+\frac{(-iA+B+iC)\log(-\tan(e+fx)+i)}{c+id}+\frac{(iA+B-iC)\log(\tan(e+fx)+i)}{c-id}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(c + d*Tan[e + f*x]),x]
```

```
[Out] ((((-I)*A + B + I*C)*Log[I - Tan[e + f*x]])/(c + I*d) + ((I*A + B - I*C)*Lo
g[I + Tan[e + f*x]])/(c - I*d) + (2*(c^2*C - B*c*d + A*d^2)*Log[c + d*Tan[e
 + f*x]])/(d*(c^2 + d^2)))/(2*f)
```

Maple [B] time = 0.037, size = 234, normalized size = 2.4

$$-\frac{\ln\left(1+\left(\tan\left(fx+e\right)\right)^2\right)Ad}{2f\left(c^2+d^2\right)}+\frac{\ln\left(1+\left(\tan\left(fx+e\right)\right)^2\right)Bc}{2f\left(c^2+d^2\right)}+\frac{\ln\left(1+\left(\tan\left(fx+e\right)\right)^2\right)Cd}{2f\left(c^2+d^2\right)}+\frac{A\arctan\left(\tan\left(fx+e\right)\right.}{f\left(c^2+d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)),x)
```

```
[Out] -1/2/f/(c^2+d^2)*ln(1+tan(f*x+e)^2)*A*d+1/2/f/(c^2+d^2)*ln(1+tan(f*x+e)^2)*
B*c+1/2/f/(c^2+d^2)*ln(1+tan(f*x+e)^2)*C*d+1/f/(c^2+d^2)*A*arctan(tan(f*x+e
))*c+1/f/(c^2+d^2)*B*arctan(tan(f*x+e))*d-1/f/(c^2+d^2)*C*arctan(tan(f*x+e)
)*c+1/f/(c^2+d^2)*d*ln(c+d*tan(f*x+e))*A-1/f/(c^2+d^2)*ln(c+d*tan(f*x+e))*B
*c+1/f/(c^2+d^2)/d*ln(c+d*tan(f*x+e))*c^2*C
```

Maxima [A] time = 1.45554, size = 143, normalized size = 1.44

$$\frac{\frac{2\left((A-C)c+Bd\right)\left(fx+e\right)}{c^2+d^2}+\frac{2\left(Cc^2-Bcd+Ad^2\right)\log\left(d\tan\left(fx+e\right)+c\right)}{c^2d+d^3}+\frac{(Bc-(A-C)d)\log\left(\tan\left(fx+e\right)^2+1\right)}{c^2+d^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] 1/2*(2*((A - C)*c + B*d)*(f*x + e)/(c^2 + d^2) + 2*(C*c^2 - B*c*d + A*d^2)*
log(d*tan(f*x + e) + c)/(c^2*d + d^3) + (B*c - (A - C)*d)*log(tan(f*x + e)^
2 + 1)/(c^2 + d^2))/f
```

Fricas [A] time = 1.22239, size = 269, normalized size = 2.72

$$\frac{2\left((A-C)cd+Bd^2\right)fx+\left(Cc^2-Bcd+Ad^2\right)\log\left(\frac{d^2\tan\left(fx+e\right)^2+2\,cd\tan\left(fx+e\right)+c^2}{\tan\left(fx+e\right)^2+1}\right)-\left(Cc^2+Cd^2\right)\log\left(\frac{1}{\tan\left(fx+e\right)^2+1}\right)}{2\left(c^2d+d^3\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] 1/2*(2*((A - C)*c*d + B*d^2)*f*x + (C*c^2 - B*c*d + A*d^2)*log((d^2*tan(f*x
 + e)^2 + 2*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) - (C*c^2 + C*d^2)
*log(1/(tan(f*x + e)^2 + 1)))/((c^2*d + d^3)*f)
```

Sympy [A] time = 14.0031, size = 966, normalized size = 9.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e)),x)
```

```
[Out] Piecewise((zoo*x*(A + B*tan(e) + C*tan(e)**2)/tan(e), Eq(c, 0) & Eq(d, 0) &
 Eq(f, 0)), ((A*x + B*log(tan(e + f*x)**2 + 1)/(2*f) - C*x + C*tan(e + f*x)
/f)/c, Eq(d, 0)), (-I*A*f*x*tan(e + f*x)/(-2*d*f*tan(e + f*x) + 2*I*d*f) -
A*f*x/(-2*d*f*tan(e + f*x) + 2*I*d*f) - I*A/(-2*d*f*tan(e + f*x) + 2*I*d*f)
 - B*f*x*tan(e + f*x)/(-2*d*f*tan(e + f*x) + 2*I*d*f) + I*B*f*x/(-2*d*f*tan
(e + f*x) + 2*I*d*f) + B/(-2*d*f*tan(e + f*x) + 2*I*d*f) - I*C*f*x*tan(e +
f*x)/(-2*d*f*tan(e + f*x) + 2*I*d*f) - C*f*x/(-2*d*f*tan(e + f*x) + 2*I*d*f
) - C*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(-2*d*f*tan(e + f*x) + 2*I*d*f)
 + I*C*log(tan(e + f*x)**2 + 1)/(-2*d*f*tan(e + f*x) + 2*I*d*f) + I*C/(-2*d
*f*tan(e + f*x) + 2*I*d*f), Eq(c, -I*d)), (-I*A*f*x*tan(e + f*x)/(2*d*f*tan
(e + f*x) + 2*I*d*f) + A*f*x/(2*d*f*tan(e + f*x) + 2*I*d*f) - I*A/(2*d*f*ta
n(e + f*x) + 2*I*d*f) + B*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) +
 I*B*f*x/(2*d*f*tan(e + f*x) + 2*I*d*f) - B/(2*d*f*tan(e + f*x) + 2*I*d*f)
- I*C*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) + C*f*x/(2*d*f*tan(e
+ f*x) + 2*I*d*f) + C*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*d*f*tan(e +
f*x) + 2*I*d*f) + I*C*log(tan(e + f*x)**2 + 1)/(2*d*f*tan(e + f*x) + 2*I*d*
f) + I*C/(2*d*f*tan(e + f*x) + 2*I*d*f), Eq(c, I*d)), (x*(A + B*tan(e) + C*
tan(e)**2)/(c + d*tan(e)), Eq(f, 0)), (2*A*c*d*f*x/(2*c**2*d*f + 2*d**3*f)
+ 2*A*d**2*log(c/d + tan(e + f*x))/(2*c**2*d*f + 2*d**3*f) - A*d**2*log(tan
(e + f*x)**2 + 1)/(2*c**2*d*f + 2*d**3*f) - 2*B*c*d*log(c/d + tan(e + f*x))
/(2*c**2*d*f + 2*d**3*f) + B*c*d*log(tan(e + f*x)**2 + 1)/(2*c**2*d*f + 2*d
**3*f) + 2*B*d**2*f*x/(2*c**2*d*f + 2*d**3*f) + 2*C*c**2*log(c/d + tan(e +
f*x))/(2*c**2*d*f + 2*d**3*f) - 2*C*c*d*f*x/(2*c**2*d*f + 2*d**3*f) + C*d**
2*log(tan(e + f*x)**2 + 1)/(2*c**2*d*f + 2*d**3*f), True))
```

Giac [A] time = 1.62131, size = 147, normalized size = 1.48

$$\frac{\frac{2\,(Ac-Cc+Bd)\left(fx+e\right)}{c^2+d^2}+\frac{(Bc-Ad+Cd)\log\left(\tan\left(fx+e\right)^2+1\right)}{c^2+d^2}+\frac{2\,\left(Cc^2-Bcd+Ad^2\right)\log\left(\left|d\tan\left(fx+e\right)+c\right|\right)}{c^2d+d^3}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] 1/2*(2*(A*c - C*c + B*d)*(f*x + e)/(c^2 + d^2) + (B*c - A*d + C*d)*log(tan(
f*x + e)^2 + 1)/(c^2 + d^2) + 2*(C*c^2 - B*c*d + A*d^2)*log(abs(d*tan(f*x +
 e) + c))/(c^2*d + d^3))/f
```

3.74 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))}\,dx$

Optimal. Leaf size=165

$$\frac{x(a(Ac+Bd-cC)+b(Bc-d(A-C)))}{(a^2+b^2)(c^2+d^2)}+\frac{(Ab^2-a(bB-aC))\log(a\cos(e+fx)+b\sin(e+fx))}{f(a^2+b^2)(bc-ad)}-\frac{(Ad^2-Bcd+c^2C)\,\mathrm{lo}}{f(c^2}$$

```
[Out] ((a*(A*c - c*C + B*d) + b*(B*c - (A - C)*d))*x)/((a^2 + b^2)*(c^2 + d^2)) +
 ((A*b^2 - a*(b*B - a*C))*Log[a*Cos[e + f*x] + b*Sin[e + f*x]])/((a^2 + b^2
)*(b*c - a*d)*f) - ((c^2*C - B*c*d + A*d^2)*Log[c*Cos[e + f*x] + d*Sin[e +
f*x]])/((b*c - a*d)*(c^2 + d^2)*f)
```

Rubi [A] time = 0.256364, antiderivative size = 164, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 2, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.044, Rules used = {3651, 3530}

$$\frac{x(a(Ac+Bd-cC)-bd(A-C)+bBc)}{(a^2+b^2)(c^2+d^2)}+\frac{(Ab^2-a(bB-aC))\log(a\cos(e+fx)+b\sin(e+fx))}{f(a^2+b^2)(bc-ad)}-\frac{(Ad^2-Bcd+c^2C)\,\mathrm{lo}}{f(c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])*(c + d*Ta
n[e + f*x])),x]
```

```
[Out] ((b*B*c - b*(A - C)*d + a*(A*c - c*C + B*d))*x)/((a^2 + b^2)*(c^2 + d^2)) +
 ((A*b^2 - a*(b*B - a*C))*Log[a*Cos[e + f*x] + b*Sin[e + f*x]])/((a^2 + b^2
)*(b*c - a*d)*f) - ((c^2*C - B*c*d + A*d^2)*Log[c*Cos[e + f*x] + d*Sin[e +
f*x]])/((b*c - a*d)*(c^2 + d^2)*f)
```

Rule 3651

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[((a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*x)
/((a^2 + b^2)*(c^2 + d^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{A + B\tan(e+fx) + C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))}\,dx = \frac{(bBc - b(A-C)d + a(Ac - cC + Bd))x}{\left(a^2+b^2\right)\left(c^2+d^2\right)} + \frac{\left(Ab^2 - a(bB - aC)\right)\int \frac{b-a\tan}{a+b\tan}}{\left(a^2+b^2\right)(bc-ad)}$$

$$= \frac{(bBc - b(A-C)d + a(Ac - cC + Bd))x}{\left(a^2+b^2\right)\left(c^2+d^2\right)} + \frac{\left(Ab^2 - a(bB - aC)\right)\log(a\cos}{\left(a^2+b^2\right)(}$$

Mathematica [A] time = 1.52495, size = 313, normalized size = 1.9

$$-\frac{\frac{\log\left(\sqrt{-b^2}-b\tan(e+fx)\right)\left(\frac{\sqrt{-b^2}(a(Ac+Bd-cC)+bd(C-A)+bBc)}{b}+aAd-aBc-aCd+Abc+bBd-bcC\right)}{\left(a^2+b^2\right)\left(c^2+d^2\right)}+\frac{\log\left(\sqrt{-b^2}+b\tan(e+fx)\right)\left(\frac{b(a(Ac+Bd-cC)+bd(C-A)+bBc)}{\sqrt{-b^2}}+aAd\right.}{\left(a^2+b^2\right)\left(c^2+d^2\right)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])*(c
+ d*Tan[e + f*x])),x]
```

```
[Out] -(((A*b*c - a*B*c - b*c*C + a*A*d + b*B*d - a*C*d + (Sqrt[-b^2]*(b*B*c + b*
(-A + C)*d + a*(A*c - c*C + B*d)))/b)*Log[Sqrt[-b^2] - b*Tan[e + f*x]])/((a
^2 + b^2)*(c^2 + d^2)) + (2*(A*b^2 + a*(-(b*B) + a*C))*Log[a + b*Tan[e + f*
x]])/((a^2 + b^2)*(-(b*c) + a*d)) + ((A*b*c - a*B*c - b*c*C + a*A*d + b*B*d
 - a*C*d + (b*(b*B*c + b*(-A + C)*d + a*(A*c - c*C + B*d)))/Sqrt[-b^2])*Log
[Sqrt[-b^2] + b*Tan[e + f*x]])/((a^2 + b^2)*(c^2 + d^2)) + (2*(c^2*C - B*c*
d + A*d^2)*Log[c + d*Tan[e + f*x]])/((b*c - a*d)*(c^2 + d^2)))/(2*f)
```

Maple [B] time = 0.076, size = 647, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e)),x)
```

```
[Out] -1/2/f/(c^2+d^2)/(a^2+b^2)*ln(1+tan(f*x+e)^2)*A*a*d-1/2/f/(c^2+d^2)/(a^2+b^
2)*ln(1+tan(f*x+e)^2)*A*b*c+1/2/f/(c^2+d^2)/(a^2+b^2)*ln(1+tan(f*x+e)^2)*B*
a*c-1/2/f/(c^2+d^2)/(a^2+b^2)*ln(1+tan(f*x+e)^2)*B*b*d+1/2/f/(c^2+d^2)/(a^2
+b^2)*ln(1+tan(f*x+e)^2)*a*C*d+1/2/f/(c^2+d^2)/(a^2+b^2)*ln(1+tan(f*x+e)^2)
*C*b*c+1/f/(c^2+d^2)/(a^2+b^2)*A*arctan(tan(f*x+e))*a*c-1/f/(c^2+d^2)/(a^2+
b^2)*A*arctan(tan(f*x+e))*b*d+1/f/(c^2+d^2)/(a^2+b^2)*B*arctan(tan(f*x+e))*
a*d+1/f/(c^2+d^2)/(a^2+b^2)*B*arctan(tan(f*x+e))*b*c-1/f/(c^2+d^2)/(a^2+b^2
)*C*arctan(tan(f*x+e))*a*c+1/f/(c^2+d^2)/(a^2+b^2)*C*arctan(tan(f*x+e))*b*d
+1/f/(a*d-b*c)/(c^2+d^2)*ln(c+d*tan(f*x+e))*A*d^2-1/f/(a*d-b*c)/(c^2+d^2)*l
n(c+d*tan(f*x+e))*B*c*d+1/f/(a*d-b*c)/(c^2+d^2)*ln(c+d*tan(f*x+e))*c^2*C-1/
f/(a*d-b*c)/(a^2+b^2)*ln(a+b*tan(f*x+e))*A*b^2+1/f/(a*d-b*c)/(a^2+b^2)*ln(a
+b*tan(f*x+e))*B*a*b-1/f/(a*d-b*c)/(a^2+b^2)*ln(a+b*tan(f*x+e))*C*a^2
```

Maxima [A] time = 1.49136, size = 328, normalized size = 1.99

$$\frac{\frac{2\,(((A-C)a+Bb)c+(Ba-(A-C)b)d)\left(fx+e\right)}{\left(a^2+b^2\right)c^2+\left(a^2+b^2\right)d^2}+\frac{2\left(Ca^2-Bab+Ab^2\right)\log\left(b\tan\left(fx+e\right)+a\right)}{\left(a^2b+b^3\right)c-\left(a^3+ab^2\right)d}-\frac{2\left(Cc^2-Bcd+Ad^2\right)\log\left(d\tan\left(fx+e\right)+c\right)}{bc^3-ac^2d+bcd^2-ad^3}+\frac{((Ba-(A-C)b)c-((A-C}{\left(a^2+b^2\right)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))
,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(((A - C)*a + B*b)*c + (B*a - (A - C)*b)*d)*(f*x + e)/((a^2 + b^2)*c
^2 + (a^2 + b^2)*d^2) + 2*(C*a^2 - B*a*b + A*b^2)*log(b*tan(f*x + e) + a)/(
(a^2*b + b^3)*c - (a^3 + a*b^2)*d) - 2*(C*c^2 - B*c*d + A*d^2)*log(d*tan(f*
x + e) + c)/(b*c^3 - a*c^2*d + b*c*d^2 - a*d^3) + ((B*a - (A - C)*b)*c - ((
A - C)*a + B*b)*d)*log(tan(f*x + e)^2 + 1)/((a^2 + b^2)*c^2 + (a^2 + b^2)*d
^2))/f
```

Fricas [A] time = 2.4913, size = 633, normalized size = 3.84

$$\frac{2\left(\left(\left((A-C)ab+Bb^2\right)c^2-\left((A-C)a^2+(A-C)b^2\right)cd-\left(Ba^2-(A-C)ab\right)d^2\right)fx+\left(\left(Ca^2-Bab+Ab^2\right)c^2+\left(Ca^2-Bab\right.\right.\right.}{2\left(\left(a^2b+b^3\right)c^3-\left(a^3+ab^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))
,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(((A - C)*a*b + B*b^2)*c^2 - ((A - C)*a^2 + (A - C)*b^2)*c*d - (B*a^
2 - (A - C)*a*b)*d^2)*f*x + ((C*a^2 - B*a*b + A*b^2)*c^2 + (C*a^2 - B*a*b +
 A*b^2)*d^2)*log((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)/(tan(f*x +
 e)^2 + 1)) - ((C*a^2 + C*b^2)*c^2 - (B*a^2 + B*b^2)*c*d + (A*a^2 + A*b^2)*
d^2)*log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 +
1)))/(((a^2*b + b^3)*c^3 - (a^3 + a*b^2)*c^2*d + (a^2*b + b^3)*c*d^2 - (a^3
 + a*b^2)*d^3)*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e)
),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.72128, size = 367, normalized size = 2.22

$$\frac{\frac{2\,(Aac-Cac+Bbc+Bad-Abd+Cbd)\left(fx+e\right)}{a^2c^2+b^2c^2+a^2d^2+b^2d^2}+\frac{(Bac-Abc+Cbc-Aad+Cad-Bbd)\log\left(\tan\left(fx+e\right)^2+1\right)}{a^2c^2+b^2c^2+a^2d^2+b^2d^2}+\frac{2\left(Ca^2b-Bab^2+Ab^3\right)\log\left(\left|b\tan\left(fx+e\right)+a\right|\right)}{a^2b^2c+b^4c-a^3bd-ab^3d}-\frac{2\left(Cc^2d-\right.}{}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))
,x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a*c - C*a*c + B*b*c + B*a*d - A*b*d + C*b*d)*(f*x + e)/(a^2*c^2 +
 b^2*c^2 + a^2*d^2 + b^2*d^2) + (B*a*c - A*b*c + C*b*c - A*a*d + C*a*d - B*
b*d)*log(tan(f*x + e)^2 + 1)/(a^2*c^2 + b^2*c^2 + a^2*d^2 + b^2*d^2) + 2*(C
*a^2*b - B*a*b^2 + A*b^3)*log(abs(b*tan(f*x + e) + a))/(a^2*b^2*c + b^4*c -
 a^3*b*d - a*b^3*d) - 2*(C*c^2*d - B*c*d^2 + A*d^3)*log(abs(d*tan(f*x + e)
+ c))/(b*c^3*d - a*c^2*d^2 + b*c*d^3 - a*d^4))/f
```

3.75 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))}\,dx$

Optimal. Leaf size=281

$$\frac{x\left(a^2(Ac+Bd-cC)+2ab(Bc-d(A-C))-b^2(Ac+Bd-cC)\right)}{\left(a^2+b^2\right)^2\left(c^2+d^2\right)}-\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))}+\frac{\left(-a^2b^2(3A\right.}{}$$

```
[Out] ((a^2*(A*c - c*C + B*d) - b^2*(A*c - c*C + B*d) + 2*a*b*(B*c - (A - C)*d))*
x)/((a^2 + b^2)^2*(c^2 + d^2)) + ((2*a*b^3*c*(A - C) + 2*a^3*b*B*d - a^4*C*
d + b^4*(B*c - A*d) - a^2*b^2*(B*c + 3*A*d - C*d))*Log[a*Cos[e + f*x] + b*S
in[e + f*x]])/((a^2 + b^2)^2*(b*c - a*d)^2*f) + (d*(c^2*C - B*c*d + A*d^2)*
Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/((b*c - a*d)^2*(c^2 + d^2)*f) - (A*b^
2 - a*(b*B - a*C))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x]))
```

Rubi [A] time = 0.795212, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3649, 3651, 3530}

$$\frac{x\left(a^2(Ac+Bd-cC)+2ab(Bc-d(A-C))-b^2(Ac+Bd-cC)\right)}{\left(a^2+b^2\right)^2\left(c^2+d^2\right)}-\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))}+\frac{\left(-a^2b^2(3A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^2*(c + d*
Tan[e + f*x])),x]
```

```
[Out] ((a^2*(A*c - c*C + B*d) - b^2*(A*c - c*C + B*d) + 2*a*b*(B*c - (A - C)*d))*
x)/((a^2 + b^2)^2*(c^2 + d^2)) + ((2*a*b^3*c*(A - C) + 2*a^3*b*B*d - a^4*C*
d + b^4*(B*c - A*d) - a^2*b^2*(B*c + 3*A*d - C*d))*Log[a*Cos[e + f*x] + b*S
in[e + f*x]])/((a^2 + b^2)^2*(b*c - a*d)^2*f) + (d*(c^2*C - B*c*d + A*d^2)*
Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/((b*c - a*d)^2*(c^2 + d^2)*f) - (A*b^
2 - a*(b*B - a*C))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x]))
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
 b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3651

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[((a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*x)
/((a^2 + b^2)*(c^2 + d^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
```

```
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))}\,dx = -\frac{Ab^2-a(bB-aC)}{\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))} - \frac{\int \frac{-abc(A-C)+a^2Ad-b^2(Bc-Ad)}{\cdots}}{\cdots}$$

$$= \frac{\left(a^2(Ac-cC+Bd)-b^2(Ac-cC+Bd)+2ab(Bc-(A-C)d)\right)x}{\left(a^2+b^2\right)^2\left(c^2+d^2\right)} - \frac{\cdots}{\left(a\cdots\right.}$$

$$= \frac{\left(a^2(Ac-cC+Bd)-b^2(Ac-cC+Bd)+2ab(Bc-(A-C)d)\right)x}{\left(a^2+b^2\right)^2\left(c^2+d^2\right)} + \frac{\left(2\cdots\right.}{\cdots}$$

Mathematica [A] time = 6.91254, size = 543, normalized size = 1.93

$$-\frac{\frac{(bc-ad)\log\left(\sqrt{-b^2}-b\tan(e+fx)\right)\left(\frac{\sqrt{-b^2}\left(a^2(Ac+Bd-cC)+2ab(d(C-A)+Bc)-b^2(Ac+Bd-cC)\right)}{b}+a^2Ad+a^2(-B)c-a^2Cd+2aAbc+2abBd-2abcC-Ab^2d+b^2Bc+b^2Cd\right)}{2\left(a^2+b^2\right)\left(c^2+d^2\right)} - \cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^2*(
c + d*Tan[e + f*x])),x]
```

```
[Out] (-((b*c - a*d)*(2*a*A*b*c - a^2*B*c + b^2*B*c - 2*a*b*c*C + a^2*A*d - A*b^2
*d + 2*a*b*B*d - a^2*C*d + b^2*C*d + (Sqrt[-b^2]*(a^2*(A*c - c*C + B*d) - b
^2*(A*c - c*C + B*d) + 2*a*b*(B*c + (-A + C)*d)))/b)*Log[Sqrt[-b^2] - b*Tan
[e + f*x]])/(2*(a^2 + b^2)*(c^2 + d^2)) + ((2*a*b^3*c*(-A + C) - 2*a^3*b*B*
d + a^4*C*d + b^4*(-(B*c) + A*d) + a^2*b^2*(B*c + 3*A*d - C*d))*Log[a + b*T
an[e + f*x]])/((a^2 + b^2)*(-(b*c) + a*d)) - ((b*c - a*d)*(2*a*A*b*c - a^2*
B*c + b^2*B*c - 2*a*b*c*C + a^2*A*d - A*b^2*d + 2*a*b*B*d - a^2*C*d + b^2*C
*d + (Sqrt[-b^2]*(-(a^2*(A*c - c*C + B*d)) + b^2*(A*c - c*C + B*d) - 2*a*b*
(B*c + (-A + C)*d)))/b)*Log[Sqrt[-b^2] + b*Tan[e + f*x]])/(2*(a^2 + b^2)*(c
^2 + d^2)) + ((a^2 + b^2)*d*(c^2*C - B*c*d + A*d^2)*Log[c + d*Tan[e + f*x]]
)/((b*c - a*d)*(c^2 + d^2)) - (A*b^2)/(a + b*Tan[e + f*x]) + (a*(b*B - a*C)
)/(a + b*Tan[e + f*x]))/((a^2 + b^2)*(b*c - a*d)*f)
```

Maple [B] time = 0.094, size = 1262, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e)),x)
```

```
[Out] -2/f/(a^2+b^2)^2/(c^2+d^2)*A*arctan(tan(f*x+e))*a*b*d+2/f/(a^2+b^2)^2/(a*d-
b*c)^2*ln(a+b*tan(f*x+e))*a^3*b*B*d-3/f/(a^2+b^2)^2/(a*d-b*c)^2*ln(a+b*tan(
f*x+e))*A*a^2*b^2*d+1/f/(a^2+b^2)^2/(a*d-b*c)^2*ln(a+b*tan(f*x+e))*C*a^2*b^
2*d-2/f/(a^2+b^2)^2/(a*d-b*c)^2*ln(a+b*tan(f*x+e))*C*a*b^3*c+2/f/(a^2+b^2)^
2/(c^2+d^2)*C*arctan(tan(f*x+e))*a*b*d-1/f/(a^2+b^2)^2/(a*d-b*c)^2*ln(a+b*t
an(f*x+e))*B*a^2*b^2*c+2/f/(a^2+b^2)^2/(c^2+d^2)*B*arctan(tan(f*x+e))*a*b*c
-1/f/(a^2+b^2)^2/(c^2+d^2)*ln(1+tan(f*x+e)^2)*A*a*b*c-1/f/(a^2+b^2)^2/(c^2+
d^2)*ln(1+tan(f*x+e)^2)*B*a*b*d+1/f/(a^2+b^2)^2/(c^2+d^2)*ln(1+tan(f*x+e)^2
)*C*a*b*c+2/f/(a^2+b^2)^2/(a*d-b*c)^2*ln(a+b*tan(f*x+e))*A*a*b^3*c+1/f/(a^2
+b^2)^2/(c^2+d^2)*A*arctan(tan(f*x+e))*a^2*c-1/f/(a^2+b^2)^2/(c^2+d^2)*A*ar
ctan(tan(f*x+e))*b^2*c+1/f/(a^2+b^2)^2/(c^2+d^2)*B*arctan(tan(f*x+e))*a^2*d
-1/f*d^2/(a*d-b*c)^2/(c^2+d^2)*ln(c+d*tan(f*x+e))*B*c+1/f*d/(a*d-b*c)^2/(c^
2+d^2)*ln(c+d*tan(f*x+e))*c^2*C-1/f/(a^2+b^2)^2/(a*d-b*c)^2*ln(a+b*tan(f*x+
e))*A*b^4*d+1/f/(a^2+b^2)^2/(a*d-b*c)^2*ln(a+b*tan(f*x+e))*B*b^4*c-1/f/(a^2
+b^2)^2/(a*d-b*c)^2*ln(a+b*tan(f*x+e))*a^4*C*d-1/f/(a^2+b^2)/(a*d-b*c)/(a+b
*tan(f*x+e))*B*a*b-1/2/f/(a^2+b^2)^2/(c^2+d^2)*ln(1+tan(f*x+e)^2)*A*a^2*d+1
/f/(a^2+b^2)^2/(c^2+d^2)*C*arctan(tan(f*x+e))*b^2*c+1/2/f/(a^2+b^2)^2/(c^2+
d^2)*ln(1+tan(f*x+e)^2)*A*b^2*d+1/2/f/(a^2+b^2)^2/(c^2+d^2)*ln(1+tan(f*x+e)
^2)*B*a^2*c-1/f/(a^2+b^2)^2/(c^2+d^2)*B*arctan(tan(f*x+e))*b^2*d-1/f/(a^2+b
^2)^2/(c^2+d^2)*C*arctan(tan(f*x+e))*a^2*c-1/2/f/(a^2+b^2)^2/(c^2+d^2)*ln(1
+tan(f*x+e)^2)*B*b^2*c+1/2/f/(a^2+b^2)^2/(c^2+d^2)*ln(1+tan(f*x+e)^2)*C*a^2
*d-1/2/f/(a^2+b^2)^2/(c^2+d^2)*ln(1+tan(f*x+e)^2)*C*b^2*d+1/f*d^3/(a*d-b*c)
^2/(c^2+d^2)*ln(c+d*tan(f*x+e))*A+1/f/(a^2+b^2)/(a*d-b*c)/(a+b*tan(f*x+e))*
A*b^2+1/f/(a^2+b^2)/(a*d-b*c)/(a+b*tan(f*x+e))*C*a^2
```

Maxima [A] time = 1.57548, size = 702, normalized size = 2.5

$$\frac{\frac{2\left(\left(\left(A-C\right)a^2+2\,Bab-\left(A-C\right)b^2\right)c+\left(Ba^2-2\,\left(A-C\right)ab-Bb^2\right)d\right)\left(fx+e\right)}{\left(a^4+2\,a^2b^2+b^4\right)c^2+\left(a^4+2\,a^2b^2+b^4\right)d^2}-\frac{2\left(\left(Ba^2b^2-2\,\left(A-C\right)ab^3-Bb^4\right)c+\left(Ca^4-2\,Ba^3b+\left(3\,A-C\right)a^2b^2+Ab^4\right)d\right)\log\left(b\tan\left(fx+e\right)+a\right)}{\left(a^4b^2+2\,a^2b^4+b^6\right)c^2-2\,\left(a^5b+2\,a^3b^3+ab^5\right)cd+\left(a^6+2\,a^4b^2+a^2b^4\right)d^2}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e
)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*c + (B*a^2 - 2*(A - C)*a*b -
B*b^2)*d)*(f*x + e)/((a^4 + 2*a^2*b^2 + b^4)*c^2 + (a^4 + 2*a^2*b^2 + b^4)*
d^2) - 2*((B*a^2*b^2 - 2*(A - C)*a*b^3 - B*b^4)*c + (C*a^4 - 2*B*a^3*b + (3
*A - C)*a^2*b^2 + A*b^4)*d)*log(b*tan(f*x + e) + a)/((a^4*b^2 + 2*a^2*b^4 +
 b^6)*c^2 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*c*d + (a^6 + 2*a^4*b^2 + a^2*b^4)
*d^2) + 2*(C*c^2*d - B*c*d^2 + A*d^3)*log(d*tan(f*x + e) + c)/(b^2*c^4 - 2*
a*b*c^3*d - 2*a*b*c*d^3 + a^2*d^4 + (a^2 + b^2)*c^2*d^2) + ((B*a^2 - 2*(A -
 C)*a*b - B*b^2)*c - ((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*d)*log(tan(f*x +
 e)^2 + 1)/((a^4 + 2*a^2*b^2 + b^4)*c^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2) - 2*
(C*a^2 - B*a*b + A*b^2)/((a^3*b + a*b^3)*c - (a^4 + a^2*b^2)*d + ((a^2*b^2
+ b^4)*c - (a^3*b + a*b^3)*d)*tan(f*x + e)))/f
```

Fricas [B] time = 7.99317, size = 2738, normalized size = 9.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e
)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(C*a^2*b^3 - B*a*b^4 + A*b^5)*c^3 - 2*(C*a^3*b^2 - B*a^2*b^3 + A*a*
b^4)*c^2*d + 2*(C*a^2*b^3 - B*a*b^4 + A*b^5)*c*d^2 - 2*(C*a^3*b^2 - B*a^2*b
^3 + A*a*b^4)*d^3 - 2*(((A - C)*a^3*b^2 + 2*B*a^2*b^3 - (A - C)*a*b^4)*c^3
- (2*(A - C)*a^4*b + 3*B*a^3*b^2 + B*a*b^4)*c^2*d + ((A - C)*a^5 + 3*(A - C
)*a^3*b^2 + 2*B*a^2*b^3)*c*d^2 + (B*a^5 - 2*(A - C)*a^4*b - B*a^3*b^2)*d^3)
*f*x + ((B*a^3*b^2 - 2*(A - C)*a^2*b^3 - B*a*b^4)*c^3 + (C*a^5 - 2*B*a^4*b
+ (3*A - C)*a^3*b^2 + A*a*b^4)*c^2*d + (B*a^3*b^2 - 2*(A - C)*a^2*b^3 - B*a
*b^4)*c*d^2 + (C*a^5 - 2*B*a^4*b + (3*A - C)*a^3*b^2 + A*a*b^4)*d^3 + ((B*a
^2*b^3 - 2*(A - C)*a*b^4 - B*b^5)*c^3 + (C*a^4*b - 2*B*a^3*b^2 + (3*A - C)*
a^2*b^3 + A*b^5)*c^2*d + (B*a^2*b^3 - 2*(A - C)*a*b^4 - B*b^5)*c*d^2 + (C*a
^4*b - 2*B*a^3*b^2 + (3*A - C)*a^2*b^3 + A*b^5)*d^3)*tan(f*x + e))*log((b^2
*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)/(tan(f*x + e)^2 + 1)) - ((C*a^5
 + 2*C*a^3*b^2 + C*a*b^4)*c^2*d - (B*a^5 + 2*B*a^3*b^2 + B*a*b^4)*c*d^2 + (
A*a^5 + 2*A*a^3*b^2 + A*a*b^4)*d^3 + ((C*a^4*b + 2*C*a^2*b^3 + C*b^5)*c^2*d
 - (B*a^4*b + 2*B*a^2*b^3 + B*b^5)*c*d^2 + (A*a^4*b + 2*A*a^2*b^3 + A*b^5)*
d^3)*tan(f*x + e))*log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) + c^2)/(tan
(f*x + e)^2 + 1)) - 2*((C*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*c^3 - (C*a^4*b - B
*a^3*b^2 + A*a^2*b^3)*c^2*d + (C*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*c*d^2 - (C*
a^4*b - B*a^3*b^2 + A*a^2*b^3)*d^3 + (((A - C)*a^2*b^3 + 2*B*a*b^4 - (A - C
)*b^5)*c^3 - (2*(A - C)*a^3*b^2 + 3*B*a^2*b^3 + B*b^5)*c^2*d + ((A - C)*a^4
*b + 3*(A - C)*a^2*b^3 + 2*B*a*b^4)*c*d^2 + (B*a^4*b - 2*(A - C)*a^3*b^2 -
B*a^2*b^3)*d^3)*f*x)*tan(f*x + e))/(((a^4*b^3 + 2*a^2*b^5 + b^7)*c^4 - 2*(a
^5*b^2 + 2*a^3*b^4 + a*b^6)*c^3*d + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*c
^2*d^2 - 2*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*c*d^3 + (a^6*b + 2*a^4*b^3 + a^2*b
^5)*d^4)*f*tan(f*x + e) + ((a^5*b^2 + 2*a^3*b^4 + a*b^6)*c^4 - 2*(a^6*b + 2
*a^4*b^3 + a^2*b^5)*c^3*d + (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*c^2*d^2 -
 2*(a^6*b + 2*a^4*b^3 + a^2*b^5)*c*d^3 + (a^7 + 2*a^5*b^2 + a^3*b^4)*d^4)*f
)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**2/(c+d*tan(f*x
+e)),x)
```

```
[Out] Exception raised: NotImplementedError
```

Giac [B] time = 1.74159, size = 1142, normalized size = 4.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e
)),x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a^2*c - C*a^2*c + 2*B*a*b*c - A*b^2*c + C*b^2*c + B*a^2*d - 2*A*a
*b*d + 2*C*a*b*d - B*b^2*d)*(f*x + e)/(a^4*c^2 + 2*a^2*b^2*c^2 + b^4*c^2 +
```

```
a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2) + (B*a^2*c - 2*A*a*b*c + 2*C*a*b*c - B*b
^2*c - A*a^2*d + C*a^2*d - 2*B*a*b*d + A*b^2*d - C*b^2*d)*log(tan(f*x + e)^
2 + 1)/(a^4*c^2 + 2*a^2*b^2*c^2 + b^4*c^2 + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d
^2) - 2*(B*a^2*b^3*c - 2*A*a*b^4*c + 2*C*a*b^4*c - B*b^5*c + C*a^4*b*d - 2*
B*a^3*b^2*d + 3*A*a^2*b^3*d - C*a^2*b^3*d + A*b^5*d)*log(abs(b*tan(f*x + e)
 + a))/(a^4*b^3*c^2 + 2*a^2*b^5*c^2 + b^7*c^2 - 2*a^5*b^2*c*d - 4*a^3*b^4*c
*d - 2*a*b^6*c*d + a^6*b*d^2 + 2*a^4*b^3*d^2 + a^2*b^5*d^2) + 2*(C*c^2*d^2
- B*c*d^3 + A*d^4)*log(abs(d*tan(f*x + e) + c))/(b^2*c^4*d - 2*a*b*c^3*d^2
+ a^2*c^2*d^3 + b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5) + 2*(B*a^2*b^3*c*tan(f
*x + e) - 2*A*a*b^4*c*tan(f*x + e) + 2*C*a*b^4*c*tan(f*x + e) - B*b^5*c*tan
(f*x + e) + C*a^4*b*d*tan(f*x + e) - 2*B*a^3*b^2*d*tan(f*x + e) + 3*A*a^2*b
^3*d*tan(f*x + e) - C*a^2*b^3*d*tan(f*x + e) + A*b^5*d*tan(f*x + e) - C*a^4
*b*c + 2*B*a^3*b^2*c - 3*A*a^2*b^3*c + C*a^2*b^3*c - A*b^5*c + 2*C*a^5*d -
3*B*a^4*b*d + 4*A*a^3*b^2*d - B*a^2*b^3*d + 2*A*a*b^4*d)/((a^4*b^2*c^2 + 2*
a^2*b^4*c^2 + b^6*c^2 - 2*a^5*b*c*d - 4*a^3*b^3*c*d - 2*a*b^5*c*d + a^6*d^2
 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(b*tan(f*x + e) + a)))/f
```

3.76 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^3(c+d\tan(e+fx))}\,dx$

Optimal. Leaf size=477

$$\frac{\left(-a^3b^3\left(8cd(A-C)+B\left(c^2-d^2\right)\right)-3a^2b^4\left(c(2Bd+cC)-A\left(c^2+d^2\right)\right)+3a^4b^2d(2Ad+Bc-Cd)-3a^5bBd^2+a^6Cd^2}{f\left(a^2+b^2\right)^3(bc-ad)^3}$$

```
[Out] ((a^3*(A*c - c*C + B*d) - 3*a*b^2*(A*c - c*C + B*d) + 3*a^2*b*(B*c - (A - C
)*d) - b^3*(B*c - (A - C)*d))*x)/((a^2 + b^2)^3*(c^2 + d^2)) + ((3*a*b^5*B*
c^2 - 3*a^5*b*B*d^2 + a^6*C*d^2 + 3*a^4*b^2*d*(B*c + 2*A*d - C*d) + b^6*(c*
(c*C - B*d) - A*(c^2 - d^2)) - a^3*b^3*(8*c*(A - C)*d + B*(c^2 - d^2)) - 3*
a^2*b^4*(c*(c*C + 2*B*d) - A*(c^2 + d^2)))*Log[a*Cos[e + f*x] + b*Sin[e + f
*x]])/((a^2 + b^2)^3*(b*c - a*d)^3*f) - (d^2*(c^2*C - B*c*d + A*d^2)*Log[c*
Cos[e + f*x] + d*Sin[e + f*x]])/((b*c - a*d)^3*(c^2 + d^2)*f) - (A*b^2 - a*
(b*B - a*C))/(2*(a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])^2) - (2*a*b^
3*c*(A - C) + 2*a^3*b*B*d - a^4*C*d + b^4*(B*c - A*d) - a^2*b^2*(B*c + 3*A*
d - C*d))/((a^2 + b^2)^2*(b*c - a*d)^2*f*(a + b*Tan[e + f*x]))
```

Rubi [A] time = 1.78584, antiderivative size = 477, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3649, 3651, 3530}

$$\frac{\left(-a^3b^3\left(8cd(A-C)+B\left(c^2-d^2\right)\right)-3a^2b^4\left(c(2Bd+cC)-A\left(c^2+d^2\right)\right)+3a^4b^2d(2Ad+Bc-Cd)-3a^5bBd^2+a^6Cd^2}{f\left(a^2+b^2\right)^3(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^3*(c + d*
Tan[e + f*x])),x]
```

```
[Out] ((a^3*(A*c - c*C + B*d) - 3*a*b^2*(A*c - c*C + B*d) + 3*a^2*b*(B*c - (A - C
)*d) - b^3*(B*c - (A - C)*d))*x)/((a^2 + b^2)^3*(c^2 + d^2)) + ((3*a*b^5*B*
c^2 - 3*a^5*b*B*d^2 + a^6*C*d^2 + 3*a^4*b^2*d*(B*c + 2*A*d - C*d) + b^6*(c*
(c*C - B*d) - A*(c^2 - d^2)) - a^3*b^3*(8*c*(A - C)*d + B*(c^2 - d^2)) - 3*
a^2*b^4*(c*(c*C + 2*B*d) - A*(c^2 + d^2)))*Log[a*Cos[e + f*x] + b*Sin[e + f
*x]])/((a^2 + b^2)^3*(b*c - a*d)^3*f) - (d^2*(c^2*C - B*c*d + A*d^2)*Log[c*
Cos[e + f*x] + d*Sin[e + f*x]])/((b*c - a*d)^3*(c^2 + d^2)*f) - (A*b^2 - a*
(b*B - a*C))/(2*(a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])^2) - (2*a*b^
3*c*(A - C) + 2*a^3*b*B*d - a^4*C*d + b^4*(B*c - A*d) - a^2*b^2*(B*c + 3*A*
d - C*d))/((a^2 + b^2)^2*(b*c - a*d)^2*f*(a + b*Tan[e + f*x]))
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
 b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3651

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[((a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*x)
/((a^2 + b^2)*(c^2 + d^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^3(c+d\tan(e+fx))}\,dx = -\frac{Ab^2-a(bB-aC)}{2\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^2} - \int \frac{-2\left(abc(A-C)-a^2Ad+b^2(Bc-A\ldots\right.}{\ldots}$$

$$= -\frac{Ab^2-a(bB-aC)}{2\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^2} - \frac{2ab^3c(A-C)+2a^3bBd-\ldots}{\left(a^2+b^2\right)^2(\ldots}$$

$$= \frac{\left(a^3(Ac-cC+Bd)-3ab^2(Ac-cC+Bd)+3a^2b(Bc-(A-C)d)-b^3(B\ldots\right.}{\left(a^2+b^2\right)^3\left(c^2+d^2\right)}$$

$$= \frac{\left(a^3(Ac-cC+Bd)-3ab^2(Ac-cC+Bd)+3a^2b(Bc-(A-C)d)-b^3(B\ldots\right.}{\left(a^2+b^2\right)^3\left(c^2+d^2\right)}$$

Mathematica [A] time = 8.87886, size = 898, normalized size = 1.88

$$-\frac{Ab^2-a(bB-aC)}{2\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^2} - \frac{-\frac{-2\left(-Ada^2+bc(A-C)a+b^2(Bc-Ad)\right)b^2-a\left(2b(Ab-Cb-aB)(bc-ad)-2a\left(Ab^2-a(bB-aC)\right)d\right)}{\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))} - \frac{2b(a\ldots}{\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^3*(
c + d*Tan[e + f*x])),x]
```

```
[Out] -(A*b^2 - a*(b*B - a*C))/(2*(a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])^
2) - (-((-((b*(b*c - a*d)^2*(3*a^2*A*b*c - A*b^3*c - a^3*B*c + 3*a*b^2*B*c
- 3*a^2*b*c*C + b^3*c*C + a^3*A*d - 3*a*A*b^2*d + 3*a^2*b*B*d - b^3*B*d - a
^3*C*d + 3*a*b^2*C*d + (Sqrt[-b^2]*(a^3*(A*c - c*C + B*d) - 3*a*b^2*(A*c -
c*C + B*d) + 3*a^2*b*(B*c - (A - C)*d) - b^3*(B*c - (A - C)*d)))/b)*Log[Sqr
t[-b^2] - b*Tan[e + f*x]])/((a^2 + b^2)*(c^2 + d^2))) + (2*b*(3*a*b^5*B*c^2
 - 3*a^5*b*B*d^2 + a^6*C*d^2 + 3*a^4*b^2*d*(B*c + 2*A*d - C*d) + b^6*(c*(c*
C - B*d) - A*(c^2 - d^2)) - a^3*b^3*(8*c*(A - C)*d + B*(c^2 - d^2)) - 3*a^2
*b^4*(c*(c*C + 2*B*d) - A*(c^2 + d^2)))*Log[a + b*Tan[e + f*x]])/((a^2 + b^
2)*(b*c - a*d)) - (b*(b*c - a*d)^2*(3*a^2*A*b*c - A*b^3*c - a^3*B*c + 3*a*b
```

```
^2*B*c - 3*a^2*b*c*C + b^3*c*C + a^3*A*d - 3*a*A*b^2*d + 3*a^2*b*B*d - b^3*
B*d - a^3*C*d + 3*a*b^2*C*d - (Sqrt[-b^2]*(a^3*(A*c - c*C + B*d) - 3*a*b^2*
(A*c - c*C + B*d) + 3*a^2*b*(B*c - (A - C)*d) - b^3*(B*c - (A - C)*d)))/b)*
Log[Sqrt[-b^2] + b*Tan[e + f*x]])/((a^2 + b^2)*(c^2 + d^2)) - (2*b*(a^2 + b
^2)^2*d^2*(c^2*C - B*c*d + A*d^2)*Log[c + d*Tan[e + f*x]])/((b*c - a*d)*(c^
2 + d^2)))/(b*(a^2 + b^2)*(b*c - a*d)*f)) - (-(a*(-2*a*(A*b^2 - a*(b*B - a*
C))*d + 2*b*(A*b - a*B - b*C)*(b*c - a*d))) - 2*b^2*(a*b*c*(A - C) - a^2*A*
d + b^2*(B*c - A*d)))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])))/(2*
(a^2 + b^2)*(b*c - a*d))
```

Maple [B] time = 0.111, size = 2298, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^3/(c+d*tan(f*x+e)),x)
```

```
[Out] 3/2/f/(a^2+b^2)^3/(c^2+d^2)*ln(1+tan(f*x+e)^2)*A*a*b^2*d+1/f/(a^2+b^2)^2/(a
*d-b*c)^2/(a+b*tan(f*x+e))*B*a^2*b^2*c-1/f/(a^2+b^2)^2/(a*d-b*c)^2/(a+b*tan
(f*x+e))*C*a^2*b^2*d-3/2/f/(a^2+b^2)^3/(c^2+d^2)*ln(1+tan(f*x+e)^2)*B*a^2*b
*d+3/f/(a^2+b^2)^3/(c^2+d^2)*B*arctan(tan(f*x+e))*a^2*b*c-3/f/(a^2+b^2)^3/(
c^2+d^2)*B*arctan(tan(f*x+e))*a*b^2*d+3/f/(a^2+b^2)^3/(a*d-b*c)^3*ln(a+b*ta
n(f*x+e))*a^4*b^2*C*d^2+3/f/(a^2+b^2)^3/(a*d-b*c)^3*ln(a+b*tan(f*x+e))*C*a^
2*b^4*c^2+3/f/(a^2+b^2)^3/(a*d-b*c)^3*ln(a+b*tan(f*x+e))*a^5*b*B*d^2+1/f/(a
^2+b^2)^3/(a*d-b*c)^3*ln(a+b*tan(f*x+e))*B*a^3*b^3*c^2-1/f/(a^2+b^2)^3/(a*d
-b*c)^3*ln(a+b*tan(f*x+e))*B*a^3*b^3*d^2-3/2/f/(a^2+b^2)^3/(c^2+d^2)*ln(1+t
an(f*x+e)^2)*B*a*b^2*c-2/f/(a^2+b^2)^2/(a*d-b*c)^2/(a+b*tan(f*x+e))*A*a*b^3
*c-3/f/(a^2+b^2)^3/(a*d-b*c)^3*ln(a+b*tan(f*x+e))*a*b^5*B*c^2+1/f/(a^2+b^2)
^3/(a*d-b*c)^3*ln(a+b*tan(f*x+e))*B*b^6*c*d+2/f/(a^2+b^2)^2/(a*d-b*c)^2/(a+
b*tan(f*x+e))*C*a*b^3*c-6/f/(a^2+b^2)^3/(a*d-b*c)^3*ln(a+b*tan(f*x+e))*A*a^
4*b^2*d^2-3/f/(a^2+b^2)^3/(a*d-b*c)^3*ln(a+b*tan(f*x+e))*A*a^2*b^4*c^2-3/f/
(a^2+b^2)^3/(a*d-b*c)^3*ln(a+b*tan(f*x+e))*A*a^2*b^4*d^2+3/f/(a^2+b^2)^2/(a
*d-b*c)^2/(a+b*tan(f*x+e))*A*a^2*b^2*d+3/f/(a^2+b^2)^3/(c^2+d^2)*C*arctan(t
an(f*x+e))*a*b^2*c+3/2/f/(a^2+b^2)^3/(c^2+d^2)*ln(1+tan(f*x+e)^2)*C*a^2*b*c
-3/2/f/(a^2+b^2)^3/(c^2+d^2)*ln(1+tan(f*x+e)^2)*C*a*b^2*d-3/2/f/(a^2+b^2)^3
/(c^2+d^2)*ln(1+tan(f*x+e)^2)*A*a^2*b*c-3/f/(a^2+b^2)^3/(c^2+d^2)*A*arctan(
tan(f*x+e))*a^2*b*d-3/f/(a^2+b^2)^3/(c^2+d^2)*A*arctan(tan(f*x+e))*a*b^2*c-
2/f/(a^2+b^2)^2/(a*d-b*c)^2/(a+b*tan(f*x+e))*a^3*b*B*d+3/f/(a^2+b^2)^3/(c^2
+d^2)*C*arctan(tan(f*x+e))*a^2*b*d+1/f*d^4/(a*d-b*c)^3/(c^2+d^2)*ln(c+d*tan
(f*x+e))*A+1/2/f/(a^2+b^2)/(a*d-b*c)/(a+b*tan(f*x+e))^2*A*b^2+1/2/f/(a^2+b^
2)/(a*d-b*c)/(a+b*tan(f*x+e))^2*C*a^2+6/f/(a^2+b^2)^3/(a*d-b*c)^3*ln(a+b*ta
n(f*x+e))*B*a^2*b^4*c*d+8/f/(a^2+b^2)^3/(a*d-b*c)^3*ln(a+b*tan(f*x+e))*A*a^
3*b^3*c*d-8/f/(a^2+b^2)^3/(a*d-b*c)^3*ln(a+b*tan(f*x+e))*C*a^3*b^3*c*d-3/f/
(a^2+b^2)^3/(a*d-b*c)^3*ln(a+b*tan(f*x+e))*B*a^4*b^2*c*d+1/f/(a^2+b^2)^3/(c
^2+d^2)*B*arctan(tan(f*x+e))*a^3*d-1/f/(a^2+b^2)^3/(c^2+d^2)*B*arctan(tan(f
*x+e))*b^3*c-1/f/(a^2+b^2)^3/(c^2+d^2)*C*arctan(tan(f*x+e))*a^3*c-1/f/(a^2+
b^2)^3/(c^2+d^2)*C*arctan(tan(f*x+e))*b^3*d-1/f*d^3/(a*d-b*c)^3/(c^2+d^2)*l
n(c+d*tan(f*x+e))*B*c+1/f*d^2/(a*d-b*c)^3/(c^2+d^2)*ln(c+d*tan(f*x+e))*c^2*
C+1/f/(a^2+b^2)^2/(a*d-b*c)^2/(a+b*tan(f*x+e))*A*b^4*d-1/f/(a^2+b^2)^2/(a*d
-b*c)^2/(a+b*tan(f*x+e))*B*b^4*c+1/f/(a^2+b^2)^2/(a*d-b*c)^2/(a+b*tan(f*x+e
))*a^4*C*d+1/f/(a^2+b^2)^3/(a*d-b*c)^3*ln(a+b*tan(f*x+e))*A*b^6*c^2-1/f/(a^
2+b^2)^3/(a*d-b*c)^3*ln(a+b*tan(f*x+e))*A*b^6*d^2-1/f/(a^2+b^2)^3/(a*d-b*c)
^3*ln(a+b*tan(f*x+e))*a^6*C*d^2-1/f/(a^2+b^2)^3/(a*d-b*c)^3*ln(a+b*tan(f*x+
e))*C*b^6*c^2-1/2/f/(a^2+b^2)^3/(c^2+d^2)*ln(1+tan(f*x+e)^2)*A*a^3*d-1/2/f/
(a^2+b^2)^3/(c^2+d^2)*ln(1+tan(f*x+e)^2)*C*b^3*c+1/f/(a^2+b^2)^3/(c^2+d^2)*
A*arctan(tan(f*x+e))*a^3*c+1/f/(a^2+b^2)^3/(c^2+d^2)*A*arctan(tan(f*x+e))*b
```

```
^3*d+1/2/f/(a^2+b^2)^3/(c^2+d^2)*ln(1+tan(f*x+e)^2)*A*b^3*c+1/2/f/(a^2+b^2)
^3/(c^2+d^2)*ln(1+tan(f*x+e)^2)*B*a^3*c+1/2/f/(a^2+b^2)^3/(c^2+d^2)*ln(1+ta
n(f*x+e)^2)*B*b^3*d+1/2/f/(a^2+b^2)^3/(c^2+d^2)*ln(1+tan(f*x+e)^2)*a^3*C*d-
1/2/f/(a^2+b^2)/(a*d-b*c)/(a+b*tan(f*x+e))^2*B*a*b
```

Maxima [B] time = 1.80378, size = 1480, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^3/(c+d*tan(f*x+e
)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(((A - C)*a^3 + 3*B*a^2*b - 3*(A - C)*a*b^2 - B*b^3)*c + (B*a^3 - 3*
(A - C)*a^2*b - 3*B*a*b^2 + (A - C)*b^3)*d)*(f*x + e)/((a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)*c^2 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2) - 2*((B*a^3*
b^3 - 3*(A - C)*a^2*b^4 - 3*B*a*b^5 + (A - C)*b^6)*c^2 - (3*B*a^4*b^2 - 8*(
A - C)*a^3*b^3 - 6*B*a^2*b^4 - B*b^6)*c*d - (C*a^6 - 3*B*a^5*b + 3*(2*A - C
)*a^4*b^2 + B*a^3*b^3 + 3*A*a^2*b^4 + A*b^6)*d^2)*log(b*tan(f*x + e) + a)/(
(a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9)*c^3 - 3*(a^7*b^2 + 3*a^5*b^4 + 3*a^
3*b^6 + a*b^8)*c^2*d + 3*(a^8*b + 3*a^6*b^3 + 3*a^4*b^5 + a^2*b^7)*c*d^2 -
(a^9 + 3*a^7*b^2 + 3*a^5*b^4 + a^3*b^6)*d^3) - 2*(C*c^2*d^2 - B*c*d^3 + A*d
^4)*log(d*tan(f*x + e) + c)/(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c*d^4 - a^3*
d^5 + (3*a^2*b + b^3)*c^3*d^2 - (a^3 + 3*a*b^2)*c^2*d^3) + ((B*a^3 - 3*(A -
 C)*a^2*b - 3*B*a*b^2 + (A - C)*b^3)*c - ((A - C)*a^3 + 3*B*a^2*b - 3*(A -
C)*a*b^2 - B*b^3)*d)*log(tan(f*x + e)^2 + 1)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4
+ b^6)*c^2 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2) - ((C*a^4*b - 3*B*a^3
*b^2 + (5*A - 3*C)*a^2*b^3 + B*a*b^4 + A*b^5)*c - (3*C*a^5 - 5*B*a^4*b + (7
*A - C)*a^3*b^2 - B*a^2*b^3 + 3*A*a*b^4)*d - 2*((B*a^2*b^3 - 2*(A - C)*a*b^
4 - B*b^5)*c + (C*a^4*b - 2*B*a^3*b^2 + (3*A - C)*a^2*b^3 + A*b^5)*d)*tan(f
*x + e))/((a^6*b^2 + 2*a^4*b^4 + a^2*b^6)*c^2 - 2*(a^7*b + 2*a^5*b^3 + a^3*
b^5)*c*d + (a^8 + 2*a^6*b^2 + a^4*b^4)*d^2 + ((a^4*b^4 + 2*a^2*b^6 + b^8)*c
^2 - 2*(a^5*b^3 + 2*a^3*b^5 + a*b^7)*c*d + (a^6*b^2 + 2*a^4*b^4 + a^2*b^6)*
d^2)*tan(f*x + e)^2 + 2*((a^5*b^3 + 2*a^3*b^5 + a*b^7)*c^2 - 2*(a^6*b^2 + 2
*a^4*b^4 + a^2*b^6)*c*d + (a^7*b + 2*a^5*b^3 + a^3*b^5)*d^2)*tan(f*x + e)))
/f
```

Fricas [B] time = 26.3859, size = 7380, normalized size = 15.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^3/(c+d*tan(f*x+e
)),x, algorithm="fricas")
```

```
[Out] -1/2*((3*C*a^4*b^4 - 5*B*a^3*b^5 + (7*A - 3*C)*a^2*b^6 + B*a*b^7 + A*b^8)*c
^4 - 4*(2*C*a^5*b^3 - 3*B*a^4*b^4 + (4*A - C)*a^3*b^5 + A*a*b^7)*c^3*d + (5
*C*a^6*b^2 - 7*B*a^5*b^3 + (9*A + 2*C)*a^4*b^4 - 6*B*a^3*b^5 + (10*A - 3*C)
*a^2*b^6 + B*a*b^7 + A*b^8)*c^2*d^2 - 4*(2*C*a^5*b^3 - 3*B*a^4*b^4 + (4*A -
 C)*a^3*b^5 + A*a*b^7)*c*d^3 + (5*C*a^6*b^2 - 7*B*a^5*b^3 + (9*A - C)*a^4*b
^4 - B*a^3*b^5 + 3*A*a^2*b^6)*d^4 - 2*(((A - C)*a^5*b^3 + 3*B*a^4*b^4 - 3*(
A - C)*a^3*b^5 - B*a^2*b^6)*c^4 - (3*(A - C)*a^6*b^2 + 8*B*a^5*b^3 - 6*(A -
 C)*a^4*b^4 - (A - C)*a^2*b^6)*c^3*d + 3*((A - C)*a^7*b + 2*B*a^6*b^2 + 2*B
```

```
*a^4*b^4 - (A - C)*a^3*b^5)*c^2*d^2 - ((A - C)*a^8 + 6*(A - C)*a^6*b^2 + 8*
B*a^5*b^3 - 3*(A - C)*a^4*b^4)*c*d^3 - (B*a^8 - 3*(A - C)*a^7*b - 3*B*a^6*b
^2 + (A - C)*a^5*b^3)*d^4)*f*x - ((C*a^4*b^4 - 3*B*a^3*b^5 + 5*(A - C)*a^2*
b^6 + 3*B*a*b^7 - A*b^8)*c^4 - 4*(C*a^5*b^3 - 2*B*a^4*b^4 + (3*A - 2*C)*a^3
*b^5 + B*a^2*b^6)*c^3*d + (3*C*a^6*b^2 - 5*B*a^5*b^3 + (7*A - 2*C)*a^4*b^4
- 2*B*a^3*b^5 + (6*A - 5*C)*a^2*b^6 + 3*B*a*b^7 - A*b^8)*c^2*d^2 - 4*(C*a^5
*b^3 - 2*B*a^4*b^4 + (3*A - 2*C)*a^3*b^5 + B*a^2*b^6)*c*d^3 + (3*C*a^6*b^2
- 5*B*a^5*b^3 + (7*A - 3*C)*a^4*b^4 + B*a^3*b^5 + A*a^2*b^6)*d^4 + 2*(((A -
 C)*a^3*b^5 + 3*B*a^2*b^6 - 3*(A - C)*a*b^7 - B*b^8)*c^4 - (3*(A - C)*a^4*b
^4 + 8*B*a^3*b^5 - 6*(A - C)*a^2*b^6 - (A - C)*b^8)*c^3*d + 3*((A - C)*a^5*
b^3 + 2*B*a^4*b^4 + 2*B*a^2*b^6 - (A - C)*a*b^7)*c^2*d^2 - ((A - C)*a^6*b^2
 + 6*(A - C)*a^4*b^4 + 8*B*a^3*b^5 - 3*(A - C)*a^2*b^6)*c*d^3 - (B*a^6*b^2
- 3*(A - C)*a^5*b^3 - 3*B*a^4*b^4 + (A - C)*a^3*b^5)*d^4)*f*x)*tan(f*x + e)
^2 + ((B*a^5*b^3 - 3*(A - C)*a^4*b^4 - 3*B*a^3*b^5 + (A - C)*a^2*b^6)*c^4 -
 (3*B*a^6*b^2 - 8*(A - C)*a^5*b^3 - 6*B*a^4*b^4 - B*a^2*b^6)*c^3*d - (C*a^8
 - 3*B*a^7*b + 3*(2*A - C)*a^6*b^2 + 3*(2*A - C)*a^4*b^4 + 3*B*a^3*b^5 + C*
a^2*b^6)*c^2*d^2 - (3*B*a^6*b^2 - 8*(A - C)*a^5*b^3 - 6*B*a^4*b^4 - B*a^2*b
^6)*c*d^3 - (C*a^8 - 3*B*a^7*b + 3*(2*A - C)*a^6*b^2 + B*a^5*b^3 + 3*A*a^4*
b^4 + A*a^2*b^6)*d^4 + ((B*a^3*b^5 - 3*(A - C)*a^2*b^6 - 3*B*a*b^7 + (A - C
)*b^8)*c^4 - (3*B*a^4*b^4 - 8*(A - C)*a^3*b^5 - 6*B*a^2*b^6 - B*b^8)*c^3*d
- (C*a^6*b^2 - 3*B*a^5*b^3 + 3*(2*A - C)*a^4*b^4 + 3*(2*A - C)*a^2*b^6 + 3*
B*a*b^7 + C*b^8)*c^2*d^2 - (3*B*a^4*b^4 - 8*(A - C)*a^3*b^5 - 6*B*a^2*b^6 -
 B*b^8)*c*d^3 - (C*a^6*b^2 - 3*B*a^5*b^3 + 3*(2*A - C)*a^4*b^4 + B*a^3*b^5
+ 3*A*a^2*b^6 + A*b^8)*d^4)*tan(f*x + e)^2 + 2*((B*a^4*b^4 - 3*(A - C)*a^3*
b^5 - 3*B*a^2*b^6 + (A - C)*a*b^7)*c^4 - (3*B*a^5*b^3 - 8*(A - C)*a^4*b^4 -
 6*B*a^3*b^5 - B*a*b^7)*c^3*d - (C*a^7*b - 3*B*a^6*b^2 + 3*(2*A - C)*a^5*b^
3 + 3*(2*A - C)*a^3*b^5 + 3*B*a^2*b^6 + C*a*b^7)*c^2*d^2 - (3*B*a^5*b^3 - 8
*(A - C)*a^4*b^4 - 6*B*a^3*b^5 - B*a*b^7)*c*d^3 - (C*a^7*b - 3*B*a^6*b^2 +
3*(2*A - C)*a^5*b^3 + B*a^4*b^4 + 3*A*a^3*b^5 + A*a*b^7)*d^4)*tan(f*x + e))
*log((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)/(tan(f*x + e)^2 + 1))
+ ((C*a^8 + 3*C*a^6*b^2 + 3*C*a^4*b^4 + C*a^2*b^6)*c^2*d^2 - (B*a^8 + 3*B*a
^6*b^2 + 3*B*a^4*b^4 + B*a^2*b^6)*c*d^3 + (A*a^8 + 3*A*a^6*b^2 + 3*A*a^4*b^
4 + A*a^2*b^6)*d^4 + ((C*a^6*b^2 + 3*C*a^4*b^4 + 3*C*a^2*b^6 + C*b^8)*c^2*d
^2 - (B*a^6*b^2 + 3*B*a^4*b^4 + 3*B*a^2*b^6 + B*b^8)*c*d^3 + (A*a^6*b^2 + 3
*A*a^4*b^4 + 3*A*a^2*b^6 + A*b^8)*d^4)*tan(f*x + e)^2 + 2*((C*a^7*b + 3*C*a
^5*b^3 + 3*C*a^3*b^5 + C*a*b^7)*c^2*d^2 - (B*a^7*b + 3*B*a^5*b^3 + 3*B*a^3*
b^5 + B*a*b^7)*c*d^3 + (A*a^7*b + 3*A*a^5*b^3 + 3*A*a^3*b^5 + A*a*b^7)*d^4)
*tan(f*x + e))*log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) + c^2)/(tan(f*x
 + e)^2 + 1)) - 2*((C*a^5*b^3 - 2*B*a^4*b^4 + 3*(A - C)*a^3*b^5 + 3*B*a^2*b
^6 - (3*A - 2*C)*a*b^7 - B*b^8)*c^4 - (3*C*a^6*b^2 - 5*B*a^5*b^3 + (7*A - 6
*C)*a^4*b^4 + 6*B*a^3*b^5 - 3*(2*A - C)*a^2*b^6 - B*a*b^7 - A*b^8)*c^3*d +
(2*C*a^7*b - 3*B*a^6*b^2 + 2*(2*A - C)*a^5*b^3 + B*a^4*b^4 - 2*C*a^3*b^5 +
3*B*a^2*b^6 - 2*(2*A - C)*a*b^7 - B*b^8)*c^2*d^2 - (3*C*a^6*b^2 - 5*B*a^5*b
^3 + (7*A - 6*C)*a^4*b^4 + 6*B*a^3*b^5 - 3*(2*A - C)*a^2*b^6 - B*a*b^7 - A*
b^8)*c*d^3 + (2*C*a^7*b - 3*B*a^6*b^2 + (4*A - 3*C)*a^5*b^3 + 3*B*a^4*b^4 -
 (3*A - C)*a^3*b^5 - A*a*b^7)*d^4 + 2*(((A - C)*a^4*b^4 + 3*B*a^3*b^5 - 3*(
A - C)*a^2*b^6 - B*a*b^7)*c^4 - (3*(A - C)*a^5*b^3 + 8*B*a^4*b^4 - 6*(A - C
)*a^3*b^5 - (A - C)*a*b^7)*c^3*d + 3*((A - C)*a^6*b^2 + 2*B*a^5*b^3 + 2*B*a
^3*b^5 - (A - C)*a^2*b^6)*c^2*d^2 - ((A - C)*a^7*b + 6*(A - C)*a^5*b^3 + 8*
B*a^4*b^4 - 3*(A - C)*a^3*b^5)*c*d^3 - (B*a^7*b - 3*(A - C)*a^6*b^2 - 3*B*a
^5*b^3 + (A - C)*a^4*b^4)*d^4)*f*x)*tan(f*x + e))/(((a^6*b^5 + 3*a^4*b^7 +
3*a^2*b^9 + b^11)*c^5 - 3*(a^7*b^4 + 3*a^5*b^6 + 3*a^3*b^8 + a*b^10)*c^4*d
+ (3*a^8*b^3 + 10*a^6*b^5 + 12*a^4*b^7 + 6*a^2*b^9 + b^11)*c^3*d^2 - (a^9*b
^2 + 6*a^7*b^4 + 12*a^5*b^6 + 10*a^3*b^8 + 3*a*b^10)*c^2*d^3 + 3*(a^8*b^3 +
 3*a^6*b^5 + 3*a^4*b^7 + a^2*b^9)*c*d^4 - (a^9*b^2 + 3*a^7*b^4 + 3*a^5*b^6
+ a^3*b^8)*d^5)*f*tan(f*x + e)^2 + 2*((a^7*b^4 + 3*a^5*b^6 + 3*a^3*b^8 + a*
b^10)*c^5 - 3*(a^8*b^3 + 3*a^6*b^5 + 3*a^4*b^7 + a^2*b^9)*c^4*d + (3*a^9*b^
2 + 10*a^7*b^4 + 12*a^5*b^6 + 6*a^3*b^8 + a*b^10)*c^3*d^2 - (a^10*b + 6*a^8
*b^3 + 12*a^6*b^5 + 10*a^4*b^7 + 3*a^2*b^9)*c^2*d^3 + 3*(a^9*b^2 + 3*a^7*b^
```

```
4 + 3*a^5*b^6 + a^3*b^8)*c*d^4 - (a^10*b + 3*a^8*b^3 + 3*a^6*b^5 + a^4*b^7)
*d^5)*f*tan(f*x + e) + ((a^8*b^3 + 3*a^6*b^5 + 3*a^4*b^7 + a^2*b^9)*c^5 - 3
*(a^9*b^2 + 3*a^7*b^4 + 3*a^5*b^6 + a^3*b^8)*c^4*d + (3*a^10*b + 10*a^8*b^3
 + 12*a^6*b^5 + 6*a^4*b^7 + a^2*b^9)*c^3*d^2 - (a^11 + 6*a^9*b^2 + 12*a^7*b
^4 + 10*a^5*b^6 + 3*a^3*b^8)*c^2*d^3 + 3*(a^10*b + 3*a^8*b^3 + 3*a^6*b^5 +
a^4*b^7)*c*d^4 - (a^11 + 3*a^9*b^2 + 3*a^7*b^4 + a^5*b^6)*d^5)*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**3/(c+d*tan(f*x
+e)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.91993, size = 2871, normalized size = 6.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^3/(c+d*tan(f*x+e
)),x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a^3*c - C*a^3*c + 3*B*a^2*b*c - 3*A*a*b^2*c + 3*C*a*b^2*c - B*b^3
*c + B*a^3*d - 3*A*a^2*b*d + 3*C*a^2*b*d - 3*B*a*b^2*d + A*b^3*d - C*b^3*d)
*(f*x + e)/(a^6*c^2 + 3*a^4*b^2*c^2 + 3*a^2*b^4*c^2 + b^6*c^2 + a^6*d^2 + 3
*a^4*b^2*d^2 + 3*a^2*b^4*d^2 + b^6*d^2) + (B*a^3*c - 3*A*a^2*b*c + 3*C*a^2*
b*c - 3*B*a*b^2*c + A*b^3*c - C*b^3*c - A*a^3*d + C*a^3*d - 3*B*a^2*b*d + 3
*A*a*b^2*d - 3*C*a*b^2*d + B*b^3*d)*log(tan(f*x + e)^2 + 1)/(a^6*c^2 + 3*a^
4*b^2*c^2 + 3*a^2*b^4*c^2 + b^6*c^2 + a^6*d^2 + 3*a^4*b^2*d^2 + 3*a^2*b^4*d
^2 + b^6*d^2) - 2*(B*a^3*b^4*c^2 - 3*A*a^2*b^5*c^2 + 3*C*a^2*b^5*c^2 - 3*B*
a*b^6*c^2 + A*b^7*c^2 - C*b^7*c^2 - 3*B*a^4*b^3*c*d + 8*A*a^3*b^4*c*d - 8*C
*a^3*b^4*c*d + 6*B*a^2*b^5*c*d + B*b^7*c*d - C*a^6*b*d^2 + 3*B*a^5*b^2*d^2
- 6*A*a^4*b^3*d^2 + 3*C*a^4*b^3*d^2 - B*a^3*b^4*d^2 - 3*A*a^2*b^5*d^2 - A*b
^7*d^2)*log(abs(b*tan(f*x + e) + a))/(a^6*b^4*c^3 + 3*a^4*b^6*c^3 + 3*a^2*b
^8*c^3 + b^10*c^3 - 3*a^7*b^3*c^2*d - 9*a^5*b^5*c^2*d - 9*a^3*b^7*c^2*d - 3
*a*b^9*c^2*d + 3*a^8*b^2*c*d^2 + 9*a^6*b^4*c*d^2 + 9*a^4*b^6*c*d^2 + 3*a^2*
b^8*c*d^2 - a^9*b*d^3 - 3*a^7*b^3*d^3 - 3*a^5*b^5*d^3 - a^3*b^7*d^3) - 2*(C
*c^2*d^3 - B*c*d^4 + A*d^5)*log(abs(d*tan(f*x + e) + c))/(b^3*c^5*d - 3*a*b
^2*c^4*d^2 + 3*a^2*b*c^3*d^3 + b^3*c^3*d^3 - a^3*c^2*d^4 - 3*a*b^2*c^2*d^4
+ 3*a^2*b*c*d^5 - a^3*d^6) + (3*B*a^3*b^5*c^2*tan(f*x + e)^2 - 9*A*a^2*b^6*
c^2*tan(f*x + e)^2 + 9*C*a^2*b^6*c^2*tan(f*x + e)^2 - 9*B*a*b^7*c^2*tan(f*x
 + e)^2 + 3*A*b^8*c^2*tan(f*x + e)^2 - 3*C*b^8*c^2*tan(f*x + e)^2 - 9*B*a^4
*b^4*c*d*tan(f*x + e)^2 + 24*A*a^3*b^5*c*d*tan(f*x + e)^2 - 24*C*a^3*b^5*c*
d*tan(f*x + e)^2 + 18*B*a^2*b^6*c*d*tan(f*x + e)^2 + 3*B*b^8*c*d*tan(f*x +
e)^2 - 3*C*a^6*b^2*d^2*tan(f*x + e)^2 + 9*B*a^5*b^3*d^2*tan(f*x + e)^2 - 18
*A*a^4*b^4*d^2*tan(f*x + e)^2 + 9*C*a^4*b^4*d^2*tan(f*x + e)^2 - 3*B*a^3*b^
5*d^2*tan(f*x + e)^2 - 9*A*a^2*b^6*d^2*tan(f*x + e)^2 - 3*A*b^8*d^2*tan(f*x
 + e)^2 + 8*B*a^4*b^4*c^2*tan(f*x + e) - 22*A*a^3*b^5*c^2*tan(f*x + e) + 22
*C*a^3*b^5*c^2*tan(f*x + e) - 18*B*a^2*b^6*c^2*tan(f*x + e) + 2*A*a*b^7*c^2
```

```
*tan(f*x + e) - 2*C*a*b^7*c^2*tan(f*x + e) - 2*B*b^8*c^2*tan(f*x + e) + 2*C
*a^6*b^2*c*d*tan(f*x + e) - 24*B*a^5*b^3*c*d*tan(f*x + e) + 58*A*a^4*b^4*c*
d*tan(f*x + e) - 52*C*a^4*b^4*c*d*tan(f*x + e) + 32*B*a^3*b^5*c*d*tan(f*x +
 e) + 12*A*a^2*b^6*c*d*tan(f*x + e) - 6*C*a^2*b^6*c*d*tan(f*x + e) + 8*B*a*
b^7*c*d*tan(f*x + e) + 2*A*b^8*c*d*tan(f*x + e) - 8*C*a^7*b*d^2*tan(f*x + e
) + 22*B*a^6*b^2*d^2*tan(f*x + e) - 42*A*a^5*b^3*d^2*tan(f*x + e) + 18*C*a^
5*b^3*d^2*tan(f*x + e) - 2*B*a^4*b^4*d^2*tan(f*x + e) - 26*A*a^3*b^5*d^2*ta
n(f*x + e) + 2*C*a^3*b^5*d^2*tan(f*x + e) - 8*A*a*b^7*d^2*tan(f*x + e) - C*
a^6*b^2*c^2 + 6*B*a^5*b^3*c^2 - 14*A*a^4*b^4*c^2 + 11*C*a^4*b^4*c^2 - 7*B*a
^3*b^5*c^2 - 3*A*a^2*b^6*c^2 - B*a*b^7*c^2 - A*b^8*c^2 + 4*C*a^7*b*c*d - 17
*B*a^6*b^2*c*d + 36*A*a^5*b^3*c*d - 24*C*a^5*b^3*c*d + 10*B*a^4*b^4*c*d + 1
6*A*a^3*b^5*c*d - 4*C*a^3*b^5*c*d + 3*B*a^2*b^6*c*d + 4*A*a*b^7*c*d - 6*C*a
^8*d^2 + 14*B*a^7*b*d^2 - 25*A*a^6*b^2*d^2 + 7*C*a^6*b^2*d^2 + 3*B*a^5*b^3*
d^2 - 19*A*a^4*b^4*d^2 + C*a^4*b^4*d^2 + B*a^3*b^5*d^2 - 6*A*a^2*b^6*d^2)/(
(a^6*b^3*c^3 + 3*a^4*b^5*c^3 + 3*a^2*b^7*c^3 + b^9*c^3 - 3*a^7*b^2*c^2*d -
9*a^5*b^4*c^2*d - 9*a^3*b^6*c^2*d - 3*a*b^8*c^2*d + 3*a^8*b*c*d^2 + 9*a^6*b
^3*c*d^2 + 9*a^4*b^5*c*d^2 + 3*a^2*b^7*c*d^2 - a^9*d^3 - 3*a^7*b^2*d^3 - 3*
a^5*b^4*d^3 - a^3*b^6*d^3)*(b*tan(f*x + e) + a)^2))/f
```

3.77 $\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^2}\,dx$

Optimal. Leaf size=579

$$\frac{\log(\cos(e+fx))\left(3a^2b\left(-A\left(c^2-d^2\right)-2Bcd+c^2C-Cd^2\right)+a^3\left(2cd(A-C)-B\left(c^2-d^2\right)\right)-3ab^2\left(2cd(A-C)-B\left(c^2-\right.\right.\right.}{f\left(c^2+d^2\right)^2}$$

```
[Out] -(((a^3*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - 3*a*b^2*(c^2*C - 2*B*c*
d - C*d^2 - A*(c^2 - d^2)) - 3*a^2*b*(2*c*(A - C)*d - B*(c^2 - d^2)) + b^3*
(2*c*(A - C)*d - B*(c^2 - d^2)))*x)/(c^2 + d^2)^2) + ((3*a^2*b*(c^2*C - 2*B
*c*d - C*d^2 - A*(c^2 - d^2)) - b^3*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2
)) + a^3*(2*c*(A - C)*d - B*(c^2 - d^2)) - 3*a*b^2*(2*c*(A - C)*d - B*(c^2
- d^2)))*Log[Cos[e + f*x]])/((c^2 + d^2)^2*f) + ((b*c - a*d)^2*(b*(3*c^4*C
- 2*B*c^3*d + c^2*(A + 5*C)*d^2 - 4*B*c*d^3 + 3*A*d^4) + a*d^2*(2*c*(A - C)
*d - B*(c^2 - d^2)))*Log[c + d*Tan[e + f*x]])/(d^4*(c^2 + d^2)^2*f) + (b^2*
(a*d*(3*c^2*C - B*c*d + (A + 2*C)*d^2) - b*(3*c^3*C - 2*B*c^2*d + c*(A + 2*
C)*d^2 - B*d^3))*Tan[e + f*x])/(d^3*(c^2 + d^2)*f) + (b*(3*c^2*C - 2*B*c*d
+ (2*A + C)*d^2)*(a + b*Tan[e + f*x])^2)/(2*d^2*(c^2 + d^2)*f) - ((c^2*C -
B*c*d + A*d^2)*(a + b*Tan[e + f*x])^3)/(d*(c^2 + d^2)*f*(c + d*Tan[e + f*x]
))
```

Rubi [A] time = 2.13382, antiderivative size = 579, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3645, 3647, 3637, 3626, 3617, 31, 3475}

$$\frac{\log(\cos(e+fx))\left(3a^2b\left(-A\left(c^2-d^2\right)-2Bcd+c^2C-Cd^2\right)+a^3\left(2cd(A-C)-B\left(c^2-d^2\right)\right)-3ab^2\left(2cd(A-C)-B\left(c^2-\right.\right.\right.}{f\left(c^2+d^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d
*Tan[e + f*x])^2,x]
```


```
[Out] -(((a^3*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - 3*a*b^2*(c^2*C - 2*B*c*
d - C*d^2 - A*(c^2 - d^2)) - 3*a^2*b*(2*c*(A - C)*d - B*(c^2 - d^2)) + b^3*
(2*c*(A - C)*d - B*(c^2 - d^2)))*x)/(c^2 + d^2)^2) + ((3*a^2*b*(c^2*C - 2*B
*c*d - C*d^2 - A*(c^2 - d^2)) - b^3*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2
)) + a^3*(2*c*(A - C)*d - B*(c^2 - d^2)) - 3*a*b^2*(2*c*(A - C)*d - B*(c^2
- d^2)))*Log[Cos[e + f*x]])/((c^2 + d^2)^2*f) + ((b*c - a*d)^2*(b*(3*c^4*C
- 2*B*c^3*d + c^2*(A + 5*C)*d^2 - 4*B*c*d^3 + 3*A*d^4) + a*d^2*(2*c*(A - C)
*d - B*(c^2 - d^2)))*Log[c + d*Tan[e + f*x]])/(d^4*(c^2 + d^2)^2*f) + (b^2*
(a*d*(3*c^2*C - B*c*d + (A + 2*C)*d^2) - b*(3*c^3*C - 2*B*c^2*d + c*(A + 2*
C)*d^2 - B*d^3))*Tan[e + f*x])/(d^3*(c^2 + d^2)*f) + (b*(3*c^2*C - 2*B*c*d
+ (2*A + C)*d^2)*(a + b*Tan[e + f*x])^2)/(2*d^2*(c^2 + d^2)*f) - ((c^2*C -
B*c*d + A*d^2)*(a + b*Tan[e + f*x])^3)/(d*(c^2 + d^2)*f*(c + d*Tan[e + f*x]
))
```


Rule 3645

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*d^2 + c*(c*C - B*d))*(a + b*Tan[e
+ f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
```

```
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3626

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((a*A + b*B -
a*C)*x)/(a^2 + b^2), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3617

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^2}dx = -\frac{\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^3}{d\left(c^2+d^2\right)f(c+d\tan(e+fx))} + \int \frac{(a+b\,\text{ta}}{}$$

$$= \frac{b\left(3c^2C-2Bcd+(2A+C)d^2\right)(a+b\tan(e+fx))^2}{2d^2\left(c^2+d^2\right)f}$$

$$= \frac{b^2\left(ad\left(3c^2C-Bcd+(A+2C)d^2\right)-b\left(3c^3C-2Bc^2d\right.\right.}{d^3\left(c^2+d^2\right)f}$$

$$= -\frac{\left(a^3\left(c^2C-2Bcd-Cd^2-A\left(c^2-d^2\right)\right)-3ab^2\left(c^2C\right.\right.}{}$$

$$= -\frac{\left(a^3\left(c^2C-2Bcd-Cd^2-A\left(c^2-d^2\right)\right)-3ab^2\left(c^2C\right.\right.}{}$$

$$= -\frac{\left(a^3\left(c^2C-2Bcd-Cd^2-A\left(c^2-d^2\right)\right)-3ab^2\left(c^2C\right.\right.}{}$$

Mathematica [C] time = 8.39404, size = 2463, normalized size = 4.25

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(c + d*Tan[e + f*x])^2,x]
```

```
[Out] ((a^3*A*c^2 - 3*a*A*b^2*c^2 - 3*a^2*b*B*c^2 + b^3*B*c^2 - a^3*c^2*C + 3*a*b
^2*c^2*C + 6*a^2*A*b*c*d - 2*A*b^3*c*d + 2*a^3*B*c*d - 6*a*b^2*B*c*d - 6*a^
2*b*c*C*d + 2*b^3*c*C*d - a^3*A*d^2 + 3*a*A*b^2*d^2 + 3*a^2*b*B*d^2 - b^3*B
*d^2 + a^3*C*d^2 - 3*a*b^2*C*d^2)*(e + f*x)*Cos[e + f*x]*(c*Cos[e + f*x] +
d*Sin[e + f*x])^2*(a + b*Tan[e + f*x])^3)/((c - I*d)^2*(c + I*d)^2*f*(a*Cos
[e + f*x] + b*Sin[e + f*x])^3*(c + d*Tan[e + f*x])^2) + (((3*I)*b^3*c^11*C*
d^3 - (2*I)*b^3*B*c^10*d^4 - (6*I)*a*b^2*c^10*C*d^4 + 3*b^3*c^10*C*d^4 + I*
A*b^3*c^9*d^5 + (3*I)*a*b^2*B*c^9*d^5 - 2*b^3*B*c^9*d^5 + (3*I)*a^2*b*c^9*C
*d^5 - 6*a*b^2*c^9*C*d^5 + (8*I)*b^3*c^9*C*d^5 + A*b^3*c^8*d^6 + 3*a*b^2*B*
c^8*d^6 - (6*I)*b^3*B*c^8*d^6 + 3*a^2*b*c^8*C*d^6 - (18*I)*a*b^2*c^8*C*d^6
+ 8*b^3*c^8*C*d^6 - (3*I)*a^2*A*b*c^7*d^7 + (4*I)*A*b^3*c^7*d^7 - I*a^3*B*c
^7*d^7 + (12*I)*a*b^2*B*c^7*d^7 - 6*b^3*B*c^7*d^7 + (12*I)*a^2*b*c^7*C*d^7
- 18*a*b^2*c^7*C*d^7 + (5*I)*b^3*c^7*C*d^7 + (2*I)*a^3*A*c^6*d^8 - 3*a^2*A*
b*c^6*d^8 - (6*I)*a*A*b^2*c^6*d^8 + 4*A*b^3*c^6*d^8 - a^3*B*c^6*d^8 - (6*I)
*a^2*b*B*c^6*d^8 + 12*a*b^2*B*c^6*d^8 - (4*I)*b^3*B*c^6*d^8 - (2*I)*a^3*c^6
*C*d^8 + 12*a^2*b*c^6*C*d^8 - (12*I)*a*b^2*c^6*C*d^8 + 5*b^3*c^6*C*d^8 + 2*
a^3*A*c^5*d^9 - 6*a*A*b^2*c^5*d^9 + (3*I)*A*b^3*c^5*d^9 - 6*a^2*b*B*c^5*d^9
 + (9*I)*a*b^2*B*c^5*d^9 - 4*b^3*B*c^5*d^9 - 2*a^3*c^5*C*d^9 + (9*I)*a^2*b*
c^5*C*d^9 - 12*a*b^2*c^5*C*d^9 + (2*I)*a^3*A*c^4*d^10 - (6*I)*a*A*b^2*c^4*d
^10 + 3*A*b^3*c^4*d^10 - (6*I)*a^2*b*B*c^4*d^10 + 9*a*b^2*B*c^4*d^10 - (2*I
)*a^3*c^4*C*d^10 + 9*a^2*b*c^4*C*d^10 + 2*a^3*A*c^3*d^11 + (3*I)*a^2*A*b*c^
3*d^11 - 6*a*A*b^2*c^3*d^11 + I*a^3*B*c^3*d^11 - 6*a^2*b*B*c^3*d^11 - 2*a^3
*c^3*C*d^11 + 3*a^2*A*b*c^2*d^12 + a^3*B*c^2*d^12)*(e + f*x)*Cos[e + f*x]*(
c*Cos[e + f*x] + d*Sin[e + f*x])^2*(a + b*Tan[e + f*x])^3)/(c^2*(c - I*d)^4
*(c + I*d)^3*d^7*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^3*(c + d*Tan[e + f*x])
^2) - (I*(3*b^3*c^6*C - 2*b^3*B*c^5*d - 6*a*b^2*c^5*C*d + A*b^3*c^4*d^2 + 3
*a*b^2*B*c^4*d^2 + 3*a^2*b*c^4*C*d^2 + 5*b^3*c^4*C*d^2 - 4*b^3*B*c^3*d^3 -
```

```
12*a*b^2*c^3*C*d^3 - 3*a^2*A*b*c^2*d^4 + 3*A*b^3*c^2*d^4 - a^3*B*c^2*d^4 +
9*a*b^2*B*c^2*d^4 + 9*a^2*b*c^2*C*d^4 + 2*a^3*A*c*d^5 - 6*a*A*b^2*c*d^5 - 6
*a^2*b*B*c*d^5 - 2*a^3*c*C*d^5 + 3*a^2*A*b*d^6 + a^3*B*d^6)*ArcTan[Tan[e +
f*x]]*Cos[e + f*x]*(c*Cos[e + f*x] + d*Sin[e + f*x])^2*(a + b*Tan[e + f*x])
^3)/(d^4*(c^2 + d^2)^2*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^3*(c + d*Tan[e +
 f*x])^2) + ((-3*b^3*c^2*C + 2*b^3*B*c*d + 6*a*b^2*c*C*d - A*b^3*d^2 - 3*a*
b^2*B*d^2 - 3*a^2*b*C*d^2 + b^3*C*d^2)*Cos[e + f*x]*Log[Cos[e + f*x]]*(c*Co
s[e + f*x] + d*Sin[e + f*x])^2*(a + b*Tan[e + f*x])^3)/(d^4*f*(a*Cos[e + f*
x] + b*Sin[e + f*x])^3*(c + d*Tan[e + f*x])^2) + ((3*b^3*c^6*C - 2*b^3*B*c^
5*d - 6*a*b^2*c^5*C*d + A*b^3*c^4*d^2 + 3*a*b^2*B*c^4*d^2 + 3*a^2*b*c^4*C*d
^2 + 5*b^3*c^4*C*d^2 - 4*b^3*B*c^3*d^3 - 12*a*b^2*c^3*C*d^3 - 3*a^2*A*b*c^2
*d^4 + 3*A*b^3*c^2*d^4 - a^3*B*c^2*d^4 + 9*a*b^2*B*c^2*d^4 + 9*a^2*b*c^2*C*
d^4 + 2*a^3*A*c*d^5 - 6*a*A*b^2*c*d^5 - 6*a^2*b*B*c*d^5 - 2*a^3*c*C*d^5 + 3
*a^2*A*b*d^6 + a^3*B*d^6)*Cos[e + f*x]*Log[(c*Cos[e + f*x] + d*Sin[e + f*x]
)^2]*(c*Cos[e + f*x] + d*Sin[e + f*x])^2*(a + b*Tan[e + f*x])^3)/(2*d^4*(c^
2 + d^2)^2*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^3*(c + d*Tan[e + f*x])^2) +
(b^3*C*Sec[e + f*x]*(c*Cos[e + f*x] + d*Sin[e + f*x])^2*(a + b*Tan[e + f*x]
)^3)/(2*d^2*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^3*(c + d*Tan[e + f*x])^2) +
 ((c*Cos[e + f*x] + d*Sin[e + f*x])^2*(-2*b^3*c*C*Sin[e + f*x] + b^3*B*d*Si
n[e + f*x] + 3*a*b^2*C*d*Sin[e + f*x])*(a + b*Tan[e + f*x])^3)/(d^3*f*(a*Co
s[e + f*x] + b*Sin[e + f*x])^3*(c + d*Tan[e + f*x])^2) + (Cos[e + f*x]*(c*C
os[e + f*x] + d*Sin[e + f*x])*(-(b^3*c^5*C*Sin[e + f*x]) + b^3*B*c^4*d*Sin[
e + f*x] + 3*a*b^2*c^4*C*d*Sin[e + f*x] - A*b^3*c^3*d^2*Sin[e + f*x] - 3*a*
b^2*B*c^3*d^2*Sin[e + f*x] - 3*a^2*b*c^3*C*d^2*Sin[e + f*x] + 3*a*A*b^2*c^2
*d^3*Sin[e + f*x] + 3*a^2*b*B*c^2*d^3*Sin[e + f*x] + a^3*c^2*C*d^3*Sin[e +
f*x] - 3*a^2*A*b*c*d^4*Sin[e + f*x] - a^3*B*c*d^4*Sin[e + f*x] + a^3*A*d^5*
Sin[e + f*x])*(a + b*Tan[e + f*x])^3)/(c*(c - I*d)*(c + I*d)*d^3*f*(a*Cos[e
 + f*x] + b*Sin[e + f*x])^3*(c + d*Tan[e + f*x])^2)
```

Maple [B] time = 0.07, size = 2250, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^2,x
)
```

```
[Out] -1/f/d/(c^2+d^2)/(c+d*tan(f*x+e))*C*c^2*a^3+1/f/d^4/(c^2+d^2)/(c+d*tan(f*x+
e))*C*c^5*b^3+2/f*d/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*A*a^3*c+3/f*d^2/(c^2+d^2
)^2*ln(c+d*tan(f*x+e))*A*a^2*b+1/f/d^2/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*A*b^3
*c^4+9/f/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*B*a*b^2*c^2+9/f/(c^2+d^2)^2*ln(c+d*
tan(f*x+e))*C*a^2*b*c^2-3/f/(c^2+d^2)^2*A*arctan(tan(f*x+e))*a*b^2*c^2+3/f/
(c^2+d^2)^2*A*arctan(tan(f*x+e))*a*b^2*d^2+3/f/(c^2+d^2)^2*ln(1+tan(f*x+e)^
2)*B*a^2*b*c*d-3/f/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*C*a*b^2*c*d+6/f/(c^2+d^2)
^2*A*arctan(tan(f*x+e))*a^2*b*c*d+3/f/d^2/(c^2+d^2)/(c+d*tan(f*x+e))*C*c^3*
a^2*b-3/f/d^3/(c^2+d^2)/(c+d*tan(f*x+e))*C*c^4*a*b^2-6/f*d/(c^2+d^2)^2*ln(c
+d*tan(f*x+e))*A*a*b^2*c-6/f*d/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*B*a^2*b*c+3/f
/d^2/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*B*a*b^2*c^4+3/f/d^2/(c^2+d^2)^2*ln(c+d*
tan(f*x+e))*C*a^2*b*c^4-12/f/d/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*C*a*b^2*c^3-3
/f/d/(c^2+d^2)/(c+d*tan(f*x+e))*A*c^2*a*b^2-3/f/d/(c^2+d^2)/(c+d*tan(f*x+e)
)*B*c^2*a^2*b+3/f/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*A*a*b^2*c*d+3/f/d^2/(c^2+d
^2)/(c+d*tan(f*x+e))*B*c^3*a*b^2-6/f/(c^2+d^2)^2*C*arctan(tan(f*x+e))*a^2*b
*c*d-6/f/d^3/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*C*a*b^2*c^5-6/f/(c^2+d^2)^2*B*a
rctan(tan(f*x+e))*a*b^2*c*d+3/2/f/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*B*a*b^2*d^
2-2/f/(c^2+d^2)^2*A*arctan(tan(f*x+e))*b^3*c*d+2/f/(c^2+d^2)^2*B*arctan(tan
(f*x+e))*a^3*c*d-3/f/(c^2+d^2)^2*B*arctan(tan(f*x+e))*a^2*b*c^2-3/2/f/(c^2+
```

```
d^2)^2*ln(1+tan(f*x+e)^2)*C*a^2*b*c^2+3/2/f/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*
C*a^2*b*d^2+3/f/(c^2+d^2)/(c+d*tan(f*x+e))*A*a^2*c*b-3/f/(c^2+d^2)^2*ln(c+d
*tan(f*x+e))*A*a^2*b*c^2-2/f/d^3/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*B*b^3*c^5-4
/f/d/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*B*b^3*c^3+1/f/d^2/(c^2+d^2)/(c+d*tan(f*
x+e))*A*c^3*b^3-1/f/d^3/(c^2+d^2)/(c+d*tan(f*x+e))*B*c^4*b^3+1/f/(c^2+d^2)^
2*B*arctan(tan(f*x+e))*b^3*c^2-1/f*d/(c^2+d^2)/(c+d*tan(f*x+e))*A*a^3+1/f*d
^2/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*B*a^3-1/f/(c^2+d^2)^2*B*arctan(tan(f*x+e)
)*b^3*d^2-1/f/(c^2+d^2)^2*C*arctan(tan(f*x+e))*a^3*c^2+1/f/(c^2+d^2)^2*C*ar
ctan(tan(f*x+e))*a^3*d^2-1/2/f/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*A*b^3*c^2+1/2
/f/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*A*b^3*d^2+1/2/f/(c^2+d^2)^2*ln(1+tan(f*x+
e)^2)*B*a^3*c^2+3/f*b^2/d^2*a*C*tan(f*x+e)-2/f*b^3/d^3*C*c*tan(f*x+e)+1/f/(
c^2+d^2)/(c+d*tan(f*x+e))*B*a^3*c+3/f/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*A*b^3*
c^2-1/f/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*B*a^3*c^2-1/2/f/(c^2+d^2)^2*ln(1+tan
(f*x+e)^2)*B*a^3*d^2+1/2/f/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*C*b^3*c^2-1/2/f/(
c^2+d^2)^2*ln(1+tan(f*x+e)^2)*C*b^3*d^2+1/f/(c^2+d^2)^2*A*arctan(tan(f*x+e)
)*a^3*c^2-1/f/(c^2+d^2)^2*A*arctan(tan(f*x+e))*a^3*d^2-2/f*d/(c^2+d^2)^2*ln
(c+d*tan(f*x+e))*C*a^3*c+3/f/d^4/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*C*b^3*c^6+5
/f/d^2/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*C*b^3*c^4-1/f/(c^2+d^2)^2*ln(1+tan(f*
x+e)^2)*B*b^3*c*d+1/f/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*C*a^3*c*d+3/f/(c^2+d^2
)^2*B*arctan(tan(f*x+e))*a^2*b*d^2+3/f/(c^2+d^2)^2*C*arctan(tan(f*x+e))*a*b
^2*c^2-3/f/(c^2+d^2)^2*C*arctan(tan(f*x+e))*a*b^2*d^2+2/f/(c^2+d^2)^2*C*arc
tan(tan(f*x+e))*b^3*c*d-1/f/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*A*a^3*c*d+3/2/f/
(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*A*a^2*b*c^2-3/2/f/(c^2+d^2)^2*ln(1+tan(f*x+e
)^2)*A*a^2*b*d^2-3/2/f/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*B*a*b^2*c^2+1/f*b^3/d
^2*B*tan(f*x+e)+1/2/f*b^3/d^2*C*tan(f*x+e)^2
```

Maxima [A] time = 1.5944, size = 923, normalized size = 1.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*c^2 + 2*(B*a^3
+ 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*c*d - ((A - C)*a^3 - 3*B*a^2*b
 - 3*(A - C)*a*b^2 + B*b^3)*d^2)*(f*x + e)/(c^4 + 2*c^2*d^2 + d^4) + 2*(3*C
*b^3*c^6 - 2*(3*C*a*b^2 + B*b^3)*c^5*d + (3*C*a^2*b + 3*B*a*b^2 + (A + 5*C)
*b^3)*c^4*d^2 - 4*(3*C*a*b^2 + B*b^3)*c^3*d^3 - (B*a^3 + 3*(A - 3*C)*a^2*b
- 9*B*a*b^2 - 3*A*b^3)*c^2*d^4 + 2*((A - C)*a^3 - 3*B*a^2*b - 3*A*a*b^2)*c*
d^5 + (B*a^3 + 3*A*a^2*b)*d^6)*log(d*tan(f*x + e) + c)/(c^4*d^4 + 2*c^2*d^6
 + d^8) + ((B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*c^2 - 2*((A
- C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*c*d - (B*a^3 + 3*(A - C)*a^
2*b - 3*B*a*b^2 - (A - C)*b^3)*d^2)*log(tan(f*x + e)^2 + 1)/(c^4 + 2*c^2*d^
2 + d^4) + 2*(C*b^3*c^5 - A*a^3*d^5 - (3*C*a*b^2 + B*b^3)*c^4*d + (3*C*a^2*
b + 3*B*a*b^2 + A*b^3)*c^3*d^2 - (C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c^2*d^3 +
(B*a^3 + 3*A*a^2*b)*c*d^4)/(c^3*d^4 + c*d^6 + (c^2*d^5 + d^7)*tan(f*x + e))
 + (C*b^3*d*tan(f*x + e)^2 - 2*(2*C*b^3*c - (3*C*a*b^2 + B*b^3)*d)*tan(f*x
+ e))/d^3)/f
```

Fricas [B] time = 8.21503, size = 3141, normalized size = 5.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(3*C*b^3*c^5*d^2 - 2*A*a^3*d^7 - 2*(3*C*a*b^2 + B*b^3)*c^4*d^3 + 2*(3*C
*a^2*b + 3*B*a*b^2 + (A + C)*b^3)*c^3*d^4 - 2*(C*a^3 + 3*B*a^2*b + 3*A*a*b^
2)*c^2*d^5 + (2*B*a^3 + 6*A*a^2*b + C*b^3)*c*d^6 + (C*b^3*c^4*d^3 + 2*C*b^3
*c^2*d^5 + C*b^3*d^7)*tan(f*x + e)^3 + 2*(((A - C)*a^3 - 3*B*a^2*b - 3*(A -
 C)*a*b^2 + B*b^3)*c^3*d^4 + 2*(B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A -
C)*b^3)*c^2*d^5 - ((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*c*d^6
)*f*x - (3*C*b^3*c^5*d^2 + 6*C*b^3*c^3*d^4 + 3*C*b^3*c*d^6 - 2*(3*C*a*b^2 +
 B*b^3)*c^4*d^3 - 4*(3*C*a*b^2 + B*b^3)*c^2*d^5 - 2*(3*C*a*b^2 + B*b^3)*d^7
)*tan(f*x + e)^2 + (3*C*b^3*c^7 - 2*(3*C*a*b^2 + B*b^3)*c^6*d + (3*C*a^2*b
+ 3*B*a*b^2 + (A + 5*C)*b^3)*c^5*d^2 - 4*(3*C*a*b^2 + B*b^3)*c^4*d^3 - (B*a
^3 + 3*(A - 3*C)*a^2*b - 9*B*a*b^2 - 3*A*b^3)*c^3*d^4 + 2*((A - C)*a^3 - 3*
B*a^2*b - 3*A*a*b^2)*c^2*d^5 + (B*a^3 + 3*A*a^2*b)*c*d^6 + (3*C*b^3*c^6*d -
 2*(3*C*a*b^2 + B*b^3)*c^5*d^2 + (3*C*a^2*b + 3*B*a*b^2 + (A + 5*C)*b^3)*c^
4*d^3 - 4*(3*C*a*b^2 + B*b^3)*c^3*d^4 - (B*a^3 + 3*(A - 3*C)*a^2*b - 9*B*a*
b^2 - 3*A*b^3)*c^2*d^5 + 2*((A - C)*a^3 - 3*B*a^2*b - 3*A*a*b^2)*c*d^6 + (B
*a^3 + 3*A*a^2*b)*d^7)*tan(f*x + e))*log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*
x + e) + c^2)/(tan(f*x + e)^2 + 1)) - (3*C*b^3*c^7 - 2*(3*C*a*b^2 + B*b^3)*
c^6*d + (3*C*a^2*b + 3*B*a*b^2 + (A + 5*C)*b^3)*c^5*d^2 - 4*(3*C*a*b^2 + B*
b^3)*c^4*d^3 + (6*C*a^2*b + 6*B*a*b^2 + (2*A + C)*b^3)*c^3*d^4 - 2*(3*C*a*b
^2 + B*b^3)*c^2*d^5 + (3*C*a^2*b + 3*B*a*b^2 + (A - C)*b^3)*c*d^6 + (3*C*b^
3*c^6*d - 2*(3*C*a*b^2 + B*b^3)*c^5*d^2 + (3*C*a^2*b + 3*B*a*b^2 + (A + 5*C
)*b^3)*c^4*d^3 - 4*(3*C*a*b^2 + B*b^3)*c^3*d^4 + (6*C*a^2*b + 6*B*a*b^2 + (
2*A + C)*b^3)*c^2*d^5 - 2*(3*C*a*b^2 + B*b^3)*c*d^6 + (3*C*a^2*b + 3*B*a*b^
2 + (A - C)*b^3)*d^7)*tan(f*x + e))*log(1/(tan(f*x + e)^2 + 1)) - (6*C*b^3*
c^6*d - C*b^3*d^7 - 4*(3*C*a*b^2 + B*b^3)*c^5*d^2 + (6*C*a^2*b + 6*B*a*b^2
+ (2*A + 7*C)*b^3)*c^4*d^3 - 2*(C*a^3 + 3*B*a^2*b + 3*(A + 2*C)*a*b^2 + 2*B
*b^3)*c^3*d^4 + 2*(B*a^3 + 3*A*a^2*b + C*b^3)*c^2*d^5 - 2*(A*a^3 + 3*C*a*b^
2 + B*b^3)*c*d^6 - 2*(((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*c
^2*d^5 + 2*(B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*c*d^6 - ((A
- C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*d^7)*f*x)*tan(f*x + e))/((c
^4*d^5 + 2*c^2*d^7 + d^9)*f*tan(f*x + e) + (c^5*d^4 + 2*c^3*d^6 + c*d^8)*f)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x
+e))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 2.43882, size = 1829, normalized size = 3.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a^3*c^2 - C*a^3*c^2 - 3*B*a^2*b*c^2 - 3*A*a*b^2*c^2 + 3*C*a*b^2*c
^2 + B*b^3*c^2 + 2*B*a^3*c*d + 6*A*a^2*b*c*d - 6*C*a^2*b*c*d - 6*B*a*b^2*c*
d - 2*A*b^3*c*d + 2*C*b^3*c*d - A*a^3*d^2 + C*a^3*d^2 + 3*B*a^2*b*d^2 + 3*A
*a*b^2*d^2 - 3*C*a*b^2*d^2 - B*b^3*d^2)*(f*x + e)/(c^4 + 2*c^2*d^2 + d^4) +
 (B*a^3*c^2 + 3*A*a^2*b*c^2 - 3*C*a^2*b*c^2 - 3*B*a*b^2*c^2 - A*b^3*c^2 + C
*b^3*c^2 - 2*A*a^3*c*d + 2*C*a^3*c*d + 6*B*a^2*b*c*d + 6*A*a*b^2*c*d - 6*C*
a*b^2*c*d - 2*B*b^3*c*d - B*a^3*d^2 - 3*A*a^2*b*d^2 + 3*C*a^2*b*d^2 + 3*B*a
*b^2*d^2 + A*b^3*d^2 - C*b^3*d^2)*log(tan(f*x + e)^2 + 1)/(c^4 + 2*c^2*d^2
+ d^4) + 2*(3*C*b^3*c^6 - 6*C*a*b^2*c^5*d - 2*B*b^3*c^5*d + 3*C*a^2*b*c^4*d
^2 + 3*B*a*b^2*c^4*d^2 + A*b^3*c^4*d^2 + 5*C*b^3*c^4*d^2 - 12*C*a*b^2*c^3*d
^3 - 4*B*b^3*c^3*d^3 - B*a^3*c^2*d^4 - 3*A*a^2*b*c^2*d^4 + 9*C*a^2*b*c^2*d^
4 + 9*B*a*b^2*c^2*d^4 + 3*A*b^3*c^2*d^4 + 2*A*a^3*c*d^5 - 2*C*a^3*c*d^5 - 6
*B*a^2*b*c*d^5 - 6*A*a*b^2*c*d^5 + B*a^3*d^6 + 3*A*a^2*b*d^6)*log(abs(d*tan
(f*x + e) + c))/(c^4*d^4 + 2*c^2*d^6 + d^8) - 2*(3*C*b^3*c^6*d*tan(f*x + e)
 - 6*C*a*b^2*c^5*d^2*tan(f*x + e) - 2*B*b^3*c^5*d^2*tan(f*x + e) + 3*C*a^2*
b*c^4*d^3*tan(f*x + e) + 3*B*a*b^2*c^4*d^3*tan(f*x + e) + A*b^3*c^4*d^3*tan
(f*x + e) + 5*C*b^3*c^4*d^3*tan(f*x + e) - 12*C*a*b^2*c^3*d^4*tan(f*x + e)
- 4*B*b^3*c^3*d^4*tan(f*x + e) - B*a^3*c^2*d^5*tan(f*x + e) - 3*A*a^2*b*c^2
*d^5*tan(f*x + e) + 9*C*a^2*b*c^2*d^5*tan(f*x + e) + 9*B*a*b^2*c^2*d^5*tan(
f*x + e) + 3*A*b^3*c^2*d^5*tan(f*x + e) + 2*A*a^3*c*d^6*tan(f*x + e) - 2*C*
a^3*c*d^6*tan(f*x + e) - 6*B*a^2*b*c*d^6*tan(f*x + e) - 6*A*a*b^2*c*d^6*tan
(f*x + e) + B*a^3*d^7*tan(f*x + e) + 3*A*a^2*b*d^7*tan(f*x + e) + 2*C*b^3*c
^7 - 3*C*a*b^2*c^6*d - B*b^3*c^6*d + 4*C*b^3*c^5*d^2 + C*a^3*c^4*d^3 + 3*B*
a^2*b*c^4*d^3 + 3*A*a*b^2*c^4*d^3 - 9*C*a*b^2*c^4*d^3 - 3*B*b^3*c^4*d^3 - 2
*B*a^3*c^3*d^4 - 6*A*a^2*b*c^3*d^4 + 6*C*a^2*b*c^3*d^4 + 6*B*a*b^2*c^3*d^4
+ 2*A*b^3*c^3*d^4 + 3*A*a^3*c^2*d^5 - C*a^3*c^2*d^5 - 3*B*a^2*b*c^2*d^5 - 3
*A*a*b^2*c^2*d^5 + A*a^3*d^7)/((c^4*d^4 + 2*c^2*d^6 + d^8)*(d*tan(f*x + e)
+ c)) + (C*b^3*d^2*tan(f*x + e)^2 - 4*C*b^3*c*d*tan(f*x + e) + 6*C*a*b^2*d^
2*tan(f*x + e) + 2*B*b^3*d^2*tan(f*x + e))/d^4)/f
```

3.78 $\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^2}\,dx$

Optimal. Leaf size=417

$$\frac{\log(\cos(e+fx))\left(a^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+2ab\left(-A\left(c^2-d^2\right)-2Bcd+c^2C-Cd^2\right)-b^2\left(2cd(A-C)-B\left(c^2-\right.\right.\right.}{f\left(c^2+d^2\right)^2}$$

```
[Out] -(((a^2*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - b^2*(c^2*C - 2*B*c*d -
C*d^2 - A*(c^2 - d^2)) - 2*a*b*(2*c*(A - C)*d - B*(c^2 - d^2)))*x)/(c^2 + d
^2)^2) + ((2*a*b*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2)) + a^2*(2*c*(A -
C)*d - B*(c^2 - d^2)) - b^2*(2*c*(A - C)*d - B*(c^2 - d^2)))*Log[Cos[e + f*
x]])/((c^2 + d^2)^2*f) - ((b*c - a*d)*(b*(2*c^4*C - B*c^3*d + 4*c^2*C*d^2 -
 3*B*c*d^3 + 2*A*d^4) + a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)))*Log[c + d*Ta
n[e + f*x]])/(d^3*(c^2 + d^2)^2*f) + (b^2*(2*c^2*C - B*c*d + (A + C)*d^2)*T
an[e + f*x])/(d^2*(c^2 + d^2)*f) - ((c^2*C - B*c*d + A*d^2)*(a + b*Tan[e +
f*x])^2)/(d*(c^2 + d^2)*f*(c + d*Tan[e + f*x]))
```

Rubi [A] time = 1.11294, antiderivative size = 417, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3645, 3637, 3626, 3617, 31, 3475}

$$\frac{\log(\cos(e+fx))\left(a^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+2ab\left(-A\left(c^2-d^2\right)-2Bcd+c^2C-Cd^2\right)-b^2\left(2cd(A-C)-B\left(c^2-\right.\right.\right.}{f\left(c^2+d^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d
*Tan[e + f*x])^2,x]
```


```
[Out] -(((a^2*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - b^2*(c^2*C - 2*B*c*d -
C*d^2 - A*(c^2 - d^2)) - 2*a*b*(2*c*(A - C)*d - B*(c^2 - d^2)))*x)/(c^2 + d
^2)^2) + ((2*a*b*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2)) + a^2*(2*c*(A -
C)*d - B*(c^2 - d^2)) - b^2*(2*c*(A - C)*d - B*(c^2 - d^2)))*Log[Cos[e + f*
x]])/((c^2 + d^2)^2*f) - ((b*c - a*d)*(b*(2*c^4*C - B*c^3*d + 4*c^2*C*d^2 -
 3*B*c*d^3 + 2*A*d^4) + a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)))*Log[c + d*Ta
n[e + f*x]])/(d^3*(c^2 + d^2)^2*f) + (b^2*(2*c^2*C - B*c*d + (A + C)*d^2)*T
an[e + f*x])/(d^2*(c^2 + d^2)*f) - ((c^2*C - B*c*d + A*d^2)*(a + b*Tan[e +
f*x])^2)/(d*(c^2 + d^2)*f*(c + d*Tan[e + f*x]))
```


Rule 3645

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*d^2 + c*(c*C - B*d))*(a + b*Tan[e
+ f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3626

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((a*A + b*B -
a*C)*x)/(a^2 + b^2), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3617

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^2}\,dx = -\frac{\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^2}{d\left(c^2+d^2\right)f(c+d\tan(e+fx))}+\frac{\int (a+b\tan\ldots}{\ldots}$$

$$=\frac{b^2\left(2c^2C-Bcd+(A+C)d^2\right)\tan(e+fx)}{d^2\left(c^2+d^2\right)f}-\frac{\left(c^2C-\ldots\right.}{d\left(c^2\ldots\right.}$$

$$=-\frac{\left(a^2\left(c^2C-2Bcd-Cd^2-A\left(c^2-d^2\right)\right)-b^2\left(c^2C-2\ldots\right.\right.}{\ldots}$$

$$=-\frac{\left(a^2\left(c^2C-2Bcd-Cd^2-A\left(c^2-d^2\right)\right)-b^2\left(c^2C-2\ldots\right.\right.}{\ldots}$$

$$=-\frac{\left(a^2\left(c^2C-2Bcd-Cd^2-A\left(c^2-d^2\right)\right)-b^2\left(c^2C-2\ldots\right.\right.}{\ldots}$$

Mathematica [C] time = 7.76262, size = 2636, normalized size = 6.32

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(c + d*Tan[e + f*x])^2,x]
```

```
[Out] (((-2*I)*b^2*c^10*C*d^2 + I*b^2*B*c^9*d^3 + (2*I)*a*b*c^9*C*d^3 - 2*b^2*c^9
*C*d^3 + b^2*B*c^8*d^4 + 2*a*b*c^8*C*d^4 - (6*I)*b^2*c^8*C*d^4 - (2*I)*a*A*
b*c^7*d^5 - I*a^2*B*c^7*d^5 + (4*I)*b^2*B*c^7*d^5 + (8*I)*a*b*c^7*C*d^5 - 6
*b^2*c^7*C*d^5 + (2*I)*a^2*A*c^6*d^6 - 2*a*A*b*c^6*d^6 - (2*I)*A*b^2*c^6*d^
6 - a^2*B*c^6*d^6 - (4*I)*a*b*B*c^6*d^6 + 4*b^2*B*c^6*d^6 - (2*I)*a^2*c^6*C
*d^6 + 8*a*b*c^6*C*d^6 - (4*I)*b^2*c^6*C*d^6 + 2*a^2*A*c^5*d^7 - 2*A*b^2*c^
5*d^7 - 4*a*b*B*c^5*d^7 + (3*I)*b^2*B*c^5*d^7 - 2*a^2*c^5*C*d^7 + (6*I)*a*b
*c^5*C*d^7 - 4*b^2*c^5*C*d^7 + (2*I)*a^2*A*c^4*d^8 - (2*I)*A*b^2*c^4*d^8 -
(4*I)*a*b*B*c^4*d^8 + 3*b^2*B*c^4*d^8 - (2*I)*a^2*c^4*C*d^8 + 6*a*b*c^4*C*d
^8 + 2*a^2*A*c^3*d^9 + (2*I)*a*A*b*c^3*d^9 - 2*A*b^2*c^3*d^9 + I*a^2*B*c^3*
d^9 - 4*a*b*B*c^3*d^9 - 2*a^2*c^3*C*d^9 + 2*a*A*b*c^2*d^10 + a^2*B*c^2*d^10
)*(e + f*x)*(c*Cos[e + f*x] + d*Sin[e + f*x])^2*(a + b*Tan[e + f*x])^2)/(c^
2*(c - I*d)^4*(c + I*d)^3*d^5*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(c + d*
Tan[e + f*x])^2) - (I*(-2*b^2*c^5*C + b^2*B*c^4*d + 2*a*b*c^4*C*d - 4*b^2*c
^3*C*d^2 - 2*a*A*b*c^2*d^3 - a^2*B*c^2*d^3 + 3*b^2*B*c^2*d^3 + 6*a*b*c^2*C*
d^3 + 2*a^2*A*c*d^4 - 2*A*b^2*c*d^4 - 4*a*b*B*c*d^4 - 2*a^2*c*C*d^4 + 2*a*A
*b*d^5 + a^2*B*d^5)*ArcTan[Tan[e + f*x]]*(c*Cos[e + f*x] + d*Sin[e + f*x])^
2*(a + b*Tan[e + f*x])^2)/(d^3*(c^2 + d^2)^2*f*(a*Cos[e + f*x] + b*Sin[e +
f*x])^2*(c + d*Tan[e + f*x])^2) + ((2*b^2*c*C - b^2*B*d - 2*a*b*C*d)*Log[Co
s[e + f*x]]*(c*Cos[e + f*x] + d*Sin[e + f*x])^2*(a + b*Tan[e + f*x])^2)/(d^
3*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(c + d*Tan[e + f*x])^2) + ((-2*b^2*
c^5*C + b^2*B*c^4*d + 2*a*b*c^4*C*d - 4*b^2*c^3*C*d^2 - 2*a*A*b*c^2*d^3 - a
^2*B*c^2*d^3 + 3*b^2*B*c^2*d^3 + 6*a*b*c^2*C*d^3 + 2*a^2*A*c*d^4 - 2*A*b^2*
c*d^4 - 4*a*b*B*c*d^4 - 2*a^2*c*C*d^4 + 2*a*A*b*d^5 + a^2*B*d^5)*Log[(c*Cos
[e + f*x] + d*Sin[e + f*x])^2]*(c*Cos[e + f*x] + d*Sin[e + f*x])^2*(a + b*T
an[e + f*x])^2)/(2*d^3*(c^2 + d^2)^2*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*
(c + d*Tan[e + f*x])^2) + (Sec[e + f*x]*(c*Cos[e + f*x] + d*Sin[e + f*x])*(
b^2*c^5*C*d + 2*b^2*c^3*C*d^3 + b^2*c*C*d^5 + a^2*A*c^4*d^2*(e + f*x) - A*b
^2*c^4*d^2*(e + f*x) - 2*a*b*B*c^4*d^2*(e + f*x) - a^2*c^4*C*d^2*(e + f*x)
+ b^2*c^4*C*d^2*(e + f*x) + 4*a*A*b*c^3*d^3*(e + f*x) + 2*a^2*B*c^3*d^3*(e
+ f*x) - 2*b^2*B*c^3*d^3*(e + f*x) - 4*a*b*c^3*C*d^3*(e + f*x) - a^2*A*c^2*
d^4*(e + f*x) + A*b^2*c^2*d^4*(e + f*x) + 2*a*b*B*c^2*d^4*(e + f*x) + a^2*c
^2*C*d^4*(e + f*x) - b^2*c^2*C*d^4*(e + f*x) - b^2*c^5*C*d*Cos[2*(e + f*x)]
 - 2*b^2*c^3*C*d^3*Cos[2*(e + f*x)] - b^2*c*C*d^5*Cos[2*(e + f*x)] + a^2*A*
c^4*d^2*(e + f*x)*Cos[2*(e + f*x)] - A*b^2*c^4*d^2*(e + f*x)*Cos[2*(e + f*x
)] - 2*a*b*B*c^4*d^2*(e + f*x)*Cos[2*(e + f*x)] - a^2*c^4*C*d^2*(e + f*x)*C
os[2*(e + f*x)] + b^2*c^4*C*d^2*(e + f*x)*Cos[2*(e + f*x)] + 4*a*A*b*c^3*d^
3*(e + f*x)*Cos[2*(e + f*x)] + 2*a^2*B*c^3*d^3*(e + f*x)*Cos[2*(e + f*x)] -
 2*b^2*B*c^3*d^3*(e + f*x)*Cos[2*(e + f*x)] - 4*a*b*c^3*C*d^3*(e + f*x)*Cos
[2*(e + f*x)] - a^2*A*c^2*d^4*(e + f*x)*Cos[2*(e + f*x)] + A*b^2*c^2*d^4*(e
 + f*x)*Cos[2*(e + f*x)] + 2*a*b*B*c^2*d^4*(e + f*x)*Cos[2*(e + f*x)] + a^2
*c^2*C*d^4*(e + f*x)*Cos[2*(e + f*x)] - b^2*c^2*C*d^4*(e + f*x)*Cos[2*(e +
f*x)] + 2*b^2*c^6*C*Sin[2*(e + f*x)] - b^2*B*c^5*d*Sin[2*(e + f*x)] - 2*a*b
*c^5*C*d*Sin[2*(e + f*x)] + A*b^2*c^4*d^2*Sin[2*(e + f*x)] + 2*a*b*B*c^4*d^
2*Sin[2*(e + f*x)] + a^2*c^4*C*d^2*Sin[2*(e + f*x)] + 3*b^2*c^4*C*d^2*Sin[2
*(e + f*x)] - 2*a*A*b*c^3*d^3*Sin[2*(e + f*x)] - a^2*B*c^3*d^3*Sin[2*(e + f
*x)] - b^2*B*c^3*d^3*Sin[2*(e + f*x)] - 2*a*b*c^3*C*d^3*Sin[2*(e + f*x)] +
a^2*A*c^2*d^4*Sin[2*(e + f*x)] + A*b^2*c^2*d^4*Sin[2*(e + f*x)] + 2*a*b*B*c
^2*d^4*Sin[2*(e + f*x)] + a^2*c^2*C*d^4*Sin[2*(e + f*x)] + b^2*c^2*C*d^4*Si
n[2*(e + f*x)] - 2*a*A*b*c*d^5*Sin[2*(e + f*x)] - a^2*B*c*d^5*Sin[2*(e + f*
x)] + a^2*A*d^6*Sin[2*(e + f*x)] + a^2*A*c^3*d^3*(e + f*x)*Sin[2*(e + f*x)]
 - A*b^2*c^3*d^3*(e + f*x)*Sin[2*(e + f*x)] - 2*a*b*B*c^3*d^3*(e + f*x)*Sin
[2*(e + f*x)] - a^2*c^3*C*d^3*(e + f*x)*Sin[2*(e + f*x)] + b^2*c^3*C*d^3*(e
 + f*x)*Sin[2*(e + f*x)] + 4*a*A*b*c^2*d^4*(e + f*x)*Sin[2*(e + f*x)] + 2*a
```

```
^2*B*c^2*d^4*(e + f*x)*Sin[2*(e + f*x)] - 2*b^2*B*c^2*d^4*(e + f*x)*Sin[2*(
e + f*x)] - 4*a*b*c^2*C*d^4*(e + f*x)*Sin[2*(e + f*x)] - a^2*A*c*d^5*(e + f
*x)*Sin[2*(e + f*x)] + A*b^2*c*d^5*(e + f*x)*Sin[2*(e + f*x)] + 2*a*b*B*c*d
^5*(e + f*x)*Sin[2*(e + f*x)] + a^2*c*C*d^5*(e + f*x)*Sin[2*(e + f*x)] - b^
2*c*C*d^5*(e + f*x)*Sin[2*(e + f*x)])*(a + b*Tan[e + f*x])^2)/(2*c*(c - I*d
)^2*(c + I*d)^2*d^2*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(c + d*Tan[e + f*
x])^2)
```

Maple [B] time = 0.072, size = 1554, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^2,x
)
```

```
[Out] 1/f/(c^2+d^2)^2*A*arctan(tan(f*x+e))*b^2*d^2-1/f/(c^2+d^2)^2*C*arctan(tan(f
*x+e))*a^2*c^2+1/f/(c^2+d^2)^2*C*arctan(tan(f*x+e))*a^2*d^2+1/f/(c^2+d^2)^2
*C*arctan(tan(f*x+e))*b^2*c^2-1/f/(c^2+d^2)^2*C*arctan(tan(f*x+e))*b^2*d^2-
1/f*d/(c^2+d^2)/(c+d*tan(f*x+e))*A*a^2+1/f*d^2/(c^2+d^2)^2*ln(c+d*tan(f*x+e
))*B*a^2+1/f/(c^2+d^2)/(c+d*tan(f*x+e))*B*a^2*c-1/2/f/(c^2+d^2)^2*ln(1+tan(
f*x+e)^2)*B*a^2*d^2-1/2/f/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*B*b^2*c^2+1/2/f/(c
^2+d^2)^2*ln(1+tan(f*x+e)^2)*B*b^2*d^2+1/f/(c^2+d^2)^2*A*arctan(tan(f*x+e))
*a^2*c^2-1/f/(c^2+d^2)^2*A*arctan(tan(f*x+e))*a^2*d^2-1/f/(c^2+d^2)^2*ln(c+
d*tan(f*x+e))*B*a^2*c^2+3/f/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*B*b^2*c^2-1/f/(c
^2+d^2)^2*A*arctan(tan(f*x+e))*b^2*c^2+1/2/f/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)
*B*a^2*c^2+2/f*d/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*A*a^2*c+1/f*b^2*C/d^2*tan(f
*x+e)+2/f/d^2/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*C*a*b*c^4+2/f/(c^2+d^2)^2*ln(1
+tan(f*x+e)^2)*B*a*b*c*d-4/f/(c^2+d^2)^2*C*arctan(tan(f*x+e))*a*b*c*d+4/f/(
c^2+d^2)^2*A*arctan(tan(f*x+e))*a*b*c*d-2/f/d/(c^2+d^2)/(c+d*tan(f*x+e))*B*
a*b*c^2+2/f/d^2/(c^2+d^2)/(c+d*tan(f*x+e))*C*c^3*a*b-4/f*d/(c^2+d^2)^2*ln(c
+d*tan(f*x+e))*B*a*b*c-1/f/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*A*a*b*d^2+1/f/(c^
2+d^2)^2*ln(1+tan(f*x+e)^2)*A*b^2*c*d+1/f/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*C*
a^2*c*d-1/f/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*C*a*b*c^2+1/f/(c^2+d^2)^2*ln(1+t
an(f*x+e)^2)*C*a*b*d^2-1/f/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*C*b^2*c*d+2/f/(c^
2+d^2)^2*B*arctan(tan(f*x+e))*a^2*c*d-2/f/(c^2+d^2)^2*B*arctan(tan(f*x+e))*
a*b*c^2+2/f*d^2/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*A*a*b-2/f*d/(c^2+d^2)^2*ln(c
+d*tan(f*x+e))*A*b^2*c+1/f/d^2/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*B*b^2*c^4-2/f
*d/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*C*a^2*c-2/f/(c^2+d^2)^2*B*arctan(tan(f*x+
e))*b^2*c*d-1/f/d/(c^2+d^2)/(c+d*tan(f*x+e))*A*b^2*c^2+1/f/d^2/(c^2+d^2)/(c
+d*tan(f*x+e))*B*c^3*b^2-1/f/d/(c^2+d^2)/(c+d*tan(f*x+e))*C*a^2*c^2-1/f/d^3
/(c^2+d^2)/(c+d*tan(f*x+e))*C*c^4*b^2+2/f/(c^2+d^2)/(c+d*tan(f*x+e))*A*a*b*
c-2/f/d^3/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*C*b^2*c^5-4/f/d/(c^2+d^2)^2*ln(c+d
*tan(f*x+e))*C*b^2*c^3-1/f/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*A*a^2*c*d+1/f/(c^
2+d^2)^2*ln(1+tan(f*x+e)^2)*A*a*b*c^2-2/f/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*A*
a*b*c^2+6/f/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*C*a*b*c^2+2/f/(c^2+d^2)^2*B*arct
an(tan(f*x+e))*a*b*d^2
```

Maxima [A] time = 1.57371, size = 666, normalized size = 1.6

$$\frac{\frac{2 Cb^2 \tan\left(fx+e\right)}{d^2} + \frac{2 \left(\left(\left((A-C)a^2-2 Bab-(A-C)b^2\right)c^2+2 \left(Ba^2+2 (A-C)ab-Bb^2\right)cd-\left((A-C)a^2-2 Bab-(A-C)b^2\right)d^2\right)\left(fx+e\right)}{c^4+2 c^2d^2+d^4} - \frac{2 \left(2 Cb^2c^5+4 Cb^2c^3d^2-\left(2 Cab+Bb^2 \right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*C*b^2*tan(f*x + e)/d^2 + 2*(((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c^
2 + 2*(B*a^2 + 2*(A - C)*a*b - B*b^2)*c*d - ((A - C)*a^2 - 2*B*a*b - (A - C
)*b^2)*d^2)*(f*x + e)/(c^4 + 2*c^2*d^2 + d^4) - 2*(2*C*b^2*c^5 + 4*C*b^2*c^
3*d^2 - (2*C*a*b + B*b^2)*c^4*d + (B*a^2 + 2*(A - 3*C)*a*b - 3*B*b^2)*c^2*d
^3 - 2*((A - C)*a^2 - 2*B*a*b - A*b^2)*c*d^4 - (B*a^2 + 2*A*a*b)*d^5)*log(d
*tan(f*x + e) + c)/(c^4*d^3 + 2*c^2*d^5 + d^7) + ((B*a^2 + 2*(A - C)*a*b -
B*b^2)*c^2 - 2*((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c*d - (B*a^2 + 2*(A -
C)*a*b - B*b^2)*d^2)*log(tan(f*x + e)^2 + 1)/(c^4 + 2*c^2*d^2 + d^4) - 2*(C
*b^2*c^4 + A*a^2*d^4 - (2*C*a*b + B*b^2)*c^3*d + (C*a^2 + 2*B*a*b + A*b^2)*
c^2*d^2 - (B*a^2 + 2*A*a*b)*c*d^3)/(c^3*d^3 + c*d^5 + (c^2*d^4 + d^6)*tan(f
*x + e)))/f
```

Fricas [B] time = 3.75543, size = 1983, normalized size = 4.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*C*b^2*c^4*d^2 + 2*A*a^2*d^6 - 2*(2*C*a*b + B*b^2)*c^3*d^3 + 2*(C*a^
2 + 2*B*a*b + A*b^2)*c^2*d^4 - 2*(B*a^2 + 2*A*a*b)*c*d^5 - 2*(((A - C)*a^2
- 2*B*a*b - (A - C)*b^2)*c^3*d^3 + 2*(B*a^2 + 2*(A - C)*a*b - B*b^2)*c^2*d^
4 - ((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c*d^5)*f*x - 2*(C*b^2*c^4*d^2 + 2
*C*b^2*c^2*d^4 + C*b^2*d^6)*tan(f*x + e)^2 + (2*C*b^2*c^6 + 4*C*b^2*c^4*d^2
 - (2*C*a*b + B*b^2)*c^5*d + (B*a^2 + 2*(A - 3*C)*a*b - 3*B*b^2)*c^3*d^3 -
2*((A - C)*a^2 - 2*B*a*b - A*b^2)*c^2*d^4 - (B*a^2 + 2*A*a*b)*c*d^5 + (2*C*
b^2*c^5*d + 4*C*b^2*c^3*d^3 - (2*C*a*b + B*b^2)*c^4*d^2 + (B*a^2 + 2*(A - 3
*C)*a*b - 3*B*b^2)*c^2*d^4 - 2*((A - C)*a^2 - 2*B*a*b - A*b^2)*c*d^5 - (B*a
^2 + 2*A*a*b)*d^6)*tan(f*x + e))*log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x +
e) + c^2)/(tan(f*x + e)^2 + 1)) - (2*C*b^2*c^6 + 4*C*b^2*c^4*d^2 + 2*C*b^2*
c^2*d^4 - (2*C*a*b + B*b^2)*c^5*d - 2*(2*C*a*b + B*b^2)*c^3*d^3 - (2*C*a*b
+ B*b^2)*c*d^5 + (2*C*b^2*c^5*d + 4*C*b^2*c^3*d^3 + 2*C*b^2*c*d^5 - (2*C*a*
b + B*b^2)*c^4*d^2 - 2*(2*C*a*b + B*b^2)*c^2*d^4 - (2*C*a*b + B*b^2)*d^6)*t
an(f*x + e))*log(1/(tan(f*x + e)^2 + 1)) - 2*(2*C*b^2*c^5*d - (2*C*a*b + B*
b^2)*c^4*d^2 + (C*a^2 + 2*B*a*b + (A + 2*C)*b^2)*c^3*d^3 - (B*a^2 + 2*A*a*b
)*c^2*d^4 + (A*a^2 + C*b^2)*c*d^5 + (((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*
c^2*d^4 + 2*(B*a^2 + 2*(A - C)*a*b - B*b^2)*c*d^5 - ((A - C)*a^2 - 2*B*a*b
- (A - C)*b^2)*d^6)*f*x)*tan(f*x + e))/((c^4*d^4 + 2*c^2*d^6 + d^8)*f*tan(f
*x + e) + (c^5*d^3 + 2*c^3*d^5 + c*d^7)*f)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x
+e))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.90879, size = 1231, normalized size = 2.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*C*b^2*tan(f*x + e)/d^2 + 2*(A*a^2*c^2 - C*a^2*c^2 - 2*B*a*b*c^2 - A*
b^2*c^2 + C*b^2*c^2 + 2*B*a^2*c*d + 4*A*a*b*c*d - 4*C*a*b*c*d - 2*B*b^2*c*d
 - A*a^2*d^2 + C*a^2*d^2 + 2*B*a*b*d^2 + A*b^2*d^2 - C*b^2*d^2)*(f*x + e)/(
c^4 + 2*c^2*d^2 + d^4) + (B*a^2*c^2 + 2*A*a*b*c^2 - 2*C*a*b*c^2 - B*b^2*c^2
 - 2*A*a^2*c*d + 2*C*a^2*c*d + 4*B*a*b*c*d + 2*A*b^2*c*d - 2*C*b^2*c*d - B*
a^2*d^2 - 2*A*a*b*d^2 + 2*C*a*b*d^2 + B*b^2*d^2)*log(tan(f*x + e)^2 + 1)/(c
^4 + 2*c^2*d^2 + d^4) - 2*(2*C*b^2*c^5 - 2*C*a*b*c^4*d - B*b^2*c^4*d + 4*C*
b^2*c^3*d^2 + B*a^2*c^2*d^3 + 2*A*a*b*c^2*d^3 - 6*C*a*b*c^2*d^3 - 3*B*b^2*c
^2*d^3 - 2*A*a^2*c*d^4 + 2*C*a^2*c*d^4 + 4*B*a*b*c*d^4 + 2*A*b^2*c*d^4 - B*
a^2*d^5 - 2*A*a*b*d^5)*log(abs(d*tan(f*x + e) + c))/(c^4*d^3 + 2*c^2*d^5 +
d^7) + 2*(2*C*b^2*c^5*d*tan(f*x + e) - 2*C*a*b*c^4*d^2*tan(f*x + e) - B*b^2
*c^4*d^2*tan(f*x + e) + 4*C*b^2*c^3*d^3*tan(f*x + e) + B*a^2*c^2*d^4*tan(f*
x + e) + 2*A*a*b*c^2*d^4*tan(f*x + e) - 6*C*a*b*c^2*d^4*tan(f*x + e) - 3*B*
b^2*c^2*d^4*tan(f*x + e) - 2*A*a^2*c*d^5*tan(f*x + e) + 2*C*a^2*c*d^5*tan(f
*x + e) + 4*B*a*b*c*d^5*tan(f*x + e) + 2*A*b^2*c*d^5*tan(f*x + e) - B*a^2*d
^6*tan(f*x + e) - 2*A*a*b*d^6*tan(f*x + e) + C*b^2*c^6 - C*a^2*c^4*d^2 - 2*
B*a*b*c^4*d^2 - A*b^2*c^4*d^2 + 3*C*b^2*c^4*d^2 + 2*B*a^2*c^3*d^3 + 4*A*a*b
*c^3*d^3 - 4*C*a*b*c^3*d^3 - 2*B*b^2*c^3*d^3 - 3*A*a^2*c^2*d^4 + C*a^2*c^2*
d^4 + 2*B*a*b*c^2*d^4 + A*b^2*c^2*d^4 - A*a^2*d^6)/((c^4*d^3 + 2*c^2*d^5 +
d^7)*(d*tan(f*x + e) + c)))/f
```

3.79 $\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^2}\,dx$

Optimal. Leaf size=292

$$\frac{(bc-ad)\left(Ad^2-Bcd+c^2C\right)}{d^2f\left(c^2+d^2\right)(c+d\tan(e+fx))}+\frac{\left(ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(-c^2d^2(A-3C)+Ad^4-2Bcd^3+c^4C\right)\right)\log(c}{d^2f\left(c^2+d^2\right)^2}$$

```
[Out] -(((a*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - b*(2*c*(A - C)*d - B*(c^2
 - d^2)))*x)/(c^2 + d^2)^2) - ((a*(B*c^2 + 2*c*C*d - B*d^2) - b*(c^2*C - 2*
B*c*d - C*d^2) - A*(2*a*c*d - b*(c^2 - d^2)))*Log[Cos[e + f*x]])/((c^2 + d^
2)^2*f) + ((b*(c^4*C - c^2*(A - 3*C)*d^2 - 2*B*c*d^3 + A*d^4) + a*d^2*(2*c*
(A - C)*d - B*(c^2 - d^2)))*Log[c + d*Tan[e + f*x]])/(d^2*(c^2 + d^2)^2*f)
+ ((b*c - a*d)*(c^2*C - B*c*d + A*d^2))/(d^2*(c^2 + d^2)*f*(c + d*Tan[e + f
*x]))
```

Rubi [A] time = 0.553893, antiderivative size = 288, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {3635, 3626, 3617, 31, 3475}

$$\frac{(bc-ad)\left(Ad^2-Bcd+c^2C\right)}{d^2f\left(c^2+d^2\right)(c+d\tan(e+fx))}+\frac{\left(ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(-c^2d^2(A-3C)+Ad^4-2Bcd^3+c^4C\right)\right)\log(c}{d^2f\left(c^2+d^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d*T
an[e + f*x])^2,x]
```

```
[Out] -(((a*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - b*(2*c*(A - C)*d - B*(c^2
 - d^2)))*x)/(c^2 + d^2)^2) + ((2*a*A*c*d - 2*a*c*C*d - A*b*(c^2 - d^2) - a
*B*(c^2 - d^2) + b*(c^2*C - 2*B*c*d - C*d^2))*Log[Cos[e + f*x]])/((c^2 + d^
2)^2*f) + ((b*(c^4*C - c^2*(A - 3*C)*d^2 - 2*B*c*d^3 + A*d^4) + a*d^2*(2*c*
(A - C)*d - B*(c^2 - d^2)))*Log[c + d*Tan[e + f*x]])/(d^2*(c^2 + d^2)^2*f)
+ ((b*c - a*d)*(c^2*C - B*c*d + A*d^2))/(d^2*(c^2 + d^2)*f*(c + d*Tan[e + f
*x]))
```

Rule 3635

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(c^2*C - B*c*d + A*d^2)*(c +
d*Tan[e + f*x])^(n + 1))/(d^2*f*(n + 1)*(c^2 + d^2)), x] + Dist[1/(d*(c^2 +
 d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(c^
2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*Ta
n[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -
1]
```

Rule 3626

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((a*A + b*B -
a*C)*x)/(a^2 + b^2), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
```

```
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3617

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^2}\,dx = \frac{(bc-ad)\left(c^2C-Bcd+Ad^2\right)}{d^2\left(c^2+d^2\right)f(c+d\tan(e+fx))} + \frac{\int \frac{ad(Ac-cC+Bd)+b(c^2}{}}{}$$

$$= -\frac{\left(a\left(c^2C-2Bcd-Cd^2-A\left(c^2-d^2\right)\right)-b\left(2c(A-C)\right.\right.}{\left(c^2+d^2\right)^2}$$

$$= -\frac{\left(a\left(c^2C-2Bcd-Cd^2-A\left(c^2-d^2\right)\right)-b\left(2c(A-C)\right.\right.}{\left(c^2+d^2\right)^2}$$

$$= -\frac{\left(a\left(c^2C-2Bcd-Cd^2-A\left(c^2-d^2\right)\right)-b\left(2c(A-C)\right.\right.}{\left(c^2+d^2\right)^2}$$

Mathematica [C] time = 6.33632, size = 606, normalized size = 2.08

$$\frac{-2ic\tan^{-1}(\tan(e+fx))(c+d\tan(e+fx))\left(ad^2\left(2cd(A-C)+B\left(d^2-c^2\right)\right)+b\left(-c^2d^2(A-3C)+Ad^4-2Bcd^3+c^4C\right)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c
 + d*Tan[e + f*x])^2,x]
```

```
[Out] (c^2*(2*(c + I*d)^2*(a*(A - I*B - C)*d^2 + b*(I*c^2*C + 2*c*C*d + ((-I)*A -
 B)*d^2))*(e + f*x) - 2*b*C*(c^2 + d^2)^2*Log[Cos[e + f*x]] + (b*(c^4*C - c
^2*(A - 3*C)*d^2 - 2*B*c*d^3 + A*d^4) + a*d^2*(2*c*(A - C)*d + B*(-c^2 + d^
2)))*Log[(c*Cos[e + f*x] + d*Sin[e + f*x])^2]) + d*(2*(c + I*d)*(b*c*(I*c^3
*C*(I + e + f*x) + d^3*((-I)*B*(e + f*x) + A*(I + e + f*x)) - I*c*d^2*(-2*C
*(e + f*x) + A*(-I + e + f*x) - I*B*(I + e + f*x)) + c^2*d*(B + C*(I + e +
f*x))) + a*d*(c^3*C - I*A*d^3 + c*d^2*(A*(1 + I*e + I*f*x) - I*C*(e + f*x)
+ B*(I + e + f*x)) - c^2*d*(B*(1 + I*e + I*f*x) - A*(e + f*x) + C*(I + e +
f*x)))) - 2*b*c*C*(c^2 + d^2)^2*Log[Cos[e + f*x]] + c*(b*(c^4*C - c^2*(A -
3*C)*d^2 - 2*B*c*d^3 + A*d^4) + a*d^2*(2*c*(A - C)*d + B*(-c^2 + d^2)))*Log
```

```
[(c*Cos[e + f*x] + d*Sin[e + f*x])^2])*Tan[e + f*x] - (2*I)*c*(b*(c^4*C - c
^2*(A - 3*C)*d^2 - 2*B*c*d^3 + A*d^4) + a*d^2*(2*c*(A - C)*d + B*(-c^2 + d^
2)))*ArcTan[Tan[e + f*x]]*(c + d*Tan[e + f*x]))/(2*c*d^2*(c^2 + d^2)^2*f*(c
 + d*Tan[e + f*x]))
```

Maple [B] time = 0.058, size = 948, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^2,x)
```

```
[Out] -1/2/f/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*A*b*d^2+1/2/f/(c^2+d^2)^2*ln(1+tan(f*
x+e)^2)*B*a*c^2-1/2/f/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*B*a*d^2-1/2/f/(c^2+d^2
)^2*ln(1+tan(f*x+e)^2)*C*b*c^2+1/2/f/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*C*b*d^2
+3/f/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*C*b*c^2+1/f/(c^2+d^2)/(c+d*tan(f*x+e))*
A*b*c+1/f/(c^2+d^2)/(c+d*tan(f*x+e))*B*a*c-1/f/(c^2+d^2)^2*ln(c+d*tan(f*x+e
))*B*a*c^2-2/f/(c^2+d^2)^2*C*arctan(tan(f*x+e))*b*c*d-1/f/d/(c^2+d^2)/(c+d*
tan(f*x+e))*B*b*c^2-1/f/d/(c^2+d^2)/(c+d*tan(f*x+e))*C*a*c^2+1/f/d^2/(c^2+d
^2)/(c+d*tan(f*x+e))*C*b*c^3+1/f/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*B*b*c*d+1/f
/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*C*a*c*d+2/f/(c^2+d^2)^2*d*ln(c+d*tan(f*x+e)
)*A*a*c-1/f/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*A*a*c*d-2/f/(c^2+d^2)^2*d*ln(c+d
*tan(f*x+e))*B*b*c-2/f/(c^2+d^2)^2*d*ln(c+d*tan(f*x+e))*C*a*c+1/f/(c^2+d^2)
^2/d^2*ln(c+d*tan(f*x+e))*C*b*c^4+2/f/(c^2+d^2)^2*B*arctan(tan(f*x+e))*a*c*
d+2/f/(c^2+d^2)^2*A*arctan(tan(f*x+e))*b*c*d-1/f*d/(c^2+d^2)/(c+d*tan(f*x+e
))*A*a+1/f/(c^2+d^2)^2*d^2*ln(c+d*tan(f*x+e))*A*b+1/f/(c^2+d^2)^2*d^2*ln(c+
d*tan(f*x+e))*B*a-1/f/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*A*b*c^2-1/f/(c^2+d^2)^
2*B*arctan(tan(f*x+e))*b*c^2+1/f/(c^2+d^2)^2*B*arctan(tan(f*x+e))*b*d^2-1/f
/(c^2+d^2)^2*C*arctan(tan(f*x+e))*a*c^2+1/f/(c^2+d^2)^2*C*arctan(tan(f*x+e)
)*a*d^2+1/f/(c^2+d^2)^2*A*arctan(tan(f*x+e))*a*c^2-1/f/(c^2+d^2)^2*A*arctan
(tan(f*x+e))*a*d^2+1/2/f/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*A*b*c^2
```

Maxima [A] time = 1.48702, size = 431, normalized size = 1.48

$$\frac{\frac{2\left(((A-C)a-Bb)c^2+2\,(Ba+(A-C)b)cd-((A-C)a-Bb)d^2\right)\left(fx+e\right)}{c^4+2\,c^2d^2+d^4}+\frac{2\left(Cbc^4-(Ba+(A-3\,C)b)c^2d^2+2\,((A-C)a-Bb)cd^3+(Ba+Ab)d^4\right)\log\left(d\tan\left(fx+e\right)+c\right)}{c^4d^2+2\,c^2d^4+d^6}+ \frac{(\cdots}{\cdots}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(((A - C)*a - B*b)*c^2 + 2*(B*a + (A - C)*b)*c*d - ((A - C)*a - B*b)
*d^2)*(f*x + e)/(c^4 + 2*c^2*d^2 + d^4) + 2*(C*b*c^4 - (B*a + (A - 3*C)*b)*
c^2*d^2 + 2*((A - C)*a - B*b)*c*d^3 + (B*a + A*b)*d^4)*log(d*tan(f*x + e) +
 c)/(c^4*d^2 + 2*c^2*d^4 + d^6) + ((B*a + (A - C)*b)*c^2 - 2*((A - C)*a - B
*b)*c*d - (B*a + (A - C)*b)*d^2)*log(tan(f*x + e)^2 + 1)/(c^4 + 2*c^2*d^2 +
 d^4) + 2*(C*b*c^3 - A*a*d^3 - (C*a + B*b)*c^2*d + (B*a + A*b)*c*d^2)/(c^3*
d^2 + c*d^4 + (c^2*d^3 + d^5)*tan(f*x + e)))/f
```

Fricas [A] time = 1.83332, size = 1095, normalized size = 3.75

$$\frac{2\,Cbc^3d^2 - 2\,Aad^5 - 2\,(Ca + Bb)c^2d^3 + 2\,(Ba + Ab)cd^4 + 2\left(((A-C)a - Bb)c^3d^2 + 2\,(Ba + (A-C)b)c^2d^3 - ((A-C)a - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*C*b*c^3*d^2 - 2*A*a*d^5 - 2*(C*a + B*b)*c^2*d^3 + 2*(B*a + A*b)*c*d^
4 + 2*(((A - C)*a - B*b)*c^3*d^2 + 2*(B*a + (A - C)*b)*c^2*d^3 - ((A - C)*a
 - B*b)*c*d^4)*f*x + (C*b*c^5 - (B*a + (A - 3*C)*b)*c^3*d^2 + 2*((A - C)*a
- B*b)*c^2*d^3 + (B*a + A*b)*c*d^4 + (C*b*c^4*d - (B*a + (A - 3*C)*b)*c^2*d
^3 + 2*((A - C)*a - B*b)*c*d^4 + (B*a + A*b)*d^5)*tan(f*x + e))*log((d^2*ta
n(f*x + e)^2 + 2*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) - (C*b*c^5 +
 2*C*b*c^3*d^2 + C*b*c*d^4 + (C*b*c^4*d + 2*C*b*c^2*d^3 + C*b*d^5)*tan(f*x
+ e))*log(1/(tan(f*x + e)^2 + 1)) - 2*(C*b*c^4*d - A*a*c*d^4 - (C*a + B*b)*
c^3*d^2 + (B*a + A*b)*c^2*d^3 - (((A - C)*a - B*b)*c^2*d^3 + 2*(B*a + (A -
C)*b)*c*d^4 - ((A - C)*a - B*b)*d^5)*f*x)*tan(f*x + e))/((c^4*d^3 + 2*c^2*d
^5 + d^7)*f*tan(f*x + e) + (c^5*d^2 + 2*c^3*d^4 + c*d^6)*f)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e)
)**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.62849, size = 713, normalized size = 2.44

$$\frac{\frac{2\left(Aac^2 - Cac^2 - Bbc^2 + 2\,Bacd + 2\,Abcd - 2\,Cbcd - Aad^2 + Cad^2 + Bbd^2\right)(fx+e)}{c^4 + 2\,c^2d^2 + d^4} + \frac{\left(Bac^2 + Abc^2 - Cbc^2 - 2\,Aacd + 2\,Cacd + 2\,Bbcd - Bad^2 - Abd^2 + Cbd^2\right)\log\left(\tan\left(fx+e\right)^2 + \right.}{c^4 + 2\,c^2d^2 + d^4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a*c^2 - C*a*c^2 - B*b*c^2 + 2*B*a*c*d + 2*A*b*c*d - 2*C*b*c*d - A
*a*d^2 + C*a*d^2 + B*b*d^2)*(f*x + e)/(c^4 + 2*c^2*d^2 + d^4) + (B*a*c^2 +
A*b*c^2 - C*b*c^2 - 2*A*a*c*d + 2*C*a*c*d + 2*B*b*c*d - B*a*d^2 - A*b*d^2 +
 C*b*d^2)*log(tan(f*x + e)^2 + 1)/(c^4 + 2*c^2*d^2 + d^4) + 2*(C*b*c^4 - B*
a*c^2*d^2 - A*b*c^2*d^2 + 3*C*b*c^2*d^2 + 2*A*a*c*d^3 - 2*C*a*c*d^3 - 2*B*b
*c*d^3 + B*a*d^4 + A*b*d^4)*log(abs(d*tan(f*x + e) + c))/(c^4*d^2 + 2*c^2*d
^4 + d^6) - 2*(C*b*c^4*tan(f*x + e) - B*a*c^2*d^2*tan(f*x + e) - A*b*c^2*d^
2*tan(f*x + e) + 3*C*b*c^2*d^2*tan(f*x + e) + 2*A*a*c*d^3*tan(f*x + e) - 2*
```

```
C*a*c*d^3*tan(f*x + e) - 2*B*b*c*d^3*tan(f*x + e) + B*a*d^4*tan(f*x + e) +
A*b*d^4*tan(f*x + e) + C*a*c^4 + B*b*c^4 - 2*B*a*c^3*d - 2*A*b*c^3*d + 2*C*
b*c^3*d + 3*A*a*c^2*d^2 - C*a*c^2*d^2 - B*b*c^2*d^2 + A*a*d^4)/((c^4*d + 2*
c^2*d^3 + d^5)*(d*tan(f*x + e) + c)))/f
```

3.80 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^2}\,dx$

Optimal. Leaf size=140

$$-\frac{Ad^2-Bcd+c^2C}{df\left(c^2+d^2\right)(c+d\tan(e+fx))}+\frac{\left(2cd(A-C)-B\left(c^2-d^2\right)\right)\log(c\cos(e+fx)+d\sin(e+fx))}{f\left(c^2+d^2\right)^2}-\frac{x\left(-A\left(c^2-d^2\right)-2B\right.}{\left(c^2+a\right.}$$

```
[Out] -(((c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2))*x)/(c^2 + d^2)^2) + ((2*c*(A -
 C)*d - B*(c^2 - d^2))*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/((c^2 + d^2)^2
*f) - (c^2*C - B*c*d + A*d^2)/(d*(c^2 + d^2)*f*(c + d*Tan[e + f*x]))
```

Rubi [A] time = 0.20904, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3628, 3531, 3530}

$$-\frac{Ad^2-Bcd+c^2C}{df\left(c^2+d^2\right)(c+d\tan(e+fx))}+\frac{\left(2cd(A-C)-B\left(c^2-d^2\right)\right)\log(c\cos(e+fx)+d\sin(e+fx))}{f\left(c^2+d^2\right)^2}-\frac{x\left(-A\left(c^2-d^2\right)-2B\right.}{\left(c^2+a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(c + d*Tan[e + f*x])^2,x]
```

```
[Out] -(((c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2))*x)/(c^2 + d^2)^2) + ((2*c*(A -
 C)*d - B*(c^2 - d^2))*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/((c^2 + d^2)^2
*f) - (c^2*C - B*c*d + A*d^2)/(d*(c^2 + d^2)*f*(c + d*Tan[e + f*x]))
```

Rule 3628

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2
- a*b*B + a^2*C)*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 + b^2)), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^2}\,dx = -\frac{c^2C-Bcd+Ad^2}{d\left(c^2+d^2\right)f(c+d\tan(e+fx))} + \frac{\int \frac{Ac-cC+Bd+(Bc-(A-C)d)\tan(e+fx)}{c+d\tan(e+fx)}\,dx}{c^2+d^2}$$

$$= -\frac{\left(c^2C-2Bcd-Cd^2-A\left(c^2-d^2\right)\right)x}{\left(c^2+d^2\right)^2} - \frac{c^2C-Bcd+Ad^2}{d\left(c^2+d^2\right)f(c+d\tan(e+fx))}$$

$$= -\frac{\left(c^2C-2Bcd-Cd^2-A\left(c^2-d^2\right)\right)x}{\left(c^2+d^2\right)^2} + \frac{\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\log(}{\left(c^2+d\right.}$$

Mathematica [C] time = 2.24105, size = 207, normalized size = 1.48

$$\frac{(d(C-A)+Bc)\left(\frac{2d\left(\frac{c^2+d^2}{c+d\tan(e+fx)}-2c\log(c+d\tan(e+fx))\right)}{\left(c^2+d^2\right)^2}+\frac{i\log(-\tan(e+fx)+i)}{(c+id)^2}-\frac{i\log(\tan(e+fx)+i)}{(c-id)^2}\right)+\frac{B((-d-ic)\log(-\tan(e+fx)+i)+i(c+i}{}}{2df}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(c + d*Tan[e + f*x])^2,x]
```

```
[Out] ((B*(((-I)*c - d)*Log[I - Tan[e + f*x]] + I*(c + I*d)*Log[I + Tan[e + f*x]]
 + 2*d*Log[c + d*Tan[e + f*x]]))/(c^2 + d^2) - (2*C)/(c + d*Tan[e + f*x]) +
 (B*c + (-A + C)*d)*((I*Log[I - Tan[e + f*x]])/(c + I*d)^2 - (I*Log[I + Tan
[e + f*x]])/(c - I*d)^2 + (2*d*(-2*c*Log[c + d*Tan[e + f*x]] + (c^2 + d^2)/
(c + d*Tan[e + f*x])))/(c^2 + d^2)^2))/(2*d*f)
```

Maple [B] time = 0.042, size = 438, normalized size = 3.1

$$-\frac{\ln\left(1+\left(\tan\left(fx+e\right)\right)^2\right)Acd}{f\left(c^2+d^2\right)^2}+\frac{\ln\left(1+\left(\tan\left(fx+e\right)\right)^2\right)Bc^2}{2f\left(c^2+d^2\right)^2}-\frac{\ln\left(1+\left(\tan\left(fx+e\right)\right)^2\right)Bd^2}{2f\left(c^2+d^2\right)^2}+\frac{\ln\left(1+\left(\tan\left(fx+e\right.\right.\right.}{f\left(c^2+d^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^2,x)
```

```
[Out] -1/f/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*A*c*d+1/2/f/(c^2+d^2)^2*ln(1+tan(f*x+e)
^2)*B*c^2-1/2/f/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*B*d^2+1/f/(c^2+d^2)^2*ln(1+t
an(f*x+e)^2)*c*C*d+1/f/(c^2+d^2)^2*A*arctan(tan(f*x+e))*c^2-1/f/(c^2+d^2)^2
*A*arctan(tan(f*x+e))*d^2+2/f/(c^2+d^2)^2*B*arctan(tan(f*x+e))*c*d-1/f/(c^2
+d^2)^2*C*arctan(tan(f*x+e))*c^2+1/f/(c^2+d^2)^2*C*arctan(tan(f*x+e))*d^2-1
/f/(c^2+d^2)*d/(c+d*tan(f*x+e))*A+1/f/(c^2+d^2)/(c+d*tan(f*x+e))*B*c-1/f/(c
^2+d^2)/d/(c+d*tan(f*x+e))*c^2*C+2/f/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*A*c*d-1
/f/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*B*c^2+1/f/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*
B*d^2-2/f/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*c*C*d
```

Maxima [A] time = 1.46227, size = 277, normalized size = 1.98

$$\frac{\frac{2\left((A-C)c^2+2Bcd-(A-C)d^2\right)(fx+e)}{c^4+2c^2d^2+d^4}-\frac{2\left(Bc^2-2(A-C)cd-Bd^2\right)\log\left(d\tan(fx+e)+c\right)}{c^4+2c^2d^2+d^4}+\frac{\left(Bc^2-2(A-C)cd-Bd^2\right)\log\left(\tan(fx+e)^2+1\right)}{c^4+2c^2d^2+d^4}-\frac{2\left(Cc^2-Bcd\right.}{c^3d+cd^3+\left(c^2d^2+d\right.}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^2,x, algorithm="
maxima")
```

```
[Out] 1/2*(2*((A - C)*c^2 + 2*B*c*d - (A - C)*d^2)*(f*x + e)/(c^4 + 2*c^2*d^2 + d
^4) - 2*(B*c^2 - 2*(A - C)*c*d - B*d^2)*log(d*tan(f*x + e) + c)/(c^4 + 2*c^
2*d^2 + d^4) + (B*c^2 - 2*(A - C)*c*d - B*d^2)*log(tan(f*x + e)^2 + 1)/(c^4
 + 2*c^2*d^2 + d^4) - 2*(C*c^2 - B*c*d + A*d^2)/(c^3*d + c*d^3 + (c^2*d^2 +
 d^4)*tan(f*x + e)))/f
```

Fricas [A] time = 1.13206, size = 566, normalized size = 4.04

$$-\frac{2\,Cc^2d-2\,Bcd^2+2\,Ad^3-2\left((A-C)c^3+2\,Bc^2d-(A-C)cd^2\right)fx+\left(Bc^3-2\,(A-C)c^2d-Bcd^2+\left(Bc^2d-2\,(A-C)cd\right.\right.}{2\left(\left(c^4d+2\,c^2d^3+d^5\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^2,x, algorithm="
fricas")
```

```
[Out] -1/2*(2*C*c^2*d - 2*B*c*d^2 + 2*A*d^3 - 2*((A - C)*c^3 + 2*B*c^2*d - (A - C
)*c*d^2)*f*x + (B*c^3 - 2*(A - C)*c^2*d - B*c*d^2 + (B*c^2*d - 2*(A - C)*c*
d^2 - B*d^3)*tan(f*x + e))*log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) + c
^2)/(tan(f*x + e)^2 + 1)) - 2*(C*c^3 - B*c^2*d + A*c*d^2 + ((A - C)*c^2*d +
 2*B*c*d^2 - (A - C)*d^3)*f*x)*tan(f*x + e))/((c^4*d + 2*c^2*d^3 + d^5)*f*t
an(f*x + e) + (c^5 + 2*c^3*d^2 + c*d^4)*f)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.58519, size = 404, normalized size = 2.89

$$\frac{\frac{2\left(Ac^2-Cc^2+2\,Bcd-Ad^2+Cd^2\right)(fx+e)}{c^4+2\,c^2d^2+d^4}+\frac{\left(Bc^2-2\,Acd+2\,Ccd-Bd^2\right)\log\left(\tan\left(fx+e\right)^2+1\right)}{c^4+2\,c^2d^2+d^4}-\frac{2\left(Bc^2d-2\,Acd^2+2\,Ccd^2-Bd^3\right)\log\left(\left|d\tan\left(fx+e\right)+c\right|\right)}{c^4d+2\,c^2d^3+d^5}+\frac{2\left(Bc^2d^2\tan\right.}{}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^2,x, algorithm="
giac")
```

```
[Out] 1/2*(2*(A*c^2 - C*c^2 + 2*B*c*d - A*d^2 + C*d^2)*(f*x + e)/(c^4 + 2*c^2*d^2
 + d^4) + (B*c^2 - 2*A*c*d + 2*C*c*d - B*d^2)*log(tan(f*x + e)^2 + 1)/(c^4
+ 2*c^2*d^2 + d^4) - 2*(B*c^2*d - 2*A*c*d^2 + 2*C*c*d^2 - B*d^3)*log(abs(d*
tan(f*x + e) + c))/(c^4*d + 2*c^2*d^3 + d^5) + 2*(B*c^2*d^2*tan(f*x + e) -
2*A*c*d^3*tan(f*x + e) + 2*C*c*d^3*tan(f*x + e) - B*d^4*tan(f*x + e) - C*c^
4 + 2*B*c^3*d - 3*A*c^2*d^2 + C*c^2*d^2 - A*d^4)/((c^4*d + 2*c^2*d^3 + d^5)
*(d*tan(f*x + e) + c)))/f
```

3.81 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^2}\,dx$

Optimal. Leaf size=293

$$-\frac{x\left(a\left(-A\left(c^2-d^2\right)-2Bcd+c^2C-Cd^2\right)+b\left(2cd(A-C)-B\left(c^2-d^2\right)\right)\right)}{\left(a^2+b^2\right)\left(c^2+d^2\right)^2}+\frac{b\left(Ab^2-a(bB-aC)\right)\log(a\cos(e+fx)+b\sin(e+fx))}{f\left(a^2+b^2\right)(bc-ad)^2}$$

```
[Out] -(((a*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2)) + b*(2*c*(A - C)*d - B*(c^2
 - d^2)))*x)/((a^2 + b^2)*(c^2 + d^2)^2)) + (b*(A*b^2 - a*(b*B - a*C))*Log[
a*Cos[e + f*x] + b*Sin[e + f*x]])/((a^2 + b^2)*(b*c - a*d)^2*f) - ((b*(c^4*
C - 2*B*c^3*d + c^2*(3*A - C)*d^2 + A*d^4) - a*d^2*(2*c*(A - C)*d - B*(c^2
- d^2)))*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/((b*c - a*d)^2*(c^2 + d^2)^2
*f) + (c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*x
]))
```

Rubi [A] time = 0.811642, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3649, 3651, 3530}

$$-\frac{x\left(a\left(-A\left(c^2-d^2\right)-2Bcd+c^2C-Cd^2\right)+b\left(2cd(A-C)-B\left(c^2-d^2\right)\right)\right)}{\left(a^2+b^2\right)\left(c^2+d^2\right)^2}+\frac{b\left(Ab^2-a(bB-aC)\right)\log(a\cos(e+fx)+b\sin(e+fx))}{f\left(a^2+b^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])*(c + d*Ta
n[e + f*x])^2),x]
```



```
[Out] -(((a*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2)) + b*(2*c*(A - C)*d - B*(c^2
 - d^2)))*x)/((a^2 + b^2)*(c^2 + d^2)^2)) + (b*(A*b^2 - a*(b*B - a*C))*Log[
a*Cos[e + f*x] + b*Sin[e + f*x]])/((a^2 + b^2)*(b*c - a*d)^2*f) - ((b*(c^4*
C - 2*B*c^3*d + c^2*(3*A - C)*d^2 + A*d^4) - a*d^2*(2*c*(A - C)*d - B*(c^2
- d^2)))*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/((b*c - a*d)^2*(c^2 + d^2)^2
*f) + (c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*x
]))
```


Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
 b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3651

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[((a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*x)
```

```
/((a^2 + b^2)*(c^2 + d^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^2}\,dx = \frac{c^2C-Bcd+Ad^2}{(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))} + \frac{\int \frac{-aAcd+ad(cC-Bd)+Ab\left(c^2+d^2\right)\cdots}{\cdots}}{\cdots}$$

$$= -\frac{\left(a\left(c^2C-2Bcd-Cd^2-A\left(c^2-d^2\right)\right)+b\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\cdots}{\left(a^2+b^2\right)\left(c^2+d^2\right)^2}$$

$$= -\frac{\left(a\left(c^2C-2Bcd-Cd^2-A\left(c^2-d^2\right)\right)+b\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\cdots}{\left(a^2+b^2\right)\left(c^2+d^2\right)^2}$$

Mathematica [B] time = 7.45192, size = 592, normalized size = 2.02

$$-\frac{\frac{b^2\left(c^2+d^2\right)\left(Ab^2-a(bB-aC)\right)\log(a+b\tan(e+fx))}{\left(a^2+b^2\right)(bc-ad)} - \frac{b(bc-ad)\log\left(\sqrt{-b^2}-b\tan(e+fx)\right)\left(-\frac{\sqrt{-b^2}\left(a\left(-A\left(c^2-d^2\right)-2Bcd+c^2C-Cd^2\right)+b\left(2cd(A-C)-B\left(c^2-d^2\right)\right)\right)}{b}+2aAcd-a\cdots\right.}{2\left(a^2+b^2\right)\left(c^2+d^2\right)}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])*(c
+ d*Tan[e + f*x])^2),x]
```

```
[Out] -((-(b*(b*c - a*d)*(A*b*c^2 - a*B*c^2 - b*c^2*C + 2*a*A*c*d + 2*b*B*c*d - 2
*a*c*C*d - A*b*d^2 + a*B*d^2 + b*C*d^2 - (Sqrt[-b^2]*(a*(c^2*C - 2*B*c*d -
C*d^2 - A*(c^2 - d^2)) + b*(2*c*(A - C)*d - B*(c^2 - d^2))))/b)*Log[Sqrt[-b
^2] - b*Tan[e + f*x]])/(2*(a^2 + b^2)*(c^2 + d^2)) + (b^2*(A*b^2 - a*(b*B -
 a*C))*(c^2 + d^2)*Log[a + b*Tan[e + f*x]])/((a^2 + b^2)*(b*c - a*d)) - (b*
(b*c - a*d)*(A*b*c^2 - a*B*c^2 - b*c^2*C + 2*a*A*c*d + 2*b*B*c*d - 2*a*c*C*
d - A*b*d^2 + a*B*d^2 + b*C*d^2 + (Sqrt[-b^2]*(a*(c^2*C - 2*B*c*d - C*d^2 -
 A*(c^2 - d^2)) + b*(2*c*(A - C)*d - B*(c^2 - d^2))))/b)*Log[Sqrt[-b^2] + b
*Tan[e + f*x]])/(2*(a^2 + b^2)*(c^2 + d^2)) - (b*(b*(c^4*C - 2*B*c^3*d + c^
2*(3*A - C)*d^2 + A*d^4) - a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)))*Log[c + d
*Tan[e + f*x]])/((b*c - a*d)*(c^2 + d^2)))/(b*(-(b*c) + a*d)*(c^2 + d^2)*f)
) - (A*d^2 - c*(-(c*C) + B*d))/((-(b*c) + a*d)*(c^2 + d^2)*f*(c + d*Tan[e +
 f*x]))
```

Maple [B] time = 0.1, size = 1263, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))^2,x)
```

```
[Out] 2/f/(a^2+b^2)/(c^2+d^2)^2*C*arctan(tan(f*x+e))*b*c*d-2/f/(a^2+b^2)/(c^2+d^2
)^2*A*arctan(tan(f*x+e))*b*c*d+2/f/(a^2+b^2)/(c^2+d^2)^2*B*arctan(tan(f*x+e
))*a*c*d+1/f/(a^2+b^2)/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*C*a*c*d-1/f/(a^2+b^2)
/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*A*a*c*d+1/f/(a*d-b*c)^2/(c^2+d^2)^2*ln(c+d*
tan(f*x+e))*C*b*c^2*d^2-2/f/(a*d-b*c)^2/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*C*a*
c*d^3-1/f/(a*d-b*c)^2/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*B*a*c^2*d^2+2/f/(a*d-b
*c)^2/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*A*a*c*d^3-1/f/(a^2+b^2)/(c^2+d^2)^2*ln
(1+tan(f*x+e)^2)*B*b*c*d-3/f/(a*d-b*c)^2/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*A*b
*c^2*d^2+2/f/(a*d-b*c)^2/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*B*b*c^3*d+1/f*b/(a*
d-b*c)^2/(a^2+b^2)*ln(a+b*tan(f*x+e))*C*a^2-1/f/(a^2+b^2)/(c^2+d^2)^2*B*arc
tan(tan(f*x+e))*b*d^2-1/f/(a^2+b^2)/(c^2+d^2)^2*C*arctan(tan(f*x+e))*a*c^2+
1/f/(a^2+b^2)/(c^2+d^2)^2*C*arctan(tan(f*x+e))*a*d^2-1/2/f/(a^2+b^2)/(c^2+d
^2)^2*ln(1+tan(f*x+e)^2)*B*a*d^2-1/f/(a*d-b*c)^2/(c^2+d^2)^2*ln(c+d*tan(f*x
+e))*A*b*d^4+1/f/(a*d-b*c)^2/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*B*a*d^4-1/f/(a*
d-b*c)^2/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*C*b*c^4-1/f/(a*d-b*c)/(c^2+d^2)/(c+
d*tan(f*x+e))*c^2*C+1/f*b^3/(a*d-b*c)^2/(a^2+b^2)*ln(a+b*tan(f*x+e))*A-1/f/
(a*d-b*c)/(c^2+d^2)/(c+d*tan(f*x+e))*A*d^2-1/2/f/(a^2+b^2)/(c^2+d^2)^2*ln(1
+tan(f*x+e)^2)*A*b*c^2+1/2/f/(a^2+b^2)/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*A*b*d
^2+1/2/f/(a^2+b^2)/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*B*a*c^2+1/2/f/(a^2+b^2)/(
c^2+d^2)^2*ln(1+tan(f*x+e)^2)*C*b*c^2-1/2/f/(a^2+b^2)/(c^2+d^2)^2*ln(1+tan(
f*x+e)^2)*C*b*d^2+1/f/(a^2+b^2)/(c^2+d^2)^2*A*arctan(tan(f*x+e))*a*c^2-1/f/
(a^2+b^2)/(c^2+d^2)^2*A*arctan(tan(f*x+e))*a*d^2+1/f/(a^2+b^2)/(c^2+d^2)^2*
B*arctan(tan(f*x+e))*b*c^2+1/f/(a*d-b*c)/(c^2+d^2)/(c+d*tan(f*x+e))*B*c*d-1
/f*b^2/(a*d-b*c)^2/(a^2+b^2)*ln(a+b*tan(f*x+e))*B*a
```

Maxima [A] time = 1.57521, size = 693, normalized size = 2.37

$$\frac{\frac{2\left(((A-C)a+Bb)c^2+2\,(Ba-(A-C)b)cd-((A-C)a+Bb)d^2\right)\left(fx+e\right)}{\left(a^2+b^2\right)c^4+2\left(a^2+b^2\right)c^2d^2+\left(a^2+b^2\right)d^4}+\frac{2\left(Ca^2b-Bab^2+Ab^3\right)\log\left(b\tan\left(fx+e\right)+a\right)}{\left(a^2b^2+b^4\right)c^2-2\left(a^3b+ab^3\right)cd+\left(a^4+a^2b^2\right)d^2}-\frac{2\left(Cbc^4-2\,Bbc^3d-2\,(A-C)acd^3+(Ba+(3\,A-C)b)\right.}{b^2c^6-2\,abc^5d-4\,abc^3d^3-2\,abcd^5+a^2d^6+}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))
^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(((A - C)*a + B*b)*c^2 + 2*(B*a - (A - C)*b)*c*d - ((A - C)*a + B*b)
*d^2)*(f*x + e)/((a^2 + b^2)*c^4 + 2*(a^2 + b^2)*c^2*d^2 + (a^2 + b^2)*d^4)
 + 2*(C*a^2*b - B*a*b^2 + A*b^3)*log(b*tan(f*x + e) + a)/((a^2*b^2 + b^4)*c
^2 - 2*(a^3*b + a*b^3)*c*d + (a^4 + a^2*b^2)*d^2) - 2*(C*b*c^4 - 2*B*b*c^3*
d - 2*(A - C)*a*c*d^3 + (B*a + (3*A - C)*b)*c^2*d^2 - (B*a - A*b)*d^4)*log(
d*tan(f*x + e) + c)/(b^2*c^6 - 2*a*b*c^5*d - 4*a*b*c^3*d^3 - 2*a*b*c*d^5 +
a^2*d^6 + (a^2 + 2*b^2)*c^4*d^2 + (2*a^2 + b^2)*c^2*d^4) + ((B*a - (A - C)*
b)*c^2 - 2*((A - C)*a + B*b)*c*d - (B*a - (A - C)*b)*d^2)*log(tan(f*x + e)^
2 + 1)/((a^2 + b^2)*c^4 + 2*(a^2 + b^2)*c^2*d^2 + (a^2 + b^2)*d^4) + 2*(C*c
^2 - B*c*d + A*d^2)/(b*c^4 - a*c^3*d + b*c^2*d^2 - a*c*d^3 + (b*c^3*d - a*c
^2*d^2 + b*c*d^3 - a*d^4)*tan(f*x + e)))/f
```

Fricas [B] time = 8.54106, size = 2620, normalized size = 8.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))
^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(C*a^2*b + C*b^3)*c^3*d^2 - 2*(C*a^3 + B*a^2*b + C*a*b^2 + B*b^3)*c^
2*d^3 + 2*(B*a^3 + A*a^2*b + B*a*b^2 + A*b^3)*c*d^4 - 2*(A*a^3 + A*a*b^2)*d
^5 + 2*(((A - C)*a*b^2 + B*b^3)*c^5 - 2*((A - C)*a^2*b + (A - C)*b^3)*c^4*d
 + ((A - C)*a^3 - 3*B*a^2*b + 3*(A - C)*a*b^2 - B*b^3)*c^3*d^2 + 2*(B*a^3 +
 B*a*b^2)*c^2*d^3 - ((A - C)*a^3 + B*a^2*b)*c*d^4)*f*x + ((C*a^2*b - B*a*b^
2 + A*b^3)*c^5 + 2*(C*a^2*b - B*a*b^2 + A*b^3)*c^3*d^2 + (C*a^2*b - B*a*b^2
 + A*b^3)*c*d^4 + ((C*a^2*b - B*a*b^2 + A*b^3)*c^4*d + 2*(C*a^2*b - B*a*b^2
 + A*b^3)*c^2*d^3 + (C*a^2*b - B*a*b^2 + A*b^3)*d^5)*tan(f*x + e))*log((b^2
*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)/(tan(f*x + e)^2 + 1)) - ((C*a^2
*b + C*b^3)*c^5 - 2*(B*a^2*b + B*b^3)*c^4*d + (B*a^3 + (3*A - C)*a^2*b + B*
a*b^2 + (3*A - C)*b^3)*c^3*d^2 - 2*((A - C)*a^3 + (A - C)*a*b^2)*c^2*d^3 -
(B*a^3 - A*a^2*b + B*a*b^2 - A*b^3)*c*d^4 + ((C*a^2*b + C*b^3)*c^4*d - 2*(B
*a^2*b + B*b^3)*c^3*d^2 + (B*a^3 + (3*A - C)*a^2*b + B*a*b^2 + (3*A - C)*b^
3)*c^2*d^3 - 2*((A - C)*a^3 + (A - C)*a*b^2)*c*d^4 - (B*a^3 - A*a^2*b + B*a
*b^2 - A*b^3)*d^5)*tan(f*x + e))*log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x +
e) + c^2)/(tan(f*x + e)^2 + 1)) - 2*((C*a^2*b + C*b^3)*c^4*d - (C*a^3 + B*a
^2*b + C*a*b^2 + B*b^3)*c^3*d^2 + (B*a^3 + A*a^2*b + B*a*b^2 + A*b^3)*c^2*d
^3 - (A*a^3 + A*a*b^2)*c*d^4 - (((A - C)*a*b^2 + B*b^3)*c^4*d - 2*((A - C)*
a^2*b + (A - C)*b^3)*c^3*d^2 + ((A - C)*a^3 - 3*B*a^2*b + 3*(A - C)*a*b^2 -
 B*b^3)*c^2*d^3 + 2*(B*a^3 + B*a*b^2)*c*d^4 - ((A - C)*a^3 + B*a^2*b)*d^5)*
f*x)*tan(f*x + e))/(((a^2*b^2 + b^4)*c^6*d - 2*(a^3*b + a*b^3)*c^5*d^2 + (a
^4 + 3*a^2*b^2 + 2*b^4)*c^4*d^3 - 4*(a^3*b + a*b^3)*c^3*d^4 + (2*a^4 + 3*a^
2*b^2 + b^4)*c^2*d^5 - 2*(a^3*b + a*b^3)*c*d^6 + (a^4 + a^2*b^2)*d^7)*f*tan
(f*x + e) + ((a^2*b^2 + b^4)*c^7 - 2*(a^3*b + a*b^3)*c^6*d + (a^4 + 3*a^2*b
^2 + 2*b^4)*c^5*d^2 - 4*(a^3*b + a*b^3)*c^4*d^3 + (2*a^4 + 3*a^2*b^2 + b^4)
*c^3*d^4 - 2*(a^3*b + a*b^3)*c^2*d^5 + (a^4 + a^2*b^2)*c*d^6)*f)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e)
)**2,x)
```

```
[Out] Exception raised: NotImplementedError
```

Giac [B] time = 1.81005, size = 1142, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))
^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a*c^2 - C*a*c^2 + B*b*c^2 + 2*B*a*c*d - 2*A*b*c*d + 2*C*b*c*d - A
*a*d^2 + C*a*d^2 - B*b*d^2)*(f*x + e)/(a^2*c^4 + b^2*c^4 + 2*a^2*c^2*d^2 +
```

```
2*b^2*c^2*d^2 + a^2*d^4 + b^2*d^4) + (B*a*c^2 - A*b*c^2 + C*b*c^2 - 2*A*a*c
*d + 2*C*a*c*d - 2*B*b*c*d - B*a*d^2 + A*b*d^2 - C*b*d^2)*log(tan(f*x + e)^
2 + 1)/(a^2*c^4 + b^2*c^4 + 2*a^2*c^2*d^2 + 2*b^2*c^2*d^2 + a^2*d^4 + b^2*d
^4) + 2*(C*a^2*b^2 - B*a*b^3 + A*b^4)*log(abs(b*tan(f*x + e) + a))/(a^2*b^3
*c^2 + b^5*c^2 - 2*a^3*b^2*c*d - 2*a*b^4*c*d + a^4*b*d^2 + a^2*b^3*d^2) - 2
*(C*b*c^4*d - 2*B*b*c^3*d^2 + B*a*c^2*d^3 + 3*A*b*c^2*d^3 - C*b*c^2*d^3 - 2
*A*a*c*d^4 + 2*C*a*c*d^4 - B*a*d^5 + A*b*d^5)*log(abs(d*tan(f*x + e) + c))/
(b^2*c^6*d - 2*a*b*c^5*d^2 + a^2*c^4*d^3 + 2*b^2*c^4*d^3 - 4*a*b*c^3*d^4 +
2*a^2*c^2*d^5 + b^2*c^2*d^5 - 2*a*b*c*d^6 + a^2*d^7) + 2*(C*b*c^4*d*tan(f*x
 + e) - 2*B*b*c^3*d^2*tan(f*x + e) + B*a*c^2*d^3*tan(f*x + e) + 3*A*b*c^2*d
^3*tan(f*x + e) - C*b*c^2*d^3*tan(f*x + e) - 2*A*a*c*d^4*tan(f*x + e) + 2*C
*a*c*d^4*tan(f*x + e) - B*a*d^5*tan(f*x + e) + A*b*d^5*tan(f*x + e) + 2*C*b
*c^5 - C*a*c^4*d - 3*B*b*c^4*d + 2*B*a*c^3*d^2 + 4*A*b*c^3*d^2 - 3*A*a*c^2*
d^3 + C*a*c^2*d^3 - B*b*c^2*d^3 + 2*A*b*c*d^4 - A*a*d^5)/((b^2*c^6 - 2*a*b*
c^5*d + a^2*c^4*d^2 + 2*b^2*c^4*d^2 - 4*a*b*c^3*d^3 + 2*a^2*c^2*d^4 + b^2*c
^2*d^4 - 2*a*b*c*d^5 + a^2*d^6)*(d*tan(f*x + e) + c)))/f
```

3.82 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^2}\,dx$

Optimal. Leaf size=509

$$-\frac{d\left(A\left(a^2d^2+b^2\left(c^2+2d^2\right)\right)+a^2\left(-Bcd+2c^2C+Cd^2\right)-abB\left(c^2+d^2\right)+b^2c(cC-Bd)\right)}{f\left(a^2+b^2\right)\left(c^2+d^2\right)(bc-ad)^2(c+d\tan(e+fx))}-\frac{x\left(a^2\left(-A\left(c^2-d^2\right)-2Bc\right.\right.}{}$$

```
[Out] -(((a^2*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - b^2*(c^2*C - 2*B*c*d -
C*d^2 - A*(c^2 - d^2)) + 2*a*b*(2*c*(A - C)*d - B*(c^2 - d^2)))*x)/((a^2 +
b^2)^2*(c^2 + d^2)^2)) + (b*(3*a^3*b*B*d - 2*a^4*C*d + b^4*(B*c - 2*A*d) -
a^2*b^2*(B*c + 4*A*d) + a*b^3*(2*A*c - 2*c*C + B*d))*Log[a*Cos[e + f*x] + b
*Sin[e + f*x]])/((a^2 + b^2)^2*(b*c - a*d)^3*f) + (d*(b*(2*c^4*C - 3*B*c^3*
d + 4*A*c^2*d^2 - B*c*d^3 + 2*A*d^4) - a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)
))*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/((b*c - a*d)^3*(c^2 + d^2)^2*f) -
(d*(b^2*c*(c*C - B*d) - a*b*B*(c^2 + d^2) + a^2*(2*c^2*C - B*c*d + C*d^2) +
 A*(a^2*d^2 + b^2*(c^2 + 2*d^2))))/((a^2 + b^2)*(b*c - a*d)^2*(c^2 + d^2)*f
*(c + d*Tan[e + f*x])) - (A*b^2 - a*(b*B - a*C))/((a^2 + b^2)*(b*c - a*d)*f
*(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x]))
```

Rubi [A] time = 2.15119, antiderivative size = 508, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3649, 3651, 3530}

$$-\frac{d\left(a^2Ad^2+a^2\left(-Bcd+2c^2C+Cd^2\right)-abB\left(c^2+d^2\right)+Ab^2\left(c^2+2d^2\right)+b^2c(cC-Bd)\right)}{f\left(a^2+b^2\right)\left(c^2+d^2\right)(bc-ad)^2(c+d\tan(e+fx))}-\frac{x\left(a^2\left(-A\left(c^2-d^2\right)-2Bcd\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^2*(c + d*
Tan[e + f*x])^2),x]
```

```
[Out] -(((a^2*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - b^2*(c^2*C - 2*B*c*d -
C*d^2 - A*(c^2 - d^2)) + 2*a*b*(2*c*(A - C)*d - B*(c^2 - d^2)))*x)/((a^2 +
b^2)^2*(c^2 + d^2)^2)) + (b*(3*a^3*b*B*d - 2*a^4*C*d + b^4*(B*c - 2*A*d) -
a^2*b^2*(B*c + 4*A*d) + a*b^3*(2*A*c - 2*c*C + B*d))*Log[a*Cos[e + f*x] + b
*Sin[e + f*x]])/((a^2 + b^2)^2*(b*c - a*d)^3*f) + (d*(b*(2*c^4*C - 3*B*c^3*
d + 4*A*c^2*d^2 - B*c*d^3 + 2*A*d^4) - a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)
))*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/((b*c - a*d)^3*(c^2 + d^2)^2*f) -
(d*(a^2*A*d^2 + b^2*c*(c*C - B*d) - a*b*B*(c^2 + d^2) + A*b^2*(c^2 + 2*d^2)
 + a^2*(2*c^2*C - B*c*d + C*d^2)))/((a^2 + b^2)*(b*c - a*d)^2*(c^2 + d^2)*f
*(c + d*Tan[e + f*x])) - (A*b^2 - a*(b*B - a*C))/((a^2 + b^2)*(b*c - a*d)*f
*(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x]))
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
 b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
```

```
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3651

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[((a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*x)
/((a^2 + b^2)*(c^2 + d^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^2}\,dx = -\frac{Ab^2-a(bB-aC)}{\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))(c+d\tan(e+fx))} - \frac{\int \frac{2Ab^2d-\ldots}{\ldots}}{\ldots}$$

$$= -\frac{d\left(a^2Ad^2+b^2c(cC-Bd)-abB\left(c^2+d^2\right)+Ab^2\left(c^2+2d^2\right)+a^2\left(2c^2C\ldots\right.\right.}{\left(a^2+b^2\right)(bc-ad)^2\left(c^2+d^2\right)f(c+d\tan(e+fx))}$$

$$= -\frac{\left(a^2\left(c^2C-2Bcd-Cd^2-A\left(c^2-d^2\right)\right)-b^2\left(c^2C-2Bcd-Cd^2-A\left(c^2\ldots\right.\right.\right.}{\left(a^2+b^2\right)^2\left(c^2+d^2\right)^2}$$

$$= -\frac{\left(a^2\left(c^2C-2Bcd-Cd^2-A\left(c^2-d^2\right)\right)-b^2\left(c^2C-2Bcd-Cd^2-A\left(c^2\ldots\right.\right.\right.}{\left(a^2+b^2\right)^2\left(c^2+d^2\right)^2}$$

Mathematica [A] time = 8.90489, size = 984, normalized size = 1.93

$$-\frac{Ab^2-a(bB-aC)}{\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))(c+d\tan(e+fx))} - \frac{-\frac{d^2\left(2Adb^2-aA(bc-ad)-(bB-aC)(bc+ad)\right)-c\left((Ab-Cb-aB)d(bc-ad)-2c\left(Ab^2\ldots\right.\right.}{(ad-bc)\left(c^2+d^2\right)f(c+d\tan(e+fx))}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^2*(
c + d*Tan[e + f*x])^2),x]
```

```
[Out] -((A*b^2 - a*(b*B - a*C))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])*(
c + d*Tan[e + f*x]))) - (-(((b*(b*c - a*d)^2*(2*a*A*b*c^2 - a^2*B*c^2 + b^2
*B*c^2 - 2*a*b*c^2*C + 2*a^2*A*c*d - 2*A*b^2*c*d + 4*a*b*B*c*d - 2*a^2*c*C*
d + 2*b^2*c*C*d - 2*a*A*b*d^2 + a^2*B*d^2 - b^2*B*d^2 + 2*a*b*C*d^2 - (Sqrt
[-b^2]*(a^2*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - b^2*(c^2*C - 2*B*c*
d - C*d^2 - A*(c^2 - d^2)) + 2*a*b*(2*c*(A - C)*d - B*(c^2 - d^2))))/b)*Log
[Sqrt[-b^2] - b*Tan[e + f*x]])/(2*(a^2 + b^2)*(c^2 + d^2)) - (b^2*(c^2 + d^
```

```
2)*(3*a^3*b*B*d - 2*a^4*C*d + b^4*(B*c - 2*A*d) - a^2*b^2*(B*c + 4*A*d) + a
*b^3*(2*A*c - 2*c*C + B*d))*Log[a + b*Tan[e + f*x]])/((a^2 + b^2)*(b*c - a*
d)) + (b*(b*c - a*d)^2*(2*a*A*b*c^2 - a^2*B*c^2 + b^2*B*c^2 - 2*a*b*c^2*C +
 2*a^2*A*c*d - 2*A*b^2*c*d + 4*a*b*B*c*d - 2*a^2*c*C*d + 2*b^2*c*C*d - 2*a*
A*b*d^2 + a^2*B*d^2 - b^2*B*d^2 + 2*a*b*C*d^2 + (Sqrt[-b^2]*(a^2*(c^2*C - 2
*B*c*d - C*d^2 - A*(c^2 - d^2)) - b^2*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d
^2)) + 2*a*b*(2*c*(A - C)*d - B*(c^2 - d^2))))/b)*Log[Sqrt[-b^2] + b*Tan[e
+ f*x]])/(2*(a^2 + b^2)*(c^2 + d^2)) - (b*(a^2 + b^2)*d*(b*(2*c^4*C - 3*B*c
^3*d + 4*A*c^2*d^2 - B*c*d^3 + 2*A*d^4) - a*d^2*(2*c*(A - C)*d - B*(c^2 - d
^2)))*Log[c + d*Tan[e + f*x]])/((b*c - a*d)*(c^2 + d^2)))/(b*(-(b*c) + a*d)
*(c^2 + d^2)*f)) - (-(c*(-2*c*(A*b^2 - a*(b*B - a*C))*d + (A*b - a*B - b*C)
*d*(b*c - a*d))) + d^2*(2*A*b^2*d - a*A*(b*c - a*d) - (b*B - a*C)*(b*c + a*
d)))/((-(b*c) + a*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*x])))/((a^2 + b^2)*(b*c
 - a*d))
```

Maple [B] time = 0.144, size = 2012, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^2,x
)
```

```
[Out] -1/2/f/(a^2+b^2)^2/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*B*a^2*d^2-1/2/f/(a^2+b^2)
^2/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*B*b^2*c^2+1/2/f/(a^2+b^2)^2/(c^2+d^2)^2*l
n(1+tan(f*x+e)^2)*B*b^2*d^2+1/f/(a^2+b^2)^2/(c^2+d^2)^2*A*arctan(tan(f*x+e)
)*a^2*c^2-1/f/(a^2+b^2)^2/(c^2+d^2)^2*A*arctan(tan(f*x+e))*a^2*d^2-4/f/(a^2
+b^2)^2/(c^2+d^2)^2*A*arctan(tan(f*x+e))*a*b*c*d+4/f/(a^2+b^2)^2/(c^2+d^2)^
2*C*arctan(tan(f*x+e))*a*b*c*d-2/f/(a^2+b^2)^2/(c^2+d^2)^2*ln(1+tan(f*x+e)^
2)*B*a*b*c*d-4/f*d^3/(a*d-b*c)^3/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*A*b*c^2-2/f
*d^4/(a*d-b*c)^3/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*C*a*c-2/f*d/(a*d-b*c)^3/(c^
2+d^2)^2*ln(c+d*tan(f*x+e))*C*b*c^4-1/f*d^3/(a*d-b*c)^3/(c^2+d^2)^2*ln(c+d*
tan(f*x+e))*B*a*c^2+2/f*d^4/(a*d-b*c)^3/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*A*a*
c+3/f*d^2/(a*d-b*c)^3/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*B*b*c^3+1/f*d^4/(a*d-b
*c)^3/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*B*b*c-1/f/(a^2+b^2)^2/(c^2+d^2)^2*ln(1
+tan(f*x+e)^2)*A*a^2*c*d-1/f/(a^2+b^2)^2/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*A*a
*b*c^2+1/f/(a^2+b^2)^2/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*A*a*b*d^2+1/f/(a^2+b^
2)^2/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*A*b^2*c*d+1/f/(a^2+b^2)^2/(c^2+d^2)^2*l
n(1+tan(f*x+e)^2)*C*a^2*c*d-1/f/(a^2+b^2)^2/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*
C*b^2*c*d+2/f/(a^2+b^2)^2/(c^2+d^2)^2*B*arctan(tan(f*x+e))*a^2*c*d+2/f/(a^2
+b^2)^2/(c^2+d^2)^2*B*arctan(tan(f*x+e))*a*b*c^2-2/f/(a^2+b^2)^2/(c^2+d^2)^
2*B*arctan(tan(f*x+e))*a*b*d^2-2/f/(a^2+b^2)^2/(c^2+d^2)^2*B*arctan(tan(f*x
+e))*b^2*c*d+1/f/(a^2+b^2)^2/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*C*a*b*c^2-1/f/(
a^2+b^2)^2/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*C*a*b*d^2+2/f*b/(a^2+b^2)^2/(a*d-
b*c)^3*ln(a+b*tan(f*x+e))*a^4*C*d+2/f*b^4/(a^2+b^2)^2/(a*d-b*c)^3*ln(a+b*ta
n(f*x+e))*C*a*c+4/f*b^3/(a^2+b^2)^2/(a*d-b*c)^3*ln(a+b*tan(f*x+e))*A*a^2*d-
2/f*b^4/(a^2+b^2)^2/(a*d-b*c)^3*ln(a+b*tan(f*x+e))*A*a*c-3/f*b^2/(a^2+b^2)^
2/(a*d-b*c)^3*ln(a+b*tan(f*x+e))*a^3*B*d+1/f*b^3/(a^2+b^2)^2/(a*d-b*c)^3*ln
(a+b*tan(f*x+e))*B*a^2*c-1/f*b^4/(a^2+b^2)^2/(a*d-b*c)^3*ln(a+b*tan(f*x+e))
*B*a*d-1/f*d^3/(a*d-b*c)^2/(c^2+d^2)/(c+d*tan(f*x+e))*A-1/f*b^3/(a^2+b^2)/(
a*d-b*c)^2/(a+b*tan(f*x+e))*A+1/f*d^2/(a*d-b*c)^2/(c^2+d^2)/(c+d*tan(f*x+e)
)*B*c-1/f/(a^2+b^2)^2/(c^2+d^2)^2*C*arctan(tan(f*x+e))*b^2*d^2-2/f*d^5/(a*d
-b*c)^3/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*A*b+1/f*d^5/(a*d-b*c)^3/(c^2+d^2)^2*
ln(c+d*tan(f*x+e))*B*a-1/f*d/(a*d-b*c)^2/(c^2+d^2)/(c+d*tan(f*x+e))*c^2*C+2
/f*b^5/(a^2+b^2)^2/(a*d-b*c)^3*ln(a+b*tan(f*x+e))*A*d-1/f*b^5/(a^2+b^2)^2/(
a*d-b*c)^3*ln(a+b*tan(f*x+e))*B*c+1/f*b^2/(a^2+b^2)/(a*d-b*c)^2/(a+b*tan(f*
```

```
x+e))*B*a+1/2/f/(a^2+b^2)^2/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*B*a^2*c^2-1/f/(a
^2+b^2)^2/(c^2+d^2)^2*A*arctan(tan(f*x+e))*b^2*c^2+1/f/(a^2+b^2)^2/(c^2+d^2
)^2*A*arctan(tan(f*x+e))*b^2*d^2-1/f/(a^2+b^2)^2/(c^2+d^2)^2*C*arctan(tan(f
*x+e))*a^2*c^2+1/f/(a^2+b^2)^2/(c^2+d^2)^2*C*arctan(tan(f*x+e))*a^2*d^2+1/f
/(a^2+b^2)^2/(c^2+d^2)^2*C*arctan(tan(f*x+e))*b^2*c^2-1/f*b/(a^2+b^2)/(a*d-
b*c)^2/(a+b*tan(f*x+e))*C*a^2
```

Maxima [B] time = 1.81216, size = 1600, normalized size = 3.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e
))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*c^2 + 2*(B*a^2 - 2*(A - C)*a*
b - B*b^2)*c*d - ((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*d^2)*(f*x + e)/((a^4
 + 2*a^2*b^2 + b^4)*c^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*c^2*d^2 + (a^4 + 2*a^2*
b^2 + b^4)*d^4) - 2*((B*a^2*b^3 - 2*(A - C)*a*b^4 - B*b^5)*c + (2*C*a^4*b -
 3*B*a^3*b^2 + 4*A*a^2*b^3 - B*a*b^4 + 2*A*b^5)*d)*log(b*tan(f*x + e) + a)/
((a^4*b^3 + 2*a^2*b^5 + b^7)*c^3 - 3*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*c^2*d +
3*(a^6*b + 2*a^4*b^3 + a^2*b^5)*c*d^2 - (a^7 + 2*a^5*b^2 + a^3*b^4)*d^3) +
2*(2*C*b*c^4*d - 3*B*b*c^3*d^2 + (B*a + 4*A*b)*c^2*d^3 - (2*(A - C)*a + B*b
)*c*d^4 - (B*a - 2*A*b)*d^5)*log(d*tan(f*x + e) + c)/(b^3*c^7 - 3*a*b^2*c^6
*d + 3*a^2*b*c*d^6 - a^3*d^7 + (3*a^2*b + 2*b^3)*c^5*d^2 - (a^3 + 6*a*b^2)*
c^4*d^3 + (6*a^2*b + b^3)*c^3*d^4 - (2*a^3 + 3*a*b^2)*c^2*d^5) + ((B*a^2 -
2*(A - C)*a*b - B*b^2)*c^2 - 2*((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*c*d -
(B*a^2 - 2*(A - C)*a*b - B*b^2)*d^2)*log(tan(f*x + e)^2 + 1)/((a^4 + 2*a^2*
b^2 + b^4)*c^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*c^2*d^2 + (a^4 + 2*a^2*b^2 + b^4
)*d^4) - 2*((C*a^2*b - B*a*b^2 + A*b^3)*c^3 + (C*a^3 + C*a*b^2)*c^2*d - (B*
a^3 - C*a^2*b + 2*B*a*b^2 - A*b^3)*c*d^2 + (A*a^3 + A*a*b^2)*d^3 + ((2*C*a^
2*b - B*a*b^2 + (A + C)*b^3)*c^2*d - (B*a^2*b + B*b^3)*c*d^2 + ((A + C)*a^2
*b - B*a*b^2 + 2*A*b^3)*d^3)*tan(f*x + e))/((a^3*b^2 + a*b^4)*c^5 - 2*(a^4*
b + a^2*b^3)*c^4*d + (a^5 + 2*a^3*b^2 + a*b^4)*c^3*d^2 - 2*(a^4*b + a^2*b^3
)*c^2*d^3 + (a^5 + a^3*b^2)*c*d^4 + ((a^2*b^3 + b^5)*c^4*d - 2*(a^3*b^2 + a
*b^4)*c^3*d^2 + (a^4*b + 2*a^2*b^3 + b^5)*c^2*d^3 - 2*(a^3*b^2 + a*b^4)*c*d
^4 + (a^4*b + a^2*b^3)*d^5)*tan(f*x + e)^2 + ((a^2*b^3 + b^5)*c^5 - (a^3*b^
2 + a*b^4)*c^4*d - (a^4*b - b^5)*c^3*d^2 + (a^5 - a*b^4)*c^2*d^3 - (a^4*b +
 a^2*b^3)*c*d^4 + (a^5 + a^3*b^2)*d^5)*tan(f*x + e)))/f
```

Fricas [B] time = 35.2913, size = 8519, normalized size = 16.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e
))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(C*a^2*b^4 - B*a*b^5 + A*b^6)*c^6 - 2*(C*a^3*b^3 - B*a^2*b^4 + A*a*
b^5)*c^5*d + 4*(C*a^2*b^4 - B*a*b^5 + A*b^6)*c^4*d^2 + 2*(C*a^5*b + 2*B*a^2
*b^4 - (2*A - C)*a*b^5)*c^3*d^3 - 2*(C*a^6 + B*a^5*b + 2*C*a^4*b^2 + 2*B*a^
3*b^3 + 2*B*a*b^5 - A*b^6)*c^2*d^4 + 2*(B*a^6 + A*a^5*b + 2*B*a^4*b^2 + (2*
A - C)*a^3*b^3 + 2*B*a^2*b^4)*c*d^5 - 2*(A*a^6 + 2*A*a^4*b^2 + A*a^2*b^4)*d
```

```
^6 - 2*(((A - C)*a^3*b^3 + 2*B*a^2*b^4 - (A - C)*a*b^5)*c^6 - (3*(A - C)*a^
4*b^2 + 4*B*a^3*b^3 + (A - C)*a^2*b^4 + 2*B*a*b^5)*c^5*d + (3*(A - C)*a^5*b
 + 8*(A - C)*a^3*b^3 + 4*B*a^2*b^4 + (A - C)*a*b^5)*c^4*d^2 - ((A - C)*a^6
- 4*B*a^5*b + 8*(A - C)*a^4*b^2 + 3*(A - C)*a^2*b^4)*c^3*d^3 - (2*B*a^6 - (
A - C)*a^5*b + 4*B*a^4*b^2 - 3*(A - C)*a^3*b^3)*c^2*d^4 + ((A - C)*a^6 + 2*
B*a^5*b - (A - C)*a^4*b^2)*c*d^5)*f*x - 2*((C*a^3*b^3 - B*a^2*b^4 + A*a*b^5
)*c^5*d + (B*a^3*b^3 - (A - 2*C)*a^2*b^4 + C*b^6)*c^4*d^2 - (C*a^5*b + B*a^
4*b^2 + 4*B*a^2*b^4 - (2*A - C)*a*b^5 + B*b^6)*c^3*d^3 + (B*a^5*b + (A - 2*
C)*a^4*b^2 + 4*B*a^3*b^3 + B*a*b^5 + A*b^6)*c^2*d^4 - (A*a^5*b + (2*A - C)*
a^3*b^3 + B*a^2*b^4)*c*d^5 - (C*a^4*b^2 - B*a^3*b^3 + A*a^2*b^4)*d^6 + (((A
 - C)*a^2*b^4 + 2*B*a*b^5 - (A - C)*b^6)*c^5*d - (3*(A - C)*a^3*b^3 + 4*B*a
^2*b^4 + (A - C)*a*b^5 + 2*B*b^6)*c^4*d^2 + (3*(A - C)*a^4*b^2 + 8*(A - C)*
a^2*b^4 + 4*B*a*b^5 + (A - C)*b^6)*c^3*d^3 - ((A - C)*a^5*b - 4*B*a^4*b^2 +
 8*(A - C)*a^3*b^3 + 3*(A - C)*a*b^5)*c^2*d^4 - (2*B*a^5*b - (A - C)*a^4*b^
2 + 4*B*a^3*b^3 - 3*(A - C)*a^2*b^4)*c*d^5 + ((A - C)*a^5*b + 2*B*a^4*b^2 -
 (A - C)*a^3*b^3)*d^6)*f*x)*tan(f*x + e)^2 + ((B*a^3*b^3 - 2*(A - C)*a^2*b^
4 - B*a*b^5)*c^6 + (2*C*a^5*b - 3*B*a^4*b^2 + 4*A*a^3*b^3 - B*a^2*b^4 + 2*A
*a*b^5)*c^5*d + 2*(B*a^3*b^3 - 2*(A - C)*a^2*b^4 - B*a*b^5)*c^4*d^2 + 2*(2*
C*a^5*b - 3*B*a^4*b^2 + 4*A*a^3*b^3 - B*a^2*b^4 + 2*A*a*b^5)*c^3*d^3 + (B*a
^3*b^3 - 2*(A - C)*a^2*b^4 - B*a*b^5)*c^2*d^4 + (2*C*a^5*b - 3*B*a^4*b^2 +
4*A*a^3*b^3 - B*a^2*b^4 + 2*A*a*b^5)*c*d^5 + ((B*a^2*b^4 - 2*(A - C)*a*b^5
- B*b^6)*c^5*d + (2*C*a^4*b^2 - 3*B*a^3*b^3 + 4*A*a^2*b^4 - B*a*b^5 + 2*A*b
^6)*c^4*d^2 + 2*(B*a^2*b^4 - 2*(A - C)*a*b^5 - B*b^6)*c^3*d^3 + 2*(2*C*a^4*
b^2 - 3*B*a^3*b^3 + 4*A*a^2*b^4 - B*a*b^5 + 2*A*b^6)*c^2*d^4 + (B*a^2*b^4 -
 2*(A - C)*a*b^5 - B*b^6)*c*d^5 + (2*C*a^4*b^2 - 3*B*a^3*b^3 + 4*A*a^2*b^4
- B*a*b^5 + 2*A*b^6)*d^6)*tan(f*x + e)^2 + ((B*a^2*b^4 - 2*(A - C)*a*b^5 -
B*b^6)*c^6 + 2*(C*a^4*b^2 - B*a^3*b^3 + (A + C)*a^2*b^4 - B*a*b^5 + A*b^6)*
c^5*d + (2*C*a^5*b - 3*B*a^4*b^2 + 4*A*a^3*b^3 + B*a^2*b^4 - 2*(A - 2*C)*a*
b^5 - 2*B*b^6)*c^4*d^2 + 4*(C*a^4*b^2 - B*a^3*b^3 + (A + C)*a^2*b^4 - B*a*b
^5 + A*b^6)*c^3*d^3 + (4*C*a^5*b - 6*B*a^4*b^2 + 8*A*a^3*b^3 - B*a^2*b^4 +
2*(A + C)*a*b^5 - B*b^6)*c^2*d^4 + 2*(C*a^4*b^2 - B*a^3*b^3 + (A + C)*a^2*b
^4 - B*a*b^5 + A*b^6)*c*d^5 + (2*C*a^5*b - 3*B*a^4*b^2 + 4*A*a^3*b^3 - B*a^
2*b^4 + 2*A*a*b^5)*d^6)*tan(f*x + e))*log((b^2*tan(f*x + e)^2 + 2*a*b*tan(f
*x + e) + a^2)/(tan(f*x + e)^2 + 1)) - (2*(C*a^5*b + 2*C*a^3*b^3 + C*a*b^5)
*c^5*d - 3*(B*a^5*b + 2*B*a^3*b^3 + B*a*b^5)*c^4*d^2 + (B*a^6 + 4*A*a^5*b +
 2*B*a^4*b^2 + 8*A*a^3*b^3 + B*a^2*b^4 + 4*A*a*b^5)*c^3*d^3 - (2*(A - C)*a^
6 + B*a^5*b + 4*(A - C)*a^4*b^2 + 2*B*a^3*b^3 + 2*(A - C)*a^2*b^4 + B*a*b^5
)*c^2*d^4 - (B*a^6 - 2*A*a^5*b + 2*B*a^4*b^2 - 4*A*a^3*b^3 + B*a^2*b^4 - 2*
A*a*b^5)*c*d^5 + (2*(C*a^4*b^2 + 2*C*a^2*b^4 + C*b^6)*c^4*d^2 - 3*(B*a^4*b^
2 + 2*B*a^2*b^4 + B*b^6)*c^3*d^3 + (B*a^5*b + 4*A*a^4*b^2 + 2*B*a^3*b^3 + 8
*A*a^2*b^4 + B*a*b^5 + 4*A*b^6)*c^2*d^4 - (2*(A - C)*a^5*b + B*a^4*b^2 + 4*
(A - C)*a^3*b^3 + 2*B*a^2*b^4 + 2*(A - C)*a*b^5 + B*b^6)*c*d^5 - (B*a^5*b -
 2*A*a^4*b^2 + 2*B*a^3*b^3 - 4*A*a^2*b^4 + B*a*b^5 - 2*A*b^6)*d^6)*tan(f*x
+ e)^2 + (2*(C*a^4*b^2 + 2*C*a^2*b^4 + C*b^6)*c^5*d + (2*C*a^5*b - 3*B*a^4*
b^2 + 4*C*a^3*b^3 - 6*B*a^2*b^4 + 2*C*a*b^5 - 3*B*b^6)*c^4*d^2 - 2*(B*a^5*b
 - 2*A*a^4*b^2 + 2*B*a^3*b^3 - 4*A*a^2*b^4 + B*a*b^5 - 2*A*b^6)*c^3*d^3 + (
B*a^6 + 2*(A + C)*a^5*b + B*a^4*b^2 + 4*(A + C)*a^3*b^3 - B*a^2*b^4 + 2*(A
+ C)*a*b^5 - B*b^6)*c^2*d^4 - 2*((A - C)*a^6 + B*a^5*b + (A - 2*C)*a^4*b^2
+ 2*B*a^3*b^3 - (A + C)*a^2*b^4 + B*a*b^5 - A*b^6)*c*d^5 - (B*a^6 - 2*A*a^5
*b + 2*B*a^4*b^2 - 4*A*a^3*b^3 + B*a^2*b^4 - 2*A*a*b^5)*d^6)*tan(f*x + e))*
log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) -
 2*((C*a^3*b^3 - B*a^2*b^4 + A*a*b^5)*c^6 - (C*a^4*b^2 - B*a^3*b^3 + (A + C
)*a^2*b^4 - B*a*b^5 + A*b^6)*c^5*d + (C*a^5*b + 5*C*a^3*b^3 - 3*B*a^2*b^4 +
 (3*A + C)*a*b^5)*c^4*d^2 - (C*a^6 + B*a^5*b + 5*C*a^4*b^2 + (2*A + 5*C)*a^
2*b^4 - B*a*b^5 + (2*A + C)*b^6)*c^3*d^3 + (B*a^6 + (A + C)*a^5*b + 3*B*a^4
*b^2 + (2*A + 5*C)*a^3*b^3 + (4*A + C)*a*b^5 + B*b^6)*c^2*d^4 - (A*a^6 + B*
a^5*b + (3*A + C)*a^4*b^2 + B*a^3*b^3 + (4*A + C)*a^2*b^4 + 2*A*b^6)*c*d^5
+ (A*a^5*b + (2*A + C)*a^3*b^3 - B*a^2*b^4 + 2*A*a*b^5)*d^6 + (((A - C)*a^2
*b^4 + 2*B*a*b^5 - (A - C)*b^6)*c^6 - 2*((A - C)*a^3*b^3 + B*a^2*b^4 + (A -
```

```
 C)*a*b^5 + B*b^6)*c^5*d - (4*B*a^3*b^3 - 7*(A - C)*a^2*b^4 - 2*B*a*b^5 - (
A - C)*b^6)*c^4*d^2 + 2*((A - C)*a^5*b + 2*B*a^4*b^2 + 2*B*a^2*b^4 - (A - C
)*a*b^5)*c^3*d^3 - ((A - C)*a^6 - 2*B*a^5*b + 7*(A - C)*a^4*b^2 + 4*B*a^3*b
^3)*c^2*d^4 - 2*(B*a^6 - (A - C)*a^5*b + B*a^4*b^2 - (A - C)*a^3*b^3)*c*d^5
 + ((A - C)*a^6 + 2*B*a^5*b - (A - C)*a^4*b^2)*d^6)*f*x)*tan(f*x + e))/(((a
^4*b^4 + 2*a^2*b^6 + b^8)*c^7*d - 3*(a^5*b^3 + 2*a^3*b^5 + a*b^7)*c^6*d^2 +
 (3*a^6*b^2 + 8*a^4*b^4 + 7*a^2*b^6 + 2*b^8)*c^5*d^3 - (a^7*b + 8*a^5*b^3 +
 13*a^3*b^5 + 6*a*b^7)*c^4*d^4 + (6*a^6*b^2 + 13*a^4*b^4 + 8*a^2*b^6 + b^8)
*c^3*d^5 - (2*a^7*b + 7*a^5*b^3 + 8*a^3*b^5 + 3*a*b^7)*c^2*d^6 + 3*(a^6*b^2
 + 2*a^4*b^4 + a^2*b^6)*c*d^7 - (a^7*b + 2*a^5*b^3 + a^3*b^5)*d^8)*f*tan(f*
x + e)^2 + ((a^4*b^4 + 2*a^2*b^6 + b^8)*c^8 - 2*(a^5*b^3 + 2*a^3*b^5 + a*b^
7)*c^7*d + 2*(a^4*b^4 + 2*a^2*b^6 + b^8)*c^6*d^2 + 2*(a^7*b - 3*a^3*b^5 - 2
*a*b^7)*c^5*d^3 - (a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*c^4*d^4 + 2*(2*a^7*b
+ 3*a^5*b^3 - a*b^7)*c^3*d^5 - 2*(a^8 + 2*a^6*b^2 + a^4*b^4)*c^2*d^6 + 2*(a
^7*b + 2*a^5*b^3 + a^3*b^5)*c*d^7 - (a^8 + 2*a^6*b^2 + a^4*b^4)*d^8)*f*tan(
f*x + e) + ((a^5*b^3 + 2*a^3*b^5 + a*b^7)*c^8 - 3*(a^6*b^2 + 2*a^4*b^4 + a^
2*b^6)*c^7*d + (3*a^7*b + 8*a^5*b^3 + 7*a^3*b^5 + 2*a*b^7)*c^6*d^2 - (a^8 +
 8*a^6*b^2 + 13*a^4*b^4 + 6*a^2*b^6)*c^5*d^3 + (6*a^7*b + 13*a^5*b^3 + 8*a^
3*b^5 + a*b^7)*c^4*d^4 - (2*a^8 + 7*a^6*b^2 + 8*a^4*b^4 + 3*a^2*b^6)*c^3*d^
5 + 3*(a^7*b + 2*a^5*b^3 + a^3*b^5)*c^2*d^6 - (a^8 + 2*a^6*b^2 + a^4*b^4)*c
*d^7)*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**2/(c+d*tan(f*x
+e))**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e
))^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.83 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^3(c+d\tan(e+fx))^2}\,dx$

Optimal. Leaf size=841

$$-\frac{\left(b\left(3Cc^4-4Bdc^3+(5A+C)d^2c^2-2Bd^3c+3Ad^4\right)-ad^2\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\right)\log(c\cos(e+fx)+d\sin(e+\ldots}{(bc-ad)^4\left(c^2+d^2\right)^2 f}$$

```
[Out] -(((a^3*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - 3*a*b^2*(c^2*C - 2*B*c*
d - C*d^2 - A*(c^2 - d^2)) + 3*a^2*b*(2*c*(A - C)*d - B*(c^2 - d^2)) - b^3*
(2*c*(A - C)*d - B*(c^2 - d^2)))*x)/((a^2 + b^2)^3*(c^2 + d^2)^2)) - (b*(6*
a^5*b*B*d^2 - 3*a^6*C*d^2 - a^4*b^2*d*(4*B*c + (10*A - C)*d) - b^6*(c*(c*C
- 2*B*d) - A*(c^2 - 3*d^2)) + a*b^5*(2*c*(A - C)*d - B*(3*c^2 - d^2)) + 3*a
^2*b^4*(c*(c*C + 2*B*d) - A*(c^2 + 3*d^2)) + a^3*b^3*(10*c*(A - C)*d + B*(c
^2 + 3*d^2)))*Log[a*Cos[e + f*x] + b*Sin[e + f*x]])/((a^2 + b^2)^3*(b*c - a
*d)^4*f) - (d^2*(b*(3*c^4*C - 4*B*c^3*d + c^2*(5*A + C)*d^2 - 2*B*c*d^3 + 3
*A*d^4) - a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)))*Log[c*Cos[e + f*x] + d*Sin
[e + f*x]])/((b*c - a*d)^4*(c^2 + d^2)^2*f) - (d*(3*a^3*b*B*d*(c^2 + d^2) +
 a*b^3*(2*A*c - 2*c*C + B*d)*(c^2 + d^2) - a^4*d*(3*c^2*C - B*c*d + (A + 2*
C)*d^2) - a^2*b^2*(B*c^3 + 4*A*c^2*d + 2*c^2*C*d - B*c*d^2 + 6*A*d^3) - b^4
*(d*(2*A*c^2 + c^2*C + 3*A*d^2) - B*(c^3 + 2*c*d^2))))/((a^2 + b^2)^2*(b*c
- a*d)^3*(c^2 + d^2)*f*(c + d*Tan[e + f*x])) - (A*b^2 - a*(b*B - a*C))/(2*(
a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])) - (5*
a^3*b*B*d - 3*a^4*C*d + b^4*(2*B*c - 3*A*d) + a*b^3*(4*A*c - 4*c*C + B*d) -
 a^2*b^2*(2*B*c + (7*A - C)*d))/(2*(a^2 + b^2)^2*(b*c - a*d)^2*f*(a + b*Tan
[e + f*x])*(c + d*Tan[e + f*x]))
```

Rubi [A] time = 4.07574, antiderivative size = 841, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3649, 3651, 3530}

$$-\frac{\left(b\left(3Cc^4-4Bdc^3+(5A+C)d^2c^2-2Bd^3c+3Ad^4\right)-ad^2\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\right)\log(c\cos(e+fx)+d\sin(e+\ldots}{(bc-ad)^4\left(c^2+d^2\right)^2 f}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^3*(c + d*
Tan[e + f*x])^2),x]
```

```
[Out] -(((a^3*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - 3*a*b^2*(c^2*C - 2*B*c*
d - C*d^2 - A*(c^2 - d^2)) + 3*a^2*b*(2*c*(A - C)*d - B*(c^2 - d^2)) - b^3*
(2*c*(A - C)*d - B*(c^2 - d^2)))*x)/((a^2 + b^2)^3*(c^2 + d^2)^2)) - (b*(6*
a^5*b*B*d^2 - 3*a^6*C*d^2 - a^4*b^2*d*(4*B*c + (10*A - C)*d) - b^6*(c*(c*C
- 2*B*d) - A*(c^2 - 3*d^2)) + a*b^5*(2*c*(A - C)*d - B*(3*c^2 - d^2)) + 3*a
^2*b^4*(c*(c*C + 2*B*d) - A*(c^2 + 3*d^2)) + a^3*b^3*(10*c*(A - C)*d + B*(c
^2 + 3*d^2)))*Log[a*Cos[e + f*x] + b*Sin[e + f*x]])/((a^2 + b^2)^3*(b*c - a
*d)^4*f) - (d^2*(b*(3*c^4*C - 4*B*c^3*d + c^2*(5*A + C)*d^2 - 2*B*c*d^3 + 3
*A*d^4) - a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)))*Log[c*Cos[e + f*x] + d*Sin
[e + f*x]])/((b*c - a*d)^4*(c^2 + d^2)^2*f) - (d*(3*a^3*b*B*d*(c^2 + d^2) +
 a*b^3*(2*A*c - 2*c*C + B*d)*(c^2 + d^2) - a^4*d*(3*c^2*C - B*c*d + (A + 2*
C)*d^2) - a^2*b^2*(B*c^3 + 4*A*c^2*d + 2*c^2*C*d - B*c*d^2 + 6*A*d^3) - b^4
*(d*(2*A*c^2 + c^2*C + 3*A*d^2) - B*(c^3 + 2*c*d^2))))/((a^2 + b^2)^2*(b*c
- a*d)^3*(c^2 + d^2)*f*(c + d*Tan[e + f*x])) - (A*b^2 - a*(b*B - a*C))/(2*(
a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])) - (5*
a^3*b*B*d - 3*a^4*C*d + b^4*(2*B*c - 3*A*d) + a*b^3*(4*A*c - 4*c*C + B*d) -
```

```
 a^2*b^2*(2*B*c + (7*A - C)*d))/(2*(a^2 + b^2)^2*(b*c - a*d)^2*f*(a + b*Tan
[e + f*x])*(c + d*Tan[e + f*x]))
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
 b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3651

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[((a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*x)
/((a^2 + b^2)*(c^2 + d^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^3(c+d\tan(e+fx))^2}\,dx = -\frac{Ab^2-a(bB-aC)}{2\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^2(c+d\tan(e+fx))} - \frac{\int \frac{3Ab^2 \cdots}{\cdots}}{\cdots}$$

$$= -\frac{Ab^2-a(bB-aC)}{2\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^2(c+d\tan(e+fx))} - \frac{5a^3bB \cdots}{\cdots}$$

$$= -\frac{d\left(3a^3bBd\left(c^2+d^2\right)+ab^3(2Ac-2cC+Bd)\left(c^2+d^2\right)-a^4d\left(3c^2C-B \cdots\right.\right.}{\left(a^2 \cdots\right.}$$

$$= -\frac{\left(a^3\left(c^2C-2Bcd-Cd^2-A\left(c^2-d^2\right)\right)-3ab^2\left(c^2C-2Bcd-Cd^2-A\left(\cdots\right.\right.\right.}{\left(a^2+\cdots\right.}$$

$$= -\frac{\left(a^3\left(c^2C-2Bcd-Cd^2-A\left(c^2-d^2\right)\right)-3ab^2\left(c^2C-2Bcd-Cd^2-A\left(\cdots\right.\right.\right.}{\left(a^2+\cdots\right.}$$

Mathematica [B] time = 8.3458, size = 1758, normalized size = 2.09

$$-\frac{Ab^2 - a(bB - aC)}{2\left(a^2 + b^2\right)(bc - ad)f(a + b\tan(e + fx))^2(c + d\tan(e + fx))} - \frac{-\frac{b^2\left(3Adb^2-2aA(bc-ad)-(bB-aC)(2bc+ad)\right)-a\left(2b(Ab-Cb-aB)(bc-a\right.}{\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))(c+d\tan(e+\right.}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^3*(
c + d*Tan[e + f*x])^2),x]
```

```
[Out] -(A*b^2 - a*(b*B - a*C))/(2*(a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])^
2*(c + d*Tan[e + f*x])) - (-((-(a*(-3*a*(A*b^2 - a*(b*B - a*C))*d + 2*b*(A*
b - a*B - b*C)*(b*c - a*d))) + b^2*(3*A*b^2*d - 2*a*A*(b*c - a*d) - (b*B -
a*C)*(2*b*c + a*d)))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])*(c + d
*Tan[e + f*x]))) - (-((-(((b*c - a*d)^3*(-(b^2*(-3*a^2*A*b*c^2 + A*b^3*c^2
+ a^3*B*c^2 - 3*a*b^2*B*c^2 + 3*a^2*b*c^2*C - b^3*c^2*C - 2*a^3*A*c*d + 6*a
*A*b^2*c*d - 6*a^2*b*B*c*d + 2*b^3*B*c*d + 2*a^3*c*C*d - 6*a*b^2*c*C*d + 3*
a^2*A*b*d^2 - A*b^3*d^2 - a^3*B*d^2 + 3*a*b^2*B*d^2 - 3*a^2*b*C*d^2 + b^3*C
*d^2)) + Sqrt[-b^2]*(a^3*A*b*c^2 - 3*a*A*b^3*c^2 + 3*a^2*b^2*B*c^2 - b^4*B*
c^2 - a^3*b*c^2*C + 3*a*b^3*c^2*C - 6*a^2*A*b^2*c*d + 2*A*b^4*c*d + 2*a^3*b
*B*c*d - 6*a*b^3*B*c*d + 6*a^2*b^2*c*C*d - 2*b^4*c*C*d - a^3*A*b*d^2 + 3*a*
A*b^3*d^2 - 3*a^2*b^2*B*d^2 + b^4*B*d^2 + a^3*b*C*d^2 - 3*a*b^3*C*d^2))*Log
[Sqrt[-b^2] - b*Tan[e + f*x]])/(b*(a^2 + b^2)*(c^2 + d^2))) - (2*b^2*(c^2 +
 d^2)*(6*a^5*b*B*d^2 - 3*a^6*C*d^2 - a^4*b^2*d*(4*B*c + (10*A - C)*d) - b^6
*(c*(c*C - 2*B*d) - A*(c^2 - 3*d^2)) + a*b^5*(2*c*(A - C)*d - B*(3*c^2 - d^
2)) + 3*a^2*b^4*(c*(c*C + 2*B*d) - A*(c^2 + 3*d^2)) + a^3*b^3*(10*c*(A - C)
*d + B*(c^2 + 3*d^2)))*Log[a + b*Tan[e + f*x]])/((a^2 + b^2)*(b*c - a*d)) +
 ((b*c - a*d)^3*(b^2*(-3*a^2*A*b*c^2 + A*b^3*c^2 + a^3*B*c^2 - 3*a*b^2*B*c^
2 + 3*a^2*b*c^2*C - b^3*c^2*C - 2*a^3*A*c*d + 6*a*A*b^2*c*d - 6*a^2*b*B*c*d
 + 2*b^3*B*c*d + 2*a^3*c*C*d - 6*a*b^2*c*C*d + 3*a^2*A*b*d^2 - A*b^3*d^2 -
a^3*B*d^2 + 3*a*b^2*B*d^2 - 3*a^2*b*C*d^2 + b^3*C*d^2) + Sqrt[-b^2]*(a^3*A*
b*c^2 - 3*a*A*b^3*c^2 + 3*a^2*b^2*B*c^2 - b^4*B*c^2 - a^3*b*c^2*C + 3*a*b^3
*c^2*C - 6*a^2*A*b^2*c*d + 2*A*b^4*c*d + 2*a^3*b*B*c*d - 6*a*b^3*B*c*d + 6*
a^2*b^2*c*C*d - 2*b^4*c*C*d - a^3*A*b*d^2 + 3*a*A*b^3*d^2 - 3*a^2*b^2*B*d^2
 + b^4*B*d^2 + a^3*b*C*d^2 - 3*a*b^3*C*d^2))*Log[Sqrt[-b^2] + b*Tan[e + f*x
]])/(b*(a^2 + b^2)*(c^2 + d^2)) - (2*b*(a^2 + b^2)^2*d^2*(b*(3*c^4*C - 4*B*
c^3*d + c^2*(5*A + C)*d^2 - 2*B*c*d^3 + 3*A*d^4) - a*d^2*(2*c*(A - C)*d - B
*(c^2 - d^2)))*Log[c + d*Tan[e + f*x]])/((b*c - a*d)*(c^2 + d^2)))/(b*(-(b*
c) + a*d)*(c^2 + d^2)*f)) - (d^2*((-(b*c) - a*d)*(-3*a*(A*b^2 - a*(b*B - a*
C))*d + 2*b*(A*b - a*B - b*C)*(b*c - a*d)) + (2*b^2*d - a*(b*c - a*d))*(3*A
*b^2*d - 2*a*A*(b*c - a*d) - (b*B - a*C)*(2*b*c + a*d))) - c*(d*(b*c - a*d)
*(-3*b*(A*b^2 - a*(b*B - a*C))*d - 2*a*(A*b - a*B - b*C)*(b*c - a*d) + b*(3
*A*b^2*d - 2*a*A*(b*c - a*d) - (b*B - a*C)*(2*b*c + a*d))) - 2*c*d*(-(a*(-3
*a*(A*b^2 - a*(b*B - a*C))*d + 2*b*(A*b - a*B - b*C)*(b*c - a*d))) + b^2*(3
*A*b^2*d - 2*a*A*(b*c - a*d) - (b*B - a*C)*(2*b*c + a*d)))))/((-(b*c) + a*d
)*(c^2 + d^2)*f*(c + d*Tan[e + f*x])))/((a^2 + b^2)*(b*c - a*d)))/(2*(a^2 +
 b^2)*(b*c - a*d))
```

Maple [B] time = 0.143, size = 3364, normalized size = 4.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^3/(c+d*tan(f*x+e))^2,x
)
```

```
[Out] -1/f*d^4/(a*d-b*c)^4/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*B*a*c^2+4/f*d^3/(a*d-b*
c)^4/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*B*b*c^3-1/f*b^4/(a^2+b^2)^3/(a*d-b*c)^4
*ln(a+b*tan(f*x+e))*B*a^3*c^2-3/f*b^4/(a^2+b^2)^3/(a*d-b*c)^4*ln(a+b*tan(f*
x+e))*B*a^3*d^2-5/f*d^4/(a*d-b*c)^4/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*A*b*c^2-
6/f*b^2/(a^2+b^2)^3/(a*d-b*c)^4*ln(a+b*tan(f*x+e))*a^5*B*d^2-3/f*b^5/(a^2+b
^2)^3/(a*d-b*c)^4*ln(a+b*tan(f*x+e))*C*a^2*c^2+1/2/f*b^2/(a^2+b^2)/(a*d-b*c
)^2/(a+b*tan(f*x+e))^2*B*a-1/2/f*b/(a^2+b^2)/(a*d-b*c)^2/(a+b*tan(f*x+e))^2
*C*a^2+1/2/f/(a^2+b^2)^3/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*A*b^3*c^2+2/f*d^5/(
a*d-b*c)^4/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*A*a*c-4/f*b^3/(a^2+b^2)^2/(a*d-b*
c)^3/(a+b*tan(f*x+e))*A*a^2*d+3/f*b/(a^2+b^2)^3/(a*d-b*c)^4*ln(a+b*tan(f*x+
e))*a^6*C*d^2-1/f*b^3/(a^2+b^2)^3/(a*d-b*c)^4*ln(a+b*tan(f*x+e))*a^4*C*d^2+
10/f*b^3/(a^2+b^2)^3/(a*d-b*c)^4*ln(a+b*tan(f*x+e))*A*a^4*d^2+3/f*b^5/(a^2+
b^2)^3/(a*d-b*c)^4*ln(a+b*tan(f*x+e))*A*a^2*c^2+9/f*b^5/(a^2+b^2)^3/(a*d-b*
c)^4*ln(a+b*tan(f*x+e))*A*a^2*d^2+3/2/f/(a^2+b^2)^3/(c^2+d^2)^2*ln(1+tan(f*
x+e)^2)*B*a*b^2*d^2+1/f/(a^2+b^2)^3/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*B*b^3*c*
d+1/f/(a^2+b^2)^3/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*C*a^3*c*d+3/2/f/(a^2+b^2)^
3/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*C*a^2*b*c^2-3/2/f/(a^2+b^2)^3/(c^2+d^2)^2*
ln(1+tan(f*x+e)^2)*C*a^2*b*d^2-3/f/(a^2+b^2)^3/(c^2+d^2)^2*A*arctan(tan(f*x
+e))*a*b^2*c^2+3/f/(a^2+b^2)^3/(c^2+d^2)^2*A*arctan(tan(f*x+e))*a*b^2*d^2+2
/f/(a^2+b^2)^3/(c^2+d^2)^2*A*arctan(tan(f*x+e))*b^3*c*d+2/f/(a^2+b^2)^3/(c^
2+d^2)^2*B*arctan(tan(f*x+e))*a^3*c*d+3/f/(a^2+b^2)^3/(c^2+d^2)^2*B*arctan(
tan(f*x+e))*a^2*b*c^2-3/f/(a^2+b^2)^3/(c^2+d^2)^2*B*arctan(tan(f*x+e))*a^2*
b*d^2+3/f/(a^2+b^2)^3/(c^2+d^2)^2*C*arctan(tan(f*x+e))*a*b^2*c^2-3/f/(a^2+b
^2)^3/(c^2+d^2)^2*C*arctan(tan(f*x+e))*a*b^2*d^2-2/f/(a^2+b^2)^3/(c^2+d^2)^
2*C*arctan(tan(f*x+e))*b^3*c*d-2/f*b/(a^2+b^2)^2/(a*d-b*c)^3/(a+b*tan(f*x+e
))*a^4*C*d-2/f*b^4/(a^2+b^2)^2/(a*d-b*c)^3/(a+b*tan(f*x+e))*C*a*c+3/f*b^2/(
a^2+b^2)^2/(a*d-b*c)^3/(a+b*tan(f*x+e))*a^3*B*d-1/f*b^3/(a^2+b^2)^2/(a*d-b*
c)^3/(a+b*tan(f*x+e))*B*a^2*c+1/f*b^4/(a^2+b^2)^2/(a*d-b*c)^3/(a+b*tan(f*x+
e))*B*a*d+2/f*b^4/(a^2+b^2)^2/(a*d-b*c)^3/(a+b*tan(f*x+e))*A*a*c-3/2/f/(a^2
+b^2)^3/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*B*a*b^2*c^2-1/f/(a^2+b^2)^3/(c^2+d^2
)^2*ln(1+tan(f*x+e)^2)*A*a^3*c*d-3/2/f/(a^2+b^2)^3/(c^2+d^2)^2*ln(1+tan(f*x
+e)^2)*A*a^2*b*c^2+3/2/f/(a^2+b^2)^3/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*A*a^2*b
*d^2-1/f*d^4/(a*d-b*c)^4/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*C*b*c^2+3/f*b^6/(a^
2+b^2)^3/(a*d-b*c)^4*ln(a+b*tan(f*x+e))*a*B*c^2-1/2/f/(a^2+b^2)^3/(c^2+d^2)
^2*ln(1+tan(f*x+e)^2)*A*b^3*d^2+1/2/f/(a^2+b^2)^3/(c^2+d^2)^2*ln(1+tan(f*x+
e)^2)*B*a^3*c^2-1/2/f/(a^2+b^2)^3/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*B*a^3*d^2-
1/2/f/(a^2+b^2)^3/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*C*b^3*c^2+1/2/f/(a^2+b^2)^
3/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*C*b^3*d^2+1/f/(a^2+b^2)^3/(c^2+d^2)^2*A*ar
ctan(tan(f*x+e))*a^3*c^2-1/f/(a^2+b^2)^3/(c^2+d^2)^2*A*arctan(tan(f*x+e))*a
^3*d^2-1/f/(a^2+b^2)^3/(c^2+d^2)^2*B*arctan(tan(f*x+e))*b^3*c^2+1/f/(a^2+b^
2)^3/(c^2+d^2)^2*B*arctan(tan(f*x+e))*b^3*d^2-1/f/(a^2+b^2)^3/(c^2+d^2)^2*C
*arctan(tan(f*x+e))*a^3*c^2-1/f*b^6/(a^2+b^2)^3/(a*d-b*c)^4*ln(a+b*tan(f*x+
e))*B*a*d^2+2/f*d^5/(a*d-b*c)^4/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*B*b*c-1/f*d^
2/(a*d-b*c)^3/(c^2+d^2)/(c+d*tan(f*x+e))*c^2*C-2/f*b^5/(a^2+b^2)^2/(a*d-b*c
)^3/(a+b*tan(f*x+e))*A*d+1/f*b^5/(a^2+b^2)^2/(a*d-b*c)^3/(a+b*tan(f*x+e))*B
*c-1/f*b^7/(a^2+b^2)^3/(a*d-b*c)^4*ln(a+b*tan(f*x+e))*A*c^2+3/f*b^7/(a^2+b^
2)^3/(a*d-b*c)^4*ln(a+b*tan(f*x+e))*A*d^2+1/f*b^7/(a^2+b^2)^3/(a*d-b*c)^4*l
n(a+b*tan(f*x+e))*C*c^2-10/f*b^4/(a^2+b^2)^3/(a*d-b*c)^4*ln(a+b*tan(f*x+e))
*A*a^3*c*d-2/f*b^6/(a^2+b^2)^3/(a*d-b*c)^4*ln(a+b*tan(f*x+e))*A*a*c*d+10/f*
b^4/(a^2+b^2)^3/(a*d-b*c)^4*ln(a+b*tan(f*x+e))*C*a^3*c*d+4/f*b^3/(a^2+b^2)^
3/(a*d-b*c)^4*ln(a+b*tan(f*x+e))*B*a^4*c*d-3/f/(a^2+b^2)^3/(c^2+d^2)^2*ln(1
+tan(f*x+e)^2)*B*a^2*b*c*d-6/f*b^5/(a^2+b^2)^3/(a*d-b*c)^4*ln(a+b*tan(f*x+e
))*B*a^2*c*d-6/f/(a^2+b^2)^3/(c^2+d^2)^2*A*arctan(tan(f*x+e))*a^2*b*c*d-6/f
/(a^2+b^2)^3/(c^2+d^2)^2*B*arctan(tan(f*x+e))*a*b^2*c*d-2/f*d^5/(a*d-b*c)^4
/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*C*a*c-3/f*d^2/(a*d-b*c)^4/(c^2+d^2)^2*ln(c+
d*tan(f*x+e))*C*b*c^4+6/f/(a^2+b^2)^3/(c^2+d^2)^2*C*arctan(tan(f*x+e))*a^2*
```

```
b*c*d+2/f*b^6/(a^2+b^2)^3/(a*d-b*c)^4*ln(a+b*tan(f*x+e))*C*a*c*d+3/f/(a^2+b
^2)^3/(c^2+d^2)^2*ln(1+tan(f*x+e)^2)*A*a*b^2*c*d+1/f/(a^2+b^2)^3/(c^2+d^2)^
2*C*arctan(tan(f*x+e))*a^3*d^2-3/f*d^6/(a*d-b*c)^4/(c^2+d^2)^2*ln(c+d*tan(f
*x+e))*A*b+1/f*d^6/(a*d-b*c)^4/(c^2+d^2)^2*ln(c+d*tan(f*x+e))*B*a+1/f*d^3/(
a*d-b*c)^3/(c^2+d^2)/(c+d*tan(f*x+e))*B*c-3/f/(a^2+b^2)^3/(c^2+d^2)^2*ln(1+
tan(f*x+e)^2)*C*a*b^2*c*d-1/2/f*b^3/(a^2+b^2)/(a*d-b*c)^2/(a+b*tan(f*x+e))^
2*A-1/f*d^4/(a*d-b*c)^3/(c^2+d^2)/(c+d*tan(f*x+e))*A-2/f*b^7/(a^2+b^2)^3/(a
*d-b*c)^4*ln(a+b*tan(f*x+e))*B*c*d
```

Maxima [B] time = 2.21069, size = 3401, normalized size = 4.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^3/(c+d*tan(f*x+e
))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(((A - C)*a^3 + 3*B*a^2*b - 3*(A - C)*a*b^2 - B*b^3)*c^2 + 2*(B*a^3
- 3*(A - C)*a^2*b - 3*B*a*b^2 + (A - C)*b^3)*c*d - ((A - C)*a^3 + 3*B*a^2*b
 - 3*(A - C)*a*b^2 - B*b^3)*d^2)*(f*x + e)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
b^6)*c^4 + 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c^2*d^2 + (a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6)*d^4) - 2*((B*a^3*b^4 - 3*(A - C)*a^2*b^5 - 3*B*a*b^6 +
(A - C)*b^7)*c^2 - 2*(2*B*a^4*b^3 - 5*(A - C)*a^3*b^4 - 3*B*a^2*b^5 - (A -
C)*a*b^6 - B*b^7)*c*d - (3*C*a^6*b - 6*B*a^5*b^2 + (10*A - C)*a^4*b^3 - 3*B
*a^3*b^4 + 9*A*a^2*b^5 - B*a*b^6 + 3*A*b^7)*d^2)*log(b*tan(f*x + e) + a)/((
a^6*b^4 + 3*a^4*b^6 + 3*a^2*b^8 + b^10)*c^4 - 4*(a^7*b^3 + 3*a^5*b^5 + 3*a^
3*b^7 + a*b^9)*c^3*d + 6*(a^8*b^2 + 3*a^6*b^4 + 3*a^4*b^6 + a^2*b^8)*c^2*d^
2 - 4*(a^9*b + 3*a^7*b^3 + 3*a^5*b^5 + a^3*b^7)*c*d^3 + (a^10 + 3*a^8*b^2 +
 3*a^6*b^4 + a^4*b^6)*d^4) - 2*(3*C*b*c^4*d^2 - 4*B*b*c^3*d^3 + (B*a + (5*A
 + C)*b)*c^2*d^4 - 2*((A - C)*a + B*b)*c*d^5 - (B*a - 3*A*b)*d^6)*log(d*tan
(f*x + e) + c)/(b^4*c^8 - 4*a*b^3*c^7*d - 4*a^3*b*c*d^7 + a^4*d^8 + 2*(3*a^
2*b^2 + b^4)*c^6*d^2 - 4*(a^3*b + 2*a*b^3)*c^5*d^3 + (a^4 + 12*a^2*b^2 + b^
4)*c^4*d^4 - 4*(2*a^3*b + a*b^3)*c^3*d^5 + 2*(a^4 + 3*a^2*b^2)*c^2*d^6) + (
(B*a^3 - 3*(A - C)*a^2*b - 3*B*a*b^2 + (A - C)*b^3)*c^2 - 2*((A - C)*a^3 +
3*B*a^2*b - 3*(A - C)*a*b^2 - B*b^3)*c*d - (B*a^3 - 3*(A - C)*a^2*b - 3*B*a
*b^2 + (A - C)*b^3)*d^2)*log(tan(f*x + e)^2 + 1)/((a^6 + 3*a^4*b^2 + 3*a^2*
b^4 + b^6)*c^4 + 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c^2*d^2 + (a^6 + 3*a
^4*b^2 + 3*a^2*b^4 + b^6)*d^4) - ((C*a^4*b^2 - 3*B*a^3*b^3 + (5*A - 3*C)*a^
2*b^4 + B*a*b^5 + A*b^6)*c^4 - (5*C*a^5*b - 7*B*a^4*b^2 + (9*A + C)*a^3*b^3
 - 3*B*a^2*b^4 + 5*A*a*b^5)*c^3*d - (2*C*a^6 + 3*C*a^4*b^2 + 3*B*a^3*b^3 -
5*(A - C)*a^2*b^4 - B*a*b^5 - A*b^6)*c^2*d^2 + (2*B*a^6 - 5*C*a^5*b + 11*B*
a^4*b^2 - (9*A + C)*a^3*b^3 + 5*B*a^2*b^4 - 5*A*a*b^5)*c*d^3 - 2*(A*a^6 + 2
*A*a^4*b^2 + A*a^2*b^4)*d^4 - 2*((B*a^2*b^4 - 2*(A - C)*a*b^5 - B*b^6)*c^3*
d + (3*C*a^4*b^2 - 3*B*a^3*b^3 + 2*(2*A + C)*a^2*b^4 - B*a*b^5 + (2*A + C)*
b^6)*c^2*d^2 - (B*a^4*b^2 + B*a^2*b^4 + 2*(A - C)*a*b^5 + 2*B*b^6)*c*d^3 +
((A + 2*C)*a^4*b^2 - 3*B*a^3*b^3 + 6*A*a^2*b^4 - B*a*b^5 + 3*A*b^6)*d^4)*ta
n(f*x + e)^2 - (2*(B*a^2*b^4 - 2*(A - C)*a*b^5 - B*b^6)*c^4 + 3*(C*a^4*b^2
- B*a^3*b^3 + (A + C)*a^2*b^4 - B*a*b^5 + A*b^6)*c^3*d + (9*C*a^5*b - 7*B*a
^4*b^2 + 9*(A + C)*a^3*b^3 - B*a^2*b^4 + (A + 8*C)*a*b^5 - 2*B*b^6)*c^2*d^2
 - (4*B*a^5*b - 3*C*a^4*b^2 + 11*B*a^3*b^3 - 3*(A + C)*a^2*b^4 + 7*B*a*b^5
- 3*A*b^6)*c*d^3 + ((4*A + 5*C)*a^5*b - 7*B*a^4*b^2 + (17*A + C)*a^3*b^3 -
3*B*a^2*b^4 + 9*A*a*b^5)*d^4)*tan(f*x + e))/((a^6*b^3 + 2*a^4*b^5 + a^2*b^7
)*c^6 - 3*(a^7*b^2 + 2*a^5*b^4 + a^3*b^6)*c^5*d + (3*a^8*b + 7*a^6*b^3 + 5*
a^4*b^5 + a^2*b^7)*c^4*d^2 - (a^9 + 5*a^7*b^2 + 7*a^5*b^4 + 3*a^3*b^6)*c^3*
d^3 + 3*(a^8*b + 2*a^6*b^3 + a^4*b^5)*c^2*d^4 - (a^9 + 2*a^7*b^2 + a^5*b^4)
*c*d^5 + ((a^4*b^5 + 2*a^2*b^7 + b^9)*c^5*d - 3*(a^5*b^4 + 2*a^3*b^6 + a*b^
```

```
8)*c^4*d^2 + (3*a^6*b^3 + 7*a^4*b^5 + 5*a^2*b^7 + b^9)*c^3*d^3 - (a^7*b^2 +
 5*a^5*b^4 + 7*a^3*b^6 + 3*a*b^8)*c^2*d^4 + 3*(a^6*b^3 + 2*a^4*b^5 + a^2*b^
7)*c*d^5 - (a^7*b^2 + 2*a^5*b^4 + a^3*b^6)*d^6)*tan(f*x + e)^3 + ((a^4*b^5
+ 2*a^2*b^7 + b^9)*c^6 - (a^5*b^4 + 2*a^3*b^6 + a*b^8)*c^5*d - (3*a^6*b^3 +
 5*a^4*b^5 + a^2*b^7 - b^9)*c^4*d^2 + (5*a^7*b^2 + 9*a^5*b^4 + 3*a^3*b^6 -
a*b^8)*c^3*d^3 - (2*a^8*b + 7*a^6*b^3 + 8*a^4*b^5 + 3*a^2*b^7)*c^2*d^4 + 5*
(a^7*b^2 + 2*a^5*b^4 + a^3*b^6)*c*d^5 - 2*(a^8*b + 2*a^6*b^3 + a^4*b^5)*d^6
)*tan(f*x + e)^2 + (2*(a^5*b^4 + 2*a^3*b^6 + a*b^8)*c^6 - 5*(a^6*b^3 + 2*a^
4*b^5 + a^2*b^7)*c^5*d + (3*a^7*b^2 + 8*a^5*b^4 + 7*a^3*b^6 + 2*a*b^8)*c^4*
d^2 + (a^8*b - 3*a^6*b^3 - 9*a^4*b^5 - 5*a^2*b^7)*c^3*d^3 - (a^9 - a^7*b^2
- 5*a^5*b^4 - 3*a^3*b^6)*c^2*d^4 + (a^8*b + 2*a^6*b^3 + a^4*b^5)*c*d^5 - (a
^9 + 2*a^7*b^2 + a^5*b^4)*d^6)*tan(f*x + e)))/f
```

Fricas [B] time = 102.058, size = 19950, normalized size = 23.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^3/(c+d*tan(f*x+e
))^2,x, algorithm="fricas")
```

```
[Out] -1/2*((3*C*a^4*b^5 - 5*B*a^3*b^6 + (7*A - 3*C)*a^2*b^7 + B*a*b^8 + A*b^9)*c
^7 - 2*(5*C*a^5*b^4 - 7*B*a^4*b^5 + (9*A - C)*a^3*b^6 - B*a^2*b^7 + 3*A*a*b
^8)*c^6*d + (7*C*a^6*b^3 - 9*B*a^5*b^4 + (11*A + 7*C)*a^4*b^5 - 13*B*a^3*b^
6 + (19*A - 6*C)*a^2*b^7 + 2*B*a*b^8 + 2*A*b^9)*c^5*d^2 - 4*(5*C*a^5*b^4 -
7*B*a^4*b^5 + (9*A - C)*a^3*b^6 - B*a^2*b^7 + 3*A*a*b^8)*c^4*d^3 - (2*C*a^8
*b - 8*C*a^6*b^3 + 18*B*a^5*b^4 - (22*A - C)*a^4*b^5 + 11*B*a^3*b^6 - (17*A
 - 5*C)*a^2*b^7 - B*a*b^8 - A*b^9)*c^3*d^4 + 2*(C*a^9 + B*a^8*b + 3*C*a^7*b
^2 + 3*B*a^6*b^3 - 2*C*a^5*b^4 + 10*B*a^4*b^5 - (9*A - 2*C)*a^3*b^6 + 2*B*a
^2*b^7 - 3*A*a*b^8)*c^2*d^5 - (2*B*a^9 + 2*A*a^8*b + 6*B*a^7*b^2 + (6*A - 7
*C)*a^6*b^3 + 15*B*a^5*b^4 - (5*A + C)*a^4*b^5 + 5*B*a^3*b^6 - 3*A*a^2*b^7)
*c*d^6 + 2*(A*a^9 + 3*A*a^7*b^2 + 3*A*a^5*b^4 + A*a^3*b^6)*d^7 - ((C*a^4*b^
5 - 3*B*a^3*b^6 + 5*(A - C)*a^2*b^7 + 3*B*a*b^8 - A*b^9)*c^6*d - 2*(3*C*a^5
*b^4 - 5*B*a^4*b^5 + (7*A - 3*C)*a^3*b^6 + B*a^2*b^7 + A*a*b^8)*c^5*d^2 + (
3*C*a^6*b^3 - 7*B*a^5*b^4 + (9*A - 5*C)*a^4*b^5 - 7*B*a^3*b^6 + (13*A - 16*
C)*a^2*b^7 + 6*B*a*b^8 - 2*(A + C)*b^9)*c^4*d^3 + 2*(C*a^7*b^2 + B*a^6*b^3
- 3*C*a^5*b^4 + 13*B*a^4*b^5 - (14*A - 9*C)*a^3*b^6 + B*a^2*b^7 - (2*A - C)
*a*b^8 + B*b^9)*c^3*d^4 - (2*B*a^7*b^2 + 2*(A - 5*C)*a^6*b^3 + 20*B*a^5*b^4
 - (12*A - C)*a^4*b^5 + 11*B*a^3*b^6 - 5*(A - C)*a^2*b^7 - B*a*b^8 + 3*A*b^
9)*c^2*d^5 + 2*(A*a^7*b^2 + 3*(A - C)*a^5*b^4 + 5*B*a^4*b^5 - (4*A - 3*C)*a
^3*b^6 - B*a^2*b^7)*c*d^6 + (5*C*a^6*b^3 - 7*B*a^5*b^4 + (9*A - C)*a^4*b^5
- B*a^3*b^6 + 3*A*a^2*b^7)*d^7 + 2*(((A - C)*a^3*b^6 + 3*B*a^2*b^7 - 3*(A -
 C)*a*b^8 - B*b^9)*c^6*d - 2*(2*(A - C)*a^4*b^5 + 5*B*a^3*b^6 - 3*(A - C)*a
^2*b^7 + B*a*b^8 - (A - C)*b^9)*c^5*d^2 + (6*(A - C)*a^5*b^4 + 10*B*a^4*b^5
 + 5*(A - C)*a^3*b^6 + 15*B*a^2*b^7 - 5*(A - C)*a*b^8 + B*b^9)*c^4*d^3 - 4*
((A - C)*a^6*b^3 + 5*(A - C)*a^4*b^5 + 5*B*a^3*b^6 + B*a*b^8)*c^3*d^4 + ((A
 - C)*a^7*b^2 - 5*B*a^6*b^3 + 15*(A - C)*a^5*b^4 + 5*B*a^4*b^5 + 10*(A - C)
*a^3*b^6 + 6*B*a^2*b^7)*c^2*d^5 + 2*(B*a^7*b^2 - (A - C)*a^6*b^3 + 3*B*a^5*
b^4 - 5*(A - C)*a^4*b^5 - 2*B*a^3*b^6)*c*d^6 - ((A - C)*a^7*b^2 + 3*B*a^6*b
^3 - 3*(A - C)*a^5*b^4 - B*a^4*b^5)*d^7)*f*x)*tan(f*x + e)^3 - 2*(((A - C)*
a^5*b^4 + 3*B*a^4*b^5 - 3*(A - C)*a^3*b^6 - B*a^2*b^7)*c^7 - 2*(2*(A - C)*a
^6*b^3 + 5*B*a^5*b^4 - 3*(A - C)*a^4*b^5 + B*a^3*b^6 - (A - C)*a^2*b^7)*c^6
*d + (6*(A - C)*a^7*b^2 + 10*B*a^6*b^3 + 5*(A - C)*a^5*b^4 + 15*B*a^4*b^5 -
 5*(A - C)*a^3*b^6 + B*a^2*b^7)*c^5*d^2 - 4*((A - C)*a^8*b + 5*(A - C)*a^6*
b^3 + 5*B*a^5*b^4 + B*a^3*b^6)*c^4*d^3 + ((A - C)*a^9 - 5*B*a^8*b + 15*(A -
 C)*a^7*b^2 + 5*B*a^6*b^3 + 10*(A - C)*a^5*b^4 + 6*B*a^4*b^5)*c^3*d^4 + 2*(
```

B*a^9 - (A - C)*a^8*b + 3*B*a^7*b^2 - 5*(A - C)*a^6*b^3 - 2*B*a^5*b^4)*c^2*
d^5 - ((A - C)*a^9 + 3*B*a^8*b - 3*(A - C)*a^7*b^2 - B*a^6*b^3)*c*d^6)*f*x
- ((C*a^4*b^5 - 3*B*a^3*b^6 + 5*(A - C)*a^2*b^7 + 3*B*a*b^8 - A*b^9)*c^7 -
2*(2*C*a^5*b^4 - 3*B*a^4*b^5 + 4*A*a^3*b^6 - 2*B*a^2*b^7 + 2*(2*A - C)*a*b^
8 + B*b^9)*c^6*d - (3*C*a^6*b^3 - 5*B*a^5*b^4 + (7*A - 13*C)*a^4*b^5 + 19*B
*a^3*b^6 - (25*A - 14*C)*a^2*b^7 - 6*B*a*b^8 - 2*A*b^9)*c^5*d^2 + 2*(C*a^7*
b^2 - 4*B*a^6*b^3 + (5*A - 13*C)*a^5*b^4 + 9*B*a^4*b^5 - (11*A + 6*C)*a^3*b
^6 + 5*B*a^2*b^7 - 2*(5*A - C)*a*b^8 - 2*B*b^9)*c^4*d^3 + (4*C*a^8*b + 4*B*
a^7*b^2 + 8*C*a^6*b^3 + 22*B*a^5*b^4 - (14*A - 41*C)*a^4*b^5 - 17*B*a^3*b^6
 + (35*A - 3*C)*a^2*b^7 + 7*B*a*b^8 + (7*A + 2*C)*b^9)*c^3*d^4 - 2*(2*B*a^8
*b + (2*A - 5*C)*a^7*b^2 + 15*B*a^6*b^3 - (4*A - 11*C)*a^5*b^4 + (16*A + 3*
C)*a^3*b^6 + B*a^2*b^7 + (10*A - C)*a*b^8 + 2*B*b^9)*c^2*d^5 + (4*A*a^8*b +
 2*B*a^7*b^2 + (14*A - 3*C)*a^6*b^3 + 11*B*a^5*b^4 + 11*(A + C)*a^4*b^5 - 7
*B*a^3*b^6 + (25*A - 4*C)*a^2*b^7 + 2*B*a*b^8 + 6*A*b^9)*c*d^6 - 2*((A - 3*
C)*a^7*b^2 + 4*B*a^6*b^3 - (2*A - 3*C)*a^5*b^4 - 3*B*a^4*b^5 + 6*A*a^3*b^6
- B*a^2*b^7 + 3*A*a*b^8)*d^7 + 2*(((A - C)*a^3*b^6 + 3*B*a^2*b^7 - 3*(A - C
)*a*b^8 - B*b^9)*c^7 - 2*((A - C)*a^4*b^5 + 2*B*a^3*b^6 + 2*B*a*b^8 - (A -
C)*b^9)*c^6*d - (2*(A - C)*a^5*b^4 + 10*B*a^4*b^5 - 17*(A - C)*a^3*b^6 - 11
*B*a^2*b^7 + (A - C)*a*b^8 - B*b^9)*c^5*d^2 + 2*(4*(A - C)*a^6*b^3 + 10*B*a
^5*b^4 - 5*(A - C)*a^4*b^5 + 5*B*a^3*b^6 - 5*(A - C)*a^2*b^7 - B*a*b^8)*c^4
d^3 - (7(A - C)*a^7*b^2 + 5*B*a^6*b^3 + 25*(A - C)*a^5*b^4 + 35*B*a^4*b^5
 - 10*(A - C)*a^3*b^6 + 2*B*a^2*b^7)*c^3*d^4 + 2*((A - C)*a^8*b - 4*B*a^7*b
^2 + 14*(A - C)*a^6*b^3 + 8*B*a^5*b^4 + 5*(A - C)*a^4*b^5 + 4*B*a^3*b^6)*c^
2*d^5 + (4*B*a^8*b - 5*(A - C)*a^7*b^2 + 9*B*a^6*b^3 - 17*(A - C)*a^5*b^4 -
 7*B*a^4*b^5)*c*d^6 - 2*((A - C)*a^8*b + 3*B*a^7*b^2 - 3*(A - C)*a^6*b^3 -
B*a^5*b^4)*d^7)*f*x)*tan(f*x + e)^2 + ((B*a^5*b^4 - 3*(A - C)*a^4*b^5 - 3*B
*a^3*b^6 + (A - C)*a^2*b^7)*c^7 - 2*(2*B*a^6*b^3 - 5*(A - C)*a^5*b^4 - 3*B*
a^4*b^5 - (A - C)*a^3*b^6 - B*a^2*b^7)*c^6*d - (3*C*a^8*b - 6*B*a^7*b^2 + (
10*A - C)*a^6*b^3 - 5*B*a^5*b^4 + 3*(5*A - 2*C)*a^4*b^5 + 5*B*a^3*b^6 + (A
+ 2*C)*a^2*b^7)*c^5*d^2 - 4*(2*B*a^6*b^3 - 5*(A - C)*a^5*b^4 - 3*B*a^4*b^5
- (A - C)*a^3*b^6 - B*a^2*b^7)*c^4*d^3 - (6*C*a^8*b - 12*B*a^7*b^2 + 2*(10*
A - C)*a^6*b^3 - 7*B*a^5*b^4 + 3*(7*A - C)*a^4*b^5 + B*a^3*b^6 + (5*A + C)*
a^2*b^7)*c^3*d^4 - 2*(2*B*a^6*b^3 - 5*(A - C)*a^5*b^4 - 3*B*a^4*b^5 - (A -
C)*a^3*b^6 - B*a^2*b^7)*c^2*d^5 - (3*C*a^8*b - 6*B*a^7*b^2 + (10*A - C)*a^6
*b^3 - 3*B*a^5*b^4 + 9*A*a^4*b^5 - B*a^3*b^6 + 3*A*a^2*b^7)*c*d^6 + ((B*a^3
b^6 - 3(A - C)*a^2*b^7 - 3*B*a*b^8 + (A - C)*b^9)*c^6*d - 2*(2*B*a^4*b^5
- 5*(A - C)*a^3*b^6 - 3*B*a^2*b^7 - (A - C)*a*b^8 - B*b^9)*c^5*d^2 - (3*C*a
^6*b^3 - 6*B*a^5*b^4 + (10*A - C)*a^4*b^5 - 5*B*a^3*b^6 + 3*(5*A - 2*C)*a^2
*b^7 + 5*B*a*b^8 + (A + 2*C)*b^9)*c^4*d^3 - 4*(2*B*a^4*b^5 - 5*(A - C)*a^3*
b^6 - 3*B*a^2*b^7 - (A - C)*a*b^8 - B*b^9)*c^3*d^4 - (6*C*a^6*b^3 - 12*B*a^
5*b^4 + 2*(10*A - C)*a^4*b^5 - 7*B*a^3*b^6 + 3*(7*A - C)*a^2*b^7 + B*a*b^8
+ (5*A + C)*b^9)*c^2*d^5 - 2*(2*B*a^4*b^5 - 5*(A - C)*a^3*b^6 - 3*B*a^2*b^7
 - (A - C)*a*b^8 - B*b^9)*c*d^6 - (3*C*a^6*b^3 - 6*B*a^5*b^4 + (10*A - C)*a
^4*b^5 - 3*B*a^3*b^6 + 9*A*a^2*b^7 - B*a*b^8 + 3*A*b^9)*d^7)*tan(f*x + e)^3
 + ((B*a^3*b^6 - 3*(A - C)*a^2*b^7 - 3*B*a*b^8 + (A - C)*b^9)*c^7 - 2*(B*a^
4*b^5 - 2*(A - C)*a^3*b^6 - 2*(A - C)*a*b^8 - B*b^9)*c^6*d - (3*C*a^6*b^3 +
 2*B*a^5*b^4 - (10*A - 19*C)*a^4*b^5 - 17*B*a^3*b^6 + (11*A - 2*C)*a^2*b^7
+ B*a*b^8 + (A + 2*C)*b^9)*c^5*d^2 - 2*(3*C*a^7*b^2 - 6*B*a^6*b^3 + (10*A -
 C)*a^5*b^4 - B*a^4*b^5 + (5*A + 4*C)*a^3*b^6 - B*a^2*b^7 - (A - 4*C)*a*b^8
 - 2*B*b^9)*c^4*d^3 - (6*C*a^6*b^3 + 4*B*a^5*b^4 - 2*(10*A - 19*C)*a^4*b^5
- 31*B*a^3*b^6 + (13*A + 5*C)*a^2*b^7 - 7*B*a*b^8 + (5*A + C)*b^9)*c^3*d^4
- 2*(6*C*a^7*b^2 - 12*B*a^6*b^3 + 2*(10*A - C)*a^5*b^4 - 5*B*a^4*b^5 + 2*(8
*A + C)*a^3*b^6 - 2*B*a^2*b^7 + 2*(2*A + C)*a*b^8 - B*b^9)*c^2*d^5 - (3*C*a
^6*b^3 + 2*B*a^5*b^4 - (10*A - 19*C)*a^4*b^5 - 15*B*a^3*b^6 + (5*A + 4*C)*a
^2*b^7 - 5*B*a*b^8 + 3*A*b^9)*c*d^6 - 2*(3*C*a^7*b^2 - 6*B*a^6*b^3 + (10*A
- C)*a^5*b^4 - 3*B*a^4*b^5 + 9*A*a^3*b^6 - B*a^2*b^7 + 3*A*a*b^8)*d^7)*tan(
f*x + e)^2 + (2*(B*a^4*b^5 - 3*(A - C)*a^3*b^6 - 3*B*a^2*b^7 + (A - C)*a*b^
8)*c^7 - (7*B*a^5*b^4 - 17*(A - C)*a^4*b^5 - 9*B*a^3*b^6 - 5*(A - C)*a^2*b^
7 - 4*B*a*b^8)*c^6*d - 2*(3*C*a^7*b^2 - 4*B*a^6*b^3 + (5*A + 4*C)*a^5*b^4 -

```
8*B*a^4*b^5 + (14*A - 5*C)*a^3*b^6 + 4*B*a^2*b^7 + (A + 2*C)*a*b^8)*c^5*d^
2 - (3*C*a^8*b - 6*B*a^7*b^2 + (10*A - C)*a^6*b^3 + 11*B*a^5*b^4 - (25*A -
34*C)*a^4*b^5 - 19*B*a^3*b^6 - (7*A - 10*C)*a^2*b^7 - 8*B*a*b^8)*c^4*d^3 -
2*(6*C*a^7*b^2 - 8*B*a^6*b^3 + 2*(5*A + 4*C)*a^5*b^4 - 13*B*a^4*b^5 + (19*A
 - C)*a^3*b^6 - B*a^2*b^7 + (5*A + C)*a*b^8)*c^3*d^4 - (6*C*a^8*b - 12*B*a^
7*b^2 + 2*(10*A - C)*a^6*b^3 + B*a^5*b^4 + (A + 17*C)*a^4*b^5 - 11*B*a^3*b^
6 + (A + 5*C)*a^2*b^7 - 4*B*a*b^8)*c^2*d^5 - 2*(3*C*a^7*b^2 - 4*B*a^6*b^3 +
 (5*A + 4*C)*a^5*b^4 - 6*B*a^4*b^5 + (8*A + C)*a^3*b^6 - 2*B*a^2*b^7 + 3*A*
a*b^8)*c*d^6 - (3*C*a^8*b - 6*B*a^7*b^2 + (10*A - C)*a^6*b^3 - 3*B*a^5*b^4
+ 9*A*a^4*b^5 - B*a^3*b^6 + 3*A*a^2*b^7)*d^7)*tan(f*x + e))*log((b^2*tan(f*
x + e)^2 + 2*a*b*tan(f*x + e) + a^2)/(tan(f*x + e)^2 + 1)) + (3*(C*a^8*b +
3*C*a^6*b^3 + 3*C*a^4*b^5 + C*a^2*b^7)*c^5*d^2 - 4*(B*a^8*b + 3*B*a^6*b^3 +
 3*B*a^4*b^5 + B*a^2*b^7)*c^4*d^3 + (B*a^9 + (5*A + C)*a^8*b + 3*B*a^7*b^2
+ 3*(5*A + C)*a^6*b^3 + 3*B*a^5*b^4 + 3*(5*A + C)*a^4*b^5 + B*a^3*b^6 + (5*
A + C)*a^2*b^7)*c^3*d^4 - 2*((A - C)*a^9 + B*a^8*b + 3*(A - C)*a^7*b^2 + 3*
B*a^6*b^3 + 3*(A - C)*a^5*b^4 + 3*B*a^4*b^5 + (A - C)*a^3*b^6 + B*a^2*b^7)*
c^2*d^5 - (B*a^9 - 3*A*a^8*b + 3*B*a^7*b^2 - 9*A*a^6*b^3 + 3*B*a^5*b^4 - 9*
A*a^4*b^5 + B*a^3*b^6 - 3*A*a^2*b^7)*c*d^6 + (3*(C*a^6*b^3 + 3*C*a^4*b^5 +
3*C*a^2*b^7 + C*b^9)*c^4*d^3 - 4*(B*a^6*b^3 + 3*B*a^4*b^5 + 3*B*a^2*b^7 + B
*b^9)*c^3*d^4 + (B*a^7*b^2 + (5*A + C)*a^6*b^3 + 3*B*a^5*b^4 + 3*(5*A + C)*
a^4*b^5 + 3*B*a^3*b^6 + 3*(5*A + C)*a^2*b^7 + B*a*b^8 + (5*A + C)*b^9)*c^2*
d^5 - 2*((A - C)*a^7*b^2 + B*a^6*b^3 + 3*(A - C)*a^5*b^4 + 3*B*a^4*b^5 + 3*
(A - C)*a^3*b^6 + 3*B*a^2*b^7 + (A - C)*a*b^8 + B*b^9)*c*d^6 - (B*a^7*b^2 -
 3*A*a^6*b^3 + 3*B*a^5*b^4 - 9*A*a^4*b^5 + 3*B*a^3*b^6 - 9*A*a^2*b^7 + B*a*
b^8 - 3*A*b^9)*d^7)*tan(f*x + e)^3 + (3*(C*a^6*b^3 + 3*C*a^4*b^5 + 3*C*a^2*
b^7 + C*b^9)*c^5*d^2 + 2*(3*C*a^7*b^2 - 2*B*a^6*b^3 + 9*C*a^5*b^4 - 6*B*a^4
*b^5 + 9*C*a^3*b^6 - 6*B*a^2*b^7 + 3*C*a*b^8 - 2*B*b^9)*c^4*d^3 - (7*B*a^7*
b^2 - (5*A + C)*a^6*b^3 + 21*B*a^5*b^4 - 3*(5*A + C)*a^4*b^5 + 21*B*a^3*b^6
 - 3*(5*A + C)*a^2*b^7 + 7*B*a*b^8 - (5*A + C)*b^9)*c^3*d^4 + 2*(B*a^8*b +
2*(2*A + C)*a^7*b^2 + 2*B*a^6*b^3 + 6*(2*A + C)*a^5*b^4 + 6*(2*A + C)*a^3*b
^6 - 2*B*a^2*b^7 + 2*(2*A + C)*a*b^8 - B*b^9)*c^2*d^5 - (4*(A - C)*a^8*b +
5*B*a^7*b^2 + 3*(3*A - 4*C)*a^6*b^3 + 15*B*a^5*b^4 + 3*(A - 4*C)*a^4*b^5 +
15*B*a^3*b^6 - (5*A + 4*C)*a^2*b^7 + 5*B*a*b^8 - 3*A*b^9)*c*d^6 - 2*(B*a^8*
b - 3*A*a^7*b^2 + 3*B*a^6*b^3 - 9*A*a^5*b^4 + 3*B*a^4*b^5 - 9*A*a^3*b^6 + B
*a^2*b^7 - 3*A*a*b^8)*d^7)*tan(f*x + e)^2 + (6*(C*a^7*b^2 + 3*C*a^5*b^4 + 3
*C*a^3*b^6 + C*a*b^8)*c^5*d^2 + (3*C*a^8*b - 8*B*a^7*b^2 + 9*C*a^6*b^3 - 24
*B*a^5*b^4 + 9*C*a^4*b^5 - 24*B*a^3*b^6 + 3*C*a^2*b^7 - 8*B*a*b^8)*c^4*d^3
- 2*(B*a^8*b - (5*A + C)*a^7*b^2 + 3*B*a^6*b^3 - 3*(5*A + C)*a^5*b^4 + 3*B*
a^4*b^5 - 3*(5*A + C)*a^3*b^6 + B*a^2*b^7 - (5*A + C)*a*b^8)*c^3*d^4 + (B*a
^9 + (A + 5*C)*a^8*b - B*a^7*b^2 + 3*(A + 5*C)*a^6*b^3 - 9*B*a^5*b^4 + 3*(A
 + 5*C)*a^4*b^5 - 11*B*a^3*b^6 + (A + 5*C)*a^2*b^7 - 4*B*a*b^8)*c^2*d^5 - 2
*((A - C)*a^9 + 2*B*a^8*b - 3*C*a^7*b^2 + 6*B*a^6*b^3 - 3*(2*A + C)*a^5*b^4
 + 6*B*a^4*b^5 - (8*A + C)*a^3*b^6 + 2*B*a^2*b^7 - 3*A*a*b^8)*c*d^6 - (B*a^
9 - 3*A*a^8*b + 3*B*a^7*b^2 - 9*A*a^6*b^3 + 3*B*a^5*b^4 - 9*A*a^4*b^5 + B*a
^3*b^6 - 3*A*a^2*b^7)*d^7)*tan(f*x + e))*log((d^2*tan(f*x + e)^2 + 2*c*d*ta
n(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) - (2*(C*a^5*b^4 - 2*B*a^4*b^5 + 3*(
A - C)*a^3*b^6 + 3*B*a^2*b^7 - (3*A - 2*C)*a*b^8 - B*b^9)*c^7 - (8*C*a^6*b^
3 - 12*B*a^5*b^4 + (16*A - 9*C)*a^4*b^5 + 7*B*a^3*b^6 - (5*A - C)*a^2*b^7 +
 B*a*b^8 - 3*A*b^9)*c^6*d + 2*(3*C*a^7*b^2 - 4*B*a^6*b^3 + (5*A + 4*C)*a^5*
b^4 - 8*B*a^4*b^5 + (12*A - 7*C)*a^3*b^6 + 6*B*a^2*b^7 - (5*A - 4*C)*a*b^8
- 2*B*b^9)*c^5*d^2 - (2*C*a^8*b + 29*C*a^6*b^3 - 33*B*a^5*b^4 + (43*A - 11*
C)*a^4*b^5 + 11*B*a^3*b^6 - (5*A - 4*C)*a^2*b^7 + 2*B*a*b^8 - 6*A*b^9)*c^4*
d^3 + 2*(C*a^9 + B*a^8*b + 11*C*a^7*b^2 - 5*B*a^6*b^3 + 2*(5*A + 7*C)*a^5*b
^4 - 7*B*a^4*b^5 + (15*A + 2*C)*a^3*b^6 + 4*B*a^2*b^7 - (A - 4*C)*a*b^8 - B
*b^9)*c^3*d^4 - (2*B*a^9 + 2*(A + 2*C)*a^8*b + 10*B*a^7*b^2 + 2*(3*A + 17*C
)*a^6*b^3 - 12*B*a^5*b^4 + (44*A + 5*C)*a^4*b^5 + 15*B*a^3*b^6 + (7*A + 5*C
)*a^2*b^7 + 5*B*a*b^8 - 3*A*b^9)*c^2*d^5 + 2*(A*a^9 + 2*B*a^8*b + (5*A + 3*
C)*a^7*b^2 + 2*B*a^6*b^3 + 2*(7*A + C)*a^5*b^4 + 2*B*a^4*b^5 + (13*A - C)*a
^3*b^6 + 2*B*a^2*b^7 + 3*A*a*b^8)*c*d^6 - (4*A*a^8*b + (12*A + 7*C)*a^6*b^3
```

```
 - 9*B*a^5*b^4 + (23*A + C)*a^4*b^5 - 3*B*a^3*b^6 + 9*A*a^2*b^7)*d^7 + 2*(2
*((A - C)*a^4*b^5 + 3*B*a^3*b^6 - 3*(A - C)*a^2*b^7 - B*a*b^8)*c^7 - (7*(A
- C)*a^5*b^4 + 17*B*a^4*b^5 - 9*(A - C)*a^3*b^6 + 5*B*a^2*b^7 - 4*(A - C)*a
*b^8)*c^6*d + 2*(4*(A - C)*a^6*b^3 + 5*B*a^5*b^4 + 8*(A - C)*a^4*b^5 + 14*B
*a^3*b^6 - 4*(A - C)*a^2*b^7 + B*a*b^8)*c^5*d^2 - (2*(A - C)*a^7*b^2 - 10*B
*a^6*b^3 + 35*(A - C)*a^5*b^4 + 25*B*a^4*b^5 + 5*(A - C)*a^3*b^6 + 7*B*a^2*
b^7)*c^4*d^3 - 2*((A - C)*a^8*b + 5*B*a^7*b^2 - 5*(A - C)*a^6*b^3 + 5*B*a^5
*b^4 - 10*(A - C)*a^4*b^5 - 4*B*a^3*b^6)*c^3*d^4 + ((A - C)*a^9 - B*a^8*b +
 11*(A - C)*a^7*b^2 + 17*B*a^6*b^3 - 10*(A - C)*a^5*b^4 - 2*B*a^4*b^5)*c^2*
d^5 + 2*(B*a^9 - 2*(A - C)*a^8*b - 2*(A - C)*a^6*b^3 - B*a^5*b^4)*c*d^6 - (
(A - C)*a^9 + 3*B*a^8*b - 3*(A - C)*a^7*b^2 - B*a^6*b^3)*d^7)*f*x)*tan(f*x
+ e))/(((a^6*b^6 + 3*a^4*b^8 + 3*a^2*b^10 + b^12)*c^8*d - 4*(a^7*b^5 + 3*a^
5*b^7 + 3*a^3*b^9 + a*b^11)*c^7*d^2 + 2*(3*a^8*b^4 + 10*a^6*b^6 + 12*a^4*b^
8 + 6*a^2*b^10 + b^12)*c^6*d^3 - 4*(a^9*b^3 + 5*a^7*b^5 + 9*a^5*b^7 + 7*a^3
*b^9 + 2*a*b^11)*c^5*d^4 + (a^10*b^2 + 15*a^8*b^4 + 40*a^6*b^6 + 40*a^4*b^8
 + 15*a^2*b^10 + b^12)*c^4*d^5 - 4*(2*a^9*b^3 + 7*a^7*b^5 + 9*a^5*b^7 + 5*a
^3*b^9 + a*b^11)*c^3*d^6 + 2*(a^10*b^2 + 6*a^8*b^4 + 12*a^6*b^6 + 10*a^4*b^
8 + 3*a^2*b^10)*c^2*d^7 - 4*(a^9*b^3 + 3*a^7*b^5 + 3*a^5*b^7 + a^3*b^9)*c*d
^8 + (a^10*b^2 + 3*a^8*b^4 + 3*a^6*b^6 + a^4*b^8)*d^9)*f*tan(f*x + e)^3 + (
(a^6*b^6 + 3*a^4*b^8 + 3*a^2*b^10 + b^12)*c^9 - 2*(a^7*b^5 + 3*a^5*b^7 + 3*
a^3*b^9 + a*b^11)*c^8*d - 2*(a^8*b^4 + 2*a^6*b^6 - 2*a^2*b^10 - b^12)*c^7*d
^2 + 4*(2*a^9*b^3 + 5*a^7*b^5 + 3*a^5*b^7 - a^3*b^9 - a*b^11)*c^6*d^3 - (7*
a^10*b^2 + 25*a^8*b^4 + 32*a^6*b^6 + 16*a^4*b^8 + a^2*b^10 - b^12)*c^5*d^4
+ 2*(a^11*b + 11*a^9*b^3 + 26*a^7*b^5 + 22*a^5*b^7 + 5*a^3*b^9 - a*b^11)*c^
4*d^5 - 2*(7*a^10*b^2 + 22*a^8*b^4 + 24*a^6*b^6 + 10*a^4*b^8 + a^2*b^10)*c^
3*d^6 + 4*(a^11*b + 5*a^9*b^3 + 9*a^7*b^5 + 7*a^5*b^7 + 2*a^3*b^9)*c^2*d^7
- 7*(a^10*b^2 + 3*a^8*b^4 + 3*a^6*b^6 + a^4*b^8)*c*d^8 + 2*(a^11*b + 3*a^9*
b^3 + 3*a^7*b^5 + a^5*b^7)*d^9)*f*tan(f*x + e)^2 + (2*(a^7*b^5 + 3*a^5*b^7
+ 3*a^3*b^9 + a*b^11)*c^9 - 7*(a^8*b^4 + 3*a^6*b^6 + 3*a^4*b^8 + a^2*b^10)*
c^8*d + 4*(2*a^9*b^3 + 7*a^7*b^5 + 9*a^5*b^7 + 5*a^3*b^9 + a*b^11)*c^7*d^2
- 2*(a^10*b^2 + 10*a^8*b^4 + 24*a^6*b^6 + 22*a^4*b^8 + 7*a^2*b^10)*c^6*d^3
- 2*(a^11*b - 5*a^9*b^3 - 22*a^7*b^5 - 26*a^5*b^7 - 11*a^3*b^9 - a*b^11)*c^
5*d^4 + (a^12 - a^10*b^2 - 16*a^8*b^4 - 32*a^6*b^6 - 25*a^4*b^8 - 7*a^2*b^1
0)*c^4*d^5 - 4*(a^11*b + a^9*b^3 - 3*a^7*b^5 - 5*a^5*b^7 - 2*a^3*b^9)*c^3*d
^6 + 2*(a^12 + 2*a^10*b^2 - 2*a^6*b^6 - a^4*b^8)*c^2*d^7 - 2*(a^11*b + 3*a^
9*b^3 + 3*a^7*b^5 + a^5*b^7)*c*d^8 + (a^12 + 3*a^10*b^2 + 3*a^8*b^4 + a^6*b
^6)*d^9)*f*tan(f*x + e) + ((a^8*b^4 + 3*a^6*b^6 + 3*a^4*b^8 + a^2*b^10)*c^9
 - 4*(a^9*b^3 + 3*a^7*b^5 + 3*a^5*b^7 + a^3*b^9)*c^8*d + 2*(3*a^10*b^2 + 10
*a^8*b^4 + 12*a^6*b^6 + 6*a^4*b^8 + a^2*b^10)*c^7*d^2 - 4*(a^11*b + 5*a^9*b
^3 + 9*a^7*b^5 + 7*a^5*b^7 + 2*a^3*b^9)*c^6*d^3 + (a^12 + 15*a^10*b^2 + 40*
a^8*b^4 + 40*a^6*b^6 + 15*a^4*b^8 + a^2*b^10)*c^5*d^4 - 4*(2*a^11*b + 7*a^9
*b^3 + 9*a^7*b^5 + 5*a^5*b^7 + a^3*b^9)*c^4*d^5 + 2*(a^12 + 6*a^10*b^2 + 12
*a^8*b^4 + 10*a^6*b^6 + 3*a^4*b^8)*c^3*d^6 - 4*(a^11*b + 3*a^9*b^3 + 3*a^7*
b^5 + a^5*b^7)*c^2*d^7 + (a^12 + 3*a^10*b^2 + 3*a^8*b^4 + a^6*b^6)*c*d^8)*f
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**3/(c+d*tan(f*x
+e))**2,x)

[Out] Timed out
```

Giac [B] time = 3.14004, size = 4288, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^3/(c+d*tan(f*x+e
))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a^3*c^2 - C*a^3*c^2 + 3*B*a^2*b*c^2 - 3*A*a*b^2*c^2 + 3*C*a*b^2*c
^2 - B*b^3*c^2 + 2*B*a^3*c*d - 6*A*a^2*b*c*d + 6*C*a^2*b*c*d - 6*B*a*b^2*c*
d + 2*A*b^3*c*d - 2*C*b^3*c*d - A*a^3*d^2 + C*a^3*d^2 - 3*B*a^2*b*d^2 + 3*A
*a*b^2*d^2 - 3*C*a*b^2*d^2 + B*b^3*d^2)*(f*x + e)/(a^6*c^4 + 3*a^4*b^2*c^4
+ 3*a^2*b^4*c^4 + b^6*c^4 + 2*a^6*c^2*d^2 + 6*a^4*b^2*c^2*d^2 + 6*a^2*b^4*c
^2*d^2 + 2*b^6*c^2*d^2 + a^6*d^4 + 3*a^4*b^2*d^4 + 3*a^2*b^4*d^4 + b^6*d^4)
 + (B*a^3*c^2 - 3*A*a^2*b*c^2 + 3*C*a^2*b*c^2 - 3*B*a*b^2*c^2 + A*b^3*c^2 -
 C*b^3*c^2 - 2*A*a^3*c*d + 2*C*a^3*c*d - 6*B*a^2*b*c*d + 6*A*a*b^2*c*d - 6*
C*a*b^2*c*d + 2*B*b^3*c*d - B*a^3*d^2 + 3*A*a^2*b*d^2 - 3*C*a^2*b*d^2 + 3*B
*a*b^2*d^2 - A*b^3*d^2 + C*b^3*d^2)*log(tan(f*x + e)^2 + 1)/(a^6*c^4 + 3*a^
4*b^2*c^4 + 3*a^2*b^4*c^4 + b^6*c^4 + 2*a^6*c^2*d^2 + 6*a^4*b^2*c^2*d^2 + 6
*a^2*b^4*c^2*d^2 + 2*b^6*c^2*d^2 + a^6*d^4 + 3*a^4*b^2*d^4 + 3*a^2*b^4*d^4
+ b^6*d^4) - 2*(B*a^3*b^5*c^2 - 3*A*a^2*b^6*c^2 + 3*C*a^2*b^6*c^2 - 3*B*a*b
^7*c^2 + A*b^8*c^2 - C*b^8*c^2 - 4*B*a^4*b^4*c*d + 10*A*a^3*b^5*c*d - 10*C*
a^3*b^5*c*d + 6*B*a^2*b^6*c*d + 2*A*a*b^7*c*d - 2*C*a*b^7*c*d + 2*B*b^8*c*d
 - 3*C*a^6*b^2*d^2 + 6*B*a^5*b^3*d^2 - 10*A*a^4*b^4*d^2 + C*a^4*b^4*d^2 + 3
*B*a^3*b^5*d^2 - 9*A*a^2*b^6*d^2 + B*a*b^7*d^2 - 3*A*b^8*d^2)*log(abs(b*tan
(f*x + e) + a))/(a^6*b^5*c^4 + 3*a^4*b^7*c^4 + 3*a^2*b^9*c^4 + b^11*c^4 - 4
*a^7*b^4*c^3*d - 12*a^5*b^6*c^3*d - 12*a^3*b^8*c^3*d - 4*a*b^10*c^3*d + 6*a
^8*b^3*c^2*d^2 + 18*a^6*b^5*c^2*d^2 + 18*a^4*b^7*c^2*d^2 + 6*a^2*b^9*c^2*d^
2 - 4*a^9*b^2*c*d^3 - 12*a^7*b^4*c*d^3 - 12*a^5*b^6*c*d^3 - 4*a^3*b^8*c*d^3
 + a^10*b*d^4 + 3*a^8*b^3*d^4 + 3*a^6*b^5*d^4 + a^4*b^7*d^4) - 2*(3*C*b*c^4
*d^3 - 4*B*b*c^3*d^4 + B*a*c^2*d^5 + 5*A*b*c^2*d^5 + C*b*c^2*d^5 - 2*A*a*c*
d^6 + 2*C*a*c*d^6 - 2*B*b*c*d^6 - B*a*d^7 + 3*A*b*d^7)*log(abs(d*tan(f*x +
e) + c))/(b^4*c^8*d - 4*a*b^3*c^7*d^2 + 6*a^2*b^2*c^6*d^3 + 2*b^4*c^6*d^3 -
 4*a^3*b*c^5*d^4 - 8*a*b^3*c^5*d^4 + a^4*c^4*d^5 + 12*a^2*b^2*c^4*d^5 + b^4
*c^4*d^5 - 8*a^3*b*c^3*d^6 - 4*a*b^3*c^3*d^6 + 2*a^4*c^2*d^7 + 6*a^2*b^2*c^
2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9) + 2*(3*C*b*c^4*d^3*tan(f*x + e) - 4*B*b*c^
3*d^4*tan(f*x + e) + B*a*c^2*d^5*tan(f*x + e) + 5*A*b*c^2*d^5*tan(f*x + e)
+ C*b*c^2*d^5*tan(f*x + e) - 2*A*a*c*d^6*tan(f*x + e) + 2*C*a*c*d^6*tan(f*x
 + e) - 2*B*b*c*d^6*tan(f*x + e) - B*a*d^7*tan(f*x + e) + 3*A*b*d^7*tan(f*x
 + e) + 4*C*b*c^5*d^2 - C*a*c^4*d^3 - 5*B*b*c^4*d^3 + 2*B*a*c^3*d^4 + 6*A*b
*c^3*d^4 + 2*C*b*c^3*d^4 - 3*A*a*c^2*d^5 + C*a*c^2*d^5 - 3*B*b*c^2*d^5 + 4*
A*b*c*d^6 - A*a*d^7)/((b^4*c^8 - 4*a*b^3*c^7*d + 6*a^2*b^2*c^6*d^2 + 2*b^4*
c^6*d^2 - 4*a^3*b*c^5*d^3 - 8*a*b^3*c^5*d^3 + a^4*c^4*d^4 + 12*a^2*b^2*c^4*
d^4 + b^4*c^4*d^4 - 8*a^3*b*c^3*d^5 - 4*a*b^3*c^3*d^5 + 2*a^4*c^2*d^6 + 6*a
^2*b^2*c^2*d^6 - 4*a^3*b*c*d^7 + a^4*d^8)*(d*tan(f*x + e) + c)) + (3*B*a^3*
b^6*c^2*tan(f*x + e)^2 - 9*A*a^2*b^7*c^2*tan(f*x + e)^2 + 9*C*a^2*b^7*c^2*t
an(f*x + e)^2 - 9*B*a*b^8*c^2*tan(f*x + e)^2 + 3*A*b^9*c^2*tan(f*x + e)^2 -
 3*C*b^9*c^2*tan(f*x + e)^2 - 12*B*a^4*b^5*c*d*tan(f*x + e)^2 + 30*A*a^3*b^
6*c*d*tan(f*x + e)^2 - 30*C*a^3*b^6*c*d*tan(f*x + e)^2 + 18*B*a^2*b^7*c*d*t
an(f*x + e)^2 + 6*A*a*b^8*c*d*tan(f*x + e)^2 - 6*C*a*b^8*c*d*tan(f*x + e)^2
 + 6*B*b^9*c*d*tan(f*x + e)^2 - 9*C*a^6*b^3*d^2*tan(f*x + e)^2 + 18*B*a^5*b
^4*d^2*tan(f*x + e)^2 - 30*A*a^4*b^5*d^2*tan(f*x + e)^2 + 3*C*a^4*b^5*d^2*t
an(f*x + e)^2 + 9*B*a^3*b^6*d^2*tan(f*x + e)^2 - 27*A*a^2*b^7*d^2*tan(f*x +
 e)^2 + 3*B*a*b^8*d^2*tan(f*x + e)^2 - 9*A*b^9*d^2*tan(f*x + e)^2 + 8*B*a^4
*b^5*c^2*tan(f*x + e) - 22*A*a^3*b^6*c^2*tan(f*x + e) + 22*C*a^3*b^6*c^2*ta
n(f*x + e) - 18*B*a^2*b^7*c^2*tan(f*x + e) + 2*A*a*b^8*c^2*tan(f*x + e) - 2
*C*a*b^8*c^2*tan(f*x + e) - 2*B*b^9*c^2*tan(f*x + e) + 4*C*a^6*b^3*c*d*tan(
```

```
f*x + e) - 32*B*a^5*b^4*c*d*tan(f*x + e) + 72*A*a^4*b^5*c*d*tan(f*x + e) -
60*C*a^4*b^5*c*d*tan(f*x + e) + 28*B*a^3*b^6*c*d*tan(f*x + e) + 28*A*a^2*b^
7*c*d*tan(f*x + e) - 16*C*a^2*b^7*c*d*tan(f*x + e) + 12*B*a*b^8*c*d*tan(f*x
 + e) + 4*A*b^9*c*d*tan(f*x + e) - 22*C*a^7*b^2*d^2*tan(f*x + e) + 42*B*a^6
*b^3*d^2*tan(f*x + e) - 68*A*a^5*b^4*d^2*tan(f*x + e) + 2*C*a^5*b^4*d^2*tan
(f*x + e) + 26*B*a^4*b^5*d^2*tan(f*x + e) - 66*A*a^3*b^6*d^2*tan(f*x + e) +
 8*B*a^2*b^7*d^2*tan(f*x + e) - 22*A*a*b^8*d^2*tan(f*x + e) - C*a^6*b^3*c^2
 + 6*B*a^5*b^4*c^2 - 14*A*a^4*b^5*c^2 + 11*C*a^4*b^5*c^2 - 7*B*a^3*b^6*c^2
- 3*A*a^2*b^7*c^2 - B*a*b^8*c^2 - A*b^9*c^2 + 6*C*a^7*b^2*c*d - 22*B*a^6*b^
3*c*d + 44*A*a^5*b^4*c*d - 26*C*a^5*b^4*c*d + 6*B*a^4*b^5*c*d + 26*A*a^3*b^
6*c*d - 8*C*a^3*b^6*c*d + 4*B*a^2*b^7*c*d + 6*A*a*b^8*c*d - 14*C*a^8*b*d^2
+ 25*B*a^7*b^2*d^2 - 39*A*a^6*b^3*d^2 - 3*C*a^6*b^3*d^2 + 19*B*a^5*b^4*d^2
- 41*A*a^4*b^5*d^2 - C*a^4*b^5*d^2 + 6*B*a^3*b^6*d^2 - 14*A*a^2*b^7*d^2)/((
a^6*b^4*c^4 + 3*a^4*b^6*c^4 + 3*a^2*b^8*c^4 + b^10*c^4 - 4*a^7*b^3*c^3*d -
12*a^5*b^5*c^3*d - 12*a^3*b^7*c^3*d - 4*a*b^9*c^3*d + 6*a^8*b^2*c^2*d^2 + 1
8*a^6*b^4*c^2*d^2 + 18*a^4*b^6*c^2*d^2 + 6*a^2*b^8*c^2*d^2 - 4*a^9*b*c*d^3
- 12*a^7*b^3*c*d^3 - 12*a^5*b^5*c*d^3 - 4*a^3*b^7*c*d^3 + a^10*d^4 + 3*a^8*
b^2*d^4 + 3*a^6*b^4*d^4 + a^4*b^6*d^4)*(b*tan(f*x + e) + a)^2))/f
```

3.84 $\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^3}\,dx$

Optimal. Leaf size=804

$$-\frac{\left(Cc^2-Bdc+Ad^2\right)(a+b\tan(e+fx))^3}{2d\left(c^2+d^2\right)f(c+d\tan(e+fx))^2}-\frac{\left(2a\left(2c(A-C)d-B\left(c^2-d^2\right)\right)d^2+b\left(3Cc^4-Bdc^3-(A-7C)d^2c^2-5Bd^3c\right.\right.}{2d^2\left(c^2+d^2\right)^2f(c+d\tan(e+fx))}$$

```
[Out] -(((3*a*b^2*(A*c^3 - c^3*C + 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 - B*d^3) + a
^3*(c^3*C - 3*B*c^2*d - 3*c*C*d^2 + B*d^3 - A*(c^3 - 3*c*d^2)) - 3*a^2*b*((
A - C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)) + b^3*((A - C)*d*(3*c^2 - d^2)
- B*(c^3 - 3*c*d^2)))*x)/(c^2 + d^2)^3) - ((3*a^2*b*(A*c^3 - c^3*C + 3*B*c^
2*d - 3*A*c*d^2 + 3*c*C*d^2 - B*d^3) - b^3*(A*c^3 - c^3*C + 3*B*c^2*d - 3*A
*c*d^2 + 3*c*C*d^2 - B*d^3) - a^3*((A - C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d
^2)) + 3*a*b^2*((A - C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)))*Log[Cos[e + f
*x]])/((c^2 + d^2)^3*f) - ((b*c - a*d)*(b^2*(3*c^6*C - B*c^5*d + 9*c^4*C*d^
2 - 3*B*c^3*d^3 - c^2*(A - 10*C)*d^4 - 6*B*c*d^5 + 3*A*d^6) + a^2*d^3*((A -
 C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)) + a*b*d^2*(8*c*(A - C)*d^3 - B*(c^
4 + 6*c^2*d^2 - 3*d^4)))*Log[c + d*Tan[e + f*x]])/(d^4*(c^2 + d^2)^3*f) + (
b^2*(b*(3*c^4*C - B*c^3*d + 6*c^2*C*d^2 - 3*B*c*d^3 + (2*A + C)*d^4) + a*d^
2*(2*c*(A - C)*d - B*(c^2 - d^2)))*Tan[e + f*x])/(d^3*(c^2 + d^2)^2*f) - ((
c^2*C - B*c*d + A*d^2)*(a + b*Tan[e + f*x])^3)/(2*d*(c^2 + d^2)*f*(c + d*Ta
n[e + f*x])^2) - ((b*(3*c^4*C - B*c^3*d - c^2*(A - 7*C)*d^2 - 5*B*c*d^3 + 3
*A*d^4) + 2*a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)))*(a + b*Tan[e + f*x])^2)/
(2*d^2*(c^2 + d^2)^2*f*(c + d*Tan[e + f*x]))
```

Rubi [A] time = 2.74732, antiderivative size = 804, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3645, 3637, 3626, 3617, 31, 3475}

$$-\frac{\left(Cc^2-Bdc+Ad^2\right)(a+b\tan(e+fx))^3}{2d\left(c^2+d^2\right)f(c+d\tan(e+fx))^2}-\frac{\left(2a\left(2c(A-C)d-B\left(c^2-d^2\right)\right)d^2+b\left(3Cc^4-Bdc^3-(A-7C)d^2c^2-5Bd^3c\right.\right.}{2d^2\left(c^2+d^2\right)^2f(c+d\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d
*Tan[e + f*x])^3,x]
```

```
[Out] -(((3*a*b^2*(A*c^3 - c^3*C + 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 - B*d^3) + a
^3*(c^3*C - 3*B*c^2*d - 3*c*C*d^2 + B*d^3 - A*(c^3 - 3*c*d^2)) - 3*a^2*b*((
A - C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)) + b^3*((A - C)*d*(3*c^2 - d^2)
- B*(c^3 - 3*c*d^2)))*x)/(c^2 + d^2)^3) - ((3*a^2*b*(A*c^3 - c^3*C + 3*B*c^
2*d - 3*A*c*d^2 + 3*c*C*d^2 - B*d^3) - b^3*(A*c^3 - c^3*C + 3*B*c^2*d - 3*A
*c*d^2 + 3*c*C*d^2 - B*d^3) - a^3*((A - C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d
^2)) + 3*a*b^2*((A - C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)))*Log[Cos[e + f
*x]])/((c^2 + d^2)^3*f) - ((b*c - a*d)*(b^2*(3*c^6*C - B*c^5*d + 9*c^4*C*d^
2 - 3*B*c^3*d^3 - c^2*(A - 10*C)*d^4 - 6*B*c*d^5 + 3*A*d^6) + a^2*d^3*((A -
 C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)) + a*b*d^2*(8*c*(A - C)*d^3 - B*(c^
4 + 6*c^2*d^2 - 3*d^4)))*Log[c + d*Tan[e + f*x]])/(d^4*(c^2 + d^2)^3*f) + (
b^2*(b*(3*c^4*C - B*c^3*d + 6*c^2*C*d^2 - 3*B*c*d^3 + (2*A + C)*d^4) + a*d^
2*(2*c*(A - C)*d - B*(c^2 - d^2)))*Tan[e + f*x])/(d^3*(c^2 + d^2)^2*f) - ((
c^2*C - B*c*d + A*d^2)*(a + b*Tan[e + f*x])^3)/(2*d*(c^2 + d^2)*f*(c + d*Ta
n[e + f*x])^2) - ((b*(3*c^4*C - B*c^3*d - c^2*(A - 7*C)*d^2 - 5*B*c*d^3 + 3
*A*d^4) + 2*a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)))*(a + b*Tan[e + f*x])^2)/
(2*d^2*(c^2 + d^2)^2*f*(c + d*Tan[e + f*x]))
```

Rule 3645

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*d^2 + c*(c*C - B*d))*(a + b*Tan[e
+ f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3626

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((a*A + b*B -
a*C)*x)/(a^2 + b^2), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3617

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^3}dx = -\frac{\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^3}{2d\left(c^2+d^2\right)f(c+d\tan(e+fx))^2} + \frac{\int \frac{(a+b\tan}{}}{}$$

$$= -\frac{\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^3}{2d\left(c^2+d^2\right)f(c+d\tan(e+fx))^2} - \frac{\left(b\left(3c^4C\right.\right.}{}$$

$$= \frac{b^2\left(b\left(3c^4C-Bc^3d+6c^2Cd^2-3Bcd^3+(2A+C)d^4\right)\right.}{d^3\left(c^2+d^2\right.}$$

$$= -\frac{\left(3ab^2\left(Ac^3-c^3C+3Bc^2d-3Acd^2+3cCd^2-Bd^3\right)\right.}{}$$

$$= -\frac{\left(3ab^2\left(Ac^3-c^3C+3Bc^2d-3Acd^2+3cCd^2-Bd^3\right)\right.}{}$$

$$= -\frac{\left(3ab^2\left(Ac^3-c^3C+3Bc^2d-3Acd^2+3cCd^2-Bd^3\right)\right.}{}$$

Mathematica [A] time = 14.9234, size = 1445, normalized size = 1.8

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(c + d*Tan[e + f*x])^3,x]
```

```
[Out] ((3*a*b^2*(-(A*c^3) + c^3*C - 3*B*c^2*d + 3*A*c*d^2 - 3*c*C*d^2 + B*d^3) +
a^3*(-(c^3*C) + 3*B*c^2*d + 3*c*C*d^2 - B*d^3 + A*(c^3 - 3*c*d^2)) - 3*a^2*
b*((A - C)*d*(-3*c^2 + d^2) + B*(c^3 - 3*c*d^2)) + b^3*((A - C)*d*(-3*c^2 +
 d^2) + B*(c^3 - 3*c*d^2)))*(e + f*x)*(c*Cos[e + f*x] + d*Sin[e + f*x])^3*(
a + b*Tan[e + f*x])^3)/((c^2 + d^2)^3*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^3
*(c + d*Tan[e + f*x])^3) - (b^2*(-3*b*c*C + b*B*d + 3*a*C*d)*Log[1 - Tan[(e
 + f*x)/2]^2]*(c*Cos[e + f*x] + d*Sin[e + f*x])^3*(a + b*Tan[e + f*x])^3)/(
d^4*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^3*(c + d*Tan[e + f*x])^3) + ((-3*a^
2*b*(-(A*c^3) + c^3*C - 3*B*c^2*d + 3*A*c*d^2 - 3*c*C*d^2 + B*d^3) + b^3*(-
(A*c^3) + c^3*C - 3*B*c^2*d + 3*A*c*d^2 - 3*c*C*d^2 + B*d^3) + a^3*((A - C)
*d*(-3*c^2 + d^2) + B*(c^3 - 3*c*d^2)) - 3*a*b^2*((A - C)*d*(-3*c^2 + d^2)
+ B*(c^3 - 3*c*d^2)))*Log[1 + Tan[(e + f*x)/2]^2]*(c*Cos[e + f*x] + d*Sin[e
 + f*x])^3*(a + b*Tan[e + f*x])^3)/((c^2 + d^2)^3*f*(a*Cos[e + f*x] + b*Sin
[e + f*x])^3*(c + d*Tan[e + f*x])^3) + ((-(b*c) + a*d)*(b^2*(3*c^6*C - B*c^
5*d + 9*c^4*C*d^2 - 3*B*c^3*d^3 - c^2*(A - 10*C)*d^4 - 6*B*c*d^5 + 3*A*d^6)
 + a^2*d^3*(-((A - C)*d*(-3*c^2 + d^2)) - B*(c^3 - 3*c*d^2)) - a*b*d^2*(8*c
*(-A + C)*d^3 + B*(c^4 + 6*c^2*d^2 - 3*d^4)))*Log[-2*d*Tan[(e + f*x)/2] + c
*(-1 + Tan[(e + f*x)/2]^2)]*(c*Cos[e + f*x] + d*Sin[e + f*x])^3*(a + b*Tan[
e + f*x])^3)/(d^4*(c^2 + d^2)^3*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^3*(c +
d*Tan[e + f*x])^3) - (2*b^3*C*(c*Cos[e + f*x] + d*Sin[e + f*x])^3*Tan[(e +
f*x)/2]*(a + b*Tan[e + f*x])^3)/(d^3*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^3*
(-1 + Tan[(e + f*x)/2]^2)*(c + d*Tan[e + f*x])^3) + (2*(b*c - a*d)^3*(c^2*C
 - B*c*d + A*d^2)*(c*Cos[e + f*x] + d*Sin[e + f*x])^3*(c + 2*d*Tan[(e + f*x
)/2])*(a + b*Tan[e + f*x])^3)/(c^3*d^2*(c^2 + d^2)*f*(a*Cos[e + f*x] + b*Si
n[e + f*x])^3*(c + 2*d*Tan[(e + f*x)/2] - c*Tan[(e + f*x)/2]^2)^2*(c + d*Ta
n[e + f*x])^3) - (2*(b*c - a*d)^2*(c*Cos[e + f*x] + d*Sin[e + f*x])^3*(a*d*
```

```
(c^2*(A + C)*d^3 + A*d^5 + c^5*C*Tan[(e + f*x)/2] + c*d^4*(-B + A*Tan[(e +
f*x)/2]) + c^4*d*(C - 2*B*Tan[(e + f*x)/2]) - c^3*d^2*(B - 3*A*Tan[(e + f*x
)/2] + C*Tan[(e + f*x)/2])) + b*c*(-(A*d^5) + 2*c^5*C*Tan[(e + f*x)/2] + c*
d^4*(B + 2*A*Tan[(e + f*x)/2]) - c^4*d*(C + B*Tan[(e + f*x)/2]) - c^2*d^3*(
A + C + 3*B*Tan[(e + f*x)/2]) + c^3*d^2*(B + 4*C*Tan[(e + f*x)/2])))*(a + b
*Tan[e + f*x])^3)/(c^3*d^3*(c^2 + d^2)^2*f*(a*Cos[e + f*x] + b*Sin[e + f*x]
)^3*(-2*d*Tan[(e + f*x)/2] + c*(-1 + Tan[(e + f*x)/2]^2))*(c + d*Tan[e + f*
x])^3)
```

Maple [B] time = 0.085, size = 3522, normalized size = 4.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^3,x
)
```

```
[Out] -3/2/f/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*C*b^3*c*d^2-3/f/(c^2+d^2)^3*A*arctan(
tan(f*x+e))*a^3*c*d^2-3/f/(c^2+d^2)^3*A*arctan(tan(f*x+e))*a^2*b*d^3-3/f/(c
^2+d^2)^3*A*arctan(tan(f*x+e))*a*b^2*c^3-3/f/(c^2+d^2)^3*A*arctan(tan(f*x+e
))*b^3*c^2*d-9/f/(c^2+d^2)^3*C*arctan(tan(f*x+e))*a^2*b*c^2*d-9/f/(c^2+d^2)
^3*C*arctan(tan(f*x+e))*a*b^2*c*d^2+12/f/d/(c^2+d^2)^2/(c+d*tan(f*x+e))*C*a
*b^2*c^3+9/f*d^2/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*A*a^2*b*c-3/2/f/d/(c^2+d^2)
/(c+d*tan(f*x+e))^2*B*a^2*b*c^2-9/f/(c^2+d^2)^3*B*arctan(tan(f*x+e))*a*b^2*
c^2*d+3/f/(c^2+d^2)^3*B*arctan(tan(f*x+e))*a^3*c^2*d-3/f/(c^2+d^2)^3*B*arct
an(tan(f*x+e))*a^2*b*c^3-3/f/d^2/(c^2+d^2)^2/(c+d*tan(f*x+e))*C*a^2*b*c^4+1
8/f*d/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*C*a*b^2*c^2+9/2/f/(c^2+d^2)^3*ln(1+tan
(f*x+e)^2)*C*a^2*b*c*d^2+3/2/f/d^2/(c^2+d^2)/(c+d*tan(f*x+e))^2*B*a*b^2*c^3
+3/2/f/d^2/(c^2+d^2)/(c+d*tan(f*x+e))^2*C*a^2*b*c^3+9/2/f/(c^2+d^2)^3*ln(1+
tan(f*x+e)^2)*B*a*b^2*c*d^2-3/2/f/d/(c^2+d^2)/(c+d*tan(f*x+e))^2*A*a*b^2*c^
2+9/2/f/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*A*a*b^2*c^2*d+6/f*d/(c^2+d^2)^2/(c+d
*tan(f*x+e))*B*a^2*b*c-3/f/d^2/(c^2+d^2)^2/(c+d*tan(f*x+e))*B*a*b^2*c^4-9/2
/f/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*A*a^2*b*c*d^2+6/f*d/(c^2+d^2)^2/(c+d*tan(
f*x+e))*A*a*b^2*c+9/2/f/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*B*a^2*b*c^2*d-9/f*d/
(c^2+d^2)^3*ln(c+d*tan(f*x+e))*A*a*b^2*c^2-9/f*d/(c^2+d^2)^3*ln(c+d*tan(f*x
+e))*B*a^2*b*c^2-9/f*d^2/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*B*a*b^2*c-9/f*d^2/(
c^2+d^2)^3*ln(c+d*tan(f*x+e))*C*a^2*b*c+3/f/d^3/(c^2+d^2)^3*ln(c+d*tan(f*x+
e))*C*a*b^2*c^6+9/f/d/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*C*a*b^2*c^4-3/2/f/d^3/
(c^2+d^2)/(c+d*tan(f*x+e))^2*C*c^4*a*b^2-9/2/f/(c^2+d^2)^3*ln(1+tan(f*x+e)^
2)*C*a*b^2*c^2*d+9/f/(c^2+d^2)^3*A*arctan(tan(f*x+e))*a^2*b*c^2*d+9/f/(c^2+
d^2)^3*A*arctan(tan(f*x+e))*a*b^2*c*d^2+9/f/(c^2+d^2)^3*B*arctan(tan(f*x+e)
)*a^2*b*c*d^2+6/f/d^3/(c^2+d^2)^2/(c+d*tan(f*x+e))*C*a*b^2*c^5-3/f*d^2/(c^2
+d^2)^2/(c+d*tan(f*x+e))*A*a^2*b+3/f*d/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*A*a^3
*c^2+3/f*d^3/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*A*a*b^2+1/f*C*b^3/d^3*tan(f*x+e
)+3/2/f/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*C*a^3*c^2*d-3/2/f/(c^2+d^2)^3*ln(1+t
an(f*x+e)^2)*C*a^2*b*c^3+3/f/(c^2+d^2)^3*B*arctan(tan(f*x+e))*a*b^2*d^3-3/f
/(c^2+d^2)^3*B*arctan(tan(f*x+e))*b^3*c*d^2+3/f/(c^2+d^2)^3*C*arctan(tan(f*
x+e))*a^3*c*d^2+3/f/(c^2+d^2)^3*C*arctan(tan(f*x+e))*a^2*b*d^3-3/2/f/(c^2+d
^2)^3*ln(1+tan(f*x+e)^2)*A*a^3*c^2*d+3/2/f/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*A
*a^2*b*c^3-3/f/d^4/(c^2+d^2)^2/(c+d*tan(f*x+e))*C*b^3*c^6+3/2/f/(c^2+d^2)^3
*ln(1+tan(f*x+e)^2)*A*b^3*c*d^2-3/2/f/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*B*a^3*
c*d^2-3/2/f/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*B*a^2*b*d^3-3/2/f/(c^2+d^2)^3*ln
(1+tan(f*x+e)^2)*B*a*b^2*c^3-3/2/f/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*B*b^3*c^2
*d-3/f/(c^2+d^2)^2/(c+d*tan(f*x+e))*A*b^3*c^2+1/2/f/(c^2+d^2)^3*ln(1+tan(f*
x+e)^2)*A*a^3*d^3-1/2/f/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*A*b^3*c^3+1/2/f/(c^2
+d^2)^3*ln(1+tan(f*x+e)^2)*B*a^3*c^3+1/2/f/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*B
```

```
*b^3*d^3-1/2/f/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*a^3*C*d^3+1/2/f/(c^2+d^2)^3*l
n(1+tan(f*x+e)^2)*C*b^3*c^3+1/f/(c^2+d^2)^3*A*arctan(tan(f*x+e))*a^3*c^3+1/
f/(c^2+d^2)^3*A*arctan(tan(f*x+e))*b^3*d^3-1/f/(c^2+d^2)^3*B*arctan(tan(f*x
+e))*a^3*d^3+1/f/(c^2+d^2)^3*B*arctan(tan(f*x+e))*b^3*c^3-1/f/(c^2+d^2)^3*C
*arctan(tan(f*x+e))*a^3*c^3-1/f/(c^2+d^2)^3*C*arctan(tan(f*x+e))*b^3*d^3-1/
f*d^3/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*A*a^3+1/f*d^3/(c^2+d^2)^3*ln(c+d*tan(f
*x+e))*C*a^3+1/2/f/(c^2+d^2)/(c+d*tan(f*x+e))^2*B*a^3*c-1/f/(c^2+d^2)^3*ln(
c+d*tan(f*x+e))*B*a^3*c^3-1/2/f*d/(c^2+d^2)/(c+d*tan(f*x+e))^2*A*a^3-1/f*d^
2/(c^2+d^2)^2/(c+d*tan(f*x+e))*B*a^3-10/f/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*C*
b^3*c^3+1/f/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*A*b^3*c^3+1/f/(c^2+d^2)^2/(c+d*t
an(f*x+e))*B*a^3*c^2-5/f/d^2/(c^2+d^2)^2/(c+d*tan(f*x+e))*C*b^3*c^4+1/f/d^3
/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*B*b^3*c^6+3/f/d/(c^2+d^2)^3*ln(c+d*tan(f*x+
e))*B*b^3*c^4+2/f/d^3/(c^2+d^2)^2/(c+d*tan(f*x+e))*B*b^3*c^5+4/f/d/(c^2+d^2
)^2/(c+d*tan(f*x+e))*B*b^3*c^3+2/f*d/(c^2+d^2)^2/(c+d*tan(f*x+e))*C*a^3*c-3
/2/f/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*A*a*b^2*d^3+3/f/(c^2+d^2)^3*C*arctan(ta
n(f*x+e))*a*b^2*c^3+3/f/(c^2+d^2)^3*C*arctan(tan(f*x+e))*b^3*c^2*d+3/2/f/(c
^2+d^2)^3*ln(1+tan(f*x+e)^2)*C*a*b^2*d^3+3/f/(c^2+d^2)^3*ln(c+d*tan(f*x+e))
*C*a^2*b*c^3-9/f/d^2/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*C*b^3*c^5-3/f*d^2/(c^2+
d^2)^3*ln(c+d*tan(f*x+e))*A*b^3*c-2/f*d/(c^2+d^2)^2/(c+d*tan(f*x+e))*A*a^3*
c-9/f/(c^2+d^2)^2/(c+d*tan(f*x+e))*C*a^2*b*c^2+6/f*d/(c^2+d^2)^3*ln(c+d*tan
(f*x+e))*B*b^3*c^2-3/f*d/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*C*a^3*c^2-3/f/d^4/(
c^2+d^2)^3*ln(c+d*tan(f*x+e))*C*b^3*c^7-1/f/d^2/(c^2+d^2)^2/(c+d*tan(f*x+e)
)*A*b^3*c^4+1/2/f/d^2/(c^2+d^2)/(c+d*tan(f*x+e))^2*A*b^3*c^3-1/2/f/d^3/(c^2
+d^2)/(c+d*tan(f*x+e))^2*B*c^4*b^3-1/2/f/d/(c^2+d^2)/(c+d*tan(f*x+e))^2*C*a
^3*c^2+1/2/f/d^4/(c^2+d^2)/(c+d*tan(f*x+e))^2*C*c^5*b^3+3/f*d^2/(c^2+d^2)^3
*ln(c+d*tan(f*x+e))*B*a^3*c+3/f*d^3/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*B*a^2*b+
3/f/(c^2+d^2)^2/(c+d*tan(f*x+e))*A*a^2*b*c^2-9/f/(c^2+d^2)^2/(c+d*tan(f*x+e
))*B*a*b^2*c^2+3/2/f/(c^2+d^2)/(c+d*tan(f*x+e))^2*A*a^2*b*c-3/f/(c^2+d^2)^3
*ln(c+d*tan(f*x+e))*A*a^2*b*c^3+3/f/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*B*a*b^2*
c^3
```

Maxima [A] time = 1.7425, size = 1499, normalized size = 1.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*C*b^3*tan(f*x + e)/d^3 + 2*(((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b
^2 + B*b^3)*c^3 + 3*(B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*c^2
*d - 3*((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*c*d^2 - (B*a^3 +
 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*d^3)*(f*x + e)/(c^6 + 3*c^4*d^2
 + 3*c^2*d^4 + d^6) - 2*(3*C*b^3*c^7 + 9*C*b^3*c^5*d^2 - (3*C*a*b^2 + B*b^3
)*c^6*d - 3*(3*C*a*b^2 + B*b^3)*c^4*d^3 + (B*a^3 + 3*(A - C)*a^2*b - 3*B*a*
b^2 - (A - 10*C)*b^3)*c^3*d^4 - 3*((A - C)*a^3 - 3*B*a^2*b - 3*(A - 2*C)*a*
b^2 + 2*B*b^3)*c^2*d^5 - 3*(B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - A*b^3)*c*
d^6 + ((A - C)*a^3 - 3*B*a^2*b - 3*A*a*b^2)*d^7)*log(d*tan(f*x + e) + c)/(c
^6*d^4 + 3*c^4*d^6 + 3*c^2*d^8 + d^10) + ((B*a^3 + 3*(A - C)*a^2*b - 3*B*a*
b^2 - (A - C)*b^3)*c^3 - 3*((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b
^3)*c^2*d - 3*(B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*c*d^2 + (
(A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*d^3)*log(tan(f*x + e)^2
+ 1)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) - (5*C*b^3*c^7 + A*a^3*d^7 - 3*(3*
C*a*b^2 + B*b^3)*c^6*d + (3*C*a^2*b + 3*B*a*b^2 + (A + 9*C)*b^3)*c^5*d^2 +
(C*a^3 + 3*B*a^2*b + 3*(A - 7*C)*a*b^2 - 7*B*b^3)*c^4*d^3 - (3*B*a^3 + 3*(3
*A - 5*C)*a^2*b - 15*B*a*b^2 - 5*A*b^3)*c^3*d^4 + ((5*A - 3*C)*a^3 - 9*B*a^
```

```
2*b - 9*A*a*b^2)*c^2*d^5 + (B*a^3 + 3*A*a^2*b)*c*d^6 + 2*(3*C*b^3*c^6*d - 2
*(3*C*a*b^2 + B*b^3)*c^5*d^2 + (3*C*a^2*b + 3*B*a*b^2 + (A + 5*C)*b^3)*c^4*
d^3 - 4*(3*C*a*b^2 + B*b^3)*c^3*d^4 - (B*a^3 + 3*(A - 3*C)*a^2*b - 9*B*a*b^
2 - 3*A*b^3)*c^2*d^5 + 2*((A - C)*a^3 - 3*B*a^2*b - 3*A*a*b^2)*c*d^6 + (B*a
^3 + 3*A*a^2*b)*d^7)*tan(f*x + e))/(c^6*d^4 + 2*c^4*d^6 + c^2*d^8 + (c^4*d^
6 + 2*c^2*d^8 + d^10)*tan(f*x + e)^2 + 2*(c^5*d^5 + 2*c^3*d^7 + c*d^9)*tan(
f*x + e)))/f
```

Fricas [B] time = 13.2101, size = 5218, normalized size = 6.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(3*C*b^3*c^7*d^2 + A*a^3*d^9 - (3*C*a*b^2 + B*b^3)*c^6*d^3 - (3*C*a^2*
b + 3*B*a*b^2 + (A - 9*C)*b^3)*c^5*d^4 + (3*C*a^3 + 9*B*a^2*b + 3*(3*A - 7*
C)*a*b^2 - 7*B*b^3)*c^4*d^5 - 5*(B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - A*b^
3)*c^3*d^6 + ((7*A - 3*C)*a^3 - 9*B*a^2*b - 9*A*a*b^2)*c^2*d^7 + (B*a^3 + 3
*A*a^2*b)*c*d^8 - 2*(C*b^3*c^6*d^3 + 3*C*b^3*c^4*d^5 + 3*C*b^3*c^2*d^7 + C*
b^3*d^9)*tan(f*x + e)^3 - 2*(((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B
*b^3)*c^5*d^4 + 3*(B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*c^4*d
^5 - 3*((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*c^3*d^6 - (B*a^3
 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*c^2*d^7)*f*x - (9*C*b^3*c^7*d
^2 - A*a^3*d^9 - 3*(3*C*a*b^2 + B*b^3)*c^6*d^3 + (3*C*a^2*b + 3*B*a*b^2 + (
A + 23*C)*b^3)*c^5*d^4 + (C*a^3 + 3*B*a^2*b + 3*(A - 9*C)*a*b^2 - 9*B*b^3)*
c^4*d^5 - (3*B*a^3 + 3*(3*A - 7*C)*a^2*b - 21*B*a*b^2 - (7*A + 12*C)*b^3)*c
^3*d^6 + 5*((A - C)*a^3 - 3*B*a^2*b - 3*A*a*b^2)*c^2*d^7 + (3*B*a^3 + 9*A*a
^2*b + 4*C*b^3)*c*d^8 + 2*(((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b
^3)*c^3*d^6 + 3*(B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*c^2*d^7
 - 3*((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*c*d^8 - (B*a^3 + 3
*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*d^9)*f*x)*tan(f*x + e)^2 + (3*C*b
^3*c^9 + 9*C*b^3*c^7*d^2 - (3*C*a*b^2 + B*b^3)*c^8*d - 3*(3*C*a*b^2 + B*b^3
)*c^6*d^3 + (B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - 10*C)*b^3)*c^5*d^4
 - 3*((A - C)*a^3 - 3*B*a^2*b - 3*(A - 2*C)*a*b^2 + 2*B*b^3)*c^4*d^5 - 3*(B*
a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - A*b^3)*c^3*d^6 + ((A - C)*a^3 - 3*B*a^2
*b - 3*A*a*b^2)*c^2*d^7 + (3*C*b^3*c^7*d^2 + 9*C*b^3*c^5*d^4 - (3*C*a*b^2 +
 B*b^3)*c^6*d^3 - 3*(3*C*a*b^2 + B*b^3)*c^4*d^5 + (B*a^3 + 3*(A - C)*a^2*b
 - 3*B*a*b^2 - (A - 10*C)*b^3)*c^3*d^6 - 3*((A - C)*a^3 - 3*B*a^2*b - 3*(A -
 2*C)*a*b^2 + 2*B*b^3)*c^2*d^7 - 3*(B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - A
*b^3)*c*d^8 + ((A - C)*a^3 - 3*B*a^2*b - 3*A*a*b^2)*d^9)*tan(f*x + e)^2 + 2
*(3*C*b^3*c^8*d + 9*C*b^3*c^6*d^3 - (3*C*a*b^2 + B*b^3)*c^7*d^2 - 3*(3*C*a*
b^2 + B*b^3)*c^5*d^4 + (B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - 10*C)*b^
3)*c^4*d^5 - 3*((A - C)*a^3 - 3*B*a^2*b - 3*(A - 2*C)*a*b^2 + 2*B*b^3)*c^3*
d^6 - 3*(B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - A*b^3)*c^2*d^7 + ((A - C)*a^
3 - 3*B*a^2*b - 3*A*a*b^2)*c*d^8)*tan(f*x + e))*log((d^2*tan(f*x + e)^2 + 2
*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) - (3*C*b^3*c^9 + 9*C*b^3*c^7
*d^2 + 9*C*b^3*c^5*d^4 + 3*C*b^3*c^3*d^6 - (3*C*a*b^2 + B*b^3)*c^8*d - 3*(3
*C*a*b^2 + B*b^3)*c^6*d^3 - 3*(3*C*a*b^2 + B*b^3)*c^4*d^5 - (3*C*a*b^2 + B*
b^3)*c^2*d^7 + (3*C*b^3*c^7*d^2 + 9*C*b^3*c^5*d^4 + 9*C*b^3*c^3*d^6 + 3*C*b
^3*c*d^8 - (3*C*a*b^2 + B*b^3)*c^6*d^3 - 3*(3*C*a*b^2 + B*b^3)*c^4*d^5 - 3*
(3*C*a*b^2 + B*b^3)*c^2*d^7 - (3*C*a*b^2 + B*b^3)*d^9)*tan(f*x + e)^2 + 2*(
3*C*b^3*c^8*d + 9*C*b^3*c^6*d^3 + 9*C*b^3*c^4*d^5 + 3*C*b^3*c^2*d^7 - (3*C*
a*b^2 + B*b^3)*c^7*d^2 - 3*(3*C*a*b^2 + B*b^3)*c^5*d^4 - 3*(3*C*a*b^2 + B*b
^3)*c^3*d^6 - (3*C*a*b^2 + B*b^3)*c*d^8)*tan(f*x + e))*log(1/(tan(f*x + e)^
```

```
2 + 1)) - 2*(3*C*b^3*c^8*d + 6*C*b^3*c^6*d^3 - (3*C*a*b^2 + B*b^3)*c^7*d^2
+ (C*a^3 + 3*B*a^2*b + 3*(A - 3*C)*a*b^2 - 3*B*b^3)*c^5*d^4 - (2*B*a^3 + 3*
(2*A - 3*C)*a^2*b - 9*B*a*b^2 - (3*A - 2*C)*b^3)*c^4*d^5 + (3*(A - C)*a^3 -
 9*B*a^2*b - 3*(3*A - 4*C)*a*b^2 + 4*B*b^3)*c^3*d^6 + (3*B*a^3 + 9*(A - C)*
a^2*b - 9*B*a*b^2 - (3*A - C)*b^3)*c^2*d^7 - ((3*A - 2*C)*a^3 - 6*B*a^2*b -
 6*A*a*b^2)*c*d^8 - (B*a^3 + 3*A*a^2*b)*d^9 + 2*(((A - C)*a^3 - 3*B*a^2*b -
 3*(A - C)*a*b^2 + B*b^3)*c^4*d^5 + 3*(B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2
- (A - C)*b^3)*c^3*d^6 - 3*((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b
^3)*c^2*d^7 - (B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*c*d^8)*f*
x)*tan(f*x + e))/((c^6*d^6 + 3*c^4*d^8 + 3*c^2*d^10 + d^12)*f*tan(f*x + e)^
2 + 2*(c^7*d^5 + 3*c^5*d^7 + 3*c^3*d^9 + c*d^11)*f*tan(f*x + e) + (c^8*d^4
+ 3*c^6*d^6 + 3*c^4*d^8 + c^2*d^10)*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x
+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.43125, size = 3382, normalized size = 4.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*C*b^3*tan(f*x + e)/d^3 + 2*(A*a^3*c^3 - C*a^3*c^3 - 3*B*a^2*b*c^3 -
3*A*a*b^2*c^3 + 3*C*a*b^2*c^3 + B*b^3*c^3 + 3*B*a^3*c^2*d + 9*A*a^2*b*c^2*d
 - 9*C*a^2*b*c^2*d - 9*B*a*b^2*c^2*d - 3*A*b^3*c^2*d + 3*C*b^3*c^2*d - 3*A*
a^3*c*d^2 + 3*C*a^3*c*d^2 + 9*B*a^2*b*c*d^2 + 9*A*a*b^2*c*d^2 - 9*C*a*b^2*c
*d^2 - 3*B*b^3*c*d^2 - B*a^3*d^3 - 3*A*a^2*b*d^3 + 3*C*a^2*b*d^3 + 3*B*a*b^
2*d^3 + A*b^3*d^3 - C*b^3*d^3)*(f*x + e)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6
) + (B*a^3*c^3 + 3*A*a^2*b*c^3 - 3*C*a^2*b*c^3 - 3*B*a*b^2*c^3 - A*b^3*c^3
+ C*b^3*c^3 - 3*A*a^3*c^2*d + 3*C*a^3*c^2*d + 9*B*a^2*b*c^2*d + 9*A*a*b^2*c
^2*d - 9*C*a*b^2*c^2*d - 3*B*b^3*c^2*d - 3*B*a^3*c*d^2 - 9*A*a^2*b*c*d^2 +
9*C*a^2*b*c*d^2 + 9*B*a*b^2*c*d^2 + 3*A*b^3*c*d^2 - 3*C*b^3*c*d^2 + A*a^3*d
^3 - C*a^3*d^3 - 3*B*a^2*b*d^3 - 3*A*a*b^2*d^3 + 3*C*a*b^2*d^3 + B*b^3*d^3)
*log(tan(f*x + e)^2 + 1)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) - 2*(3*C*b^3*c
^7 - 3*C*a*b^2*c^6*d - B*b^3*c^6*d + 9*C*b^3*c^5*d^2 - 9*C*a*b^2*c^4*d^3 -
3*B*b^3*c^4*d^3 + B*a^3*c^3*d^4 + 3*A*a^2*b*c^3*d^4 - 3*C*a^2*b*c^3*d^4 - 3
*B*a*b^2*c^3*d^4 - A*b^3*c^3*d^4 + 10*C*b^3*c^3*d^4 - 3*A*a^3*c^2*d^5 + 3*C
*a^3*c^2*d^5 + 9*B*a^2*b*c^2*d^5 + 9*A*a*b^2*c^2*d^5 - 18*C*a*b^2*c^2*d^5 -
 6*B*b^3*c^2*d^5 - 3*B*a^3*c*d^6 - 9*A*a^2*b*c*d^6 + 9*C*a^2*b*c*d^6 + 9*B*
a*b^2*c*d^6 + 3*A*b^3*c*d^6 + A*a^3*d^7 - C*a^3*d^7 - 3*B*a^2*b*d^7 - 3*A*a
*b^2*d^7)*log(abs(d*tan(f*x + e) + c))/(c^6*d^4 + 3*c^4*d^6 + 3*c^2*d^8 + d
^10) + (9*C*b^3*c^7*d^2*tan(f*x + e)^2 - 9*C*a*b^2*c^6*d^3*tan(f*x + e)^2 -
 3*B*b^3*c^6*d^3*tan(f*x + e)^2 + 27*C*b^3*c^5*d^4*tan(f*x + e)^2 - 27*C*a*
```

```
b^2*c^4*d^5*tan(f*x + e)^2 - 9*B*b^3*c^4*d^5*tan(f*x + e)^2 + 3*B*a^3*c^3*d
^6*tan(f*x + e)^2 + 9*A*a^2*b*c^3*d^6*tan(f*x + e)^2 - 9*C*a^2*b*c^3*d^6*ta
n(f*x + e)^2 - 9*B*a*b^2*c^3*d^6*tan(f*x + e)^2 - 3*A*b^3*c^3*d^6*tan(f*x +
 e)^2 + 30*C*b^3*c^3*d^6*tan(f*x + e)^2 - 9*A*a^3*c^2*d^7*tan(f*x + e)^2 +
9*C*a^3*c^2*d^7*tan(f*x + e)^2 + 27*B*a^2*b*c^2*d^7*tan(f*x + e)^2 + 27*A*a
*b^2*c^2*d^7*tan(f*x + e)^2 - 54*C*a*b^2*c^2*d^7*tan(f*x + e)^2 - 18*B*b^3*
c^2*d^7*tan(f*x + e)^2 - 9*B*a^3*c*d^8*tan(f*x + e)^2 - 27*A*a^2*b*c*d^8*ta
n(f*x + e)^2 + 27*C*a^2*b*c*d^8*tan(f*x + e)^2 + 27*B*a*b^2*c*d^8*tan(f*x +
 e)^2 + 9*A*b^3*c*d^8*tan(f*x + e)^2 + 3*A*a^3*d^9*tan(f*x + e)^2 - 3*C*a^3
*d^9*tan(f*x + e)^2 - 9*B*a^2*b*d^9*tan(f*x + e)^2 - 9*A*a*b^2*d^9*tan(f*x
+ e)^2 + 12*C*b^3*c^8*d*tan(f*x + e) - 6*C*a*b^2*c^7*d^2*tan(f*x + e) - 2*B
*b^3*c^7*d^2*tan(f*x + e) - 6*C*a^2*b*c^6*d^3*tan(f*x + e) - 6*B*a*b^2*c^6*
d^3*tan(f*x + e) - 2*A*b^3*c^6*d^3*tan(f*x + e) + 38*C*b^3*c^6*d^3*tan(f*x
+ e) - 18*C*a*b^2*c^5*d^4*tan(f*x + e) - 6*B*b^3*c^5*d^4*tan(f*x + e) + 8*B
*a^3*c^4*d^5*tan(f*x + e) + 24*A*a^2*b*c^4*d^5*tan(f*x + e) - 42*C*a^2*b*c^
4*d^5*tan(f*x + e) - 42*B*a*b^2*c^4*d^5*tan(f*x + e) - 14*A*b^3*c^4*d^5*tan
(f*x + e) + 50*C*b^3*c^4*d^5*tan(f*x + e) - 22*A*a^3*c^3*d^6*tan(f*x + e) +
 22*C*a^3*c^3*d^6*tan(f*x + e) + 66*B*a^2*b*c^3*d^6*tan(f*x + e) + 66*A*a*b
^2*c^3*d^6*tan(f*x + e) - 84*C*a*b^2*c^3*d^6*tan(f*x + e) - 28*B*b^3*c^3*d^
6*tan(f*x + e) - 18*B*a^3*c^2*d^7*tan(f*x + e) - 54*A*a^2*b*c^2*d^7*tan(f*x
 + e) + 36*C*a^2*b*c^2*d^7*tan(f*x + e) + 36*B*a*b^2*c^2*d^7*tan(f*x + e) +
 12*A*b^3*c^2*d^7*tan(f*x + e) + 2*A*a^3*c*d^8*tan(f*x + e) - 2*C*a^3*c*d^8
*tan(f*x + e) - 6*B*a^2*b*c*d^8*tan(f*x + e) - 6*A*a*b^2*c*d^8*tan(f*x + e)
 - 2*B*a^3*d^9*tan(f*x + e) - 6*A*a^2*b*d^9*tan(f*x + e) + 4*C*b^3*c^9 - 3*
C*a^2*b*c^7*d^2 - 3*B*a*b^2*c^7*d^2 - A*b^3*c^7*d^2 + 13*C*b^3*c^7*d^2 - C*
a^3*c^6*d^3 - 3*B*a^2*b*c^6*d^3 - 3*A*a*b^2*c^6*d^3 + 3*C*a*b^2*c^6*d^3 + B
*b^3*c^6*d^3 + 6*B*a^3*c^5*d^4 + 18*A*a^2*b*c^5*d^4 - 27*C*a^2*b*c^5*d^4 -
27*B*a*b^2*c^5*d^4 - 9*A*b^3*c^5*d^4 + 21*C*b^3*c^5*d^4 - 14*A*a^3*c^4*d^5
+ 11*C*a^3*c^4*d^5 + 33*B*a^2*b*c^4*d^5 + 33*A*a*b^2*c^4*d^5 - 33*C*a*b^2*c
^4*d^5 - 11*B*b^3*c^4*d^5 - 7*B*a^3*c^3*d^6 - 21*A*a^2*b*c^3*d^6 + 12*C*a^2
*b*c^3*d^6 + 12*B*a*b^2*c^3*d^6 + 4*A*b^3*c^3*d^6 - 3*A*a^3*c^2*d^7 - B*a^3
*c*d^8 - 3*A*a^2*b*c*d^8 - A*a^3*d^9)/((c^6*d^4 + 3*c^4*d^6 + 3*c^2*d^8 + d
^10)*(d*tan(f*x + e) + c)^2))/f
```

3.85 $\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^3}\,dx$

Optimal. Leaf size=597

$$-\frac{\left(-a^2d^3\left(d(A-C)\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)+2abd^3\left(Ac^3-3Acd^2+3Bc^2d-Bd^3-c^3C+3cCd^2\right)-b^2\left(-3c^2d^4(A-2\right.}{d^3f\left(c^2+d^2\right)^3}$$

```
[Out] -((((b^2*(A*c^3 - c^3*C + 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 - B*d^3) + a^2*(
c^3*C - 3*B*c^2*d - 3*c*C*d^2 + B*d^3 - A*(c^3 - 3*c*d^2)) - 2*a*b*((A - C)
*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)))*x)/(c^2 + d^2)^3) - ((2*a*b*(A*c^3 -
 c^3*C + 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 - B*d^3) - a^2*((A - C)*d*(3*c^2
 - d^2) - B*(c^3 - 3*c*d^2)) + b^2*((A - C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*
d^2)))*Log[Cos[e + f*x]])/((c^2 + d^2)^3*f) - ((2*a*b*d^3*(A*c^3 - c^3*C +
3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 - B*d^3) - b^2*(c^6*C + 3*c^4*C*d^2 + B*c
^3*d^3 - 3*c^2*(A - 2*C)*d^4 - 3*B*c*d^5 + A*d^6) - a^2*d^3*((A - C)*d*(3*c
^2 - d^2) - B*(c^3 - 3*c*d^2)))*Log[c + d*Tan[e + f*x]])/(d^3*(c^2 + d^2)^3
*f) - ((c^2*C - B*c*d + A*d^2)*(a + b*Tan[e + f*x])^2)/(2*d*(c^2 + d^2)*f*(
c + d*Tan[e + f*x])^2) + ((b*c - a*d)*(b*(c^4*C - c^2*(A - 3*C)*d^2 - 2*B*c
*d^3 + A*d^4) + a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2))))/(d^3*(c^2 + d^2)^2*
f*(c + d*Tan[e + f*x]))
```

Rubi [A] time = 1.38508, antiderivative size = 597, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3645, 3635, 3626, 3617, 31, 3475}

$$-\frac{\left(-a^2d^3\left(d(A-C)\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)+2abd^3\left(Ac^3-3Acd^2+3Bc^2d-Bd^3-c^3C+3cCd^2\right)-b^2\left(-3c^2d^4(A-2\right.}{d^3f\left(c^2+d^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d
*Tan[e + f*x])^3,x]
```


```
[Out] -((((b^2*(A*c^3 - c^3*C + 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 - B*d^3) + a^2*(
c^3*C - 3*B*c^2*d - 3*c*C*d^2 + B*d^3 - A*(c^3 - 3*c*d^2)) - 2*a*b*((A - C)
*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)))*x)/(c^2 + d^2)^3) - ((2*a*b*(A*c^3 -
 c^3*C + 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 - B*d^3) - a^2*((A - C)*d*(3*c^2
 - d^2) - B*(c^3 - 3*c*d^2)) + b^2*((A - C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*
d^2)))*Log[Cos[e + f*x]])/((c^2 + d^2)^3*f) - ((2*a*b*d^3*(A*c^3 - c^3*C +
3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 - B*d^3) - b^2*(c^6*C + 3*c^4*C*d^2 + B*c
^3*d^3 - 3*c^2*(A - 2*C)*d^4 - 3*B*c*d^5 + A*d^6) - a^2*d^3*((A - C)*d*(3*c
^2 - d^2) - B*(c^3 - 3*c*d^2)))*Log[c + d*Tan[e + f*x]])/(d^3*(c^2 + d^2)^3
*f) - ((c^2*C - B*c*d + A*d^2)*(a + b*Tan[e + f*x])^2)/(2*d*(c^2 + d^2)*f*(
c + d*Tan[e + f*x])^2) + ((b*c - a*d)*(b*(c^4*C - c^2*(A - 3*C)*d^2 - 2*B*c
*d^3 + A*d^4) + a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2))))/(d^3*(c^2 + d^2)^2*
f*(c + d*Tan[e + f*x]))
```


Rule 3645

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*d^2 + c*(c*C - B*d))*(a + b*Tan[e
+ f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
```

```
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3635

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(c^2*C - B*c*d + A*d^2)*(c +
d*Tan[e + f*x])^(n + 1))/(d^2*f*(n + 1)*(c^2 + d^2)), x] + Dist[1/(d*(c^2 +
 d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(c^
2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*Ta
n[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -
1]
```

Rule 3626

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((a*A + b*B -
a*C)*x)/(a^2 + b^2), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3617

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^3}dx = -\frac{\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^2}{2d\left(c^2+d^2\right)f(c+d\tan(e+fx))^2} + \int \frac{(a+b\,\mathrm{ta}}{}$$

$$= -\frac{\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^2}{2d\left(c^2+d^2\right)f(c+d\tan(e+fx))^2} + \frac{(bc-ad}{}$$

$$= \frac{\left(a^2\left(Ac^3-c^3C+3Bc^2d-3Acd^2+3cCd^2-Bd^3\right)+b}{}$$

$$= \frac{\left(a^2\left(Ac^3-c^3C+3Bc^2d-3Acd^2+3cCd^2-Bd^3\right)+b}{}$$

$$= \frac{\left(a^2\left(Ac^3-c^3C+3Bc^2d-3Acd^2+3cCd^2-Bd^3\right)+b}{}$$

Mathematica [C] time = 7.86294, size = 2499, normalized size = 4.19

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(c + d*Tan[e + f*x])^3,x]
```

```
[Out] ((-(b^2*c^4*C) + b^2*B*c^3*d + 2*a*b*c^3*C*d - A*b^2*c^2*d^2 - 2*a*b*B*c^2*
d^2 - a^2*c^2*C*d^2 + 2*a*A*b*c*d^3 + a^2*B*c*d^3 - a^2*A*d^4)*Sec[e + f*x]
*(c*Cos[e + f*x] + d*Sin[e + f*x])*(a + b*Tan[e + f*x])^2)/(2*(c - I*d)^2*(
c + I*d)^2*d*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(c + d*Tan[e + f*x])^3)
+ ((a^2*A*c^3 - A*b^2*c^3 - 2*a*b*B*c^3 - a^2*c^3*C + b^2*c^3*C + 6*a*A*b*c
^2*d + 3*a^2*B*c^2*d - 3*b^2*B*c^2*d - 6*a*b*c^2*C*d - 3*a^2*A*c*d^2 + 3*A*
b^2*c*d^2 + 6*a*b*B*c*d^2 + 3*a^2*c*C*d^2 - 3*b^2*c*C*d^2 - 2*a*A*b*d^3 - a
^2*B*d^3 + b^2*B*d^3 + 2*a*b*C*d^3)*(e + f*x)*Sec[e + f*x]*(c*Cos[e + f*x]
+ d*Sin[e + f*x])^3*(a + b*Tan[e + f*x])^2)/((c - I*d)^3*(c + I*d)^3*f*(a*C
os[e + f*x] + b*Sin[e + f*x])^2*(c + d*Tan[e + f*x])^3) + ((I*b^2*c^13*C*d^
2 + b^2*c^12*C*d^3 + (5*I)*b^2*c^11*C*d^4 - (2*I)*a*A*b*c^10*d^5 - I*a^2*B*
c^10*d^5 + I*b^2*B*c^10*d^5 + (2*I)*a*b*c^10*C*d^5 + 5*b^2*c^10*C*d^5 + (3*
I)*a^2*A*c^9*d^6 - 2*a*A*b*c^9*d^6 - (3*I)*A*b^2*c^9*d^6 - a^2*B*c^9*d^6 -
(6*I)*a*b*B*c^9*d^6 + b^2*B*c^9*d^6 - (3*I)*a^2*c^9*C*d^6 + 2*a*b*c^9*C*d^6
 + (13*I)*b^2*c^9*C*d^6 + 3*a^2*A*c^8*d^7 + (2*I)*a*A*b*c^8*d^7 - 3*A*b^2*c
^8*d^7 + I*a^2*B*c^8*d^7 - 6*a*b*B*c^8*d^7 - I*b^2*B*c^8*d^7 - 3*a^2*c^8*C*
d^7 - (2*I)*a*b*c^8*C*d^7 + 13*b^2*c^8*C*d^7 + (5*I)*a^2*A*c^7*d^8 + 2*a*A*
b*c^7*d^8 - (5*I)*A*b^2*c^7*d^8 + a^2*B*c^7*d^8 - (10*I)*a*b*B*c^7*d^8 - b^
2*B*c^7*d^8 - (5*I)*a^2*c^7*C*d^8 - 2*a*b*c^7*C*d^8 + (15*I)*b^2*c^7*C*d^8
+ 5*a^2*A*c^6*d^9 + (10*I)*a*A*b*c^6*d^9 - 5*A*b^2*c^6*d^9 + (5*I)*a^2*B*c^
6*d^9 - 10*a*b*B*c^6*d^9 - (5*I)*b^2*B*c^6*d^9 - 5*a^2*c^6*C*d^9 - (10*I)*a
*b*c^6*C*d^9 + 15*b^2*c^6*C*d^9 + I*a^2*A*c^5*d^10 + 10*a*A*b*c^5*d^10 - I*
A*b^2*c^5*d^10 + 5*a^2*B*c^5*d^10 - (2*I)*a*b*B*c^5*d^10 - 5*b^2*B*c^5*d^10
 - I*a^2*c^5*C*d^10 - 10*a*b*c^5*C*d^10 + (6*I)*b^2*c^5*C*d^10 + a^2*A*c^4*
d^11 + (6*I)*a*A*b*c^4*d^11 - A*b^2*c^4*d^11 + (3*I)*a^2*B*c^4*d^11 - 2*a*b
*B*c^4*d^11 - (3*I)*b^2*B*c^4*d^11 - a^2*c^4*C*d^11 - (6*I)*a*b*c^4*C*d^11
+ 6*b^2*c^4*C*d^11 - I*a^2*A*c^3*d^12 + 6*a*A*b*c^3*d^12 + I*A*b^2*c^3*d^12
 + 3*a^2*B*c^3*d^12 + (2*I)*a*b*B*c^3*d^12 - 3*b^2*B*c^3*d^12 + I*a^2*c^3*C
*d^12 - 6*a*b*c^3*C*d^12 - a^2*A*c^2*d^13 + A*b^2*c^2*d^13 + 2*a*b*B*c^2*d^
13 + a^2*c^2*C*d^13)*(e + f*x)*Sec[e + f*x]*(c*Cos[e + f*x] + d*Sin[e + f*x
```

```
])^3*(a + b*Tan[e + f*x])^2)/(c^2*(c - I*d)^6*(c + I*d)^5*d^5*f*(a*Cos[e +
f*x] + b*Sin[e + f*x])^2*(c + d*Tan[e + f*x])^3) - (I*(b^2*c^6*C + 3*b^2*c^
4*C*d^2 - 2*a*A*b*c^3*d^3 - a^2*B*c^3*d^3 + b^2*B*c^3*d^3 + 2*a*b*c^3*C*d^3
 + 3*a^2*A*c^2*d^4 - 3*A*b^2*c^2*d^4 - 6*a*b*B*c^2*d^4 - 3*a^2*c^2*C*d^4 +
6*b^2*c^2*C*d^4 + 6*a*A*b*c*d^5 + 3*a^2*B*c*d^5 - 3*b^2*B*c*d^5 - 6*a*b*c*C
*d^5 - a^2*A*d^6 + A*b^2*d^6 + 2*a*b*B*d^6 + a^2*C*d^6)*ArcTan[Tan[e + f*x]
]*Sec[e + f*x]*(c*Cos[e + f*x] + d*Sin[e + f*x])^3*(a + b*Tan[e + f*x])^2)/
(d^3*(c^2 + d^2)^3*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(c + d*Tan[e + f*x
])^3) - (b^2*C*Log[Cos[e + f*x]]*Sec[e + f*x]*(c*Cos[e + f*x] + d*Sin[e + f
*x])^3*(a + b*Tan[e + f*x])^2)/(d^3*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(
c + d*Tan[e + f*x])^3) + ((b^2*c^6*C + 3*b^2*c^4*C*d^2 - 2*a*A*b*c^3*d^3 -
a^2*B*c^3*d^3 + b^2*B*c^3*d^3 + 2*a*b*c^3*C*d^3 + 3*a^2*A*c^2*d^4 - 3*A*b^2
*c^2*d^4 - 6*a*b*B*c^2*d^4 - 3*a^2*c^2*C*d^4 + 6*b^2*c^2*C*d^4 + 6*a*A*b*c*
d^5 + 3*a^2*B*c*d^5 - 3*b^2*B*c*d^5 - 6*a*b*c*C*d^5 - a^2*A*d^6 + A*b^2*d^6
 + 2*a*b*B*d^6 + a^2*C*d^6)*Log[(c*Cos[e + f*x] + d*Sin[e + f*x])^2]*Sec[e
+ f*x]*(c*Cos[e + f*x] + d*Sin[e + f*x])^3*(a + b*Tan[e + f*x])^2)/(2*d^3*(
c^2 + d^2)^3*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(c + d*Tan[e + f*x])^3)
+ (Sec[e + f*x]*(c*Cos[e + f*x] + d*Sin[e + f*x])^2*(-(b^2*c^5*C*Sin[e + f*
x]) + A*b^2*c^3*d^2*Sin[e + f*x] + 2*a*b*B*c^3*d^2*Sin[e + f*x] + a^2*c^3*C
*d^2*Sin[e + f*x] - 4*b^2*c^3*C*d^2*Sin[e + f*x] - 4*a*A*b*c^2*d^3*Sin[e +
f*x] - 2*a^2*B*c^2*d^3*Sin[e + f*x] + 3*b^2*B*c^2*d^3*Sin[e + f*x] + 6*a*b*
c^2*C*d^3*Sin[e + f*x] + 3*a^2*A*c*d^4*Sin[e + f*x] - 2*A*b^2*c*d^4*Sin[e +
 f*x] - 4*a*b*B*c*d^4*Sin[e + f*x] - 2*a^2*c*C*d^4*Sin[e + f*x] + 2*a*A*b*d
^5*Sin[e + f*x] + a^2*B*d^5*Sin[e + f*x])*(a + b*Tan[e + f*x])^2)/(c*(c - I
*d)^2*(c + I*d)^2*d^2*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(c + d*Tan[e +
f*x])^3)
```

Maple [B] time = 0.085, size = 2465, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^3,x
)
```

```
[Out] -2/f/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*A*a*b*c^3+2/f/(c^2+d^2)^3*ln(c+d*tan(f*
x+e))*C*a*b*c^3+1/f/(c^2+d^2)/(c+d*tan(f*x+e))^2*A*a*c*b+2/f/(c^2+d^2)^2/(c
+d*tan(f*x+e))*A*a*b*c^2+3/f/(c^2+d^2)^3/d*ln(c+d*tan(f*x+e))*C*b^2*c^4+6/f
/(c^2+d^2)^3*d*ln(c+d*tan(f*x+e))*C*b^2*c^2-3/f/(c^2+d^2)^3*d^2*ln(c+d*tan(
f*x+e))*B*b^2*c-3/f/(c^2+d^2)^3*d*ln(c+d*tan(f*x+e))*C*a^2*c^2-3/2/f/(c^2+d
^2)^3*ln(1+tan(f*x+e)^2)*A*a^2*c^2*d+1/f/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*A*a
*b*c^3+3/2/f/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*A*b^2*c^2*d+1/f/(c^2+d^2)^3*d^3
*ln(c+d*tan(f*x+e))*C*a^2-1/2/f*d/(c^2+d^2)/(c+d*tan(f*x+e))^2*a^2*A-1/f*d^
2/(c^2+d^2)^2/(c+d*tan(f*x+e))*B*a^2-1/f/(c^2+d^2)^3*d^3*ln(c+d*tan(f*x+e))
*A*a^2+1/f/(c^2+d^2)^3*d^3*ln(c+d*tan(f*x+e))*A*b^2-1/f/(c^2+d^2)^3*ln(c+d*
tan(f*x+e))*B*a^2*c^3+1/f/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*B*b^2*c^3+1/2/f/(c
^2+d^2)/(c+d*tan(f*x+e))^2*B*a^2*c-1/f/(c^2+d^2)^3*B*arctan(tan(f*x+e))*a^2
*d^3-3/2/f/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*B*a^2*c*d^2-1/f/(c^2+d^2)^3*ln(1+
tan(f*x+e)^2)*B*a*b*d^3-2/f*d/(c^2+d^2)^2/(c+d*tan(f*x+e))*A*a^2*c-2/f*d^2/
(c^2+d^2)^2/(c+d*tan(f*x+e))*A*a*b+2/f*d/(c^2+d^2)^2/(c+d*tan(f*x+e))*A*b^2
*c-1/f/d^2/(c^2+d^2)^2/(c+d*tan(f*x+e))*B*b^2*c^4+2/f*d/(c^2+d^2)^2/(c+d*ta
n(f*x+e))*C*a^2*c+2/f/d^3/(c^2+d^2)^2/(c+d*tan(f*x+e))*C*b^2*c^5+4/f/d/(c^2
+d^2)^2/(c+d*tan(f*x+e))*C*b^2*c^3+3/f/(c^2+d^2)^3*d*ln(c+d*tan(f*x+e))*A*a
^2*c^2-3/f/(c^2+d^2)^3*d*ln(c+d*tan(f*x+e))*A*b^2*c^2+3/f/(c^2+d^2)^3*d^2*l
n(c+d*tan(f*x+e))*B*a^2*c+2/f/(c^2+d^2)^3*d^3*ln(c+d*tan(f*x+e))*B*a*b+1/f/
(c^2+d^2)^3/d^3*ln(c+d*tan(f*x+e))*C*b^2*c^6-2/f/(c^2+d^2)^3*B*arctan(tan(f
```

```
*x+e))*a*b*c^3-3/f/(c^2+d^2)^3*B*arctan(tan(f*x+e))*b^2*c^2*d+3/f/(c^2+d^2)
^3*C*arctan(tan(f*x+e))*a^2*c*d^2+2/f/(c^2+d^2)^3*C*arctan(tan(f*x+e))*a*b*
d^3-3/f/(c^2+d^2)^3*C*arctan(tan(f*x+e))*b^2*c*d^2-1/2/f/d/(c^2+d^2)/(c+d*t
an(f*x+e))^2*A*c^2*b^2+1/2/f/d^2/(c^2+d^2)/(c+d*tan(f*x+e))^2*B*b^2*c^3+3/f
/(c^2+d^2)^3*B*arctan(tan(f*x+e))*a^2*c^2*d+6/f/(c^2+d^2)^3*B*arctan(tan(f*
x+e))*a*b*c*d^2-6/f/(c^2+d^2)^3*C*arctan(tan(f*x+e))*a*b*c^2*d+3/f/(c^2+d^2
)^3*ln(1+tan(f*x+e)^2)*B*a*b*c^2*d-6/f/(c^2+d^2)^3*d^2*ln(c+d*tan(f*x+e))*C
*a*b*c+4/f*d/(c^2+d^2)^2/(c+d*tan(f*x+e))*B*a*b*c-3/f/(c^2+d^2)^3*ln(1+tan(
f*x+e)^2)*A*a*b*c*d^2+3/f/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*C*a*b*c*d^2+6/f/(c
^2+d^2)^3*d^2*ln(c+d*tan(f*x+e))*A*a*b*c+1/f/d^2/(c^2+d^2)/(c+d*tan(f*x+e))
^2*C*a*b*c^3-2/f/d^2/(c^2+d^2)^2/(c+d*tan(f*x+e))*C*a*b*c^4-1/f/d/(c^2+d^2)
/(c+d*tan(f*x+e))^2*B*c^2*a*b+6/f/(c^2+d^2)^3*A*arctan(tan(f*x+e))*a*b*c^2*
d-6/f/(c^2+d^2)^3*d*ln(c+d*tan(f*x+e))*B*a*b*c^2+3/2/f/(c^2+d^2)^3*ln(1+tan
(f*x+e)^2)*B*b^2*c*d^2+3/2/f/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*C*a^2*c^2*d-1/f
/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*C*a*b*c^3-3/2/f/(c^2+d^2)^3*ln(1+tan(f*x+e)
^2)*C*b^2*c^2*d-3/f/(c^2+d^2)^3*A*arctan(tan(f*x+e))*a^2*c*d^2-2/f/(c^2+d^2
)^3*A*arctan(tan(f*x+e))*a*b*d^3+3/f/(c^2+d^2)^3*A*arctan(tan(f*x+e))*b^2*c
*d^2-1/2/f/d/(c^2+d^2)/(c+d*tan(f*x+e))^2*C*c^2*a^2-1/2/f/d^3/(c^2+d^2)/(c+
d*tan(f*x+e))^2*b^2*C*c^4-6/f/(c^2+d^2)^2/(c+d*tan(f*x+e))*C*a*b*c^2+1/f/(c
^2+d^2)^3*B*arctan(tan(f*x+e))*b^2*d^3-1/f/(c^2+d^2)^3*C*arctan(tan(f*x+e))
*a^2*c^3+1/f/(c^2+d^2)^3*C*arctan(tan(f*x+e))*b^2*c^3+1/2/f/(c^2+d^2)^3*ln(
1+tan(f*x+e)^2)*C*b^2*d^3+1/f/(c^2+d^2)^3*A*arctan(tan(f*x+e))*a^2*c^3-1/f/
(c^2+d^2)^3*A*arctan(tan(f*x+e))*b^2*c^3+1/2/f/(c^2+d^2)^3*ln(1+tan(f*x+e)^
2)*A*a^2*d^3-1/2/f/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*A*b^2*d^3+1/2/f/(c^2+d^2)
^3*ln(1+tan(f*x+e)^2)*B*a^2*c^3+1/f/(c^2+d^2)^2/(c+d*tan(f*x+e))*B*a^2*c^2-
3/f/(c^2+d^2)^2/(c+d*tan(f*x+e))*B*b^2*c^2-1/2/f/(c^2+d^2)^3*ln(1+tan(f*x+e
)^2)*B*b^2*c^3-1/2/f/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*C*a^2*d^3
```

Maxima [A] time = 1.63636, size = 1116, normalized size = 1.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c^3 + 3*(B*a^2 + 2*(A - C)*a*
b - B*b^2)*c^2*d - 3*((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c*d^2 - (B*a^2 +
 2*(A - C)*a*b - B*b^2)*d^3)*(f*x + e)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6)
+ 2*(C*b^2*c^6 + 3*C*b^2*c^4*d^2 - (B*a^2 + 2*(A - C)*a*b - B*b^2)*c^3*d^3
+ 3*((A - C)*a^2 - 2*B*a*b - (A - 2*C)*b^2)*c^2*d^4 + 3*(B*a^2 + 2*(A - C)*
a*b - B*b^2)*c*d^5 - ((A - C)*a^2 - 2*B*a*b - A*b^2)*d^6)*log(d*tan(f*x + e
) + c)/(c^6*d^3 + 3*c^4*d^5 + 3*c^2*d^7 + d^9) + ((B*a^2 + 2*(A - C)*a*b -
B*b^2)*c^3 - 3*((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c^2*d - 3*(B*a^2 + 2*(
A - C)*a*b - B*b^2)*c*d^2 + ((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*d^3)*log(
tan(f*x + e)^2 + 1)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) + (3*C*b^2*c^6 - A*
a^2*d^6 - (2*C*a*b + B*b^2)*c^5*d - (C*a^2 + 2*B*a*b + (A - 7*C)*b^2)*c^4*d
^2 + (3*B*a^2 + 2*(3*A - 5*C)*a*b - 5*B*b^2)*c^3*d^3 - ((5*A - 3*C)*a^2 - 6
*B*a*b - 3*A*b^2)*c^2*d^4 - (B*a^2 + 2*A*a*b)*c*d^5 + 2*(2*C*b^2*c^5*d + 4*
C*b^2*c^3*d^3 - (2*C*a*b + B*b^2)*c^4*d^2 + (B*a^2 + 2*(A - 3*C)*a*b - 3*B*
b^2)*c^2*d^4 - 2*((A - C)*a^2 - 2*B*a*b - A*b^2)*c*d^5 - (B*a^2 + 2*A*a*b)*
d^6)*tan(f*x + e))/(c^6*d^3 + 2*c^4*d^5 + c^2*d^7 + (c^4*d^5 + 2*c^2*d^7 +
d^9)*tan(f*x + e)^2 + 2*(c^5*d^4 + 2*c^3*d^6 + c*d^8)*tan(f*x + e)))/f
```

Fricas [B] time = 5.45139, size = 3366, normalized size = 5.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(C*b^2*c^6*d^2 - A*a^2*d^8 + (2*C*a*b + B*b^2)*c^5*d^3 - (3*C*a^2 + 6*B
*a*b + (3*A - 7*C)*b^2)*c^4*d^4 + 5*(B*a^2 + 2*(A - C)*a*b - B*b^2)*c^3*d^5
 - ((7*A - 3*C)*a^2 - 6*B*a*b - 3*A*b^2)*c^2*d^6 - (B*a^2 + 2*A*a*b)*c*d^7
+ 2*(((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c^5*d^3 + 3*(B*a^2 + 2*(A - C)*a
*b - B*b^2)*c^4*d^4 - 3*((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c^3*d^5 - (B*
a^2 + 2*(A - C)*a*b - B*b^2)*c^2*d^6)*f*x - (3*C*b^2*c^6*d^2 + A*a^2*d^8 -
(2*C*a*b + B*b^2)*c^5*d^3 - (C*a^2 + 2*B*a*b + (A - 9*C)*b^2)*c^4*d^4 + (3*
B*a^2 + 2*(3*A - 7*C)*a*b - 7*B*b^2)*c^3*d^5 - 5*((A - C)*a^2 - 2*B*a*b - A
*b^2)*c^2*d^6 - 3*(B*a^2 + 2*A*a*b)*c*d^7 - 2*(((A - C)*a^2 - 2*B*a*b - (A
- C)*b^2)*c^3*d^5 + 3*(B*a^2 + 2*(A - C)*a*b - B*b^2)*c^2*d^6 - 3*((A - C)*
a^2 - 2*B*a*b - (A - C)*b^2)*c*d^7 - (B*a^2 + 2*(A - C)*a*b - B*b^2)*d^8)*f
*x)*tan(f*x + e)^2 + (C*b^2*c^8 + 3*C*b^2*c^6*d^2 - (B*a^2 + 2*(A - C)*a*b
- B*b^2)*c^5*d^3 + 3*((A - C)*a^2 - 2*B*a*b - (A - 2*C)*b^2)*c^4*d^4 + 3*(B
*a^2 + 2*(A - C)*a*b - B*b^2)*c^3*d^5 - ((A - C)*a^2 - 2*B*a*b - A*b^2)*c^2
*d^6 + (C*b^2*c^6*d^2 + 3*C*b^2*c^4*d^4 - (B*a^2 + 2*(A - C)*a*b - B*b^2)*c
^3*d^5 + 3*((A - C)*a^2 - 2*B*a*b - (A - 2*C)*b^2)*c^2*d^6 + 3*(B*a^2 + 2*(
A - C)*a*b - B*b^2)*c*d^7 - ((A - C)*a^2 - 2*B*a*b - A*b^2)*d^8)*tan(f*x +
e)^2 + 2*(C*b^2*c^7*d + 3*C*b^2*c^5*d^3 - (B*a^2 + 2*(A - C)*a*b - B*b^2)*c
^4*d^4 + 3*((A - C)*a^2 - 2*B*a*b - (A - 2*C)*b^2)*c^3*d^5 + 3*(B*a^2 + 2*(
A - C)*a*b - B*b^2)*c^2*d^6 - ((A - C)*a^2 - 2*B*a*b - A*b^2)*c*d^7)*tan(f*
x + e))*log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2
 + 1)) - (C*b^2*c^8 + 3*C*b^2*c^6*d^2 + 3*C*b^2*c^4*d^4 + C*b^2*c^2*d^6 + (
C*b^2*c^6*d^2 + 3*C*b^2*c^4*d^4 + 3*C*b^2*c^2*d^6 + C*b^2*d^8)*tan(f*x + e)
^2 + 2*(C*b^2*c^7*d + 3*C*b^2*c^5*d^3 + 3*C*b^2*c^3*d^5 + C*b^2*c*d^7)*tan(
f*x + e))*log(1/(tan(f*x + e)^2 + 1)) - 2*(C*b^2*c^7*d - (C*a^2 + 2*B*a*b +
 (A - 3*C)*b^2)*c^5*d^3 + (2*B*a^2 + 2*(2*A - 3*C)*a*b - 3*B*b^2)*c^4*d^4 -
 (3*(A - C)*a^2 - 6*B*a*b - (3*A - 4*C)*b^2)*c^3*d^5 - 3*(B*a^2 + 2*(A - C)
*a*b - B*b^2)*c^2*d^6 + ((3*A - 2*C)*a^2 - 4*B*a*b - 2*A*b^2)*c*d^7 + (B*a^
2 + 2*A*a*b)*d^8 - 2*(((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c^4*d^4 + 3*(B*
a^2 + 2*(A - C)*a*b - B*b^2)*c^3*d^5 - 3*((A - C)*a^2 - 2*B*a*b - (A - C)*b
^2)*c^2*d^6 - (B*a^2 + 2*(A - C)*a*b - B*b^2)*c*d^7)*f*x)*tan(f*x + e))/((c
^6*d^5 + 3*c^4*d^7 + 3*c^2*d^9 + d^11)*f*tan(f*x + e)^2 + 2*(c^7*d^4 + 3*c^
5*d^6 + 3*c^3*d^8 + c*d^10)*f*tan(f*x + e) + (c^8*d^3 + 3*c^6*d^5 + 3*c^4*d
^7 + c^2*d^9)*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x
+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.96929, size = 2307, normalized size = 3.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a^2*c^3 - C*a^2*c^3 - 2*B*a*b*c^3 - A*b^2*c^3 + C*b^2*c^3 + 3*B*a
^2*c^2*d + 6*A*a*b*c^2*d - 6*C*a*b*c^2*d - 3*B*b^2*c^2*d - 3*A*a^2*c*d^2 +
3*C*a^2*c*d^2 + 6*B*a*b*c*d^2 + 3*A*b^2*c*d^2 - 3*C*b^2*c*d^2 - B*a^2*d^3 -
 2*A*a*b*d^3 + 2*C*a*b*d^3 + B*b^2*d^3)*(f*x + e)/(c^6 + 3*c^4*d^2 + 3*c^2*
d^4 + d^6) + (B*a^2*c^3 + 2*A*a*b*c^3 - 2*C*a*b*c^3 - B*b^2*c^3 - 3*A*a^2*c
^2*d + 3*C*a^2*c^2*d + 6*B*a*b*c^2*d + 3*A*b^2*c^2*d - 3*C*b^2*c^2*d - 3*B*
a^2*c*d^2 - 6*A*a*b*c*d^2 + 6*C*a*b*c*d^2 + 3*B*b^2*c*d^2 + A*a^2*d^3 - C*a
^2*d^3 - 2*B*a*b*d^3 - A*b^2*d^3 + C*b^2*d^3)*log(tan(f*x + e)^2 + 1)/(c^6
+ 3*c^4*d^2 + 3*c^2*d^4 + d^6) + 2*(C*b^2*c^6 + 3*C*b^2*c^4*d^2 - B*a^2*c^3
*d^3 - 2*A*a*b*c^3*d^3 + 2*C*a*b*c^3*d^3 + B*b^2*c^3*d^3 + 3*A*a^2*c^2*d^4
- 3*C*a^2*c^2*d^4 - 6*B*a*b*c^2*d^4 - 3*A*b^2*c^2*d^4 + 6*C*b^2*c^2*d^4 + 3
*B*a^2*c*d^5 + 6*A*a*b*c*d^5 - 6*C*a*b*c*d^5 - 3*B*b^2*c*d^5 - A*a^2*d^6 +
C*a^2*d^6 + 2*B*a*b*d^6 + A*b^2*d^6)*log(abs(d*tan(f*x + e) + c))/(c^6*d^3
+ 3*c^4*d^5 + 3*c^2*d^7 + d^9) - (3*C*b^2*c^6*d*tan(f*x + e)^2 + 9*C*b^2*c^
4*d^3*tan(f*x + e)^2 - 3*B*a^2*c^3*d^4*tan(f*x + e)^2 - 6*A*a*b*c^3*d^4*tan
(f*x + e)^2 + 6*C*a*b*c^3*d^4*tan(f*x + e)^2 + 3*B*b^2*c^3*d^4*tan(f*x + e)
^2 + 9*A*a^2*c^2*d^5*tan(f*x + e)^2 - 9*C*a^2*c^2*d^5*tan(f*x + e)^2 - 18*B
*a*b*c^2*d^5*tan(f*x + e)^2 - 9*A*b^2*c^2*d^5*tan(f*x + e)^2 + 18*C*b^2*c^2
*d^5*tan(f*x + e)^2 + 9*B*a^2*c*d^6*tan(f*x + e)^2 + 18*A*a*b*c*d^6*tan(f*x
 + e)^2 - 18*C*a*b*c*d^6*tan(f*x + e)^2 - 9*B*b^2*c*d^6*tan(f*x + e)^2 - 3*
A*a^2*d^7*tan(f*x + e)^2 + 3*C*a^2*d^7*tan(f*x + e)^2 + 6*B*a*b*d^7*tan(f*x
 + e)^2 + 3*A*b^2*d^7*tan(f*x + e)^2 + 2*C*b^2*c^7*tan(f*x + e) + 4*C*a*b*c
^6*d*tan(f*x + e) + 2*B*b^2*c^6*d*tan(f*x + e) + 6*C*b^2*c^5*d^2*tan(f*x +
e) - 8*B*a^2*c^4*d^3*tan(f*x + e) - 16*A*a*b*c^4*d^3*tan(f*x + e) + 28*C*a*
b*c^4*d^3*tan(f*x + e) + 14*B*b^2*c^4*d^3*tan(f*x + e) + 22*A*a^2*c^3*d^4*t
an(f*x + e) - 22*C*a^2*c^3*d^4*tan(f*x + e) - 44*B*a*b*c^3*d^4*tan(f*x + e)
 - 22*A*b^2*c^3*d^4*tan(f*x + e) + 28*C*b^2*c^3*d^4*tan(f*x + e) + 18*B*a^2
*c^2*d^5*tan(f*x + e) + 36*A*a*b*c^2*d^5*tan(f*x + e) - 24*C*a*b*c^2*d^5*ta
n(f*x + e) - 12*B*b^2*c^2*d^5*tan(f*x + e) - 2*A*a^2*c*d^6*tan(f*x + e) + 2
*C*a^2*c*d^6*tan(f*x + e) + 4*B*a*b*c*d^6*tan(f*x + e) + 2*A*b^2*c*d^6*tan(
f*x + e) + 2*B*a^2*d^7*tan(f*x + e) + 4*A*a*b*d^7*tan(f*x + e) + 2*C*a*b*c^
7 + B*b^2*c^7 + C*a^2*c^6*d + 2*B*a*b*c^6*d + A*b^2*c^6*d - C*b^2*c^6*d - 6
*B*a^2*c^5*d^2 - 12*A*a*b*c^5*d^2 + 18*C*a*b*c^5*d^2 + 9*B*b^2*c^5*d^2 + 14
*A*a^2*c^4*d^3 - 11*C*a^2*c^4*d^3 - 22*B*a*b*c^4*d^3 - 11*A*b^2*c^4*d^3 + 1
1*C*b^2*c^4*d^3 + 7*B*a^2*c^3*d^4 + 14*A*a*b*c^3*d^4 - 8*C*a*b*c^3*d^4 - 4*
B*b^2*c^3*d^4 + 3*A*a^2*c^2*d^5 + B*a^2*c*d^6 + 2*A*a*b*c*d^6 + A*a^2*d^7)/
((c^6*d^2 + 3*c^4*d^4 + 3*c^2*d^6 + d^8)*(d*tan(f*x + e) + c)^2))/f
```

3.86 $\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^3}\,dx$

Optimal. Leaf size=352

$$\frac{(bc-ad)\left(Ad^2-Bcd+c^2C\right)}{2d^2f\left(c^2+d^2\right)(c+d\tan(e+fx))^2}-\frac{ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(-c^2d^2(A-3C)+Ad^4-2Bcd^3+c^4C\right)}{d^2f\left(c^2+d^2\right)^2(c+d\tan(e+fx))}+\frac{(A\cdots}{\cdots}$$

```
[Out] -(((a*(c^3*C - 3*B*c^2*d - 3*c*C*d^2 + B*d^3 - A*(c^3 - 3*c*d^2)) - b*((A -
 C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)))*x)/(c^2 + d^2)^3) + ((b*(c^3*C -
3*B*c^2*d - 3*c*C*d^2 + B*d^3) - a*(B*c^3 + 3*c^2*C*d - 3*B*c*d^2 - C*d^3)
+ A*(a*d*(3*c^2 - d^2) - b*(c^3 - 3*c*d^2)))*Log[c*Cos[e + f*x] + d*Sin[e +
 f*x]])/((c^2 + d^2)^3*f) + ((b*c - a*d)*(c^2*C - B*c*d + A*d^2))/(2*d^2*(c
^2 + d^2)*f*(c + d*Tan[e + f*x])^2) - (b*(c^4*C - c^2*(A - 3*C)*d^2 - 2*B*c
*d^3 + A*d^4) + a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)))/(d^2*(c^2 + d^2)^2*f
*(c + d*Tan[e + f*x]))
```

Rubi [A] time = 0.711263, antiderivative size = 349, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {3635, 3628, 3531, 3530}

$$\frac{(bc-ad)\left(Ad^2-Bcd+c^2C\right)}{2d^2f\left(c^2+d^2\right)(c+d\tan(e+fx))^2}-\frac{ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(-c^2d^2(A-3C)+Ad^4-2Bcd^3+c^4C\right)}{d^2f\left(c^2+d^2\right)^2(c+d\tan(e+fx))}+\frac{(a\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d*T
an[e + f*x])^3,x]
```

```
[Out] ((b*(A - C)*d*(3*c^2 - d^2) - b*B*(c^3 - 3*c*d^2) - a*(c^3*C - 3*B*c^2*d -
3*c*C*d^2 + B*d^3 - A*(c^3 - 3*c*d^2)))*x)/(c^2 + d^2)^3 + ((a*A*d*(3*c^2 -
 d^2) - A*b*(c^3 - 3*c*d^2) + b*(c^3*C - 3*B*c^2*d - 3*c*C*d^2 + B*d^3) - a
*(B*c^3 + 3*c^2*C*d - 3*B*c*d^2 - C*d^3))*Log[c*Cos[e + f*x] + d*Sin[e + f*
x]])/((c^2 + d^2)^3*f) + ((b*c - a*d)*(c^2*C - B*c*d + A*d^2))/(2*d^2*(c^2
+ d^2)*f*(c + d*Tan[e + f*x])^2) - (b*(c^4*C - c^2*(A - 3*C)*d^2 - 2*B*c*d^
3 + A*d^4) + a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)))/(d^2*(c^2 + d^2)^2*f*(c
 + d*Tan[e + f*x]))
```

Rule 3635

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(c^2*C - B*c*d + A*d^2)*(c +
d*Tan[e + f*x])^(n + 1))/(d^2*f*(n + 1)*(c^2 + d^2)), x] + Dist[1/(d*(c^2 +
 d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(c^
2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*Ta
n[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -
1]
```

Rule 3628

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2
- a*b*B + a^2*C)*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 + b^2)), x
```

```
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^3}\,dx = \frac{(bc-ad)\left(c^2C-Bcd+Ad^2\right)}{2d^2\left(c^2+d^2\right)f(c+d\tan(e+fx))^2}+\int \frac{ad(Ac-cC+Bd)+b\ldots}{\ldots}$$

$$= \frac{(bc-ad)\left(c^2C-Bcd+Ad^2\right)}{2d^2\left(c^2+d^2\right)f(c+d\tan(e+fx))^2}-\frac{b\left(c^4C-c^2(A-\ldots\right.}{\ldots}$$

$$= \frac{\left(b(A-C)d\left(3c^2-d^2\right)-bB\left(c^3-3cd^2\right)-a\left(c^3C-3Bc\ldots\right.\right.}{\left(c^2+d^2\right)^3}$$

$$= \frac{\left(b(A-C)d\left(3c^2-d^2\right)-bB\left(c^3-3cd^2\right)-a\left(c^3C-3Bc\ldots\right.\right.}{\left(c^2+d^2\right)^3}$$

Mathematica [C] time = 6.02578, size = 331, normalized size = 0.94

$$\frac{2d(aB+Ab-bC)\left(\frac{d\left(2c\log(c+d\tan(e+fx))-\frac{c^2+d^2}{c+d\tan(e+fx)}\right)}{\left(c^2+d^2\right)^2}-\frac{i\log(-\tan(e+fx)+i)}{2(c+id)^2}+\frac{i\log(\tan(e+fx)+i)}{2(c-id)^2}\right)-d(-aAd+aBc+aCd+Abc+\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c
 + d*Tan[e + f*x])^3,x]
```

```
[Out] ((a*C*d - b*(c*C + B*d))/(c + d*Tan[e + f*x])^2 - (2*C*d*(a + b*Tan[e + f*x
]))/(c + d*Tan[e + f*x])^2 + 2*(A*b + a*B - b*C)*d*(((-I/2)*Log[I - Tan[e +
 f*x]])/(c + I*d)^2 + ((I/2)*Log[I + Tan[e + f*x]])/(c - I*d)^2 + (d*(2*c*L
og[c + d*Tan[e + f*x]] - (c^2 + d^2)/(c + d*Tan[e + f*x])))/(c^2 + d^2)^2)
- d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*(Log[I - Tan[e + f*x]]/
((-I)*c + d)^3 + Log[I + Tan[e + f*x]]/(I*c + d)^3 + (d*((6*c^2 - 2*d^2)*Lo
g[c + d*Tan[e + f*x]] - ((c^2 + d^2)*(5*c^2 + d^2 + 4*c*d*Tan[e + f*x]))/(c
 + d*Tan[e + f*x])^2))/(c^2 + d^2)^3))/(2*d^2*f)
```

Maple [B] time = 0.07, size = 1513, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^3,x)
```

```
[Out] 3/2/f/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*C*b*c*d^2-2/f/(c^2+d^2)^2*d/(c+d*tan(f
*x+e))*A*a*c+2/f/(c^2+d^2)^2*d/(c+d*tan(f*x+e))*B*b*c+2/f/(c^2+d^2)^2*d/(c+
d*tan(f*x+e))*C*a*c-1/f/(c^2+d^2)^2/d^2/(c+d*tan(f*x+e))*C*b*c^4-3/f/(c^2+d
^2)^3*A*arctan(tan(f*x+e))*a*c*d^2+3/f/(c^2+d^2)^3*A*arctan(tan(f*x+e))*b*c
^2*d+3/f/(c^2+d^2)^3*B*arctan(tan(f*x+e))*a*c^2*d+3/f/(c^2+d^2)^3*B*arctan(
tan(f*x+e))*b*c*d^2+3/f/(c^2+d^2)^3*C*arctan(tan(f*x+e))*a*c*d^2-3/f/(c^2+d
^2)^3*ln(c+d*tan(f*x+e))*C*b*c*d^2-1/2/f/d/(c^2+d^2)/(c+d*tan(f*x+e))^2*B*b
*c^2-1/2/f/d/(c^2+d^2)/(c+d*tan(f*x+e))^2*C*a*c^2-3/2/f/(c^2+d^2)^3*ln(1+ta
n(f*x+e)^2)*A*a*c^2*d+3/f/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*A*a*c^2*d+3/f/(c^2
+d^2)^3*ln(c+d*tan(f*x+e))*A*b*c*d^2+3/f/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*B*a
*c*d^2-3/f/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*B*b*c^2*d+1/2/f/d^2/(c^2+d^2)/(c+
d*tan(f*x+e))^2*C*b*c^3-3/2/f/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*A*b*c*d^2-3/2/
f/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*B*a*c*d^2+3/2/f/(c^2+d^2)^3*ln(1+tan(f*x+e
)^2)*B*b*c^2*d+3/2/f/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*C*a*c^2*d-1/2/f*d/(c^2+
d^2)/(c+d*tan(f*x+e))^2*A*a-1/f/(c^2+d^2)^3*B*arctan(tan(f*x+e))*b*c^3-1/f/
(c^2+d^2)^3*C*arctan(tan(f*x+e))*a*c^3+1/f/(c^2+d^2)^3*C*arctan(tan(f*x+e))
*b*d^3+1/2/f/(c^2+d^2)/(c+d*tan(f*x+e))^2*A*b*c+1/2/f/(c^2+d^2)/(c+d*tan(f*
x+e))^2*B*a*c+1/2/f/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*A*a*d^3+1/2/f/(c^2+d^2)^
3*ln(1+tan(f*x+e)^2)*A*b*c^3+1/2/f/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*B*a*c^3-1
/f/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*A*a*d^3-1/f/(c^2+d^2)^3*ln(c+d*tan(f*x+e)
)*A*b*c^3-1/f/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*B*a*c^3+1/f/(c^2+d^2)^3*ln(c+d
*tan(f*x+e))*B*b*d^3+1/f/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*C*a*d^3+1/f/(c^2+d^
2)^2/(c+d*tan(f*x+e))*A*b*c^2+1/f/(c^2+d^2)^2/(c+d*tan(f*x+e))*B*a*c^2-3/f/
(c^2+d^2)^2/(c+d*tan(f*x+e))*C*b*c^2+1/f/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*C*b
*c^3-1/f/(c^2+d^2)^2*d^2/(c+d*tan(f*x+e))*A*b-1/f/(c^2+d^2)^2*d^2/(c+d*tan(
f*x+e))*B*a-1/2/f/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*B*b*d^3-1/2/f/(c^2+d^2)^3*
ln(1+tan(f*x+e)^2)*C*a*d^3-1/2/f/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*C*b*c^3+1/f
/(c^2+d^2)^3*A*arctan(tan(f*x+e))*a*c^3-1/f/(c^2+d^2)^3*A*arctan(tan(f*x+e)
)*b*d^3-1/f/(c^2+d^2)^3*B*arctan(tan(f*x+e))*a*d^3-3/f/(c^2+d^2)^3*C*arctan
(tan(f*x+e))*b*c^2*d-3/f/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*C*a*c^2*d
```

Maxima [A] time = 1.53331, size = 733, normalized size = 2.08

$$\frac{\frac{2\left(((A-C)a-Bb)c^3+3(Ba+(A-C)b)c^2d-3((A-C)a-Bb)cd^2-(Ba+(A-C)b)d^3\right)(fx+e)}{c^6+3c^4d^2+3c^2d^4+d^6} - \frac{2\left((Ba+(A-C)b)c^3-3((A-C)a-Bb)c^2d-3(Ba+(A-C)b)cd^2+((A-C)a\cdots\right.}{c^6+3c^4d^2+3c^2d^4+d^6}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(((A - C)*a - B*b)*c^3 + 3*(B*a + (A - C)*b)*c^2*d - 3*((A - C)*a -
B*b)*c*d^2 - (B*a + (A - C)*b)*d^3)*(f*x + e)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4
+ d^6) - 2*((B*a + (A - C)*b)*c^3 - 3*((A - C)*a - B*b)*c^2*d - 3*(B*a + (A
 - C)*b)*c*d^2 + ((A - C)*a - B*b)*d^3)*log(d*tan(f*x + e) + c)/(c^6 + 3*c^
```

```
4*d^2 + 3*c^2*d^4 + d^6) + ((B*a + (A - C)*b)*c^3 - 3*((A - C)*a - B*b)*c^2
*d - 3*(B*a + (A - C)*b)*c*d^2 + ((A - C)*a - B*b)*d^3)*log(tan(f*x + e)^2
+ 1)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) - (C*b*c^5 + A*a*d^5 + (C*a + B*b)
*c^4*d - (3*B*a + (3*A - 5*C)*b)*c^3*d^2 + ((5*A - 3*C)*a - 3*B*b)*c^2*d^3
+ (B*a + A*b)*c*d^4 + 2*(C*b*c^4*d - (B*a + (A - 3*C)*b)*c^2*d^3 + 2*((A -
C)*a - B*b)*c*d^4 + (B*a + A*b)*d^5)*tan(f*x + e))/(c^6*d^2 + 2*c^4*d^4 + c
^2*d^6 + (c^4*d^4 + 2*c^2*d^6 + d^8)*tan(f*x + e)^2 + 2*(c^5*d^3 + 2*c^3*d^
5 + c*d^7)*tan(f*x + e)))/f
```

Fricas [B] time = 1.63896, size = 1905, normalized size = 5.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^3,x, algorithm="fricas")
```

```
[Out] 1/2*(C*b*c^5 - A*a*d^5 - 3*(C*a + B*b)*c^4*d + 5*(B*a + (A - C)*b)*c^3*d^2
- ((7*A - 3*C)*a - 3*B*b)*c^2*d^3 - (B*a + A*b)*c*d^4 + 2*(((A - C)*a - B*b
)*c^5 + 3*(B*a + (A - C)*b)*c^4*d - 3*((A - C)*a - B*b)*c^3*d^2 - (B*a + (A
 - C)*b)*c^2*d^3)*f*x + (C*b*c^5 - A*a*d^5 + (C*a + B*b)*c^4*d - (3*B*a + (
3*A - 7*C)*b)*c^3*d^2 + 5*((A - C)*a - B*b)*c^2*d^3 + 3*(B*a + A*b)*c*d^4 +
 2*(((A - C)*a - B*b)*c^3*d^2 + 3*(B*a + (A - C)*b)*c^2*d^3 - 3*((A - C)*a
- B*b)*c*d^4 - (B*a + (A - C)*b)*d^5)*f*x)*tan(f*x + e)^2 - ((B*a + (A - C)
*b)*c^5 - 3*((A - C)*a - B*b)*c^4*d - 3*(B*a + (A - C)*b)*c^3*d^2 + ((A - C
)*a - B*b)*c^2*d^3 + ((B*a + (A - C)*b)*c^3*d^2 - 3*((A - C)*a - B*b)*c^2*d
^3 - 3*(B*a + (A - C)*b)*c*d^4 + ((A - C)*a - B*b)*d^5)*tan(f*x + e)^2 + 2*
((B*a + (A - C)*b)*c^4*d - 3*((A - C)*a - B*b)*c^3*d^2 - 3*(B*a + (A - C)*b
)*c^2*d^3 + ((A - C)*a - B*b)*c*d^4)*tan(f*x + e))*log((d^2*tan(f*x + e)^2
+ 2*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) + 2*((C*a + B*b)*c^5 - (2
*B*a + (2*A - 3*C)*b)*c^4*d + 3*((A - C)*a - B*b)*c^3*d^2 + 3*(B*a + (A - C
)*b)*c^2*d^3 - ((3*A - 2*C)*a - 2*B*b)*c*d^4 - (B*a + A*b)*d^5 + 2*(((A - C
)*a - B*b)*c^4*d + 3*(B*a + (A - C)*b)*c^3*d^2 - 3*((A - C)*a - B*b)*c^2*d^
3 - (B*a + (A - C)*b)*c*d^4)*f*x)*tan(f*x + e))/((c^6*d^2 + 3*c^4*d^4 + 3*c
^2*d^6 + d^8)*f*tan(f*x + e)^2 + 2*(c^7*d + 3*c^5*d^3 + 3*c^3*d^5 + c*d^7)*
f*tan(f*x + e) + (c^8 + 3*c^6*d^2 + 3*c^4*d^4 + c^2*d^6)*f)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e)
)**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.82195, size = 1400, normalized size = 3.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a*c^3 - C*a*c^3 - B*b*c^3 + 3*B*a*c^2*d + 3*A*b*c^2*d - 3*C*b*c^2
*d - 3*A*a*c*d^2 + 3*C*a*c*d^2 + 3*B*b*c*d^2 - B*a*d^3 - A*b*d^3 + C*b*d^3)
*(f*x + e)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) + (B*a*c^3 + A*b*c^3 - C*b*c
^3 - 3*A*a*c^2*d + 3*C*a*c^2*d + 3*B*b*c^2*d - 3*B*a*c*d^2 - 3*A*b*c*d^2 +
3*C*b*c*d^2 + A*a*d^3 - C*a*d^3 - B*b*d^3)*log(tan(f*x + e)^2 + 1)/(c^6 + 3
*c^4*d^2 + 3*c^2*d^4 + d^6) - 2*(B*a*c^3*d + A*b*c^3*d - C*b*c^3*d - 3*A*a*
c^2*d^2 + 3*C*a*c^2*d^2 + 3*B*b*c^2*d^2 - 3*B*a*c*d^3 - 3*A*b*c*d^3 + 3*C*b
*c*d^3 + A*a*d^4 - C*a*d^4 - B*b*d^4)*log(abs(d*tan(f*x + e) + c))/(c^6*d +
 3*c^4*d^3 + 3*c^2*d^5 + d^7) + (3*B*a*c^3*d^4*tan(f*x + e)^2 + 3*A*b*c^3*d
^4*tan(f*x + e)^2 - 3*C*b*c^3*d^4*tan(f*x + e)^2 - 9*A*a*c^2*d^5*tan(f*x +
e)^2 + 9*C*a*c^2*d^5*tan(f*x + e)^2 + 9*B*b*c^2*d^5*tan(f*x + e)^2 - 9*B*a*
c*d^6*tan(f*x + e)^2 - 9*A*b*c*d^6*tan(f*x + e)^2 + 9*C*b*c*d^6*tan(f*x + e
)^2 + 3*A*a*d^7*tan(f*x + e)^2 - 3*C*a*d^7*tan(f*x + e)^2 - 3*B*b*d^7*tan(f
*x + e)^2 - 2*C*b*c^6*d*tan(f*x + e) + 8*B*a*c^4*d^3*tan(f*x + e) + 8*A*b*c
^4*d^3*tan(f*x + e) - 14*C*b*c^4*d^3*tan(f*x + e) - 22*A*a*c^3*d^4*tan(f*x
+ e) + 22*C*a*c^3*d^4*tan(f*x + e) + 22*B*b*c^3*d^4*tan(f*x + e) - 18*B*a*c
^2*d^5*tan(f*x + e) - 18*A*b*c^2*d^5*tan(f*x + e) + 12*C*b*c^2*d^5*tan(f*x
+ e) + 2*A*a*c*d^6*tan(f*x + e) - 2*C*a*c*d^6*tan(f*x + e) - 2*B*b*c*d^6*ta
n(f*x + e) - 2*B*a*d^7*tan(f*x + e) - 2*A*b*d^7*tan(f*x + e) - C*b*c^7 - C*
a*c^6*d - B*b*c^6*d + 6*B*a*c^5*d^2 + 6*A*b*c^5*d^2 - 9*C*b*c^5*d^2 - 14*A*
a*c^4*d^3 + 11*C*a*c^4*d^3 + 11*B*b*c^4*d^3 - 7*B*a*c^3*d^4 - 7*A*b*c^3*d^4
 + 4*C*b*c^3*d^4 - 3*A*a*c^2*d^5 - B*a*c*d^6 - A*b*c*d^6 - A*a*d^7)/((c^6*d
^2 + 3*c^4*d^4 + 3*c^2*d^6 + d^8)*(d*tan(f*x + e) + c)^2))/f
```

3.87 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^3}\,dx$

Optimal. Leaf size=209

$$-\frac{Ad^2-Bcd+c^2C}{2df\left(c^2+d^2\right)(c+d\tan(e+fx))^2}-\frac{2cd(A-C)-B\left(c^2-d^2\right)}{f\left(c^2+d^2\right)^2(c+d\tan(e+fx))}+\frac{\left(d(A-C)\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)\log(c\cos(}{f\left(c^2+d^2\right)^3}$$

```
[Out] -(((c^3*C - 3*B*c^2*d - 3*c*C*d^2 + B*d^3 - A*(c^3 - 3*c*d^2))*x)/(c^2 + d^
2)^3) + (((A - C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2))*Log[c*Cos[e + f*x] +
 d*Sin[e + f*x]])/((c^2 + d^2)^3*f) - (c^2*C - B*c*d + A*d^2)/(2*d*(c^2 + d
^2)*f*(c + d*Tan[e + f*x])^2) - (2*c*(A - C)*d - B*(c^2 - d^2))/((c^2 + d^2
)^2*f*(c + d*Tan[e + f*x]))
```

Rubi [A] time = 0.375888, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {3628, 3529, 3531, 3530}

$$-\frac{Ad^2-Bcd+c^2C}{2df\left(c^2+d^2\right)(c+d\tan(e+fx))^2}-\frac{2cd(A-C)-B\left(c^2-d^2\right)}{f\left(c^2+d^2\right)^2(c+d\tan(e+fx))}+\frac{\left(d(A-C)\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)\log(c\cos(}{f\left(c^2+d^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(c + d*Tan[e + f*x])^3,x]
```

```
[Out] -(((c^3*C - 3*B*c^2*d - 3*c*C*d^2 + B*d^3 - A*(c^3 - 3*c*d^2))*x)/(c^2 + d^
2)^3) + (((A - C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2))*Log[c*Cos[e + f*x] +
 d*Sin[e + f*x]])/((c^2 + d^2)^3*f) - (c^2*C - B*c*d + A*d^2)/(2*d*(c^2 + d
^2)*f*(c + d*Tan[e + f*x])^2) - (2*c*(A - C)*d - B*(c^2 - d^2))/((c^2 + d^2
)^2*f*(c + d*Tan[e + f*x]))
```

Rule 3628

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2
- a*b*B + a^2*C)*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 + b^2)), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^3}\,dx = -\frac{c^2C-Bcd+Ad^2}{2d\left(c^2+d^2\right)f(c+d\tan(e+fx))^2}+\frac{\int \frac{Ac-cC+Bd+(Bc-(A-C)d)\tan(e+fx)}{(c+d\tan(e+fx))^2}\,dx}{c^2+d^2}$$

$$= -\frac{c^2C-Bcd+Ad^2}{2d\left(c^2+d^2\right)f(c+d\tan(e+fx))^2}-\frac{2c(A-C)d-B\left(c^2-d^2\right)}{\left(c^2+d^2\right)^2f(c+d\tan(e+fx))}+$$

$$= \frac{\left(Ac^3-c^3C+3Bc^2d-3Acd^2+3cCd^2-Bd^3\right)x}{\left(c^2+d^2\right)^3}-\frac{c^2C-Bcd+}{2d\left(c^2+d^2\right)f(c+d\tan}$$

$$= \frac{\left(Ac^3-c^3C+3Bc^2d-3Acd^2+3cCd^2-Bd^3\right)x}{\left(c^2+d^2\right)^3}+\frac{\left((A-C)d\left(3c^2-d^2\right)\right.}{}$$

Mathematica [C] time = 4.63189, size = 261, normalized size = 1.25

$$-\frac{-(d(C-A)+Bc)\left(\frac{d\left(\frac{\left(c^2+d^2\right)\left(5c^2+4cd\tan(e+fx)+d^2\right)}{(c+d\tan(e+fx))^2}+\left(2d^2-6c^2\right)\log(c+d\tan(e+fx))\right)}{\left(c^2+d^2\right)^3}+\frac{i\log(-\tan(e+fx)+i)}{(c+id)^3}-\frac{\log(\tan(e+fx)+i)}{(d+ic)^3}\right)+B\left(\frac{2d\left(\frac{}{c+d}\right.}{}\right.}{2df}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(c + d*Tan[e + f*x])^3,x]
```

```
[Out] -(C/(c + d*Tan[e + f*x])^2 + B*((I*Log[I - Tan[e + f*x]])/(c + I*d)^2 - (I*
Log[I + Tan[e + f*x]])/(c - I*d)^2 + (2*d*(-2*c*Log[c + d*Tan[e + f*x]] + (
c^2 + d^2)/(c + d*Tan[e + f*x])))/(c^2 + d^2)^2) - (B*c + (-A + C)*d)*((I*L
og[I - Tan[e + f*x]])/(c + I*d)^3 - Log[I + Tan[e + f*x]]/(I*c + d)^3 + (d*
((-6*c^2 + 2*d^2)*Log[c + d*Tan[e + f*x]] + ((c^2 + d^2)*(5*c^2 + d^2 + 4*c
*d*Tan[e + f*x]))/(c + d*Tan[e + f*x])^2))/(c^2 + d^2)^3))/(2*d*f)
```

Maple [B] time = 0.055, size = 713, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^3,x)
```

```
[Out] -2/f/(c^2+d^2)^2/(c+d*tan(f*x+e))*A*c*d+2/f/(c^2+d^2)^2/(c+d*tan(f*x+e))*c*
C*d-3/f/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*C*c^2*d-1/2/f/(c^2+d^2)/d/(c+d*tan(f
*x+e))^2*c^2*C-1/f/(c^2+d^2)^3*C*arctan(tan(f*x+e))*c^3+1/2/f/(c^2+d^2)^3*l
n(1+tan(f*x+e)^2)*B*c^3-1/2/f/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*C*d^3+1/f/(c^2
+d^2)^3*A*arctan(tan(f*x+e))*c^3-1/f/(c^2+d^2)^3*B*arctan(tan(f*x+e))*d^3-1
```

```
/2/f/(c^2+d^2)*d/(c+d*tan(f*x+e))^2*A+1/2/f/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*
A*d^3+3/f/(c^2+d^2)^3*C*arctan(tan(f*x+e))*c*d^2-3/2/f/(c^2+d^2)^3*ln(1+tan
(f*x+e)^2)*A*c^2*d-3/2/f/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*B*c*d^2+1/f/(c^2+d^
2)^2/(c+d*tan(f*x+e))*B*c^2-1/f/(c^2+d^2)^2/(c+d*tan(f*x+e))*B*d^2-1/f/(c^2
+d^2)^3*ln(c+d*tan(f*x+e))*A*d^3-1/f/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*B*c^3+1
/f/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*C*d^3+1/2/f/(c^2+d^2)/(c+d*tan(f*x+e))^2*
B*c+3/f/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*B*c*d^2+3/2/f/(c^2+d^2)^3*ln(1+tan(f
*x+e)^2)*C*c^2*d+3/f/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*A*c^2*d-3/f/(c^2+d^2)^3
*A*arctan(tan(f*x+e))*c*d^2+3/f/(c^2+d^2)^3*B*arctan(tan(f*x+e))*c^2*d
```

Maxima [A] time = 1.51475, size = 495, normalized size = 2.37

$$\frac{\frac{2\left((A-C)c^3+3Bc^2d-3(A-C)cd^2-Bd^3\right)(fx+e)}{c^6+3c^4d^2+3c^2d^4+d^6}-\frac{2\left(Bc^3-3(A-C)c^2d-3Bcd^2+(A-C)d^3\right)\log\left(d\tan\left(fx+e\right)+c\right)}{c^6+3c^4d^2+3c^2d^4+d^6}+\frac{\left(Bc^3-3(A-C)c^2d-3Bcd^2+(A-C)d^3\right)\log\left(\tan\left(\right.\right.}{c^6+3c^4d^2+3c^2d^4+d^6}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^3,x, algorithm="
maxima")
```

```
[Out] 1/2*(2*((A - C)*c^3 + 3*B*c^2*d - 3*(A - C)*c*d^2 - B*d^3)*(f*x + e)/(c^6 +
 3*c^4*d^2 + 3*c^2*d^4 + d^6) - 2*(B*c^3 - 3*(A - C)*c^2*d - 3*B*c*d^2 + (A
 - C)*d^3)*log(d*tan(f*x + e) + c)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) + (B
*c^3 - 3*(A - C)*c^2*d - 3*B*c*d^2 + (A - C)*d^3)*log(tan(f*x + e)^2 + 1)/(
c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) - (C*c^4 - 3*B*c^3*d + (5*A - 3*C)*c^2*d
^2 + B*c*d^3 + A*d^4 - 2*(B*c^2*d^2 - 2*(A - C)*c*d^3 - B*d^4)*tan(f*x + e)
)/(c^6*d + 2*c^4*d^3 + c^2*d^5 + (c^4*d^3 + 2*c^2*d^5 + d^7)*tan(f*x + e)^2
 + 2*(c^5*d^2 + 2*c^3*d^4 + c*d^6)*tan(f*x + e)))/f
```

Fricas [B] time = 1.35637, size = 1215, normalized size = 5.81

$$-\frac{3Cc^4d-5Bc^3d^2+(7A-3C)c^2d^3+Bcd^4+Ad^5-2\left((A-C)c^5+3Bc^4d-3(A-C)c^3d^2-Bc^2d^3\right)fx-\left(Cc^4d-3Bc^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^3,x, algorithm="
fricas")
```

```
[Out] -1/2*(3*C*c^4*d - 5*B*c^3*d^2 + (7*A - 3*C)*c^2*d^3 + B*c*d^4 + A*d^5 - 2*(
(A - C)*c^5 + 3*B*c^4*d - 3*(A - C)*c^3*d^2 - B*c^2*d^3)*f*x - (C*c^4*d - 3
*B*c^3*d^2 + 5*(A - C)*c^2*d^3 + 3*B*c*d^4 - A*d^5 + 2*((A - C)*c^3*d^2 + 3
*B*c^2*d^3 - 3*(A - C)*c*d^4 - B*d^5)*f*x)*tan(f*x + e)^2 + (B*c^5 - 3*(A -
 C)*c^4*d - 3*B*c^3*d^2 + (A - C)*c^2*d^3 + (B*c^3*d^2 - 3*(A - C)*c^2*d^3
- 3*B*c*d^4 + (A - C)*d^5)*tan(f*x + e)^2 + 2*(B*c^4*d - 3*(A - C)*c^3*d^2
- 3*B*c^2*d^3 + (A - C)*c*d^4)*tan(f*x + e))*log((d^2*tan(f*x + e)^2 + 2*c*
d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) - 2*(C*c^5 - 2*B*c^4*d + 3*(A -
 C)*c^3*d^2 + 3*B*c^2*d^3 - (3*A - 2*C)*c*d^4 - B*d^5 + 2*((A - C)*c^4*d +
3*B*c^3*d^2 - 3*(A - C)*c^2*d^3 - B*c*d^4)*f*x)*tan(f*x + e))/((c^6*d^2 + 3
*c^4*d^4 + 3*c^2*d^6 + d^8)*f*tan(f*x + e)^2 + 2*(c^7*d + 3*c^5*d^3 + 3*c^3
*d^5 + c*d^7)*f*tan(f*x + e) + (c^8 + 3*c^6*d^2 + 3*c^4*d^4 + c^2*d^6)*f)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.96573, size = 740, normalized size = 3.54

$$\frac{\frac{2\,(Ac^3-Cc^3+3\,Bc^2d-3\,Acd^2+3\,Ccd^2-Bd^3)(fx+e)}{c^6+3\,c^4d^2+3\,c^2d^4+d^6}+\frac{(Bc^3-3\,Ac^2d+3\,Cc^2d-3\,Bcd^2+Ad^3-Cd^3)\log\left(\tan(fx+e)^2+1\right)}{c^6+3\,c^4d^2+3\,c^2d^4+d^6}-\frac{2\,(Bc^3d-3\,Ac^2d^2+3\,Cc^2d^2-3\,Bcd^3+\cdots}{c^6d+3\,c^4d^3+3\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^3,x, algorithm="
giac")
```

```
[Out] 1/2*(2*(A*c^3 - C*c^3 + 3*B*c^2*d - 3*A*c*d^2 + 3*C*c*d^2 - B*d^3)*(f*x + e
)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) + (B*c^3 - 3*A*c^2*d + 3*C*c^2*d - 3*
B*c*d^2 + A*d^3 - C*d^3)*log(tan(f*x + e)^2 + 1)/(c^6 + 3*c^4*d^2 + 3*c^2*d
^4 + d^6) - 2*(B*c^3*d - 3*A*c^2*d^2 + 3*C*c^2*d^2 - 3*B*c*d^3 + A*d^4 - C*
d^4)*log(abs(d*tan(f*x + e) + c))/(c^6*d + 3*c^4*d^3 + 3*c^2*d^5 + d^7) + (
3*B*c^3*d^3*tan(f*x + e)^2 - 9*A*c^2*d^4*tan(f*x + e)^2 + 9*C*c^2*d^4*tan(f
*x + e)^2 - 9*B*c*d^5*tan(f*x + e)^2 + 3*A*d^6*tan(f*x + e)^2 - 3*C*d^6*tan
(f*x + e)^2 + 8*B*c^4*d^2*tan(f*x + e) - 22*A*c^3*d^3*tan(f*x + e) + 22*C*c
^3*d^3*tan(f*x + e) - 18*B*c^2*d^4*tan(f*x + e) + 2*A*c*d^5*tan(f*x + e) -
2*C*c*d^5*tan(f*x + e) - 2*B*d^6*tan(f*x + e) - C*c^6 + 6*B*c^5*d - 14*A*c^
4*d^2 + 11*C*c^4*d^2 - 7*B*c^3*d^3 - 3*A*c^2*d^4 - B*c*d^5 - A*d^6)/((c^6*d
 + 3*c^4*d^3 + 3*c^2*d^5 + d^7)*(d*tan(f*x + e) + c)^2))/f
```

3.88 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^3}\,dx$

Optimal. Leaf size=487

$$-\frac{\left(a^2d^3\left(d(A-C)\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)-abd^2\left(8c^3d(A-C)-B\left(-6c^2d^2+3c^4-d^4\right)\right)+b^2\left(3c^4d^2(2A-C)+3Ac^2\cdots\right.\right.}{f\left(c^2+d^2\right)^3(bc-ad)^3}$$

```
[Out] -(((a*(c^3*C - 3*B*c^2*d - 3*c*C*d^2 + B*d^3 - A*(c^3 - 3*c*d^2)) + b*((A -
 C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)))*x)/((a^2 + b^2)*(c^2 + d^2)^3)) +
 (b^2*(A*b^2 - a*(b*B - a*C))*Log[a*Cos[e + f*x] + b*Sin[e + f*x]])/((a^2 +
 b^2)*(b*c - a*d)^3*f) - ((b^2*(c^6*C - 3*B*c^5*d + 3*c^4*(2*A - C)*d^2 + B
*c^3*d^3 + 3*A*c^2*d^4 + A*d^6) + a^2*d^3*((A - C)*d*(3*c^2 - d^2) - B*(c^3
 - 3*c*d^2)) - a*b*d^2*(8*c^3*(A - C)*d - B*(3*c^4 - 6*c^2*d^2 - d^4)))*Log
[c*Cos[e + f*x] + d*Sin[e + f*x]])/((b*c - a*d)^3*(c^2 + d^2)^3*f) + (c^2*C
 - B*c*d + A*d^2)/(2*(b*c - a*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^2) + (b
*(c^4*C - 2*B*c^3*d + c^2*(3*A - C)*d^2 + A*d^4) - a*d^2*(2*c*(A - C)*d - B
*(c^2 - d^2)))/((b*c - a*d)^2*(c^2 + d^2)^2*f*(c + d*Tan[e + f*x]))
```

Rubi [A] time = 1.82961, antiderivative size = 487, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3649, 3651, 3530}

$$-\frac{\left(a^2d^3\left(d(A-C)\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)-abd^2\left(8c^3d(A-C)-B\left(-6c^2d^2+3c^4-d^4\right)\right)+b^2\left(3c^4d^2(2A-C)+3Ac^2\cdots\right.\right.}{f\left(c^2+d^2\right)^3(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])*(c + d*Ta
n[e + f*x])^3),x]
```

```
[Out] -(((a*(c^3*C - 3*B*c^2*d - 3*c*C*d^2 + B*d^3 - A*(c^3 - 3*c*d^2)) + b*((A -
 C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)))*x)/((a^2 + b^2)*(c^2 + d^2)^3)) +
 (b^2*(A*b^2 - a*(b*B - a*C))*Log[a*Cos[e + f*x] + b*Sin[e + f*x]])/((a^2 +
 b^2)*(b*c - a*d)^3*f) - ((b^2*(c^6*C - 3*B*c^5*d + 3*c^4*(2*A - C)*d^2 + B
*c^3*d^3 + 3*A*c^2*d^4 + A*d^6) + a^2*d^3*((A - C)*d*(3*c^2 - d^2) - B*(c^3
 - 3*c*d^2)) - a*b*d^2*(8*c^3*(A - C)*d - B*(3*c^4 - 6*c^2*d^2 - d^4)))*Log
[c*Cos[e + f*x] + d*Sin[e + f*x]])/((b*c - a*d)^3*(c^2 + d^2)^3*f) + (c^2*C
 - B*c*d + A*d^2)/(2*(b*c - a*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^2) + (b
*(c^4*C - 2*B*c^3*d + c^2*(3*A - C)*d^2 + A*d^4) - a*d^2*(2*c*(A - C)*d - B
*(c^2 - d^2)))/((b*c - a*d)^2*(c^2 + d^2)^2*f*(c + d*Tan[e + f*x]))
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
 b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3651

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[((a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*x)
/((a^2 + b^2)*(c^2 + d^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^3}\,dx = \frac{c^2C-Bcd+Ad^2}{2(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))^2} + \int \frac{-2\left(aAcd-ad(cC-Bd)-Ab\left(c^2\cdots\right.\right.}{\cdots}$$

$$= \frac{c^2C-Bcd+Ad^2}{2(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))^2} + \frac{b\left(c^4C-2Bc^3d+c^2(3A\cdots\right.}{(bc-ad)\cdots}$$

$$= -\frac{\left(b(A-C)d\left(3c^2-d^2\right)-bB\left(c^3-3cd^2\right)-a\left(Ac^3-c^3C+3Bc^2d-3\cdots\right.\right.}{\left(a^2+b^2\right)\left(c^2+d^2\right)^3}$$

$$= -\frac{\left(b(A-C)d\left(3c^2-d^2\right)-bB\left(c^3-3cd^2\right)-a\left(Ac^3-c^3C+3Bc^2d-3\cdots\right.\right.}{\left(a^2+b^2\right)\left(c^2+d^2\right)^3}$$

Mathematica [A] time = 8.88245, size = 912, normalized size = 1.87

$$-\frac{Ad^2-c(Bd-cC)}{2(ad-bc)\left(c^2+d^2\right)f(c+d\tan(e+fx))^2} - \frac{-\frac{-2\left(aAcd-a(cC-Bd)d-Ab\left(c^2+d^2\right)\right)d^2-c\left(2d(bc-ad)(Bc-(A-C)d)-2bc\left(Cc^2-Bdc+Ad^2\right)\right)}{(ad-bc)\left(c^2+d^2\right)f(c+d\tan(e+fx))} - \frac{2(\cdots}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])*(c
+ d*Tan[e + f*x])^3),x]
```

```
[Out] -(A*d^2 - c*(-(c*C) + B*d))/(2*(-(b*c) + a*d)*(c^2 + d^2)*f*(c + d*Tan[e +
f*x])^2) - (-((-((b*(b*c - a*d)^2*(A*b*c^3 - a*B*c^3 - b*c^3*C + 3*a*A*c^2*
d + 3*b*B*c^2*d - 3*a*c^2*C*d - 3*A*b*c*d^2 + 3*a*B*c*d^2 + 3*b*c*C*d^2 - a
*A*d^3 - b*B*d^3 + a*C*d^3 - (Sqrt[-b^2]*(a*(c^3*C - 3*B*c^2*d - 3*c*C*d^2
+ B*d^3 - A*(c^3 - 3*c*d^2)) + b*((A - C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^
2))))/b)*Log[Sqrt[-b^2] - b*Tan[e + f*x]])/((a^2 + b^2)*(c^2 + d^2))) + (2*
b^3*(A*b^2 - a*(b*B - a*C))*(c^2 + d^2)^2*Log[a + b*Tan[e + f*x]])/((a^2 +
b^2)*(b*c - a*d)) - (b*(b*c - a*d)^2*(A*b*c^3 - a*B*c^3 - b*c^3*C + 3*a*A*c
^2*d + 3*b*B*c^2*d - 3*a*c^2*C*d - 3*A*b*c*d^2 + 3*a*B*c*d^2 + 3*b*c*C*d^2
- a*A*d^3 - b*B*d^3 + a*C*d^3 + (Sqrt[-b^2]*(b*(A - C)*d*(3*c^2 - d^2) - b*
```

```
B*(c^3 - 3*c*d^2) - a*(A*c^3 - c^3*C + 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 -
B*d^3)))/b)*Log[Sqrt[-b^2] + b*Tan[e + f*x]])/((a^2 + b^2)*(c^2 + d^2)) - (
2*b*(b^2*(c^6*C - 3*B*c^5*d + 3*c^4*(2*A - C)*d^2 + B*c^3*d^3 + 3*A*c^2*d^4
 + A*d^6) + a^2*d^3*((A - C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)) - a*b*d^2
*(8*c^3*(A - C)*d - B*(3*c^4 - 6*c^2*d^2 - d^4)))*Log[c + d*Tan[e + f*x]])/
((b*c - a*d)*(c^2 + d^2)))/(b*(-(b*c) + a*d)*(c^2 + d^2)*f)) - (-2*d^2*(a*A
*c*d - a*d*(c*C - B*d) - A*b*(c^2 + d^2)) - c*(2*d*(b*c - a*d)*(B*c - (A -
C)*d) - 2*b*c*(c^2*C - B*c*d + A*d^2)))/((-(b*c) + a*d)*(c^2 + d^2)*f*(c +
d*Tan[e + f*x])))/(2*(-(b*c) + a*d)*(c^2 + d^2))
```

Maple [B] time = 0.117, size = 2298, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))^3,x)
```

```
[Out] -3/f/(a*d-b*c)^3/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*C*a^2*c^2*d^4-3/f/(a*d-b*c)
^3/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*C*b^2*c^4*d^2+1/f/(a*d-b*c)^3/(c^2+d^2)^3
*ln(c+d*tan(f*x+e))*B*b^2*c^3*d^3+3/f/(a*d-b*c)^3/(c^2+d^2)^3*ln(c+d*tan(f*
x+e))*A*b^2*c^2*d^4-1/f/(a*d-b*c)^3/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*B*a^2*c^
3*d^3+1/f/(a*d-b*c)^2/(c^2+d^2)^2/(c+d*tan(f*x+e))*B*a*c^2*d^2-2/f/(a*d-b*c
)^2/(c^2+d^2)^2/(c+d*tan(f*x+e))*B*b*c^3*d+2/f/(a*d-b*c)^2/(c^2+d^2)^2/(c+d
*tan(f*x+e))*C*a*c*d^3-2/f/(a*d-b*c)^2/(c^2+d^2)^2/(c+d*tan(f*x+e))*A*a*c*d
^3+3/f/(a*d-b*c)^2/(c^2+d^2)^2/(c+d*tan(f*x+e))*A*b*c^2*d^2+3/f/(a^2+b^2)/(
c^2+d^2)^3*B*arctan(tan(f*x+e))*a*c^2*d-3/f/(a^2+b^2)/(c^2+d^2)^3*B*arctan(
tan(f*x+e))*b*c*d^2-3/2/f/(a^2+b^2)/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*B*a*c*d^
2+1/2/f/(a^2+b^2)/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*B*b*d^3-1/2/f/(a^2+b^2)/(c
^2+d^2)^3*ln(1+tan(f*x+e)^2)*C*a*d^3+3/f/(a*d-b*c)^3/(c^2+d^2)^3*ln(c+d*tan
(f*x+e))*A*a^2*c^2*d^4-3/2/f/(a^2+b^2)/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*B*b*c
^2*d+3/2/f/(a^2+b^2)/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*C*a*c^2*d-3/2/f/(a^2+b^
2)/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*C*b*c*d^2-3/f/(a^2+b^2)/(c^2+d^2)^3*A*arc
tan(tan(f*x+e))*a*c*d^2+3/f/(a^2+b^2)/(c^2+d^2)^3*C*arctan(tan(f*x+e))*b*c^
2*d+3/f/(a^2+b^2)/(c^2+d^2)^3*C*arctan(tan(f*x+e))*a*c*d^2-3/f/(a^2+b^2)/(c
^2+d^2)^3*A*arctan(tan(f*x+e))*b*c^2*d+6/f/(a*d-b*c)^3/(c^2+d^2)^3*ln(c+d*t
an(f*x+e))*A*b^2*c^4*d^2+3/f/(a*d-b*c)^3/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*B*a
^2*c*d^5-1/f/(a*d-b*c)^3/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*B*a*b*d^6-1/f*b^4/(
a*d-b*c)^3/(a^2+b^2)*ln(a+b*tan(f*x+e))*A-1/2/f/(a*d-b*c)/(c^2+d^2)/(c+d*ta
n(f*x+e))^2*A*d^2-1/2/f/(a*d-b*c)/(c^2+d^2)/(c+d*tan(f*x+e))^2*c^2*C-3/f/(a
*d-b*c)^3/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*B*b^2*c^5*d+8/f/(a*d-b*c)^3/(c^2+d
^2)^3*ln(c+d*tan(f*x+e))*C*a*b*c^3*d^3-1/f/(a*d-b*c)^2/(c^2+d^2)^2/(c+d*tan
(f*x+e))*C*b*c^2*d^2-3/2/f/(a^2+b^2)/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*A*a*c^2
*d+3/2/f/(a^2+b^2)/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*A*b*c*d^2-1/f*b^2/(a*d-b*
c)^3/(a^2+b^2)*ln(a+b*tan(f*x+e))*C*a^2-1/2/f/(a^2+b^2)/(c^2+d^2)^3*ln(1+ta
n(f*x+e)^2)*A*b*c^3+1/2/f/(a^2+b^2)/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*B*a*c^3+
1/2/f/(a^2+b^2)/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*C*b*c^3+1/f/(a^2+b^2)/(c^2+d
^2)^3*A*arctan(tan(f*x+e))*a*c^3+1/f/(a^2+b^2)/(c^2+d^2)^3*A*arctan(tan(f*x
+e))*b*d^3-1/f/(a^2+b^2)/(c^2+d^2)^3*B*arctan(tan(f*x+e))*a*d^3+1/f/(a^2+b^
2)/(c^2+d^2)^3*B*arctan(tan(f*x+e))*b*c^3-1/f/(a^2+b^2)/(c^2+d^2)^3*C*arcta
n(tan(f*x+e))*a*c^3-1/f/(a^2+b^2)/(c^2+d^2)^3*C*arctan(tan(f*x+e))*b*d^3+1/
f/(a*d-b*c)^2/(c^2+d^2)^2/(c+d*tan(f*x+e))*A*b*d^4-6/f/(a*d-b*c)^3/(c^2+d^2
)^3*ln(c+d*tan(f*x+e))*B*a*b*c^2*d^4+3/f/(a*d-b*c)^3/(c^2+d^2)^3*ln(c+d*tan
(f*x+e))*B*a*b*c^4*d^2-1/f/(a*d-b*c)^2/(c^2+d^2)^2/(c+d*tan(f*x+e))*B*a*d^4
+1/f/(a*d-b*c)^2/(c^2+d^2)^2/(c+d*tan(f*x+e))*C*b*c^4-1/f/(a*d-b*c)^3/(c^2+
d^2)^3*ln(c+d*tan(f*x+e))*A*a^2*d^6+1/2/f/(a*d-b*c)/(c^2+d^2)/(c+d*tan(f*x+
e))^2*B*c*d+1/f*b^3/(a*d-b*c)^3/(a^2+b^2)*ln(a+b*tan(f*x+e))*B*a+1/f/(a*d-b
```

```
*c)^3/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*A*b^2*d^6+1/f/(a*d-b*c)^3/(c^2+d^2)^3*
ln(c+d*tan(f*x+e))*C*a^2*d^6+1/f/(a*d-b*c)^3/(c^2+d^2)^3*ln(c+d*tan(f*x+e))
*C*b^2*c^6+1/2/f/(a^2+b^2)/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*A*a*d^3-8/f/(a*d-
b*c)^3/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*A*a*b*c^3*d^3
```

Maxima [B] time = 1.84013, size = 1455, normalized size = 2.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))
^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(((A - C)*a + B*b)*c^3 + 3*(B*a - (A - C)*b)*c^2*d - 3*((A - C)*a +
B*b)*c*d^2 - (B*a - (A - C)*b)*d^3)*(f*x + e)/((a^2 + b^2)*c^6 + 3*(a^2 + b
^2)*c^4*d^2 + 3*(a^2 + b^2)*c^2*d^4 + (a^2 + b^2)*d^6) + 2*(C*a^2*b^2 - B*a
*b^3 + A*b^4)*log(b*tan(f*x + e) + a)/((a^2*b^3 + b^5)*c^3 - 3*(a^3*b^2 + a
*b^4)*c^2*d + 3*(a^4*b + a^2*b^3)*c*d^2 - (a^5 + a^3*b^2)*d^3) - 2*(C*b^2*c
^6 - 3*B*b^2*c^5*d + 3*B*a^2*c*d^5 + 3*(B*a*b + (2*A - C)*b^2)*c^4*d^2 - (B
*a^2 + 8*(A - C)*a*b - B*b^2)*c^3*d^3 + 3*((A - C)*a^2 - 2*B*a*b + A*b^2)*c
^2*d^4 - ((A - C)*a^2 + B*a*b - A*b^2)*d^6)*log(d*tan(f*x + e) + c)/(b^3*c^
9 - 3*a*b^2*c^8*d + 3*a^2*b*c*d^8 - a^3*d^9 + 3*(a^2*b + b^3)*c^7*d^2 - (a^
3 + 9*a*b^2)*c^6*d^3 + 3*(3*a^2*b + b^3)*c^5*d^4 - 3*(a^3 + 3*a*b^2)*c^4*d^
5 + (9*a^2*b + b^3)*c^3*d^6 - 3*(a^3 + a*b^2)*c^2*d^7) + ((B*a - (A - C)*b)
*c^3 - 3*((A - C)*a + B*b)*c^2*d - 3*(B*a - (A - C)*b)*c*d^2 + ((A - C)*a +
 B*b)*d^3)*log(tan(f*x + e)^2 + 1)/((a^2 + b^2)*c^6 + 3*(a^2 + b^2)*c^4*d^2
 + 3*(a^2 + b^2)*c^2*d^4 + (a^2 + b^2)*d^6) + (3*C*b*c^5 - A*a*d^5 - (C*a +
 5*B*b)*c^4*d + (3*B*a + (7*A - C)*b)*c^3*d^2 - ((5*A - 3*C)*a + B*b)*c^2*d
^3 - (B*a - 3*A*b)*c*d^4 + 2*(C*b*c^4*d - 2*B*b*c^3*d^2 - 2*(A - C)*a*c*d^4
 + (B*a + (3*A - C)*b)*c^2*d^3 - (B*a - A*b)*d^5)*tan(f*x + e))/(b^2*c^8 -
2*a*b*c^7*d - 4*a*b*c^5*d^3 - 2*a*b*c^3*d^5 + a^2*c^2*d^6 + (a^2 + 2*b^2)*c
^6*d^2 + (2*a^2 + b^2)*c^4*d^4 + (b^2*c^6*d^2 - 2*a*b*c^5*d^3 - 4*a*b*c^3*d
^5 - 2*a*b*c*d^7 + a^2*d^8 + (a^2 + 2*b^2)*c^4*d^4 + (2*a^2 + b^2)*c^2*d^6)
*tan(f*x + e)^2 + 2*(b^2*c^7*d - 2*a*b*c^6*d^2 - 4*a*b*c^4*d^4 - 2*a*b*c^2*
d^6 + a^2*c*d^7 + (a^2 + 2*b^2)*c^5*d^3 + (2*a^2 + b^2)*c^3*d^5)*tan(f*x +
e)))/f
```

Fricas [B] time = 38.5076, size = 7109, normalized size = 14.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))
^3,x, algorithm="fricas")
```

```
[Out] 1/2*(5*(C*a^2*b^2 + C*b^4)*c^6*d^2 - (8*C*a^3*b + 7*B*a^2*b^2 + 8*C*a*b^3 +
 7*B*b^4)*c^5*d^3 + (3*C*a^4 + 12*B*a^3*b + (9*A + 2*C)*a^2*b^2 + 12*B*a*b^
3 + (9*A - C)*b^4)*c^4*d^4 - (5*B*a^4 + 4*(4*A - C)*a^3*b + 6*B*a^2*b^2 + 4
*(4*A - C)*a*b^3 + B*b^4)*c^3*d^5 + ((7*A - 3*C)*a^4 + (10*A - 3*C)*a^2*b^2
 + 3*A*b^4)*c^2*d^6 + (B*a^4 - 4*A*a^3*b + B*a^2*b^2 - 4*A*a*b^3)*c*d^7 + (
A*a^4 + A*a^2*b^2)*d^8 + 2*(((A - C)*a*b^3 + B*b^4)*c^8 - 3*((A - C)*a^2*b^
2 + (A - C)*b^4)*c^7*d + 3*((A - C)*a^3*b - 2*B*a^2*b^2 + 2*(A - C)*a*b^3 -
 B*b^4)*c^6*d^2 - ((A - C)*a^4 - 8*B*a^3*b - 8*B*a*b^3 - (A - C)*b^4)*c^5*d
```

^3 - 3*(B*a^4 + 2*(A - C)*a^3*b + 2*B*a^2*b^2 + (A - C)*a*b^3)*c^4*d^4 + 3*
((A - C)*a^4 + (A - C)*a^2*b^2)*c^3*d^5 + (B*a^4 - (A - C)*a^3*b)*c^2*d^6)*
f*x - (3*(C*a^2*b^2 + C*b^4)*c^6*d^2 - (4*C*a^3*b + 5*B*a^2*b^2 + 4*C*a*b^3
 + 5*B*b^4)*c^5*d^3 + (C*a^4 + 8*B*a^3*b + (7*A - 2*C)*a^2*b^2 + 8*B*a*b^3
+ (7*A - 3*C)*b^4)*c^4*d^4 - (3*B*a^4 + 4*(3*A - 2*C)*a^3*b + 2*B*a^2*b^2 +
 4*(3*A - 2*C)*a*b^3 - B*b^4)*c^3*d^5 + (5*(A - C)*a^4 - 4*B*a^3*b + (6*A -
 5*C)*a^2*b^2 - 4*B*a*b^3 + A*b^4)*c^2*d^6 + 3*(B*a^4 + B*a^2*b^2)*c*d^7 -
(A*a^4 + A*a^2*b^2)*d^8 - 2*(((A - C)*a*b^3 + B*b^4)*c^6*d^2 - 3*((A - C)*a
^2*b^2 + (A - C)*b^4)*c^5*d^3 + 3*((A - C)*a^3*b - 2*B*a^2*b^2 + 2*(A - C)*
a*b^3 - B*b^4)*c^4*d^4 - ((A - C)*a^4 - 8*B*a^3*b - 8*B*a*b^3 - (A - C)*b^4
)*c^3*d^5 - 3*(B*a^4 + 2*(A - C)*a^3*b + 2*B*a^2*b^2 + (A - C)*a*b^3)*c^2*d
^6 + 3*((A - C)*a^4 + (A - C)*a^2*b^2)*c*d^7 + (B*a^4 - (A - C)*a^3*b)*d^8)
*f*x)*tan(f*x + e)^2 + ((C*a^2*b^2 - B*a*b^3 + A*b^4)*c^8 + 3*(C*a^2*b^2 -
B*a*b^3 + A*b^4)*c^6*d^2 + 3*(C*a^2*b^2 - B*a*b^3 + A*b^4)*c^4*d^4 + (C*a^2
*b^2 - B*a*b^3 + A*b^4)*c^2*d^6 + ((C*a^2*b^2 - B*a*b^3 + A*b^4)*c^6*d^2 +
3*(C*a^2*b^2 - B*a*b^3 + A*b^4)*c^4*d^4 + 3*(C*a^2*b^2 - B*a*b^3 + A*b^4)*c
^2*d^6 + (C*a^2*b^2 - B*a*b^3 + A*b^4)*d^8)*tan(f*x + e)^2 + 2*((C*a^2*b^2
- B*a*b^3 + A*b^4)*c^7*d + 3*(C*a^2*b^2 - B*a*b^3 + A*b^4)*c^5*d^3 + 3*(C*a
^2*b^2 - B*a*b^3 + A*b^4)*c^3*d^5 + (C*a^2*b^2 - B*a*b^3 + A*b^4)*c*d^7)*ta
n(f*x + e))*log((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)/(tan(f*x +
e)^2 + 1)) - ((C*a^2*b^2 + C*b^4)*c^8 - 3*(B*a^2*b^2 + B*b^4)*c^7*d + 3*(B*
a^3*b + (2*A - C)*a^2*b^2 + B*a*b^3 + (2*A - C)*b^4)*c^6*d^2 - (B*a^4 + 8*(
A - C)*a^3*b + 8*(A - C)*a*b^3 - B*b^4)*c^5*d^3 + 3*((A - C)*a^4 - 2*B*a^3*
b + (2*A - C)*a^2*b^2 - 2*B*a*b^3 + A*b^4)*c^4*d^4 + 3*(B*a^4 + B*a^2*b^2)*
c^3*d^5 - ((A - C)*a^4 + B*a^3*b - C*a^2*b^2 + B*a*b^3 - A*b^4)*c^2*d^6 + (
(C*a^2*b^2 + C*b^4)*c^6*d^2 - 3*(B*a^2*b^2 + B*b^4)*c^5*d^3 + 3*(B*a^3*b +
(2*A - C)*a^2*b^2 + B*a*b^3 + (2*A - C)*b^4)*c^4*d^4 - (B*a^4 + 8*(A - C)*a
^3*b + 8*(A - C)*a*b^3 - B*b^4)*c^3*d^5 + 3*((A - C)*a^4 - 2*B*a^3*b + (2*A
 - C)*a^2*b^2 - 2*B*a*b^3 + A*b^4)*c^2*d^6 + 3*(B*a^4 + B*a^2*b^2)*c*d^7 -
((A - C)*a^4 + B*a^3*b - C*a^2*b^2 + B*a*b^3 - A*b^4)*d^8)*tan(f*x + e)^2 +
 2*((C*a^2*b^2 + C*b^4)*c^7*d - 3*(B*a^2*b^2 + B*b^4)*c^6*d^2 + 3*(B*a^3*b
+ (2*A - C)*a^2*b^2 + B*a*b^3 + (2*A - C)*b^4)*c^5*d^3 - (B*a^4 + 8*(A - C)
*a^3*b + 8*(A - C)*a*b^3 - B*b^4)*c^4*d^4 + 3*((A - C)*a^4 - 2*B*a^3*b + (2
*A - C)*a^2*b^2 - 2*B*a*b^3 + A*b^4)*c^3*d^5 + 3*(B*a^4 + B*a^2*b^2)*c^2*d^
6 - ((A - C)*a^4 + B*a^3*b - C*a^2*b^2 + B*a*b^3 - A*b^4)*c*d^7)*tan(f*x +
e))*log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1
)) - 2*(2*(C*a^2*b^2 + C*b^4)*c^7*d - 3*(C*a^3*b + B*a^2*b^2 + C*a*b^3 + B*
b^4)*c^6*d^2 + (C*a^4 + 5*B*a^3*b + 2*(2*A - C)*a^2*b^2 + 5*B*a*b^3 + (4*A
- 3*C)*b^4)*c^5*d^3 - (2*B*a^4 + (7*A - 6*C)*a^3*b - B*a^2*b^2 + (7*A - 6*C
)*a*b^3 - 3*B*b^4)*c^4*d^4 + (3*(A - C)*a^4 - 6*B*a^3*b - 2*C*a^2*b^2 - 6*B
*a*b^3 - (3*A - C)*b^4)*c^3*d^5 + 3*(B*a^4 + (2*A - C)*a^3*b + B*a^2*b^2 +
(2*A - C)*a*b^3)*c^2*d^6 - ((3*A - 2*C)*a^4 - B*a^3*b + 2*(2*A - C)*a^2*b^2
 - B*a*b^3 + A*b^4)*c*d^7 - (B*a^4 - A*a^3*b + B*a^2*b^2 - A*a*b^3)*d^8 - 2
*(((A - C)*a*b^3 + B*b^4)*c^7*d - 3*((A - C)*a^2*b^2 + (A - C)*b^4)*c^6*d^2
 + 3*((A - C)*a^3*b - 2*B*a^2*b^2 + 2*(A - C)*a*b^3 - B*b^4)*c^5*d^3 - ((A
- C)*a^4 - 8*B*a^3*b - 8*B*a*b^3 - (A - C)*b^4)*c^4*d^4 - 3*(B*a^4 + 2*(A -
 C)*a^3*b + 2*B*a^2*b^2 + (A - C)*a*b^3)*c^3*d^5 + 3*((A - C)*a^4 + (A - C)
*a^2*b^2)*c^2*d^6 + (B*a^4 - (A - C)*a^3*b)*c*d^7)*f*x)*tan(f*x + e))/(((a^
2*b^3 + b^5)*c^9*d^2 - 3*(a^3*b^2 + a*b^4)*c^8*d^3 + 3*(a^4*b + 2*a^2*b^3 +
 b^5)*c^7*d^4 - (a^5 + 10*a^3*b^2 + 9*a*b^4)*c^6*d^5 + 3*(3*a^4*b + 4*a^2*b
^3 + b^5)*c^5*d^6 - 3*(a^5 + 4*a^3*b^2 + 3*a*b^4)*c^4*d^7 + (9*a^4*b + 10*a
^2*b^3 + b^5)*c^3*d^8 - 3*(a^5 + 2*a^3*b^2 + a*b^4)*c^2*d^9 + 3*(a^4*b + a^
2*b^3)*c*d^10 - (a^5 + a^3*b^2)*d^11)*f*tan(f*x + e)^2 + 2*((a^2*b^3 + b^5)
*c^10*d - 3*(a^3*b^2 + a*b^4)*c^9*d^2 + 3*(a^4*b + 2*a^2*b^3 + b^5)*c^8*d^3
 - (a^5 + 10*a^3*b^2 + 9*a*b^4)*c^7*d^4 + 3*(3*a^4*b + 4*a^2*b^3 + b^5)*c^6
d^5 - 3(a^5 + 4*a^3*b^2 + 3*a*b^4)*c^5*d^6 + (9*a^4*b + 10*a^2*b^3 + b^5)
*c^4*d^7 - 3*(a^5 + 2*a^3*b^2 + a*b^4)*c^3*d^8 + 3*(a^4*b + a^2*b^3)*c^2*d^
9 - (a^5 + a^3*b^2)*c*d^10)*f*tan(f*x + e) + ((a^2*b^3 + b^5)*c^11 - 3*(a^3
*b^2 + a*b^4)*c^10*d + 3*(a^4*b + 2*a^2*b^3 + b^5)*c^9*d^2 - (a^5 + 10*a^3*

```
b^2 + 9*a*b^4)*c^8*d^3 + 3*(3*a^4*b + 4*a^2*b^3 + b^5)*c^7*d^4 - 3*(a^5 + 4
*a^3*b^2 + 3*a*b^4)*c^6*d^5 + (9*a^4*b + 10*a^2*b^3 + b^5)*c^5*d^6 - 3*(a^5
 + 2*a^3*b^2 + a*b^4)*c^4*d^7 + 3*(a^4*b + a^2*b^3)*c^3*d^8 - (a^5 + a^3*b^
2)*c^2*d^9)*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e)
)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.61727, size = 2869, normalized size = 5.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))
^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a*c^3 - C*a*c^3 + B*b*c^3 + 3*B*a*c^2*d - 3*A*b*c^2*d + 3*C*b*c^2
*d - 3*A*a*c*d^2 + 3*C*a*c*d^2 - 3*B*b*c*d^2 - B*a*d^3 + A*b*d^3 - C*b*d^3)
*(f*x + e)/(a^2*c^6 + b^2*c^6 + 3*a^2*c^4*d^2 + 3*b^2*c^4*d^2 + 3*a^2*c^2*d
^4 + 3*b^2*c^2*d^4 + a^2*d^6 + b^2*d^6) + (B*a*c^3 - A*b*c^3 + C*b*c^3 - 3*
A*a*c^2*d + 3*C*a*c^2*d - 3*B*b*c^2*d - 3*B*a*c*d^2 + 3*A*b*c*d^2 - 3*C*b*c
*d^2 + A*a*d^3 - C*a*d^3 + B*b*d^3)*log(tan(f*x + e)^2 + 1)/(a^2*c^6 + b^2*
c^6 + 3*a^2*c^4*d^2 + 3*b^2*c^4*d^2 + 3*a^2*c^2*d^4 + 3*b^2*c^2*d^4 + a^2*d
^6 + b^2*d^6) + 2*(C*a^2*b^3 - B*a*b^4 + A*b^5)*log(abs(b*tan(f*x + e) + a)
)/(a^2*b^4*c^3 + b^6*c^3 - 3*a^3*b^3*c^2*d - 3*a*b^5*c^2*d + 3*a^4*b^2*c*d^
2 + 3*a^2*b^4*c*d^2 - a^5*b*d^3 - a^3*b^3*d^3) - 2*(C*b^2*c^6*d - 3*B*b^2*c
^5*d^2 + 3*B*a*b*c^4*d^3 + 6*A*b^2*c^4*d^3 - 3*C*b^2*c^4*d^3 - B*a^2*c^3*d^
4 - 8*A*a*b*c^3*d^4 + 8*C*a*b*c^3*d^4 + B*b^2*c^3*d^4 + 3*A*a^2*c^2*d^5 - 3
*C*a^2*c^2*d^5 - 6*B*a*b*c^2*d^5 + 3*A*b^2*c^2*d^5 + 3*B*a^2*c*d^6 - A*a^2*
d^7 + C*a^2*d^7 - B*a*b*d^7 + A*b^2*d^7)*log(abs(d*tan(f*x + e) + c))/(b^3*
c^9*d - 3*a*b^2*c^8*d^2 + 3*a^2*b*c^7*d^3 + 3*b^3*c^7*d^3 - a^3*c^6*d^4 - 9
*a*b^2*c^6*d^4 + 9*a^2*b*c^5*d^5 + 3*b^3*c^5*d^5 - 3*a^3*c^4*d^6 - 9*a*b^2*
c^4*d^6 + 9*a^2*b*c^3*d^7 + b^3*c^3*d^7 - 3*a^3*c^2*d^8 - 3*a*b^2*c^2*d^8 +
 3*a^2*b*c*d^9 - a^3*d^10) + (3*C*b^2*c^6*d^2*tan(f*x + e)^2 - 9*B*b^2*c^5*
d^3*tan(f*x + e)^2 + 9*B*a*b*c^4*d^4*tan(f*x + e)^2 + 18*A*b^2*c^4*d^4*tan(
f*x + e)^2 - 9*C*b^2*c^4*d^4*tan(f*x + e)^2 - 3*B*a^2*c^3*d^5*tan(f*x + e)^
2 - 24*A*a*b*c^3*d^5*tan(f*x + e)^2 + 24*C*a*b*c^3*d^5*tan(f*x + e)^2 + 3*B
*b^2*c^3*d^5*tan(f*x + e)^2 + 9*A*a^2*c^2*d^6*tan(f*x + e)^2 - 9*C*a^2*c^2*
d^6*tan(f*x + e)^2 - 18*B*a*b*c^2*d^6*tan(f*x + e)^2 + 9*A*b^2*c^2*d^6*tan(
f*x + e)^2 + 9*B*a^2*c*d^7*tan(f*x + e)^2 - 3*A*a^2*d^8*tan(f*x + e)^2 + 3*
C*a^2*d^8*tan(f*x + e)^2 - 3*B*a*b*d^8*tan(f*x + e)^2 + 3*A*b^2*d^8*tan(f*x
 + e)^2 + 8*C*b^2*c^7*d*tan(f*x + e) - 2*C*a*b*c^6*d^2*tan(f*x + e) - 22*B*
b^2*c^6*d^2*tan(f*x + e) + 24*B*a*b*c^5*d^3*tan(f*x + e) + 42*A*b^2*c^5*d^3
*tan(f*x + e) - 18*C*b^2*c^5*d^3*tan(f*x + e) - 8*B*a^2*c^4*d^4*tan(f*x + e
) - 58*A*a*b*c^4*d^4*tan(f*x + e) + 52*C*a*b*c^4*d^4*tan(f*x + e) + 2*B*b^2
```

```
*c^4*d^4*tan(f*x + e) + 22*A*a^2*c^3*d^5*tan(f*x + e) - 22*C*a^2*c^3*d^5*ta
n(f*x + e) - 32*B*a*b*c^3*d^5*tan(f*x + e) + 26*A*b^2*c^3*d^5*tan(f*x + e)
- 2*C*b^2*c^3*d^5*tan(f*x + e) + 18*B*a^2*c^2*d^6*tan(f*x + e) - 12*A*a*b*c
^2*d^6*tan(f*x + e) + 6*C*a*b*c^2*d^6*tan(f*x + e) - 2*A*a^2*c*d^7*tan(f*x
+ e) + 2*C*a^2*c*d^7*tan(f*x + e) - 8*B*a*b*c*d^7*tan(f*x + e) + 8*A*b^2*c*
d^7*tan(f*x + e) + 2*B*a^2*d^8*tan(f*x + e) - 2*A*a*b*d^8*tan(f*x + e) + 6*
C*b^2*c^8 - 4*C*a*b*c^7*d - 14*B*b^2*c^7*d + C*a^2*c^6*d^2 + 17*B*a*b*c^6*d
^2 + 25*A*b^2*c^6*d^2 - 7*C*b^2*c^6*d^2 - 6*B*a^2*c^5*d^3 - 36*A*a*b*c^5*d^
3 + 24*C*a*b*c^5*d^3 - 3*B*b^2*c^5*d^3 + 14*A*a^2*c^4*d^4 - 11*C*a^2*c^4*d^
4 - 10*B*a*b*c^4*d^4 + 19*A*b^2*c^4*d^4 - C*b^2*c^4*d^4 + 7*B*a^2*c^3*d^5 -
 16*A*a*b*c^3*d^5 + 4*C*a*b*c^3*d^5 - B*b^2*c^3*d^5 + 3*A*a^2*c^2*d^6 - 3*B
*a*b*c^2*d^6 + 6*A*b^2*c^2*d^6 + B*a^2*c*d^7 - 4*A*a*b*c*d^7 + A*a^2*d^8)/(
(b^3*c^9 - 3*a*b^2*c^8*d + 3*a^2*b*c^7*d^2 + 3*b^3*c^7*d^2 - a^3*c^6*d^3 -
9*a*b^2*c^6*d^3 + 9*a^2*b*c^5*d^4 + 3*b^3*c^5*d^4 - 3*a^3*c^4*d^5 - 9*a*b^2
*c^4*d^5 + 9*a^2*b*c^3*d^6 + b^3*c^3*d^6 - 3*a^3*c^2*d^7 - 3*a*b^2*c^2*d^7
+ 3*a^2*b*c*d^8 - a^3*d^9)*(d*tan(f*x + e) + c)^2))/f
```

3.89 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}\,dx$

Optimal. Leaf size=861

$$\frac{\left(-3Cda^4+4bBda^3-b^2(Bc+(5A+C)d)a^2+2b^3(Ac-Cc+Bd)a+b^4(Bc-3Ad)\right)\log(a\cos(e+fx)+b\sin(e+fx))}{\left(a^2+b^2\right)^2(bc-ad)^4f}$$

```
[Out] -(((b^2*(A*c^3 - c^3*C + 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 - B*d^3) + a^2*(
c^3*C - 3*B*c^2*d - 3*c*C*d^2 + B*d^3 - A*(c^3 - 3*c*d^2)) + 2*a*b*((A - C)
*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)))*x)/((a^2 + b^2)^2*(c^2 + d^2)^3)) +
(b^2*(4*a^3*b*B*d - 3*a^4*C*d + b^4*(B*c - 3*A*d) + 2*a*b^3*(A*c - c*C + B*
d) - a^2*b^2*(B*c + (5*A + C)*d))*Log[a*Cos[e + f*x] + b*Sin[e + f*x]])/((a
^2 + b^2)^2*(b*c - a*d)^4*f) + (d*(b^2*(3*c^6*C - 6*B*c^5*d + c^4*(10*A - C
)*d^2 - 3*B*c^3*d^3 + 9*A*c^2*d^4 - B*c*d^5 + 3*A*d^6) + a^2*d^3*((A - C)*d
*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)) - 2*a*b*d^2*(c*(A - C)*d*(5*c^2 + d^2)
- B*(2*c^4 - 3*c^2*d^2 - d^4)))*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/((b*c
 - a*d)^4*(c^2 + d^2)^3*f) - (d*(b^2*c*(c*C - B*d) - 2*a*b*B*(c^2 + d^2) +
a^2*(3*c^2*C - B*c*d + 2*C*d^2) + A*(a^2*d^2 + b^2*(2*c^2 + 3*d^2))))/(2*(a
^2 + b^2)*(b*c - a*d)^2*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^2) - (A*b^2 - a*
(b*B - a*C))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])*(c + d*Tan[e +
 f*x])^2) - (d*(b^3*c*(2*c^3*C - 3*B*c^2*d - B*d^3) + a^2*b*(3*c^4*C - 3*B*
c^3*d + 2*c^2*C*d^2 - B*c*d^3 + C*d^4) + a^3*d^2*(2*c*C*d + B*(c^2 - d^2))
+ a*b^2*(2*c*C*d^3 - B*(c^4 + c^2*d^2 + 2*d^4)) - A*(2*a^3*c*d^3 + 2*a*b^2*
c*d^3 - 2*a^2*b*d^2*(2*c^2 + d^2) - b^3*(c^4 + 6*c^2*d^2 + 3*d^4))))/((a^2
+ b^2)*(b*c - a*d)^3*(c^2 + d^2)^2*f*(c + d*Tan[e + f*x]))
```

Rubi [A] time = 4.27601, antiderivative size = 860, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3649, 3651, 3530}

$$\frac{\left(-3Cda^4+4bBda^3-b^2(Bc+(5A+C)d)a^2+2b^3(Ac-Cc+Bd)a+b^4(Bc-3Ad)\right)\log(a\cos(e+fx)+b\sin(e+fx))}{\left(a^2+b^2\right)^2(bc-ad)^4f}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^2*(c + d*
Tan[e + f*x])^3),x]
```

```
[Out] -(((b^2*(A*c^3 - c^3*C + 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 - B*d^3) + a^2*(
c^3*C - 3*B*c^2*d - 3*c*C*d^2 + B*d^3 - A*(c^3 - 3*c*d^2)) + 2*a*b*((A - C)
*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)))*x)/((a^2 + b^2)^2*(c^2 + d^2)^3)) +
(b^2*(4*a^3*b*B*d - 3*a^4*C*d + b^4*(B*c - 3*A*d) + 2*a*b^3*(A*c - c*C + B*
d) - a^2*b^2*(B*c + (5*A + C)*d))*Log[a*Cos[e + f*x] + b*Sin[e + f*x]])/((a
^2 + b^2)^2*(b*c - a*d)^4*f) + (d*(b^2*(3*c^6*C - 6*B*c^5*d + c^4*(10*A - C
)*d^2 - 3*B*c^3*d^3 + 9*A*c^2*d^4 - B*c*d^5 + 3*A*d^6) + a^2*d^3*((A - C)*d
*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)) - 2*a*b*d^2*(c*(A - C)*d*(5*c^2 + d^2)
- B*(2*c^4 - 3*c^2*d^2 - d^4)))*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/((b*c
 - a*d)^4*(c^2 + d^2)^3*f) - (d*(a^2*A*d^2 + b^2*c*(c*C - B*d) - 2*a*b*B*(c
^2 + d^2) + A*b^2*(2*c^2 + 3*d^2) + a^2*(3*c^2*C - B*c*d + 2*C*d^2)))/(2*(a
^2 + b^2)*(b*c - a*d)^2*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^2) - (A*b^2 - a*
(b*B - a*C))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])*(c + d*Tan[e +
 f*x])^2) - (d*(b^3*c*(2*c^3*C - 3*B*c^2*d - B*d^3) + a^2*b*(3*c^4*C - 3*B*
c^3*d + 2*c^2*C*d^2 - B*c*d^3 + C*d^4) + a^3*d^2*(2*c*C*d + B*(c^2 - d^2))
+ a*b^2*(2*c*C*d^3 - B*(c^4 + c^2*d^2 + 2*d^4)) - A*(2*a^3*c*d^3 + 2*a*b^2*
```

```
c*d^3 - 2*a^2*b*d^2*(2*c^2 + d^2) - b^3*(c^4 + 6*c^2*d^2 + 3*d^4))))/((a^2
+ b^2)*(b*c - a*d)^3*(c^2 + d^2)^2*f*(c + d*Tan[e + f*x]))
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
 b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3651

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[((a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*x)
/((a^2 + b^2)*(c^2 + d^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}\,dx = -\frac{Ab^2-a(bB-aC)}{\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))(c+d\tan(e+fx))^2} - \frac{\int \frac{3Ab^2d\cdots}{\cdots}}{\cdots}$$

$$= -\frac{d\left(a^2Ad^2+b^2c(cC-Bd)-2abB\left(c^2+d^2\right)+Ab^2\left(2c^2+3d^2\right)+a^2\left(3c^2\cdots\right.\right.}{2\left(a^2+b^2\right)(bc-ad)^2\left(c^2+d^2\right)f(c+d\tan(e+fx))^2\cdots}$$

$$= -\frac{d\left(a^2Ad^2+b^2c(cC-Bd)-2abB\left(c^2+d^2\right)+Ab^2\left(2c^2+3d^2\right)+a^2\left(3c^2\cdots\right.\right.}{2\left(a^2+b^2\right)(bc-ad)^2\left(c^2+d^2\right)f(c+d\tan(e+fx))^2\cdots}$$

$$= -\frac{\left(b^2\left(Ac^3-c^3C+3Bc^2d-3Acd^2+3cCd^2-Bd^3\right)+a^2\left(c^3C-3Bc^2d-\cdots\right.\right.}{\left(a^2+b\cdots\right.}$$

$$= -\frac{\left(b^2\left(Ac^3-c^3C+3Bc^2d-3Acd^2+3cCd^2-Bd^3\right)+a^2\left(c^3C-3Bc^2d-\cdots\right.\right.}{\left(a^2+b\cdots\right.}$$

Mathematica [B] time = 8.20003, size = 1732, normalized size = 2.01

$$-\frac{Ab^2-a(bB-aC)}{\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))(c+d\tan(e+fx))^2}-\frac{-\frac{d^2\left(3Adb^2-aA(bc-ad)-(bB-aC)(bc+2ad)\right)-c\left((Ab-Cb-aB)d(bc-ad)-3\right.}{2(ad-bc)\left(c^2+d^2\right)f(c+d\tan(e+fx))^2}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^2*(
c + d*Tan[e + f*x])^3),x]
```

```
[Out] -((A*b^2 - a*(b*B - a*C))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])*(
c + d*Tan[e + f*x])^2)) - (-(-(c*(-3*c*(A*b^2 - a*(b*B - a*C))*d + (A*b - a
*B - b*C)*d*(b*c - a*d))) + d^2*(3*A*b^2*d - a*A*(b*c - a*d) - (b*B - a*C)*
(b*c + 2*a*d)))/(2*(-(b*c) + a*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^2) - (
-((-(((b*c - a*d)^3*(-(b^2*(2*a*A*b*c^3 - a^2*B*c^3 + b^2*B*c^3 - 2*a*b*c^3
*C + 3*a^2*A*c^2*d - 3*A*b^2*c^2*d + 6*a*b*B*c^2*d - 3*a^2*c^2*C*d + 3*b^2*
c^2*C*d - 6*a*A*b*c*d^2 + 3*a^2*B*c*d^2 - 3*b^2*B*c*d^2 + 6*a*b*c*C*d^2 - a
^2*A*d^3 + A*b^2*d^3 - 2*a*b*B*d^3 + a^2*C*d^3 - b^2*C*d^3)) + Sqrt[-b^2]*(
-(a^2*A*b*c^3) + A*b^3*c^3 - 2*a*b^2*B*c^3 + a^2*b*c^3*C - b^3*c^3*C + 6*a*
A*b^2*c^2*d - 3*a^2*b*B*c^2*d + 3*b^3*B*c^2*d - 6*a*b^2*c^2*C*d + 3*a^2*A*b
*c*d^2 - 3*A*b^3*c*d^2 + 6*a*b^2*B*c*d^2 - 3*a^2*b*c*C*d^2 + 3*b^3*c*C*d^2
- 2*a*A*b^2*d^3 + a^2*b*B*d^3 - b^3*B*d^3 + 2*a*b^2*C*d^3))*Log[Sqrt[-b^2]
- b*Tan[e + f*x]])/(b*(a^2 + b^2)*(c^2 + d^2))) - (2*b^3*(c^2 + d^2)^2*(4*a
^3*b*B*d - 3*a^4*C*d + b^4*(B*c - 3*A*d) + 2*a*b^3*(A*c - c*C + B*d) - a^2*
b^2*(B*c + (5*A + C)*d))*Log[a + b*Tan[e + f*x]])/((a^2 + b^2)*(b*c - a*d))
 + ((b*c - a*d)^3*(b^2*(2*a*A*b*c^3 - a^2*B*c^3 + b^2*B*c^3 - 2*a*b*c^3*C +
 3*a^2*A*c^2*d - 3*A*b^2*c^2*d + 6*a*b*B*c^2*d - 3*a^2*c^2*C*d + 3*b^2*c^2*
C*d - 6*a*A*b*c*d^2 + 3*a^2*B*c*d^2 - 3*b^2*B*c*d^2 + 6*a*b*c*C*d^2 - a^2*A
*d^3 + A*b^2*d^3 - 2*a*b*B*d^3 + a^2*C*d^3 - b^2*C*d^3) + Sqrt[-b^2]*(-(a^2
*A*b*c^3) + A*b^3*c^3 - 2*a*b^2*B*c^3 + a^2*b*c^3*C - b^3*c^3*C + 6*a*A*b^2
*c^2*d - 3*a^2*b*B*c^2*d + 3*b^3*B*c^2*d - 6*a*b^2*c^2*C*d + 3*a^2*A*b*c*d^
2 - 3*A*b^3*c*d^2 + 6*a*b^2*B*c*d^2 - 3*a^2*b*c*C*d^2 + 3*b^3*c*C*d^2 - 2*a
*A*b^2*d^3 + a^2*b*B*d^3 - b^3*B*d^3 + 2*a*b^2*C*d^3))*Log[Sqrt[-b^2] + b*T
an[e + f*x]])/(b*(a^2 + b^2)*(c^2 + d^2)) - (2*b*(a^2 + b^2)*d*(b^2*(3*c^6*
C - 6*B*c^5*d + c^4*(10*A - C)*d^2 - 3*B*c^3*d^3 + 9*A*c^2*d^4 - B*c*d^5 +
3*A*d^6) + a^2*d^3*((A - C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)) - 2*a*b*d^
2*(c*(A - C)*d*(5*c^2 + d^2) - B*(2*c^4 - 3*c^2*d^2 - d^4)))*Log[c + d*Tan[
e + f*x]])/((b*c - a*d)*(c^2 + d^2)))/(b*(-(b*c) + a*d)*(c^2 + d^2)*f)) - (
d^2*(-2*a*d*(-3*c*(A*b^2 - a*(b*B - a*C))*d + (A*b - a*B - b*C)*d*(b*c - a*
d)) + (2*b*d^2 - 2*c*(-(b*c) + a*d))*(3*A*b^2*d - a*A*(b*c - a*d) - (b*B -
a*C)*(b*c + 2*a*d))) - c*(2*d*(-(b*c) + a*d)*(-3*(A*b^2 - a*(b*B - a*C))*d^
2 - c*(A*b - a*B - b*C)*(b*c - a*d) + d*(3*A*b^2*d - a*A*(b*c - a*d) - (b*B
 - a*C)*(b*c + 2*a*d))) - 2*b*c*(-(c*(-3*c*(A*b^2 - a*(b*B - a*C))*d + (A*b
 - a*B - b*C)*d*(b*c - a*d))) + d^2*(3*A*b^2*d - a*A*(b*c - a*d) - (b*B - a
*C)*(b*c + 2*a*d)))))/((-(b*c) + a*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*x])))/
(2*(-(b*c) + a*d)*(c^2 + d^2)))/((a^2 + b^2)*(b*c - a*d))
```

Maple [B] time = 0.14, size = 3364, normalized size = 3.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^3,x
```

)

[Out] -1/2/f/(a^2+b^2)^2/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*B*b^2*c^3-1/2/f/(a^2+b^2)^2/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*C*a^2*d^3+1/2/f/(a^2+b^2)^2/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*C*b^2*d^3-1/f*d^5/(a*d-b*c)^3/(c^2+d^2)^2/(c+d*tan(f*x+e))*B*a+1/f/(a^2+b^2)^2/(c^2+d^2)^3*A*arctan(tan(f*x+e))*a^2*c^3-1/f/(a^2+b^2)^2/(c^2+d^2)^3*A*arctan(tan(f*x+e))*b^2*c^3-1/f/(a^2+b^2)^2/(c^2+d^2)^3*B*arctan(tan(f*x+e))*a^2*d^3+1/f/(a^2+b^2)^2/(c^2+d^2)^3*B*arctan(tan(f*x+e))*b^2*d^3-1/f/(a^2+b^2)^2/(c^2+d^2)^3*C*arctan(tan(f*x+e))*a^2*c^3+1/f/(a^2+b^2)^2/(c^2+d^2)^3*C*arctan(tan(f*x+e))*b^2*c^3+1/2/f/(a^2+b^2)^2/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*A*a^2*d^3-1/2/f/(a^2+b^2)^2/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*A*b^2*d^3+3/f*d^5/(a*d-b*c)^4/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*A*a^2*c^2+10/f*d^3/(a*d-b*c)^4/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*A*b^2*c^4+9/f*d^5/(a*d-b*c)^4/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*A*b^2*c^2-1/f*d^4/(a*d-b*c)^4/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*B*a^2*c^3-5/f*b^4/(a^2+b^2)^2/(a*d-b*c)^4*ln(a+b*tan(f*x+e))*A*a^2*d+2/f*b^5/(a^2+b^2)^2/(a*d-b*c)^4*ln(a+b*tan(f*x+e))*A*a*c+4/f*b^3/(a^2+b^2)^2/(a*d-b*c)^4*ln(a+b*tan(f*x+e))*a^3*B*d-1/f*d^4/(a*d-b*c)^3/(c^2+d^2)^2/(c+d*tan(f*x+e))*B*b*c+2/f*d^4/(a*d-b*c)^3/(c^2+d^2)^2/(c+d*tan(f*x+e))*C*a*c+2/f*d/(a*d-b*c)^3/(c^2+d^2)^2/(c+d*tan(f*x+e))*C*b*c^4-3/f*b^2/(a^2+b^2)^2/(a*d-b*c)^4*ln(a+b*tan(f*x+e))*a^4*C*d-3/f*d^2/(a*d-b*c)^3/(c^2+d^2)^2/(c+d*tan(f*x+e))*B*b*c^3-3/f/(a^2+b^2)^2/(c^2+d^2)^3*B*arctan(tan(f*x+e))*b^2*c^2*d+2/f/(a^2+b^2)^2/(c^2+d^2)^3*B*arctan(tan(f*x+e))*a*b*c^3-1/f*b^4/(a^2+b^2)^2/(a*d-b*c)^4*ln(a+b*tan(f*x+e))*B*a^2*c+2/f*b^5/(a^2+b^2)^2/(a*d-b*c)^4*ln(a+b*tan(f*x+e))*B*a*d-3/f/(a^2+b^2)^2/(c^2+d^2)^3*A*arctan(tan(f*x+e))*a^2*c*d^2+3/2/f/(a^2+b^2)^2/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*B*b^2*c*d^2+3/2/f/(a^2+b^2)^2/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*C*a^2*c^2*d+1/f/(a^2+b^2)^2/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*C*a*b*c^3-3/2/f/(a^2+b^2)^2/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*C*b^2*c^2*d+2/f/(a^2+b^2)^2/(c^2+d^2)^3*A*arctan(tan(f*x+e))*a*b*d^3+3/f/(a^2+b^2)^2/(c^2+d^2)^3*A*arctan(tan(f*x+e))*b^2*c*d^2+3/f/(a^2+b^2)^2/(c^2+d^2)^3*B*arctan(tan(f*x+e))*a^2*c^2*d-1/f*b^4/(a^2+b^2)^2/(a*d-b*c)^4*ln(a+b*tan(f*x+e))*C*a^2*d-2/f*b^5/(a^2+b^2)^2/(a*d-b*c)^4*ln(a+b*tan(f*x+e))*C*a*c+3/f/(a^2+b^2)^2/(c^2+d^2)^3*C*arctan(tan(f*x+e))*a^2*c*d^2-2/f/(a^2+b^2)^2/(c^2+d^2)^3*C*arctan(tan(f*x+e))*a*b*d^3-3/f/(a^2+b^2)^2/(c^2+d^2)^3*C*arctan(tan(f*x+e))*b^2*c*d^2-3/2/f/(a^2+b^2)^2/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*A*a^2*c^2*d-1/f/(a^2+b^2)^2/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*A*a*b*c^3+1/f/(a^2+b^2)^2/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*B*a*b*d^3+1/f*d^3/(a*d-b*c)^3/(c^2+d^2)^2/(c+d*tan(f*x+e))*B*a*c^2+4/f*d^3/(a*d-b*c)^3/(c^2+d^2)^2/(c+d*tan(f*x+e))*A*b*c^2+3/f*d/(a*d-b*c)^4/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*C*b^2*c^6-1/f*d^3/(a*d-b*c)^4/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*C*b^2*c^4-2/f*d^4/(a*d-b*c)^3/(c^2+d^2)^2/(c+d*tan(f*x+e))*A*a*c-1/f*d^6/(a*d-b*c)^4/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*B*b^2*c-3/f*d^5/(a*d-b*c)^4/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*C*a^2*c^2-6/f*d^2/(a*d-b*c)^4/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*B*b^2*c^5-3/f*d^4/(a*d-b*c)^4/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*B*b^2*c^3+3/f*d^6/(a*d-b*c)^4/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*B*a^2*c-2/f*d^7/(a*d-b*c)^4/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*B*a*b+3/2/f/(a^2+b^2)^2/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*A*b^2*c^2*d-3/2/f/(a^2+b^2)^2/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*B*a^2*c*d^2-3/f/(a^2+b^2)^2/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*B*a*b*c^2*d+1/2/f/(a^2+b^2)^2/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*B*a^2*c^3-6/f*d^5/(a*d-b*c)^4/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*B*a*b*c^2+10/f*d^4/(a*d-b*c)^4/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*C*a*b*c^3+2/f*d^6/(a*d-b*c)^4/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*C*a*b*c-6/f/(a^2+b^2)^2/(c^2+d^2)^3*A*arctan(tan(f*x+e))*a*b*c^2*d+6/f/(a^2+b^2)^2/(c^2+d^2)^3*C*arctan(tan(f*x+e))*a*b*c^2*d-3/f/(a^2+b^2)^2/(c^2+d^2)^3*ln(1+tan(f*x+e)^2)*C*a*b*c*d^2-6/f/(a^2+b^2)^2/(c^2+d^2)^3*B*arctan(tan(f*x+e))*a*b*c*d^2-1/f*d^7/(a*d-b*c)^4/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*A*a^2+3/f*d^7/(a*d-b*c)^4/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*A*b^2+1/f*d^7/(a*d-b*c)^4/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*C*a^2+1/2/f*d^2/(a*d-b*c)^2/(c^2+d^2)/(c+d*tan(f*x+e))^2*B*c-1/2/f*d/(a*d-b*c)^2/(c^2+d^2)/(c+d*tan(f*x+e))^2*c^2*C-3/f*b^6/(a^2+b^2)^2/(a*d-b*c)^4*ln(a+b*tan(f*x+e))*A*d+1/f*b^6/(a^2+b^2)^2/(a*d-b*c)^4*ln(a+b*tan(f*x+e))*B*c-1/f*b^3/(a^2+b^

```
2)/(a*d-b*c)^3/(a+b*tan(f*x+e))*B*a+1/f*b^2/(a^2+b^2)/(a*d-b*c)^3/(a+b*tan(
f*x+e))*C*a^2+2/f*d^5/(a*d-b*c)^3/(c^2+d^2)^2/(c+d*tan(f*x+e))*A*b-10/f*d^4
/(a*d-b*c)^4/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*A*a*b*c^3+3/f/(a^2+b^2)^2/(c^2+
d^2)^3*ln(1+tan(f*x+e)^2)*A*a*b*c*d^2-2/f*d^6/(a*d-b*c)^4/(c^2+d^2)^3*ln(c+
d*tan(f*x+e))*A*a*b*c+4/f*d^3/(a*d-b*c)^4/(c^2+d^2)^3*ln(c+d*tan(f*x+e))*B*
a*b*c^4-1/2/f*d^3/(a*d-b*c)^2/(c^2+d^2)/(c+d*tan(f*x+e))^2*A+1/f*b^4/(a^2+b
^2)/(a*d-b*c)^3/(a+b*tan(f*x+e))*A
```

Maxima [B] time = 2.27326, size = 3425, normalized size = 3.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e
))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*c^3 + 3*(B*a^2 - 2*(A - C)*a*
b - B*b^2)*c^2*d - 3*((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*c*d^2 - (B*a^2 -
 2*(A - C)*a*b - B*b^2)*d^3)*(f*x + e)/((a^4 + 2*a^2*b^2 + b^4)*c^6 + 3*(a^
4 + 2*a^2*b^2 + b^4)*c^4*d^2 + 3*(a^4 + 2*a^2*b^2 + b^4)*c^2*d^4 + (a^4 + 2
*a^2*b^2 + b^4)*d^6) - 2*((B*a^2*b^4 - 2*(A - C)*a*b^5 - B*b^6)*c + (3*C*a^
4*b^2 - 4*B*a^3*b^3 + (5*A + C)*a^2*b^4 - 2*B*a*b^5 + 3*A*b^6)*d)*log(b*tan
(f*x + e) + a)/((a^4*b^4 + 2*a^2*b^6 + b^8)*c^4 - 4*(a^5*b^3 + 2*a^3*b^5 +
a*b^7)*c^3*d + 6*(a^6*b^2 + 2*a^4*b^4 + a^2*b^6)*c^2*d^2 - 4*(a^7*b + 2*a^5
*b^3 + a^3*b^5)*c*d^3 + (a^8 + 2*a^6*b^2 + a^4*b^4)*d^4) + 2*(3*C*b^2*c^6*d
 - 6*B*b^2*c^5*d^2 + (4*B*a*b + (10*A - C)*b^2)*c^4*d^3 - (B*a^2 + 10*(A -
C)*a*b + 3*B*b^2)*c^3*d^4 + 3*((A - C)*a^2 - 2*B*a*b + 3*A*b^2)*c^2*d^5 + (
3*B*a^2 - 2*(A - C)*a*b - B*b^2)*c*d^6 - ((A - C)*a^2 + 2*B*a*b - 3*A*b^2)*
d^7)*log(d*tan(f*x + e) + c)/(b^4*c^10 - 4*a*b^3*c^9*d - 4*a^3*b*c*d^9 + a^
4*d^10 + 3*(2*a^2*b^2 + b^4)*c^8*d^2 - 4*(a^3*b + 3*a*b^3)*c^7*d^3 + (a^4 +
 18*a^2*b^2 + 3*b^4)*c^6*d^4 - 12*(a^3*b + a*b^3)*c^5*d^5 + (3*a^4 + 18*a^2
*b^2 + b^4)*c^4*d^6 - 4*(3*a^3*b + a*b^3)*c^3*d^7 + 3*(a^4 + 2*a^2*b^2)*c^2
*d^8) + ((B*a^2 - 2*(A - C)*a*b - B*b^2)*c^3 - 3*((A - C)*a^2 + 2*B*a*b - (
A - C)*b^2)*c^2*d - 3*(B*a^2 - 2*(A - C)*a*b - B*b^2)*c*d^2 + ((A - C)*a^2
+ 2*B*a*b - (A - C)*b^2)*d^3)*log(tan(f*x + e)^2 + 1)/((a^4 + 2*a^2*b^2 + b
^4)*c^6 + 3*(a^4 + 2*a^2*b^2 + b^4)*c^4*d^2 + 3*(a^4 + 2*a^2*b^2 + b^4)*c^2
*d^4 + (a^4 + 2*a^2*b^2 + b^4)*d^6) - (2*(C*a^2*b^2 - B*a*b^3 + A*b^4)*c^6
+ 5*(C*a^3*b + C*a*b^3)*c^5*d - (C*a^4 + 7*B*a^3*b - 3*C*a^2*b^2 + 11*B*a*b
^3 - 4*A*b^4)*c^4*d^2 + (3*B*a^4 + (9*A + C)*a^3*b + 3*B*a^2*b^2 + (9*A + C
)*a*b^3)*c^3*d^3 - ((5*A - 3*C)*a^4 + 3*B*a^3*b + 5*(A - C)*a^2*b^2 + 5*B*a
*b^3 - 2*A*b^4)*c^2*d^4 - (B*a^4 - 5*A*a^3*b + B*a^2*b^2 - 5*A*a*b^3)*c*d^5
 - (A*a^4 + A*a^2*b^2)*d^6 + 2*((3*C*a^2*b^2 - B*a*b^3 + (A + 2*C)*b^4)*c^4
*d^2 - 3*(B*a^2*b^2 + B*b^4)*c^3*d^3 + (B*a^3*b + 2*(2*A + C)*a^2*b^2 - B*a
*b^3 + 6*A*b^4)*c^2*d^4 - (2*(A - C)*a^3*b + B*a^2*b^2 + 2*(A - C)*a*b^3 +
B*b^4)*c*d^5 - (B*a^3*b - (2*A + C)*a^2*b^2 + 2*B*a*b^3 - 3*A*b^4)*d^6)*tan
(f*x + e)^2 + ((9*C*a^2*b^2 - 4*B*a*b^3 + (4*A + 5*C)*b^4)*c^5*d + (3*C*a^3
*b - 7*B*a^2*b^2 + 3*C*a*b^3 - 7*B*b^4)*c^4*d^2 - (3*B*a^3*b - 9*(A + C)*a^
2*b^2 + 11*B*a*b^3 - (17*A + C)*b^4)*c^3*d^3 + (2*B*a^4 + 3*(A + C)*a^3*b -
 B*a^2*b^2 + 3*(A + C)*a*b^3 - 3*B*b^4)*c^2*d^4 - (4*(A - C)*a^4 + 3*B*a^3*
b - (A + 8*C)*a^2*b^2 + 7*B*a*b^3 - 9*A*b^4)*c*d^5 - (2*B*a^4 - 3*A*a^3*b +
 2*B*a^2*b^2 - 3*A*a*b^3)*d^6)*tan(f*x + e))/((a^3*b^3 + a*b^5)*c^9 - 3*(a^
4*b^2 + a^2*b^4)*c^8*d + (3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*c^7*d^2 - (a^6 + 7
*a^4*b^2 + 6*a^2*b^4)*c^6*d^3 + (6*a^5*b + 7*a^3*b^3 + a*b^5)*c^5*d^4 - (2*
a^6 + 5*a^4*b^2 + 3*a^2*b^4)*c^4*d^5 + 3*(a^5*b + a^3*b^3)*c^3*d^6 - (a^6 +
 a^4*b^2)*c^2*d^7 + ((a^2*b^4 + b^6)*c^7*d^2 - 3*(a^3*b^3 + a*b^5)*c^6*d^3
+ (3*a^4*b^2 + 5*a^2*b^4 + 2*b^6)*c^5*d^4 - (a^5*b + 7*a^3*b^3 + 6*a*b^5)*c
```

```
^4*d^5 + (6*a^4*b^2 + 7*a^2*b^4 + b^6)*c^3*d^6 - (2*a^5*b + 5*a^3*b^3 + 3*a
*b^5)*c^2*d^7 + 3*(a^4*b^2 + a^2*b^4)*c*d^8 - (a^5*b + a^3*b^3)*d^9)*tan(f*
x + e)^3 + (2*(a^2*b^4 + b^6)*c^8*d - 5*(a^3*b^3 + a*b^5)*c^7*d^2 + (3*a^4*
b^2 + 7*a^2*b^4 + 4*b^6)*c^6*d^3 + (a^5*b - 9*a^3*b^3 - 10*a*b^5)*c^5*d^4 -
 (a^6 - 5*a^4*b^2 - 8*a^2*b^4 - 2*b^6)*c^4*d^5 + (2*a^5*b - 3*a^3*b^3 - 5*a
*b^5)*c^3*d^6 - (2*a^6 - a^4*b^2 - 3*a^2*b^4)*c^2*d^7 + (a^5*b + a^3*b^3)*c
*d^8 - (a^6 + a^4*b^2)*d^9)*tan(f*x + e)^2 + ((a^2*b^4 + b^6)*c^9 - (a^3*b^
3 + a*b^5)*c^8*d - (3*a^4*b^2 + a^2*b^4 - 2*b^6)*c^7*d^2 + (5*a^5*b + 3*a^3
*b^3 - 2*a*b^5)*c^6*d^3 - (2*a^6 + 8*a^4*b^2 + 5*a^2*b^4 - b^6)*c^5*d^4 + (
10*a^5*b + 9*a^3*b^3 - a*b^5)*c^4*d^5 - (4*a^6 + 7*a^4*b^2 + 3*a^2*b^4)*c^3
*d^6 + 5*(a^5*b + a^3*b^3)*c^2*d^7 - 2*(a^6 + a^4*b^2)*c*d^8)*tan(f*x + e))
)/f
```

Fricas [B] time = 98.9495, size = 20006, normalized size = 23.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e
))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(C*a^2*b^5 - B*a*b^6 + A*b^7)*c^9 - 2*(C*a^3*b^4 - B*a^2*b^5 + A*a*
b^6)*c^8*d + 6*(C*a^2*b^5 - B*a*b^6 + A*b^7)*c^7*d^2 + (7*C*a^5*b^2 + 8*C*a
^3*b^4 + 6*B*a^2*b^5 - (6*A - 7*C)*a*b^6)*c^6*d^3 - (10*C*a^6*b + 9*B*a^5*b
^2 + 20*C*a^4*b^3 + 18*B*a^3*b^4 + 4*C*a^2*b^5 + 15*B*a*b^6 - 6*A*b^7)*c^5*
d^4 + (3*C*a^7 + 14*B*a^6*b + (11*A + 7*C)*a^5*b^2 + 28*B*a^4*b^3 + (22*A -
 C)*a^3*b^4 + 20*B*a^2*b^5 + (5*A + C)*a*b^6)*c^4*d^5 - (5*B*a^7 + 2*(9*A -
 C)*a^6*b + 13*B*a^5*b^2 + 4*(9*A - C)*a^4*b^3 + 11*B*a^3*b^4 + 2*(9*A - 2*
C)*a^2*b^5 + 5*B*a*b^6 - 2*A*b^7)*c^3*d^6 + ((7*A - 3*C)*a^7 + 2*B*a^6*b +
(19*A - 6*C)*a^5*b^2 + 4*B*a^4*b^3 + (17*A - 5*C)*a^3*b^4 + 4*B*a^2*b^5 + 3
*A*a*b^6)*c^2*d^7 + (B*a^7 - 6*A*a^6*b + 2*B*a^5*b^2 - 12*A*a^4*b^3 + B*a^3
*b^4 - 6*A*a^2*b^5)*c*d^8 + (A*a^7 + 2*A*a^5*b^2 + A*a^3*b^4)*d^9 - (2*(C*a
^3*b^4 - B*a^2*b^5 + A*a*b^6)*c^7*d^2 + (3*C*a^4*b^3 + 2*B*a^3*b^4 - 2*(A -
 5*C)*a^2*b^5 + 5*C*b^7)*c^6*d^3 - (6*C*a^5*b^2 + 7*B*a^4*b^3 + 6*C*a^3*b^4
 + 20*B*a^2*b^5 - 6*(A - C)*a*b^6 + 7*B*b^7)*c^5*d^4 + (C*a^6*b + 10*B*a^5*
b^2 + (9*A - 5*C)*a^4*b^3 + 26*B*a^3*b^4 + (12*A - C)*a^2*b^5 + 10*B*a*b^6
+ (9*A - C)*b^7)*c^4*d^5 - (3*B*a^6*b + 2*(7*A - 3*C)*a^5*b^2 + 7*B*a^4*b^3
 + 2*(14*A - 9*C)*a^3*b^4 + 11*B*a^2*b^5 + 2*(4*A - 3*C)*a*b^6 + B*b^7)*c^3
*d^6 + (5*(A - C)*a^6*b - 2*B*a^5*b^2 + (13*A - 16*C)*a^4*b^3 + 2*B*a^3*b^4
 + 5*(A - C)*a^2*b^5 - 2*B*a*b^6 + 3*A*b^7)*c^2*d^7 + (3*B*a^6*b - 2*A*a^5*
b^2 + 6*B*a^4*b^3 - 2*(2*A - C)*a^3*b^4 + B*a^2*b^5)*c*d^8 - (A*a^6*b + 2*(
A + C)*a^4*b^3 - 2*B*a^3*b^4 + 3*A*a^2*b^5)*d^9 + 2*(((A - C)*a^2*b^5 + 2*B
*a*b^6 - (A - C)*b^7)*c^7*d^2 - (4*(A - C)*a^3*b^4 + 5*B*a^2*b^5 + 2*(A - C
)*a*b^6 + 3*B*b^7)*c^6*d^3 + 3*(2*(A - C)*a^4*b^3 + 5*(A - C)*a^2*b^5 + 2*B
*a*b^6 + (A - C)*b^7)*c^5*d^4 - (4*(A - C)*a^5*b^2 - 10*B*a^4*b^3 + 20*(A -
 C)*a^3*b^4 - 5*B*a^2*b^5 + 10*(A - C)*a*b^6 - B*b^7)*c^4*d^5 + ((A - C)*a^
6*b - 10*B*a^5*b^2 + 5*(A - C)*a^4*b^3 - 20*B*a^3*b^4 + 10*(A - C)*a^2*b^5
- 4*B*a*b^6)*c^3*d^6 + 3*(B*a^6*b + 2*(A - C)*a^5*b^2 + 5*B*a^4*b^3 + 2*B*a
^2*b^5)*c^2*d^7 - (3*(A - C)*a^6*b + 2*B*a^5*b^2 + 5*(A - C)*a^4*b^3 + 4*B*
a^3*b^4)*c*d^8 - (B*a^6*b - 2*(A - C)*a^5*b^2 - B*a^4*b^3)*d^9)*f*x)*tan(f*
x + e)^3 - 2*(((A - C)*a^3*b^4 + 2*B*a^2*b^5 - (A - C)*a*b^6)*c^9 - (4*(A -
 C)*a^4*b^3 + 5*B*a^3*b^4 + 2*(A - C)*a^2*b^5 + 3*B*a*b^6)*c^8*d + 3*(2*(A
- C)*a^5*b^2 + 5*(A - C)*a^3*b^4 + 2*B*a^2*b^5 + (A - C)*a*b^6)*c^7*d^2 - (
4*(A - C)*a^6*b - 10*B*a^5*b^2 + 20*(A - C)*a^4*b^3 - 5*B*a^3*b^4 + 10*(A -
 C)*a^2*b^5 - B*a*b^6)*c^6*d^3 + ((A - C)*a^7 - 10*B*a^6*b + 5*(A - C)*a^5*
b^2 - 20*B*a^4*b^3 + 10*(A - C)*a^3*b^4 - 4*B*a^2*b^5)*c^5*d^4 + 3*(B*a^7 +
```

```
2*(A - C)*a^6*b + 5*B*a^5*b^2 + 2*B*a^3*b^4)*c^4*d^5 - (3*(A - C)*a^7 + 2*
B*a^6*b + 5*(A - C)*a^5*b^2 + 4*B*a^4*b^3)*c^3*d^6 - (B*a^7 - 2*(A - C)*a^6
*b - B*a^5*b^2)*c^2*d^7)*f*x - (4*(C*a^3*b^4 - B*a^2*b^5 + A*a*b^6)*c^8*d +
 2*(C*a^4*b^3 + 2*B*a^3*b^4 - (2*A - 5*C)*a^2*b^5 + B*a*b^6 - (A - 3*C)*b^7
)*c^7*d^2 - (3*C*a^5*b^2 + 8*B*a^4*b^3 - 8*C*a^3*b^4 + 30*B*a^2*b^5 - (14*A
 - 3*C)*a*b^6 + 8*B*b^7)*c^6*d^3 - (4*C*a^6*b - 5*B*a^5*b^2 - 2*(5*A - 13*C
)*a^4*b^3 - 22*B*a^3*b^4 - 2*(4*A - 11*C)*a^2*b^5 - 11*B*a*b^6 - 2*(2*A - 3
*C)*b^7)*c^5*d^4 + (C*a^7 + 6*B*a^6*b - (7*A - 13*C)*a^5*b^2 + 18*B*a^4*b^3
 - (14*A - 41*C)*a^3*b^4 + 11*(A + C)*a*b^6 + 6*B*b^7)*c^4*d^5 - (3*B*a^7 +
 8*A*a^6*b + 19*B*a^5*b^2 + 2*(11*A + 6*C)*a^4*b^3 + 17*B*a^3*b^4 + 2*(16*A
 + 3*C)*a^2*b^5 + 7*B*a*b^6 + 12*A*b^7)*c^3*d^6 + (5*(A - C)*a^7 + 4*B*a^6*
b + (25*A - 14*C)*a^5*b^2 + 10*B*a^4*b^3 + (35*A - 3*C)*a^3*b^4 - 2*B*a^2*b
^5 + (25*A - 4*C)*a*b^6 + 2*B*b^7)*c^2*d^7 + (3*B*a^7 - 4*(2*A - C)*a^6*b +
 6*B*a^5*b^2 - 4*(5*A - C)*a^4*b^3 + 7*B*a^3*b^4 - 2*(10*A - C)*a^2*b^5 + 2
*B*a*b^6 - 6*A*b^7)*c*d^8 - (A*a^7 + 2*B*a^6*b - 2*A*a^5*b^2 + 4*B*a^4*b^3
- (7*A + 2*C)*a^3*b^4 + 4*B*a^2*b^5 - 6*A*a*b^6)*d^9 + 2*(2*((A - C)*a^2*b^
5 + 2*B*a*b^6 - (A - C)*b^7)*c^8*d - (7*(A - C)*a^3*b^4 + 8*B*a^2*b^5 + 5*(
A - C)*a*b^6 + 6*B*b^7)*c^7*d^2 + (8*(A - C)*a^4*b^3 - 5*B*a^3*b^4 + 28*(A
- C)*a^2*b^5 + 9*B*a*b^6 + 6*(A - C)*b^7)*c^6*d^3 - (2*(A - C)*a^5*b^2 - 20
*B*a^4*b^3 + 25*(A - C)*a^3*b^4 - 16*B*a^2*b^5 + 17*(A - C)*a*b^6 - 2*B*b^7
)*c^5*d^4 - (2*(A - C)*a^6*b + 10*B*a^5*b^2 + 10*(A - C)*a^4*b^3 + 35*B*a^3
*b^4 - 10*(A - C)*a^2*b^5 + 7*B*a*b^6)*c^4*d^5 + ((A - C)*a^7 - 4*B*a^6*b +
 17*(A - C)*a^5*b^2 + 10*B*a^4*b^3 + 10*(A - C)*a^3*b^4 + 8*B*a^2*b^5)*c^3*
d^6 + (3*B*a^7 + 11*B*a^5*b^2 - 10*(A - C)*a^4*b^3 - 2*B*a^3*b^4)*c^2*d^7 -
 (3*(A - C)*a^7 + 4*B*a^6*b + (A - C)*a^5*b^2 + 2*B*a^4*b^3)*c*d^8 - (B*a^7
 - 2*(A - C)*a^6*b - B*a^5*b^2)*d^9)*f*x)*tan(f*x + e)^2 + ((B*a^3*b^4 - 2*
(A - C)*a^2*b^5 - B*a*b^6)*c^9 + (3*C*a^5*b^2 - 4*B*a^4*b^3 + (5*A + C)*a^3
*b^4 - 2*B*a^2*b^5 + 3*A*a*b^6)*c^8*d + 3*(B*a^3*b^4 - 2*(A - C)*a^2*b^5 -
B*a*b^6)*c^7*d^2 + 3*(3*C*a^5*b^2 - 4*B*a^4*b^3 + (5*A + C)*a^3*b^4 - 2*B*a
^2*b^5 + 3*A*a*b^6)*c^6*d^3 + 3*(B*a^3*b^4 - 2*(A - C)*a^2*b^5 - B*a*b^6)*c
^5*d^4 + 3*(3*C*a^5*b^2 - 4*B*a^4*b^3 + (5*A + C)*a^3*b^4 - 2*B*a^2*b^5 + 3
*A*a*b^6)*c^4*d^5 + (B*a^3*b^4 - 2*(A - C)*a^2*b^5 - B*a*b^6)*c^3*d^6 + (3*
C*a^5*b^2 - 4*B*a^4*b^3 + (5*A + C)*a^3*b^4 - 2*B*a^2*b^5 + 3*A*a*b^6)*c^2*
d^7 + ((B*a^2*b^5 - 2*(A - C)*a*b^6 - B*b^7)*c^7*d^2 + (3*C*a^4*b^3 - 4*B*a
^3*b^4 + (5*A + C)*a^2*b^5 - 2*B*a*b^6 + 3*A*b^7)*c^6*d^3 + 3*(B*a^2*b^5 -
2*(A - C)*a*b^6 - B*b^7)*c^5*d^4 + 3*(3*C*a^4*b^3 - 4*B*a^3*b^4 + (5*A + C)
*a^2*b^5 - 2*B*a*b^6 + 3*A*b^7)*c^4*d^5 + 3*(B*a^2*b^5 - 2*(A - C)*a*b^6 -
B*b^7)*c^3*d^6 + 3*(3*C*a^4*b^3 - 4*B*a^3*b^4 + (5*A + C)*a^2*b^5 - 2*B*a*b
^6 + 3*A*b^7)*c^2*d^7 + (B*a^2*b^5 - 2*(A - C)*a*b^6 - B*b^7)*c*d^8 + (3*C*
a^4*b^3 - 4*B*a^3*b^4 + (5*A + C)*a^2*b^5 - 2*B*a*b^6 + 3*A*b^7)*d^9)*tan(f
*x + e)^3 + (2*(B*a^2*b^5 - 2*(A - C)*a*b^6 - B*b^7)*c^8*d + (6*C*a^4*b^3 -
 7*B*a^3*b^4 + 4*(2*A + C)*a^2*b^5 - 5*B*a*b^6 + 6*A*b^7)*c^7*d^2 + (3*C*a^
5*b^2 - 4*B*a^4*b^3 + (5*A + C)*a^3*b^4 + 4*B*a^2*b^5 - 3*(3*A - 4*C)*a*b^6
 - 6*B*b^7)*c^6*d^3 + 3*(6*C*a^4*b^3 - 7*B*a^3*b^4 + 4*(2*A + C)*a^2*b^5 -
5*B*a*b^6 + 6*A*b^7)*c^5*d^4 + 3*(3*C*a^5*b^2 - 4*B*a^4*b^3 + (5*A + C)*a^3
*b^4 - (A - 4*C)*a*b^6 - 2*B*b^7)*c^4*d^5 + 3*(6*C*a^4*b^3 - 7*B*a^3*b^4 +
4*(2*A + C)*a^2*b^5 - 5*B*a*b^6 + 6*A*b^7)*c^3*d^6 + (9*C*a^5*b^2 - 12*B*a^
4*b^3 + 3*(5*A + C)*a^3*b^4 - 4*B*a^2*b^5 + (5*A + 4*C)*a*b^6 - 2*B*b^7)*c^
2*d^7 + (6*C*a^4*b^3 - 7*B*a^3*b^4 + 4*(2*A + C)*a^2*b^5 - 5*B*a*b^6 + 6*A*
b^7)*c*d^8 + (3*C*a^5*b^2 - 4*B*a^4*b^3 + (5*A + C)*a^3*b^4 - 2*B*a^2*b^5 +
 3*A*a*b^6)*d^9)*tan(f*x + e)^2 + ((B*a^2*b^5 - 2*(A - C)*a*b^6 - B*b^7)*c^
9 + (3*C*a^4*b^3 - 2*B*a^3*b^4 + (A + 5*C)*a^2*b^5 - 4*B*a*b^6 + 3*A*b^7)*c
^8*d + (6*C*a^5*b^2 - 8*B*a^4*b^3 + 2*(5*A + C)*a^3*b^4 - B*a^2*b^5 + 6*C*a
*b^6 - 3*B*b^7)*c^7*d^2 + 3*(3*C*a^4*b^3 - 2*B*a^3*b^4 + (A + 5*C)*a^2*b^5
- 4*B*a*b^6 + 3*A*b^7)*c^6*d^3 + 3*(6*C*a^5*b^2 - 8*B*a^4*b^3 + 2*(5*A + C)
*a^3*b^4 - 3*B*a^2*b^5 + 2*(2*A + C)*a*b^6 - B*b^7)*c^5*d^4 + 3*(3*C*a^4*b^
3 - 2*B*a^3*b^4 + (A + 5*C)*a^2*b^5 - 4*B*a*b^6 + 3*A*b^7)*c^4*d^5 + (18*C*
a^5*b^2 - 24*B*a^4*b^3 + 6*(5*A + C)*a^3*b^4 - 11*B*a^2*b^5 + 2*(8*A + C)*a
*b^6 - B*b^7)*c^3*d^6 + (3*C*a^4*b^3 - 2*B*a^3*b^4 + (A + 5*C)*a^2*b^5 - 4*
```

B*a*b^6 + 3*A*b^7)*c^2*d^7 + 2*(3*C*a^5*b^2 - 4*B*a^4*b^3 + (5*A + C)*a^3*b
^4 - 2*B*a^2*b^5 + 3*A*a*b^6)*c*d^8)*tan(f*x + e))*log((b^2*tan(f*x + e)^2
+ 2*a*b*tan(f*x + e) + a^2)/(tan(f*x + e)^2 + 1)) - (3*(C*a^5*b^2 + 2*C*a^3
*b^4 + C*a*b^6)*c^8*d - 6*(B*a^5*b^2 + 2*B*a^3*b^4 + B*a*b^6)*c^7*d^2 + (4*
B*a^6*b + (10*A - C)*a^5*b^2 + 8*B*a^4*b^3 + 2*(10*A - C)*a^3*b^4 + 4*B*a^2
*b^5 + (10*A - C)*a*b^6)*c^6*d^3 - (B*a^7 + 10*(A - C)*a^6*b + 5*B*a^5*b^2
+ 20*(A - C)*a^4*b^3 + 7*B*a^3*b^4 + 10*(A - C)*a^2*b^5 + 3*B*a*b^6)*c^5*d^
4 + 3*((A - C)*a^7 - 2*B*a^6*b + (5*A - 2*C)*a^5*b^2 - 4*B*a^4*b^3 + (7*A -
C)*a^3*b^4 - 2*B*a^2*b^5 + 3*A*a*b^6)*c^4*d^5 + (3*B*a^7 - 2*(A - C)*a^6*b
+ 5*B*a^5*b^2 - 4*(A - C)*a^4*b^3 + B*a^3*b^4 - 2*(A - C)*a^2*b^5 - B*a*b^
6)*c^3*d^6 - ((A - C)*a^7 + 2*B*a^6*b - (A + 2*C)*a^5*b^2 + 4*B*a^4*b^3 - (
5*A + C)*a^3*b^4 + 2*B*a^2*b^5 - 3*A*a*b^6)*c^2*d^7 + (3*(C*a^4*b^3 + 2*C*a
^2*b^5 + C*b^7)*c^6*d^3 - 6*(B*a^4*b^3 + 2*B*a^2*b^5 + B*b^7)*c^5*d^4 + (4*
B*a^5*b^2 + (10*A - C)*a^4*b^3 + 8*B*a^3*b^4 + 2*(10*A - C)*a^2*b^5 + 4*B*a
*b^6 + (10*A - C)*b^7)*c^4*d^5 - (B*a^6*b + 10*(A - C)*a^5*b^2 + 5*B*a^4*b^
3 + 20*(A - C)*a^3*b^4 + 7*B*a^2*b^5 + 10*(A - C)*a*b^6 + 3*B*b^7)*c^3*d^6
+ 3*((A - C)*a^6*b - 2*B*a^5*b^2 + (5*A - 2*C)*a^4*b^3 - 4*B*a^3*b^4 + (7*A
- C)*a^2*b^5 - 2*B*a*b^6 + 3*A*b^7)*c^2*d^7 + (3*B*a^6*b - 2*(A - C)*a^5*b
^2 + 5*B*a^4*b^3 - 4*(A - C)*a^3*b^4 + B*a^2*b^5 - 2*(A - C)*a*b^6 - B*b^7)
*c*d^8 - ((A - C)*a^6*b + 2*B*a^5*b^2 - (A + 2*C)*a^4*b^3 + 4*B*a^3*b^4 - (
5*A + C)*a^2*b^5 + 2*B*a*b^6 - 3*A*b^7)*d^9)*tan(f*x + e)^3 + (6*(C*a^4*b^3
+ 2*C*a^2*b^5 + C*b^7)*c^7*d^2 + 3*(C*a^5*b^2 - 4*B*a^4*b^3 + 2*C*a^3*b^4
- 8*B*a^2*b^5 + C*a*b^6 - 4*B*b^7)*c^6*d^3 + 2*(B*a^5*b^2 + (10*A - C)*a^4*
b^3 + 2*B*a^3*b^4 + 2*(10*A - C)*a^2*b^5 + B*a*b^6 + (10*A - C)*b^7)*c^5*d^
4 + (2*B*a^6*b - (10*A - 19*C)*a^5*b^2 - 2*B*a^4*b^3 - 2*(10*A - 19*C)*a^3*
b^4 - 10*B*a^2*b^5 - (10*A - 19*C)*a*b^6 - 6*B*b^7)*c^4*d^5 - (B*a^7 + 4*(A
- C)*a^6*b + 17*B*a^5*b^2 - 2*(5*A + 4*C)*a^4*b^3 + 31*B*a^3*b^4 - 4*(8*A
+ C)*a^2*b^5 + 15*B*a*b^6 - 18*A*b^7)*c^3*d^6 + (3*(A - C)*a^7 + (11*A - 2*
C)*a^5*b^2 - 2*B*a^4*b^3 + (13*A + 5*C)*a^3*b^4 - 4*B*a^2*b^5 + (5*A + 4*C)
*a*b^6 - 2*B*b^7)*c^2*d^7 + (3*B*a^7 - 4*(A - C)*a^6*b + B*a^5*b^2 - 2*(A -
4*C)*a^4*b^3 - 7*B*a^3*b^4 + 4*(2*A + C)*a^2*b^5 - 5*B*a*b^6 + 6*A*b^7)*c*
d^8 - ((A - C)*a^7 + 2*B*a^6*b - (A + 2*C)*a^5*b^2 + 4*B*a^4*b^3 - (5*A + C
)*a^3*b^4 + 2*B*a^2*b^5 - 3*A*a*b^6)*d^9)*tan(f*x + e)^2 + (3*(C*a^4*b^3 +
2*C*a^2*b^5 + C*b^7)*c^8*d + 6*(C*a^5*b^2 - B*a^4*b^3 + 2*C*a^3*b^4 - 2*B*a
^2*b^5 + C*a*b^6 - B*b^7)*c^7*d^2 - (8*B*a^5*b^2 - (10*A - C)*a^4*b^3 + 16*
B*a^3*b^4 - 2*(10*A - C)*a^2*b^5 + 8*B*a*b^6 - (10*A - C)*b^7)*c^6*d^3 + (7
*B*a^6*b + 2*(5*A + 4*C)*a^5*b^2 + 11*B*a^4*b^3 + 4*(5*A + 4*C)*a^3*b^4 + B
*a^2*b^5 + 2*(5*A + 4*C)*a*b^6 - 3*B*b^7)*c^5*d^4 - (2*B*a^7 + 17*(A - C)*a
^6*b + 16*B*a^5*b^2 + (25*A - 34*C)*a^4*b^3 + 26*B*a^3*b^4 - (A + 17*C)*a^2
*b^5 + 12*B*a*b^6 - 9*A*b^7)*c^4*d^5 + (6*(A - C)*a^7 - 9*B*a^6*b + 2*(14*A
- 5*C)*a^5*b^2 - 19*B*a^4*b^3 + 2*(19*A - C)*a^3*b^4 - 11*B*a^2*b^5 + 2*(8
*A + C)*a*b^6 - B*b^7)*c^3*d^6 + (6*B*a^7 - 5*(A - C)*a^6*b + 8*B*a^5*b^2 -
(7*A - 10*C)*a^4*b^3 - 2*B*a^3*b^4 + (A + 5*C)*a^2*b^5 - 4*B*a*b^6 + 3*A*b
^7)*c^2*d^7 - 2*((A - C)*a^7 + 2*B*a^6*b - (A + 2*C)*a^5*b^2 + 4*B*a^4*b^3
- (5*A + C)*a^3*b^4 + 2*B*a^2*b^5 - 3*A*a*b^6)*c*d^8)*tan(f*x + e))*log((d^
2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) - (2*(C*
a^3*b^4 - B*a^2*b^5 + A*a*b^6)*c^9 - 2*(C*a^4*b^3 - B*a^3*b^4 + (A + 2*C)*a
^2*b^5 - 2*B*a*b^6 + 2*A*b^7)*c^8*d + 2*(3*C*a^5*b^2 + 11*C*a^3*b^4 - 5*B*a
^2*b^5 + (5*A + 3*C)*a*b^6)*c^7*d^2 - (8*C*a^6*b + 8*B*a^5*b^2 + 29*C*a^4*b
^3 + 10*B*a^3*b^4 + 2*(3*A + 17*C)*a^2*b^5 - 4*B*a*b^6 + (12*A + 7*C)*b^7)*
c^6*d^3 + (2*C*a^7 + 12*B*a^6*b + 2*(5*A + 4*C)*a^5*b^2 + 33*B*a^4*b^3 + 4*
(5*A + 7*C)*a^3*b^4 + 12*B*a^2*b^5 + 4*(7*A + C)*a*b^6 + 9*B*b^7)*c^5*d^4 -
(4*B*a^7 + (16*A - 9*C)*a^6*b + 16*B*a^5*b^2 + (43*A - 11*C)*a^4*b^3 + 14*
B*a^3*b^4 + (44*A + 5*C)*a^2*b^5 - 4*B*a*b^6 + (23*A + C)*b^7)*c^4*d^5 + (6
*(A - C)*a^7 - 7*B*a^6*b + 2*(12*A - 7*C)*a^5*b^2 - 11*B*a^4*b^3 + 2*(15*A
+ 2*C)*a^3*b^4 - 15*B*a^2*b^5 + 2*(13*A - C)*a*b^6 + 3*B*b^7)*c^3*d^6 + (6*
B*a^7 + (5*A - C)*a^6*b + 12*B*a^5*b^2 + (5*A - 4*C)*a^4*b^3 + 8*B*a^3*b^4
- (7*A + 5*C)*a^2*b^5 + 4*B*a*b^6 - 9*A*b^7)*c^2*d^7 - (2*(3*A - 2*C)*a^7 +
B*a^6*b + 2*(5*A - 4*C)*a^5*b^2 + 2*B*a^4*b^3 + 2*(A - 4*C)*a^3*b^4 + 5*B*

```
a^2*b^5 - 6*A*a*b^6)*c*d^8 - (2*B*a^7 - 3*A*a^6*b + 4*B*a^5*b^2 - 6*A*a^4*b
^3 + 2*B*a^3*b^4 - 3*A*a^2*b^5)*d^9 + 2*(((A - C)*a^2*b^5 + 2*B*a*b^6 - (A
- C)*b^7)*c^9 - (2*(A - C)*a^3*b^4 + B*a^2*b^5 + 4*(A - C)*a*b^6 + 3*B*b^7)
*c^8*d - (2*(A - C)*a^4*b^3 + 10*B*a^3*b^4 - 11*(A - C)*a^2*b^5 - 3*(A - C)
*b^7)*c^7*d^2 + (8*(A - C)*a^5*b^2 + 10*B*a^4*b^3 + 10*(A - C)*a^3*b^4 + 17
*B*a^2*b^5 - 4*(A - C)*a*b^6 + B*b^7)*c^6*d^3 - (7*(A - C)*a^6*b - 10*B*a^5
*b^2 + 35*(A - C)*a^4*b^3 + 10*B*a^3*b^4 + 10*(A - C)*a^2*b^5 + 2*B*a*b^6)*
c^5*d^4 + (2*(A - C)*a^7 - 17*B*a^6*b + 16*(A - C)*a^5*b^2 - 25*B*a^4*b^3 +
 20*(A - C)*a^3*b^4 - 2*B*a^2*b^5)*c^4*d^5 + (6*B*a^7 + 9*(A - C)*a^6*b + 2
8*B*a^5*b^2 - 5*(A - C)*a^4*b^3 + 8*B*a^3*b^4)*c^3*d^6 - (6*(A - C)*a^7 + 5
*B*a^6*b + 8*(A - C)*a^5*b^2 + 7*B*a^4*b^3)*c^2*d^7 - 2*(B*a^7 - 2*(A - C)*
a^6*b - B*a^5*b^2)*c*d^8)*f*x)*tan(f*x + e))/(((a^4*b^5 + 2*a^2*b^7 + b^9)*
c^10*d^2 - 4*(a^5*b^4 + 2*a^3*b^6 + a*b^8)*c^9*d^3 + 3*(2*a^6*b^3 + 5*a^4*b
^5 + 4*a^2*b^7 + b^9)*c^8*d^4 - 4*(a^7*b^2 + 5*a^5*b^4 + 7*a^3*b^6 + 3*a*b^
8)*c^7*d^5 + (a^8*b + 20*a^6*b^3 + 40*a^4*b^5 + 24*a^2*b^7 + 3*b^9)*c^6*d^6
 - 12*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*c^5*d^7 + (3*a^8*b + 24*a^6
*b^3 + 40*a^4*b^5 + 20*a^2*b^7 + b^9)*c^4*d^8 - 4*(3*a^7*b^2 + 7*a^5*b^4 +
5*a^3*b^6 + a*b^8)*c^3*d^9 + 3*(a^8*b + 4*a^6*b^3 + 5*a^4*b^5 + 2*a^2*b^7)*
c^2*d^10 - 4*(a^7*b^2 + 2*a^5*b^4 + a^3*b^6)*c*d^11 + (a^8*b + 2*a^6*b^3 +
a^4*b^5)*d^12)*f*tan(f*x + e)^3 + (2*(a^4*b^5 + 2*a^2*b^7 + b^9)*c^11*d - 7
*(a^5*b^4 + 2*a^3*b^6 + a*b^8)*c^10*d^2 + 2*(4*a^6*b^3 + 11*a^4*b^5 + 10*a^
2*b^7 + 3*b^9)*c^9*d^3 - (2*a^7*b^2 + 25*a^5*b^4 + 44*a^3*b^6 + 21*a*b^8)*c
^8*d^4 - 2*(a^8*b - 10*a^6*b^3 - 26*a^4*b^5 - 18*a^2*b^7 - 3*b^9)*c^7*d^5 +
 (a^9 - 4*a^7*b^2 - 32*a^5*b^4 - 48*a^3*b^6 - 21*a*b^8)*c^6*d^6 - 2*(3*a^8*
b - 6*a^6*b^3 - 22*a^4*b^5 - 14*a^2*b^7 - b^9)*c^5*d^7 + (3*a^9 - 16*a^5*b^
4 - 20*a^3*b^6 - 7*a*b^8)*c^4*d^8 - 2*(3*a^8*b + 2*a^6*b^3 - 5*a^4*b^5 - 4*
a^2*b^7)*c^3*d^9 + (3*a^9 + 4*a^7*b^2 - a^5*b^4 - 2*a^3*b^6)*c^2*d^10 - 2*(
a^8*b + 2*a^6*b^3 + a^4*b^5)*c*d^11 + (a^9 + 2*a^7*b^2 + a^5*b^4)*d^12)*f*t
an(f*x + e)^2 + ((a^4*b^5 + 2*a^2*b^7 + b^9)*c^12 - 2*(a^5*b^4 + 2*a^3*b^6
+ a*b^8)*c^11*d - (2*a^6*b^3 + a^4*b^5 - 4*a^2*b^7 - 3*b^9)*c^10*d^2 + 2*(4
*a^7*b^2 + 5*a^5*b^4 - 2*a^3*b^6 - 3*a*b^8)*c^9*d^3 - (7*a^8*b + 20*a^6*b^3
 + 16*a^4*b^5 - 3*b^9)*c^8*d^4 + 2*(a^9 + 14*a^7*b^2 + 22*a^5*b^4 + 6*a^3*b
^6 - 3*a*b^8)*c^7*d^5 - (21*a^8*b + 48*a^6*b^3 + 32*a^4*b^5 + 4*a^2*b^7 - b
^9)*c^6*d^6 + 2*(3*a^9 + 18*a^7*b^2 + 26*a^5*b^4 + 10*a^3*b^6 - a*b^8)*c^5*
d^7 - (21*a^8*b + 44*a^6*b^3 + 25*a^4*b^5 + 2*a^2*b^7)*c^4*d^8 + 2*(3*a^9 +
 10*a^7*b^2 + 11*a^5*b^4 + 4*a^3*b^6)*c^3*d^9 - 7*(a^8*b + 2*a^6*b^3 + a^4*
b^5)*c^2*d^10 + 2*(a^9 + 2*a^7*b^2 + a^5*b^4)*c*d^11)*f*tan(f*x + e) + ((a^
5*b^4 + 2*a^3*b^6 + a*b^8)*c^12 - 4*(a^6*b^3 + 2*a^4*b^5 + a^2*b^7)*c^11*d
+ 3*(2*a^7*b^2 + 5*a^5*b^4 + 4*a^3*b^6 + a*b^8)*c^10*d^2 - 4*(a^8*b + 5*a^6
*b^3 + 7*a^4*b^5 + 3*a^2*b^7)*c^9*d^3 + (a^9 + 20*a^7*b^2 + 40*a^5*b^4 + 24
*a^3*b^6 + 3*a*b^8)*c^8*d^4 - 12*(a^8*b + 3*a^6*b^3 + 3*a^4*b^5 + a^2*b^7)*
c^7*d^5 + (3*a^9 + 24*a^7*b^2 + 40*a^5*b^4 + 20*a^3*b^6 + a*b^8)*c^6*d^6 -
4*(3*a^8*b + 7*a^6*b^3 + 5*a^4*b^5 + a^2*b^7)*c^5*d^7 + 3*(a^9 + 4*a^7*b^2
+ 5*a^5*b^4 + 2*a^3*b^6)*c^4*d^8 - 4*(a^8*b + 2*a^6*b^3 + a^4*b^5)*c^3*d^9
+ (a^9 + 2*a^7*b^2 + a^5*b^4)*c^2*d^10)*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**2/(c+d*tan(f*x
+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.95277, size = 4288, normalized size = 4.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e
))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a^2*c^3 - C*a^2*c^3 + 2*B*a*b*c^3 - A*b^2*c^3 + C*b^2*c^3 + 3*B*a
^2*c^2*d - 6*A*a*b*c^2*d + 6*C*a*b*c^2*d - 3*B*b^2*c^2*d - 3*A*a^2*c*d^2 +
3*C*a^2*c*d^2 - 6*B*a*b*c*d^2 + 3*A*b^2*c*d^2 - 3*C*b^2*c*d^2 - B*a^2*d^3 +
 2*A*a*b*d^3 - 2*C*a*b*d^3 + B*b^2*d^3)*(f*x + e)/(a^4*c^6 + 2*a^2*b^2*c^6
+ b^4*c^6 + 3*a^4*c^4*d^2 + 6*a^2*b^2*c^4*d^2 + 3*b^4*c^4*d^2 + 3*a^4*c^2*d
^4 + 6*a^2*b^2*c^2*d^4 + 3*b^4*c^2*d^4 + a^4*d^6 + 2*a^2*b^2*d^6 + b^4*d^6)
 + (B*a^2*c^3 - 2*A*a*b*c^3 + 2*C*a*b*c^3 - B*b^2*c^3 - 3*A*a^2*c^2*d + 3*C
*a^2*c^2*d - 6*B*a*b*c^2*d + 3*A*b^2*c^2*d - 3*C*b^2*c^2*d - 3*B*a^2*c*d^2
+ 6*A*a*b*c*d^2 - 6*C*a*b*c*d^2 + 3*B*b^2*c*d^2 + A*a^2*d^3 - C*a^2*d^3 + 2
*B*a*b*d^3 - A*b^2*d^3 + C*b^2*d^3)*log(tan(f*x + e)^2 + 1)/(a^4*c^6 + 2*a^
2*b^2*c^6 + b^4*c^6 + 3*a^4*c^4*d^2 + 6*a^2*b^2*c^4*d^2 + 3*b^4*c^4*d^2 + 3
*a^4*c^2*d^4 + 6*a^2*b^2*c^2*d^4 + 3*b^4*c^2*d^4 + a^4*d^6 + 2*a^2*b^2*d^6
+ b^4*d^6) - 2*(B*a^2*b^5*c - 2*A*a*b^6*c + 2*C*a*b^6*c - B*b^7*c + 3*C*a^4
*b^3*d - 4*B*a^3*b^4*d + 5*A*a^2*b^5*d + C*a^2*b^5*d - 2*B*a*b^6*d + 3*A*b^
7*d)*log(abs(b*tan(f*x + e) + a))/(a^4*b^5*c^4 + 2*a^2*b^7*c^4 + b^9*c^4 -
4*a^5*b^4*c^3*d - 8*a^3*b^6*c^3*d - 4*a*b^8*c^3*d + 6*a^6*b^3*c^2*d^2 + 12*
a^4*b^5*c^2*d^2 + 6*a^2*b^7*c^2*d^2 - 4*a^7*b^2*c*d^3 - 8*a^5*b^4*c*d^3 - 4
*a^3*b^6*c*d^3 + a^8*b*d^4 + 2*a^6*b^3*d^4 + a^4*b^5*d^4) + 2*(3*C*b^2*c^6*
d^2 - 6*B*b^2*c^5*d^3 + 4*B*a*b*c^4*d^4 + 10*A*b^2*c^4*d^4 - C*b^2*c^4*d^4
- B*a^2*c^3*d^5 - 10*A*a*b*c^3*d^5 + 10*C*a*b*c^3*d^5 - 3*B*b^2*c^3*d^5 + 3
*A*a^2*c^2*d^6 - 3*C*a^2*c^2*d^6 - 6*B*a*b*c^2*d^6 + 9*A*b^2*c^2*d^6 + 3*B*
a^2*c*d^7 - 2*A*a*b*c*d^7 + 2*C*a*b*c*d^7 - B*b^2*c*d^7 - A*a^2*d^8 + C*a^2
*d^8 - 2*B*a*b*d^8 + 3*A*b^2*d^8)*log(abs(d*tan(f*x + e) + c))/(b^4*c^10*d
- 4*a*b^3*c^9*d^2 + 6*a^2*b^2*c^8*d^3 + 3*b^4*c^8*d^3 - 4*a^3*b*c^7*d^4 - 1
2*a*b^3*c^7*d^4 + a^4*c^6*d^5 + 18*a^2*b^2*c^6*d^5 + 3*b^4*c^6*d^5 - 12*a^3
*b*c^5*d^6 - 12*a*b^3*c^5*d^6 + 3*a^4*c^4*d^7 + 18*a^2*b^2*c^4*d^7 + b^4*c^
4*d^7 - 12*a^3*b*c^3*d^8 - 4*a*b^3*c^3*d^8 + 3*a^4*c^2*d^9 + 6*a^2*b^2*c^2*
d^9 - 4*a^3*b*c*d^10 + a^4*d^11) + 2*(B*a^2*b^5*c*tan(f*x + e) - 2*A*a*b^6*
c*tan(f*x + e) + 2*C*a*b^6*c*tan(f*x + e) - B*b^7*c*tan(f*x + e) + 3*C*a^4*
b^3*d*tan(f*x + e) - 4*B*a^3*b^4*d*tan(f*x + e) + 5*A*a^2*b^5*d*tan(f*x + e
) + C*a^2*b^5*d*tan(f*x + e) - 2*B*a*b^6*d*tan(f*x + e) + 3*A*b^7*d*tan(f*x
 + e) - C*a^4*b^3*c + 2*B*a^3*b^4*c - 3*A*a^2*b^5*c + C*a^2*b^5*c - A*b^7*c
 + 4*C*a^5*b^2*d - 5*B*a^4*b^3*d + 6*A*a^3*b^4*d + 2*C*a^3*b^4*d - 3*B*a^2*
b^5*d + 4*A*a*b^6*d)/((a^4*b^4*c^4 + 2*a^2*b^6*c^4 + b^8*c^4 - 4*a^5*b^3*c^
3*d - 8*a^3*b^5*c^3*d - 4*a*b^7*c^3*d + 6*a^6*b^2*c^2*d^2 + 12*a^4*b^4*c^2*
d^2 + 6*a^2*b^6*c^2*d^2 - 4*a^7*b*c*d^3 - 8*a^5*b^3*c*d^3 - 4*a^3*b^5*c*d^3
 + a^8*d^4 + 2*a^6*b^2*d^4 + a^4*b^4*d^4)*(b*tan(f*x + e) + a)) - (9*C*b^2*
c^6*d^3*tan(f*x + e)^2 - 18*B*b^2*c^5*d^4*tan(f*x + e)^2 + 12*B*a*b*c^4*d^5
*tan(f*x + e)^2 + 30*A*b^2*c^4*d^5*tan(f*x + e)^2 - 3*C*b^2*c^4*d^5*tan(f*x
 + e)^2 - 3*B*a^2*c^3*d^6*tan(f*x + e)^2 - 30*A*a*b*c^3*d^6*tan(f*x + e)^2
+ 30*C*a*b*c^3*d^6*tan(f*x + e)^2 - 9*B*b^2*c^3*d^6*tan(f*x + e)^2 + 9*A*a^
2*c^2*d^7*tan(f*x + e)^2 - 9*C*a^2*c^2*d^7*tan(f*x + e)^2 - 18*B*a*b*c^2*d^
7*tan(f*x + e)^2 + 27*A*b^2*c^2*d^7*tan(f*x + e)^2 + 9*B*a^2*c*d^8*tan(f*x
+ e)^2 - 6*A*a*b*c*d^8*tan(f*x + e)^2 + 6*C*a*b*c*d^8*tan(f*x + e)^2 - 3*B*
b^2*c*d^8*tan(f*x + e)^2 - 3*A*a^2*d^9*tan(f*x + e)^2 + 3*C*a^2*d^9*tan(f*x
 + e)^2 - 6*B*a*b*d^9*tan(f*x + e)^2 + 9*A*b^2*d^9*tan(f*x + e)^2 + 22*C*b^
2*c^7*d^2*tan(f*x + e) - 4*C*a*b*c^6*d^3*tan(f*x + e) - 42*B*b^2*c^6*d^3*ta
n(f*x + e) + 32*B*a*b*c^5*d^4*tan(f*x + e) + 68*A*b^2*c^5*d^4*tan(f*x + e)
- 2*C*b^2*c^5*d^4*tan(f*x + e) - 8*B*a^2*c^4*d^5*tan(f*x + e) - 72*A*a*b*c^
```

```
4*d^5*tan(f*x + e) + 60*C*a*b*c^4*d^5*tan(f*x + e) - 26*B*b^2*c^4*d^5*tan(f
*x + e) + 22*A*a^2*c^3*d^6*tan(f*x + e) - 22*C*a^2*c^3*d^6*tan(f*x + e) - 2
8*B*a*b*c^3*d^6*tan(f*x + e) + 66*A*b^2*c^3*d^6*tan(f*x + e) + 18*B*a^2*c^2
*d^7*tan(f*x + e) - 28*A*a*b*c^2*d^7*tan(f*x + e) + 16*C*a*b*c^2*d^7*tan(f*
x + e) - 8*B*b^2*c^2*d^7*tan(f*x + e) - 2*A*a^2*c*d^8*tan(f*x + e) + 2*C*a^
2*c*d^8*tan(f*x + e) - 12*B*a*b*c*d^8*tan(f*x + e) + 22*A*b^2*c*d^8*tan(f*x
 + e) + 2*B*a^2*d^9*tan(f*x + e) - 4*A*a*b*d^9*tan(f*x + e) + 14*C*b^2*c^8*
d - 6*C*a*b*c^7*d^2 - 25*B*b^2*c^7*d^2 + C*a^2*c^6*d^3 + 22*B*a*b*c^6*d^3 +
 39*A*b^2*c^6*d^3 + 3*C*b^2*c^6*d^3 - 6*B*a^2*c^5*d^4 - 44*A*a*b*c^5*d^4 +
26*C*a*b*c^5*d^4 - 19*B*b^2*c^5*d^4 + 14*A*a^2*c^4*d^5 - 11*C*a^2*c^4*d^5 -
 6*B*a*b*c^4*d^5 + 41*A*b^2*c^4*d^5 + C*b^2*c^4*d^5 + 7*B*a^2*c^3*d^6 - 26*
A*a*b*c^3*d^6 + 8*C*a*b*c^3*d^6 - 6*B*b^2*c^3*d^6 + 3*A*a^2*c^2*d^7 - 4*B*a
*b*c^2*d^7 + 14*A*b^2*c^2*d^7 + B*a^2*c*d^8 - 6*A*a*b*c*d^8 + A*a^2*d^9)/((
b^4*c^10 - 4*a*b^3*c^9*d + 6*a^2*b^2*c^8*d^2 + 3*b^4*c^8*d^2 - 4*a^3*b*c^7*
d^3 - 12*a*b^3*c^7*d^3 + a^4*c^6*d^4 + 18*a^2*b^2*c^6*d^4 + 3*b^4*c^6*d^4 -
 12*a^3*b*c^5*d^5 - 12*a*b^3*c^5*d^5 + 3*a^4*c^4*d^6 + 18*a^2*b^2*c^4*d^6 +
 b^4*c^4*d^6 - 12*a^3*b*c^3*d^7 - 4*a*b^3*c^3*d^7 + 3*a^4*c^2*d^8 + 6*a^2*b
^2*c^2*d^8 - 4*a^3*b*c*d^9 + a^4*d^10)*(d*tan(f*x + e) + c)^2))/f
```

3.90 $\int (a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+\right.$

Optimal. Leaf size=464

$$\frac{2(c+d\tan(e+fx))^{3/2}\left(-6a^2bd^2(16cC-45Bd)+40a^3Cd^3+9ab^2d\left(35d^2(A-C)-14Bcd+8c^2C\right)+b^3\left(-\left(42cd^2(A-C\right.\right.\right.}{315d^4f}$$

```
[Out] -(((a - I*b)^3*(I*A + B - I*C)*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x
]]/Sqrt[c - I*d]])/f) + ((a + I*b)^3*(I*A - B - I*C)*Sqrt[c + I*d]*ArcTanh[
Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/f + (2*(a^3*B - 3*a*b^2*B + 3*a^2*
b*(A - C) - b^3*(A - C))*Sqrt[c + d*Tan[e + f*x]])/f + (2*(40*a^3*C*d^3 - 6
*a^2*b*d^2*(16*c*C - 45*B*d) + 9*a*b^2*d*(8*c^2*C - 14*B*c*d + 35*(A - C)*d
^2) - b^3*(16*c^3*C - 24*B*c^2*d + 42*c*(A - C)*d^2 + 105*B*d^3))*(c + d*Ta
n[e + f*x])^(3/2))/(315*d^4*f) + (2*b*(21*b*(A*b + a*B - b*C)*d^2 + 4*(b*c
- a*d)*(2*b*c*C - 3*b*B*d - 2*a*C*d))*Tan[e + f*x]*(c + d*Tan[e + f*x])^(3/
2))/(105*d^3*f) - (2*(2*b*c*C - 3*b*B*d - 2*a*C*d)*(a + b*Tan[e + f*x])^2*(
c + d*Tan[e + f*x])^(3/2))/(21*d^2*f) + (2*C*(a + b*Tan[e + f*x])^3*(c + d*
Tan[e + f*x])^(3/2))/(9*d*f)
```

Rubi [A] time = 2.08898, antiderivative size = 464, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.17, Rules used = {3647, 3637, 3630, 3528, 3539, 3537, 63, 208}

$$\frac{2(c+d\tan(e+fx))^{3/2}\left(-6a^2bd^2(16cC-45Bd)+40a^3Cd^3+9ab^2d\left(35d^2(A-C)-14Bcd+8c^2C\right)+b^3\left(-\left(42cd^2(A-C\right.\right.\right.}{315d^4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^3*Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C
*Tan[e + f*x]^2),x]
```


```
[Out] -(((a - I*b)^3*(I*A + B - I*C)*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x
]]/Sqrt[c - I*d]])/f) + ((a + I*b)^3*(I*A - B - I*C)*Sqrt[c + I*d]*ArcTanh[
Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/f + (2*(a^3*B - 3*a*b^2*B + 3*a^2*
b*(A - C) - b^3*(A - C))*Sqrt[c + d*Tan[e + f*x]])/f + (2*(40*a^3*C*d^3 - 6
*a^2*b*d^2*(16*c*C - 45*B*d) + 9*a*b^2*d*(8*c^2*C - 14*B*c*d + 35*(A - C)*d
^2) - b^3*(16*c^3*C - 24*B*c^2*d + 42*c*(A - C)*d^2 + 105*B*d^3))*(c + d*Ta
n[e + f*x])^(3/2))/(315*d^4*f) + (2*b*(21*b*(A*b + a*B - b*C)*d^2 + 4*(b*c
- a*d)*(2*b*c*C - 3*b*B*d - 2*a*C*d))*Tan[e + f*x]*(c + d*Tan[e + f*x])^(3/
2))/(105*d^3*f) - (2*(2*b*c*C - 3*b*B*d - 2*a*C*d)*(a + b*Tan[e + f*x])^2*(
c + d*Tan[e + f*x])^(3/2))/(21*d^2*f) + (2*C*(a + b*Tan[e + f*x])^3*(c + d*
Tan[e + f*x])^(3/2))/(9*d*f)
```


Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int (a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx = \frac{2C(a+b\tan(e+fx))^3(c+d\tan(}{9df}$$

$$= -\frac{2(2bcC-3bBd-2aCd)(a+b\,\mathrm{ta}}{21}$$

$$= \frac{2b\left(21b(Ab+aB-bC)d^2+4(bc-\right.}{}$$

$$= \frac{2\left(40a^3Cd^3-6a^2bd^2(16cC-45B\right.}{}$$

$$= \frac{2\left(a^3B-3ab^2B+3a^2b(A-C)-\right.}{f}$$

$$= \frac{2\left(a^3B-3ab^2B+3a^2b(A-C)-\right.}{f}$$

$$= \frac{2\left(a^3B-3ab^2B+3a^2b(A-C)-\right.}{f}$$

$$= \frac{2\left(a^3B-3ab^2B+3a^2b(A-C)-\right.}{f}$$

$$= -\frac{(a-ib)^3(iA+B-iC)\sqrt{c-id}\,\mathrm{ta}}{f}$$

Mathematica [B] time = 6.42582, size = 1232, normalized size = 2.66

$$\frac{2C(c+d\tan(e+fx))^{3/2}(a+b\tan(e+fx))^3}{9df} + \frac{2\left(2\left(\frac{3b\left(21b(Ab-Cb+aB)d^2+4(bc-ad)(2bcC-2adC-3bBd)\right)\tan(e+fx)(c+d\tan(e+fx))^{3/2}}{10df} - \frac{2\left(\frac{2\left(b\left(\frac{3}{4}c\left(21b(Ab-C\right.\right.\right.}{}\right.}{}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^3*Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*
```

```
x] + C*Tan[e + f*x]^2),x]
```

```
[Out] (2*C*(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^(3/2))/(9*d*f) + (2*((-3*(
2*b*c*C - 3*b*B*d - 2*a*C*d)*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(3
/2))/(7*d*f) + (2*((3*b*(21*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(2*b*c*
C - 3*b*B*d - 2*a*C*d))*Tan[e + f*x]*(c + d*Tan[e + f*x])^(3/2))/(10*d*f) -
 (2*((2*((-15*a*d*(21*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(2*b*c*C - 3*
b*B*d - 2*a*C*d)))/8 + b*((-315*(a^2*B - b^2*B + 2*a*b*(A - C))*d^3)/8 + (3
*c*(21*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(2*b*c*C - 3*b*B*d - 2*a*C*d
)))/4))*(c + d*Tan[e + f*x])^(3/2))/(3*d*f) + ((I/2)*((-15*a*d*(a^2*(21*A -
 13*C)*d^2 + 4*b^2*c*(2*c*C - 3*B*d) - a*b*d*(16*c*C + 9*B*d)))/8 + (3*b*c*
(21*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(2*b*c*C - 3*b*B*d - 2*a*C*d)))
/4 + (15*a*d*(21*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(2*b*c*C - 3*b*B*d
 - 2*a*C*d)))/8 + ((5*I)/2)*d*((63*a*(a^2*B - b^2*B + 2*a*b*(A - C))*d^2)/4
 + (3*b*(a^2*(21*A - 13*C)*d^2 + 4*b^2*c*(2*c*C - 3*B*d) - a*b*d*(16*c*C +
9*B*d)))/4 - (3*b*(21*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(2*b*c*C - 3*
b*B*d - 2*a*C*d)))/4) - b*((-315*(a^2*B - b^2*B + 2*a*b*(A - C))*d^3)/8 + (
3*c*(21*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(2*b*c*C - 3*b*B*d - 2*a*C*
d)))/4))*((2*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]
])/(-c + I*d) + 2*Sqrt[c + d*Tan[e + f*x]]))/f - ((I/2)*((-15*a*d*(a^2*(21*
A - 13*C)*d^2 + 4*b^2*c*(2*c*C - 3*B*d) - a*b*d*(16*c*C + 9*B*d)))/8 + (3*b
*c*(21*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(2*b*c*C - 3*b*B*d - 2*a*C*d
)))/4 + (15*a*d*(21*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(2*b*c*C - 3*b*
B*d - 2*a*C*d)))/8 - ((5*I)/2)*d*((63*a*(a^2*B - b^2*B + 2*a*b*(A - C))*d^2
)/4 + (3*b*(a^2*(21*A - 13*C)*d^2 + 4*b^2*c*(2*c*C - 3*B*d) - a*b*d*(16*c*C
 + 9*B*d)))/4 - (3*b*(21*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(2*b*c*C -
 3*b*B*d - 2*a*C*d)))/4) - b*((-315*(a^2*B - b^2*B + 2*a*b*(A - C))*d^3)/8
+ (3*c*(21*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(2*b*c*C - 3*b*B*d - 2*a
*C*d)))/4))*((2*(c + I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I
*d]])/(-c - I*d) + 2*Sqrt[c + d*Tan[e + f*x]]))/f))/(5*d)))/(7*d)))/(9*d)
```

Maple [B] time = 0.227, size = 6661, normalized size = 14.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f
*x+e)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f
*x+e)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tan\left(e + fx\right)\right)^3 \sqrt{c + d\tan\left(e + fx\right)} \left(A + B\tan\left(e + fx\right) + C\tan^2\left(e + fx\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)*(a+b*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan
(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**3*sqrt(c + d*tan(e + f*x))*(A + B*tan(e + f*
x) + C*tan(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\tan\left(fx + e\right)^2 + B\tan\left(fx + e\right) + A\right)\left(b\tan\left(fx + e\right) + a\right)^3 \sqrt{d\tan\left(fx + e\right) + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f
*x+e)^2),x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^3*sq
rt(d*tan(f*x + e) + c), x)
```

3.91 $\int (a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)\right.$

Optimal. Leaf size=325

$$\frac{2(c+d\tan(e+fx))^{3/2}\left(20a^2Cd^2-14abd(2cC-5Bd)+b^2\left(35d^2(A-C)-14Bcd+8c^2C\right)\right)}{105d^3f}+\frac{2\left(a^2B+2ab(A-C)-\right.}{}$$

```
[Out] -(((a - I*b)^2*(B + I*(A - C))*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x
]]/Sqrt[c - I*d]])/f) - ((a + I*b)^2*(B - I*(A - C))*Sqrt[c + I*d]*ArcTanh[
Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/f + (2*(a^2*B - b^2*B + 2*a*b*(A -
 C))*Sqrt[c + d*Tan[e + f*x]])/f + (2*(20*a^2*C*d^2 - 14*a*b*d*(2*c*C - 5*B
*d) + b^2*(8*c^2*C - 14*B*c*d + 35*(A - C)*d^2))*(c + d*Tan[e + f*x])^(3/2)
)/(105*d^3*f) - (2*b*(4*b*c*C - 7*b*B*d - 4*a*C*d)*Tan[e + f*x]*(c + d*Tan[
e + f*x])^(3/2))/(35*d^2*f) + (2*C*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*
x])^(3/2))/(7*d*f)
```

Rubi [A] time = 1.30632, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.17, Rules used = {3647, 3637, 3630, 3528, 3539, 3537, 63, 208}

$$\frac{2(c+d\tan(e+fx))^{3/2}\left(20a^2Cd^2-14abd(2cC-5Bd)+b^2\left(35d^2(A-C)-14Bcd+8c^2C\right)\right)}{105d^3f}+\frac{2\left(a^2B+2ab(A-C)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^2*Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C
*Tan[e + f*x]^2),x]
```

```
[Out] -(((a - I*b)^2*(B + I*(A - C))*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x
]]/Sqrt[c - I*d]])/f) - ((a + I*b)^2*(B - I*(A - C))*Sqrt[c + I*d]*ArcTanh[
Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/f + (2*(a^2*B - b^2*B + 2*a*b*(A -
 C))*Sqrt[c + d*Tan[e + f*x]])/f + (2*(20*a^2*C*d^2 - 14*a*b*d*(2*c*C - 5*B
*d) + b^2*(8*c^2*C - 14*B*c*d + 35*(A - C)*d^2))*(c + d*Tan[e + f*x])^(3/2)
)/(105*d^3*f) - (2*b*(4*b*c*C - 7*b*B*d - 4*a*C*d)*Tan[e + f*x]*(c + d*Tan[
e + f*x])^(3/2))/(35*d^2*f) + (2*C*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*
x])^(3/2))/(7*d*f)
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
```

```
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int (a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \frac{2C(a+b\tan(e+fx))^2(c+d\,\mathrm{t}}{7df}$$

$$= -\frac{2b(4bcC-7bBd-4aCd)\tan}{35}$$

$$= \frac{2\left(20a^2Cd^2-14abd(2cC-5B\right.}{}$$

$$= \frac{2\left(a^2B-b^2B+2ab(A-C)\right)\sqrt{}}{f}$$

$$= \frac{2\left(a^2B-b^2B+2ab(A-C)\right)\sqrt{}}{f}$$

$$= \frac{2\left(a^2B-b^2B+2ab(A-C)\right)\sqrt{}}{f}$$

$$= \frac{2\left(a^2B-b^2B+2ab(A-C)\right)\sqrt{}}{f}$$

$$= -\frac{(a-ib)^2(B+i(A-C))\sqrt{c-i}}{f}$$

Mathematica [A] time = 4.77124, size = 314, normalized size = 0.97

$$\frac{2\left((c+d\tan(e+fx))^{3/2}\left(20a^2Cd^2+14abd(5Bd-2cC)+b^2\left(35d^2(A-C)-14Bcd+8c^2C\right)\right)+\frac{105}{2}d^3(a-ib)^2(iA+B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^2*Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*
x] + C*Tan[e + f*x]^2),x]
```

```
[Out] (2*((20*a^2*C*d^2 + 14*a*b*d*(-2*c*C + 5*B*d) + b^2*(8*c^2*C - 14*B*c*d + 3
5*(A - C)*d^2))*(c + d*Tan[e + f*x])^(3/2) + 3*b*d*(-4*b*c*C + 7*b*B*d + 4*
a*C*d)*Tan[e + f*x]*(c + d*Tan[e + f*x])^(3/2) + 15*C*d^2*(a + b*Tan[e + f*
x])^2*(c + d*Tan[e + f*x])^(3/2) + (105*(a - I*b)^2*(I*A + B - I*C)*d^3*(-(
Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]]) + Sqrt[c + d
*Tan[e + f*x]]))/2 + (105*(a + I*b)^2*((-I)*A + B + I*C)*d^3*(-(Sqrt[c + I*
d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]]) + Sqrt[c + d*Tan[e + f*
x]]))/2))/(105*d^3*f)
```

Maple [B] time = 0.172, size = 4775, normalized size = 14.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x)
```

```
[Out] 1/2/f/d*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e
)-c-(c^2+d^2)^(1/2))*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c-1/2/f/d*ln((c+d*
tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1
/2))*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*b+1/2/f/d*ln(d*tan(f
*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2
))*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*b-1/2/f/d*ln(d*tan(f*x
+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))
*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c+2/f*B*a^2*(c+d*tan(f*x+e))^(1/2)-2/f
*B*b^2*(c+d*tan(f*x+e))^(1/2)+1/4/f/d*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*C*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*(c^2+d^2)^(1/2)*b^2+1/4/f/d*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2
)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*(c^2+d^2)^(1/2)*b^2+1/4/f/d*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)*a^2*c-1/4/f/d*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c+1/4/f/
d*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c
^2+d^2)^(1/2))*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^2-1/4/f/d*
ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2
+d^2)^(1/2))*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b^2-1/4/f/d*ln
(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d
^2)^(1/2))*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c-2/f/(2*(c^2+d^2)^(1/2)-2*c
)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*
(c^2+d^2)^(1/2)-2*c)^(1/2))*C*a*b*c+1/4/f/d*ln((c+d*tan(f*x+e))^(1/2)*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^2-1/4/f/d*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*(c^2+d^2)^(1/2)*b^2+1/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2
*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2
*c)^(1/2))*B*(c^2+d^2)^(1/2)*b^2+1/2/f*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)*a*b-1/2/f*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-
d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b+1/f/(2*
(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1
/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*B*a^2*c-1/f/(2*(c^2+d^2)^(1/
2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2
))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*B*b^2*c-1/2/f*ln(d*tan(f*x+e)+c+(c+d*tan(
f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*A*(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*a*b+1/2/f*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*
b+1/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2
*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*B*(c^2+d^2)^(1/2)*a
^2-1/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-
2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*B*(c^2+d^2)^(1/2)*
b^2-1/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)
-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*B*a^2*c+1/f/(2*(c
^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f
*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*B*b^2*c-4/5/f/d^3*C*(c+d*tan(f
*x+e))^(5/2)*b^2*c-1/f*d/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f
*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*
A*b^2+1/f*d/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(
1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*C*a^2-1/f*d/(
2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*t
an(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*C*b^2+2/3/f/d^3*C*(c+d*tan
(f*x+e))^(3/2)*b^2*c^2-1/f*d/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+
d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/
2))*A*a^2+1/f*d/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*
c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*A*b^2-1/f
*d/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d
```

```
^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*C*a^2+1/f*d/(2*(c^2+d^
2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c
)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*C*b^2+1/f*d/(2*(c^2+d^2)^(1/2)-2*c)
^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(
c^2+d^2)^(1/2)-2*c)^(1/2))*A*a^2+4/5/f/d^2*C*(c+d*tan(f*x+e))^(5/2)*a*b+4/3
/f/d*B*(c+d*tan(f*x+e))^(3/2)*a*b-2/3/f/d^2*B*(c+d*tan(f*x+e))^(3/2)*b^2*c-
1/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+
d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*B*(c^2+d^2)^(1/2)*a^2
+2/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2
+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*C*(c^2+d^2)^(1/2)*a*
b+1/4/f/d*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x
+e)-c-(c^2+d^2)^(1/2))*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c-1/4/f/d*ln((c+
d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^
(1/2))*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c-4/3/f/d^2*C*(c+d*tan(f*x+e))^(
3/2)*a*b*c+1/4/f/d*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c-2/f*d
/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2
)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*B*a*b-1/4/f/d*ln((c+d*ta
n(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2
))*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c+2/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*
arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2
)^(1/2)-2*c)^(1/2))*A*a*b*c+2/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c
^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^
(1/2))*C*a*b*c+2/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)
+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*A*(c^2
+d^2)^(1/2)*a*b-2/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2
)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*C*(c^
2+d^2)^(1/2)*a*b-2/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/
2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*A*a*
b*c-2/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(
c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*A*(c^2+d^2)^(1/2)
*a*b+2/f*d/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1
/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*B*a*b-1/4/f/d*
ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2
+d^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^2+1/4/f/d*ln
((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d
^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c-1/4/f/d*ln((c+d*tan(f*x+e)
)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*C*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^2+2/5/f/d^2*B*(c+d*tan(f*x+e))
^(5/2)*b^2+2/3/f/d*A*(c+d*tan(f*x+e))^(3/2)*b^2+2/3/f/d*C*(c+d*tan(f*x+e))^
(3/2)*a^2+2/7/f/d^3*b^2*C*(c+d*tan(f*x+e))^(7/2)-2/3/f/d*C*(c+d*tan(f*x+e))
^(3/2)*b^2-1/4/f*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2
)+2*c)^(1/2)+(c^2+d^2)^(1/2))*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2+1/4/f*ln(
d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^
2)^(1/2))*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2-4/f*C*a*b*(c+d*tan(f*x+e))^(1
/2)+1/4/f*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x
+e)-c-(c^2+d^2)^(1/2))*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2-1/4/f*ln((c+d*ta
n(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2
))*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2+4/f*A*a*b*(c+d*tan(f*x+e))^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f
*x+e)^2),x, algorithm="maxima")
```

[Out] Timed out

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f
*x+e)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b \tan \left(e + fx\right)\right)^2 \sqrt{c + d \tan \left(e + fx\right)} \left(A + B \tan \left(e + fx\right) + C \tan^2 \left(e + fx\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)*(a+b*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan
(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**2*sqrt(c + d*tan(e + f*x))*(A + B*tan(e + f*
x) + C*tan(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \tan \left(fx + e\right)^2 + B \tan \left(fx + e\right) + A\right)\left(b \tan \left(fx + e\right) + a\right)^2 \sqrt{d \tan \left(fx + e\right) + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f
*x+e)^2),x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^2*sq
rt(d*tan(f*x + e) + c), x)
```

3.92 $\int (a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)$

Optimal. Leaf size=224

$$\frac{2(aB+Ab-bC)\sqrt{c+d\tan(e+fx)}}{f}-\frac{(b+ia)\sqrt{c-id}(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{(-b+ia)\sqrt{c+id}(A+iB}{}$$

```
[Out] -(((I*a + b)*(A - I*B - C)*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/S
qrt[c - I*d]])/f) + ((I*a - b)*(A + I*B - C)*Sqrt[c + I*d]*ArcTanh[Sqrt[c +
 d*Tan[e + f*x]]/Sqrt[c + I*d]])/f + (2*(A*b + a*B - b*C)*Sqrt[c + d*Tan[e
+ f*x]])/f - (2*(2*b*c*C - 5*b*B*d - 5*a*C*d)*(c + d*Tan[e + f*x])^(3/2))/(
15*d^2*f) + (2*b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(3/2))/(5*d*f)
```

Rubi [A] time = 0.628311, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3637, 3630, 3528, 3539, 3537, 63, 208}

$$\frac{2(aB+Ab-bC)\sqrt{c+d\tan(e+fx)}}{f}-\frac{(b+ia)\sqrt{c-id}(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{(-b+ia)\sqrt{c+id}(A+iB}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])*Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*T
an[e + f*x]^2),x]
```


```
[Out] -(((I*a + b)*(A - I*B - C)*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/S
qrt[c - I*d]])/f) + ((I*a - b)*(A + I*B - C)*Sqrt[c + I*d]*ArcTanh[Sqrt[c +
 d*Tan[e + f*x]]/Sqrt[c + I*d]])/f + (2*(A*b + a*B - b*C)*Sqrt[c + d*Tan[e
+ f*x]])/f - (2*(2*b*c*C - 5*b*B*d - 5*a*C*d)*(c + d*Tan[e + f*x])^(3/2))/(
15*d^2*f) + (2*b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(3/2))/(5*d*f)
```


Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
```

```
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int (a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx=\frac{2bC\tan(e+fx)(c+d\tan(e+fx)}{5df}$$

$$=-\frac{2(2bcC-5bBd-5aCd)(c+d\tan}{15d^2f}$$

$$=\frac{2(Ab+aB-bC)\sqrt{c+d\tan(e+fx}}{f}$$

$$=\frac{2(Ab+aB-bC)\sqrt{c+d\tan(e+fx}}{f}$$

$$=\frac{2(Ab+aB-bC)\sqrt{c+d\tan(e+fx}}{f}$$

$$=\frac{2(Ab+aB-bC)\sqrt{c+d\tan(e+fx}}{f}$$

$$=-\frac{(ia+b)(A-iB-C)\sqrt{c-id}\tanh}{f}$$

Mathematica [A] time = 1.96048, size = 220, normalized size = 0.98

$$\frac{15d(b+ia)(A-iB-C)\left(\sqrt{c+d\tan(e+fx)}-\sqrt{c-id}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)\right)+15d(b-ia)(A+iB-C)\left(\sqrt{c+d\tan\right.}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])*Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x]
 + C*Tan[e + f*x]^2),x]
```

```
[Out] ((2*(-2*b*c*C + 5*b*B*d + 5*a*C*d)*(c + d*Tan[e + f*x])^(3/2))/d + 6*b*C*Ta
n[e + f*x]*(c + d*Tan[e + f*x])^(3/2) + 15*(I*a + b)*(A - I*B - C)*d*(-(Sqr
t[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]]) + Sqrt[c + d*Ta
n[e + f*x]]) + 15*((-I)*a + b)*(A + I*B - C)*d*(-(Sqrt[c + I*d]*ArcTanh[Sqr
t[c + d*Tan[e + f*x]]/Sqrt[c + I*d]]) + Sqrt[c + d*Tan[e + f*x]]))/(15*d*f)
```

Maple [B] time = 0.148, size = 3028, normalized size = 13.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)
,x)
```

```
[Out] 1/4/f*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-
c-(c^2+d^2)^(1/2))*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a-1/4/f*ln((c+d*tan(f*x+
e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*C*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b+1/4/f/d*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)*(c^2+d^2)^(1/2)*a-1/4/f/d*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(
1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*a*c-1/4/f/d*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*
tan(f*x+e)-c-(c^2+d^2)^(1/2))*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/
2)*b+1/4/f/d*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)+(c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c+1/4/f/d*ln(d*
tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)
^(1/2))*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b-1/4/f/d*ln(d*tan(
f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/
2))*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c+2/f*B*(c+d*tan(f*x+e))^(1/2)*a+2/3/
f/d*B*(c+d*tan(f*x+e))^(3/2)*b+2/3/f/d*C*(c+d*tan(f*x+e))^(3/2)*a-1/4/f*ln(
d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^
2)^(1/2))*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a+1/4/f*ln(d*tan(f*x+e)+c+(c+d*ta
n(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*C*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*b-1/4/f*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b+
1/4/f*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-
c-(c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b-1/f*d/(2*(c^2+d^2)^(1/
2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2
))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*A*a-2/f*C*b*(c+d*tan(f*x+e))^(1/2)+2/f*A*
(c+d*tan(f*x+e))^(1/2)*b+1/4/f/d*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*(c^2+d^2)^(1/2)*a-1/4/f/d*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a
*c-1/4/f/d*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)
^(1/2)+(c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a+1
```

```
/4/f/d*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)
-c-(c^2+d^2)^(1/2))*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c-1/4/f/d*ln((c+d*tan
(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2)
)*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a+1/4/f/d*ln((c+d*tan(f*x
+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*C*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c+2/5/f/d^2*C*b*(c+d*tan(f*x+e))^(5/2)+1/f*
d/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+
d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*B*b+1/f*d/(2*(c^2+d^2)^
(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(
1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*C*a+1/f*d/(2*(c^2+d^2)^(1/2)-2*c)^(1/2
)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d
^2)^(1/2)-2*c)^(1/2))*A*a-1/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2
+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1
/2))*B*a*c+1/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c
)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*C*b*c+1/f/
(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*
tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*B*(c^2+d^2)^(1/2)*a-1/f/(
2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*t
an(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*C*(c^2+d^2)^(1/2)*b-1/f/(2
*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*ta
n(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*A*b*c+1/f/(2*(c^2+d^2)^(1/2
)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2)
)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*A*(c^2+d^2)^(1/2)*b+1/f/(2*(c^2+d^2)^(1/2)
-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))
/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*A*b*c+1/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arc
tan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(
1/2)-2*c)^(1/2))*B*a*c+1/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan
(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)
)*C*(c^2+d^2)^(1/2)*b-1/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(
f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))
*A*(c^2+d^2)^(1/2)*b-1/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f
*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*
B*(c^2+d^2)^(1/2)*a-1/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*
x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*C
*b*c-1/f*d/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(
2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*B*b-1/f*d/(2*(
c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/
2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*C*a-2/3/f/d^2*C*(c+d*tan(f*x+
e))^(3/2)*b*c
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\tan\left(e+fx\right)\right)\sqrt{c+d\tan\left(e+fx\right)}\left(A+B\tan\left(e+fx\right)+C\tan^2\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)*(a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*
x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))*sqrt(c + d*tan(e + f*x))*(A + B*tan(e + f*x)
+ C*tan(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\tan\left(fx+e\right)^2+B\tan\left(fx+e\right)+A\right)\left(b\tan\left(fx+e\right)+a\right)\sqrt{d\tan\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)*sqrt
(d*tan(f*x + e) + c), x)
```

3.93 $\int \sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)$

Optimal. Leaf size=155

$$-\frac{\sqrt{c-id}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}-\frac{\sqrt{c+id}(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f}+\frac{2B\sqrt{c+d\tan(e+fx)}}{f}$$

```
[Out] -(((I*A + B - I*C)*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c -
I*d]])/f) - ((B - I*(A - C))*Sqrt[c + I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]
/Sqrt[c + I*d]])/f + (2*B*Sqrt[c + d*Tan[e + f*x]])/f + (2*C*(c + d*Tan[e +
 f*x])^(3/2))/(3*d*f)
```

Rubi [A] time = 0.305837, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3630, 3528, 3539, 3537, 63, 208}

$$-\frac{\sqrt{c-id}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}-\frac{\sqrt{c+id}(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f}+\frac{2B\sqrt{c+d\tan(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2),x]
```


```
[Out] -(((I*A + B - I*C)*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c -
I*d]])/f) - ((B - I*(A - C))*Sqrt[c + I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]
/Sqrt[c + I*d]])/f + (2*B*Sqrt[c + d*Tan[e + f*x]])/f + (2*C*(c + d*Tan[e +
 f*x])^(3/2))/(3*d*f)
```


Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
```

```
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx = \frac{2C(c+d\tan(e+fx))^{3/2}}{3df}+\int(A-C+B\tan(e+$$

$$=\frac{2B\sqrt{c+d\tan(e+fx)}}{f}+\frac{2C(c+d\tan(e+fx))^{3/2}}{3df}$$

$$=\frac{2B\sqrt{c+d\tan(e+fx)}}{f}+\frac{2C(c+d\tan(e+fx))^{3/2}}{3df}$$

$$=\frac{2B\sqrt{c+d\tan(e+fx)}}{f}+\frac{2C(c+d\tan(e+fx))^{3/2}}{3df}$$

$$=\frac{2B\sqrt{c+d\tan(e+fx)}}{f}+\frac{2C(c+d\tan(e+fx))^{3/2}}{3df}$$

$$=-\frac{(B+i(A-C))\sqrt{c-id}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}-$$

Mathematica [A] time = 0.552001, size = 150, normalized size = 0.97

$$\frac{-3id\sqrt{c-id}(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)+3id\sqrt{c+id}(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)+2\sqrt{c+d\tan(e+}{3df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2),
x]
```

```
[Out] ((-3*I)*(A - I*B - C)*Sqrt[c - I*d]*d*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt
[c - I*d]] + (3*I)*(A + I*B - C)*Sqrt[c + I*d]*d*ArcTanh[Sqrt[c + d*Tan[e +
 f*x]]/Sqrt[c + I*d]] + 2*Sqrt[c + d*Tan[e + f*x]]*(c*C + 3*B*d + C*d*Tan[e
 + f*x]))/(3*d*f)
```

Maple [B] time = 0.128, size = 1472, normalized size = 9.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x)
```

```
[Out] 2/3*C*(c+d*tan(f*x+e))^(3/2)/f/d+2*B*(c+d*tan(f*x+e))^(1/2)/f+1/4/f*ln((c+d
*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(
1/2))*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+1/4/d/f*ln((c+d*tan(f*x+e))^(1/2)*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^(
1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)-1/4/d/f*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)*c-1/4/f*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d/f/(2*(c^2+
d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+
e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*A+d/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/
2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+
d^2)^(1/2)-2*c)^(1/2))*C+d/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*t
an(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/
2))*A-d/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2
*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*C-1/4/d/f*ln(d*
tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)
^(1/2))*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)+1/4/d/f*ln(d*tan(f*
x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2)
)*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c+1/4/d/f*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+
e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*C*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*(c^2+d^2)^(1/2)-1/4/d/f*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1
/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*C*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*c-1/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2
)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*B*(c^2+d^2)
^(1/2)+1/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(
2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*B*c-1/4/d/f*ln
((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d
^2)^(1/2))*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)+1/4/d/f*ln((c+d*
tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1
/2))*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c+1/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*ar
ctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^
(1/2)-2*c)^(1/2))*B*(c^2+d^2)^(1/2)-1/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arcta
n(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/
2)-2*c)^(1/2))*B*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\tan\left(fx+e\right)^2 + B\tan\left(fx+e\right) + A\right)\sqrt{d\tan\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*sqrt(d*tan(f*x + e) + c),
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c+d\tan\left(e+fx\right)}\left(A+B\tan\left(e+fx\right)+C\tan^2\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2),x)
```

```
[Out] Integral(sqrt(c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2),
 x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\tan\left(fx+e\right)^2+B\tan\left(fx+e\right)+A\right)\sqrt{d\tan\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*sqrt(d*tan(f*x + e) + c),
 x)
```

3.94 $\int \frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=234

$$-\frac{2\sqrt{bc-ad}\left(Ab^2-a(bB-aC)\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{b^{3/2}f\left(a^2+b^2\right)}-\frac{\sqrt{c-id}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(a-ib)}+\frac{\sqrt{c+id}(iA\cdots}{}$$

```
[Out] -((((I*A + B - I*C)*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c -
I*d]])/((a - I*b)*f)) + ((I*A - B - I*C)*Sqrt[c + I*d]*ArcTanh[Sqrt[c + d*T
an[e + f*x]]/Sqrt[c + I*d]])/((a + I*b)*f) - (2*(A*b^2 - a*(b*B - a*C))*Sqr
t[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(
b^(3/2)*(a^2 + b^2)*f) + (2*C*Sqrt[c + d*Tan[e + f*x]])/(b*f)
```

Rubi [A] time = 1.08733, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.149, Rules used = {3647, 3653, 3539, 3537, 63, 208, 3634}

$$-\frac{2\sqrt{bc-ad}\left(Ab^2-a(bB-aC)\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{b^{3/2}f\left(a^2+b^2\right)}-\frac{\sqrt{c-id}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(a-ib)}+\frac{\sqrt{c+id}(iA\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a +
 b*Tan[e + f*x]),x]
```

```
[Out] -((((I*A + B - I*C)*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c -
I*d]])/((a - I*b)*f)) + ((I*A - B - I*C)*Sqrt[c + I*d]*ArcTanh[Sqrt[c + d*T
an[e + f*x]]/Sqrt[c + I*d]])/((a + I*b)*f) - (2*(A*b^2 - a*(b*B - a*C))*Sqr
t[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(
b^(3/2)*(a^2 + b^2)*f) + (2*C*Sqrt[c + d*Tan[e + f*x]])/(b*f)
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3653

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[e + f*x])^n*(1 + Tan[e
+ f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
```

```
!GtQ[n, 0] &&  !LeQ[n, -1]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3634

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx = \frac{2C\sqrt{c+d\tan(e+fx)}}{bf} + \frac{2\int \frac{\frac{1}{2}(Abc-aCd)+\frac{1}{2}b(Bc+(A-C)d)\mathrm{t}}{(a+b\tan(e\cdots}}{\cdots}$$

$$= \frac{2C\sqrt{c+d\tan(e+fx)}}{bf} + \frac{2\int \frac{\frac{1}{2}b(bBc+b(A-C)d+a(Ac-cC-B\cdots}{\cdots}}{\cdots}$$

$$= \frac{2C\sqrt{c+d\tan(e+fx)}}{bf} + \frac{((A-iB-C)(c-id))\int \frac{1+i\cdots}{\sqrt{c+\cdots}}}{2(a-ib)}$$

$$= \frac{2C\sqrt{c+d\tan(e+fx)}}{bf} - \frac{(i(A+iB-C)(c+id))\,\mathrm{Subs}\cdots}{\cdots}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e\cdots}}{\sqrt{bc-ad}}\cdots\right.}{b^{3/2}\left(a^2+b^2\right)f}$$

$$= \frac{(A-iB-C)\sqrt{c-id}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(ia+b)f} - \frac{(A+i\cdots}{\cdots}$$

Mathematica [A] time = 0.673089, size = 233, normalized size = 1.

$$\frac{2\sqrt{b}C\left(a^2+b^2\right)\sqrt{c+d\tan(e+fx)}+b^{3/2}(b-ia)\sqrt{c-id}(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)+b^{3/2}(b+ia)\sqrt{c+id}(A+i\cdots}{b^{3/2}f\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)
)/(a + b*Tan[e + f*x]),x]
```

```
[Out] (b^(3/2)*((-I)*a + b)*(A - I*B - C)*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e
+ f*x]]/Sqrt[c - I*d]] + b^(3/2)*(I*a + b)*(A + I*B - C)*Sqrt[c + I*d]*ArcT
anh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]] - 2*(A*b^2 + a*(-(b*B) + a*C))*
Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]]
 + 2*Sqrt[b]*(a^2 + b^2)*C*Sqrt[c + d*Tan[e + f*x]])/(b^(3/2)*(a^2 + b^2)*f
)
```

Maple [B] time = 0.191, size = 3576, normalized size = 15.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
,x)
```

```
[Out] 1/4/f/(a^2+b^2)/d*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1
/2)*a-1/4/f/(a^2+b^2)/d*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c+1/
```

```
4/f/(a^2+b^2)/d*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)+(c^2+d^2)^(1/2))*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c+1/4/f/(a^2
+b^2)/d*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e
)-c-(c^2+d^2)^(1/2))*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b-1/4/
f/(a^2+b^2)/d*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan
(f*x+e)-c-(c^2+d^2)^(1/2))*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*
a+1/4/f/(a^2+b^2)/d*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c-1/4/f/
(a^2+b^2)/d*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)+(c^2+d^2)^(1/2))*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c-1/4/f/(a^2+b^2
)/d*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+
(c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a+1/4/f/(a
^2+b^2)/d*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)+(c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c-1/4/f/(a^2+b^2)/
d*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c
^2+d^2)^(1/2))*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b+1/4/f/(a^2
+b^2)*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-
c-(c^2+d^2)^(1/2))*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b-1/4/f/(a^2+b^2)*ln(d*t
an(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^
(1/2))*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a-1/4/f/(a^2+b^2)*ln(d*tan(f*x+e)+c+
(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*C*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*b+1/4/f/(a^2+b^2)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x
+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)*b-1/4/f/(a^2+b^2)*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*b-1/4/f/(a^2+b^2)/d*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c-2/f
*b/(a^2+b^2)/((a*d-b*c)*b)^(1/2)*arctan((c+d*tan(f*x+e))^(1/2)*b/((a*d-b*c)
*b)^(1/2))*A*a*d+1/4/f/(a^2+b^2)/d*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(
1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*(c^2+d^2)^(1/2)*a-2/f/b/(a^2+b^2)/((a*d-b*c)*b)^(1/2)*arctan((c+d*tan(
f*x+e))^(1/2)*b/((a*d-b*c)*b)^(1/2))*a^3*C*d-2/f*b/(a^2+b^2)/((a*d-b*c)*b)^
(1/2)*arctan((c+d*tan(f*x+e))^(1/2)*b/((a*d-b*c)*b)^(1/2))*B*a*c-1/f/(a^2+b
^2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+
d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*B*(c^2+d^2)^(1/2)*a-1
/f/(a^2+b^2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)
+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*C*(c^2+d^2)^
(1/2)*b+1/f/(a^2+b^2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+
e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*C*b
*c-1/f/(a^2+b^2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(
1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*A*b*c+1/
f/(a^2+b^2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+
(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*B*a*c+1/f/(a^
2+b^2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-
2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*A*b*c-1/f/(a^2+b^2
)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+
d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*A*(c^2+d^2)^(1/2)*b+1/f
/(a^2+b^2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1
/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*B*(c^2+d^2)^(1
/2)*a-1/f/(a^2+b^2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2
)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*C*b*c
+1/f/(a^2+b^2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c
)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*C*(c^2+d^2
)^(1/2)*b+2/f/(a^2+b^2)/((a*d-b*c)*b)^(1/2)*arctan((c+d*tan(f*x+e))^(1/2)*b
/((a*d-b*c)*b)^(1/2))*B*a^2*d+2/f/(a^2+b^2)/((a*d-b*c)*b)^(1/2)*arctan((c+d
*tan(f*x+e))^(1/2)*b/((a*d-b*c)*b)^(1/2))*C*a^2*c+1/f/(a^2+b^2)*d/(2*(c^2+d
^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*
c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*A*a+1/f/(a^2+b^2)*d/(2*(c^2+d^2)^(
1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1
```

```
/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*B*b-1/f/(a^2+b^2)*d/(2*(c^2+d^2)^(1/2)-
2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/
(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*C*a+1/f/(a^2+b^2)*d/(2*(c^2+d^2)^(1/2)-2*c)^
(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c
^2+d^2)^(1/2)-2*c)^(1/2))*C*a-1/f/(a^2+b^2)*d/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)
*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^
2)^(1/2)-2*c)^(1/2))*A*a-1/f/(a^2+b^2)*d/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arct
an(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1
/2)-2*c)^(1/2))*B*b+2*C*(c+d*tan(f*x+e))^(1/2)/b/f-1/f/(a^2+b^2)/(2*(c^2+d^
2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e)
)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*B*a*c+2/f*b^2/(a^2+b^2)/((a*d-b*c)*
b)^(1/2)*arctan((c+d*tan(f*x+e))^(1/2)*b/((a*d-b*c)*b)^(1/2))*A*c+1/f/(a^2+
b^2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2
+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*A*(c^2+d^2)^(1/2)*b+
1/4/f/(a^2+b^2)*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*t
an(f*x+e)-c-(c^2+d^2)^(1/2))*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + d\tan\left(e + fx\right)}\left(A + B\tan\left(e + fx\right) + C\tan^2\left(e + fx\right)\right)}{a + b\tan\left(e + fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e)),x)
```

```
[Out] Integral(sqrt(c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/
(a + b*tan(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\tan\left(fx+e\right)^2+B\tan\left(fx+e\right)+A\right)\sqrt{d\tan\left(fx+e\right)+c}}{b\tan\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e)),x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*sqrt(d*tan(f*x + e) + c)/
(b*tan(f*x + e) + a), x)
```

3.95 $\int \frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=317

$$-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{\left(-a^2b^2(3Ad+2Bc-5Cd)+a^3bBd+a^4Cd+ab^3(4Ac-3Bd-4cC)+b^4(Ac\right.}{b^{3/2}f\left(a^2+b^2\right)^2\sqrt{bc-ad}}$$

```
[Out] -(((I*A + B - I*C)*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c -
I*d]])/((a - I*b)^2*f)) - ((B - I*(A - C))*Sqrt[c + I*d]*ArcTanh[Sqrt[c + d
*Tan[e + f*x]]/Sqrt[c + I*d]])/((a + I*b)^2*f) - ((a^3*b*B*d + a^4*C*d + b^
4*(2*B*c + A*d) + a*b^3*(4*A*c - 4*c*C - 3*B*d) - a^2*b^2*(2*B*c + 3*A*d -
5*C*d))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(b^(3/
2)*(a^2 + b^2)^2*Sqrt[b*c - a*d]*f) - ((A*b^2 - a*(b*B - a*C))*Sqrt[c + d*T
an[e + f*x]])/(b*(a^2 + b^2)*f*(a + b*Tan[e + f*x]))
```

Rubi [A] time = 1.43942, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.149, Rules used = {3645, 3653, 3539, 3537, 63, 208, 3634}

$$-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{\left(-a^2b^2(3Ad+2Bc-5Cd)+a^3bBd+a^4Cd+ab^3(4Ac-3Bd-4cC)+b^4(Ac\right.}{b^{3/2}f\left(a^2+b^2\right)^2\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a +
 b*Tan[e + f*x])^2,x]
```

```
[Out] -(((I*A + B - I*C)*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c -
I*d]])/((a - I*b)^2*f)) - ((B - I*(A - C))*Sqrt[c + I*d]*ArcTanh[Sqrt[c + d
*Tan[e + f*x]]/Sqrt[c + I*d]])/((a + I*b)^2*f) - ((a^3*b*B*d + a^4*C*d + b^
4*(2*B*c + A*d) + a*b^3*(4*A*c - 4*c*C - 3*B*d) - a^2*b^2*(2*B*c + 3*A*d -
5*C*d))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(b^(3/
2)*(a^2 + b^2)^2*Sqrt[b*c - a*d]*f) - ((A*b^2 - a*(b*B - a*C))*Sqrt[c + d*T
an[e + f*x]])/(b*(a^2 + b^2)*f*(a + b*Tan[e + f*x]))
```

Rule 3645

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*d^2 + c*(c*C - B*d))*(a + b*Tan[e
+ f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3653

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
```

```
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[e + f*x])^n*(1 + Tan[e
+ f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3634

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{b\left(a^2+b^2\right)f(a+b\tan(e+fx))} + \int \frac{\frac{1}{2}\left(2(bB-\ldots\right.}{\ldots}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{b\left(a^2+b^2\right)f(a+b\tan(e+fx))} + \int \frac{b\left(a^2(Ac-\ldots\right.}{\ldots}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{b\left(a^2+b^2\right)f(a+b\tan(e+fx))} + \frac{((A-iB\ldots}{\ldots}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{b\left(a^2+b^2\right)f(a+b\tan(e+fx))} - \frac{(i(A+iB\ldots}{\ldots}$$

$$= -\frac{\left(a^3bBd+a^4Cd+b^4(2Bc+Ad)+ab^3(4Ac-4cC-\ldots\right.}{b^{3/2}\left(a^2+\ldots\right.}$$

$$= -\frac{(B+i(A-C))\sqrt{c-id}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)^2f} - \frac{(B\ldots}{\ldots}$$

Mathematica [B] time = 6.39603, size = 764, normalized size = 2.41

$$-\frac{2C\sqrt{c+d\tan(e+fx)}}{bf(a+b\tan(e+fx))} - \frac{2\left(-\frac{\sqrt{c+d\tan(e+fx)}\left(\frac{1}{2}b^2(-aCd-Abc+2bcC)-a\left(-\frac{1}{2}a(-aCd-bBd+bcC)-\frac{1}{2}b^2(d(A-C)+Bc)\right)\right)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))} - \frac{2\sqrt{bc-ad}\left(-\frac{1}{4}a^2d(bc-ad)\left(a^2(-C\ldots\right.\right.}{\ldots}\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)
)/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (-2*C*Sqrt[c + d*Tan[e + f*x]])/(b*f*(a + b*Tan[e + f*x])) - (2*(-((((I*Sqr
t[c - I*d]*((b*(b*c - a*d)*(a^2*(A*c - c*C - B*d) - b^2*(A*c - c*C - B*d) +
 2*a*b*(B*c + (A - C)*d)))/2 + (I/2)*b*(b*c - a*d)*(2*a*b*(A*c - c*C - B*d)
 - a^2*(B*c + (A - C)*d) + b^2*(B*c + (A - C)*d)))*ArcTanh[Sqrt[c + d*Tan[e
 + f*x]]/Sqrt[c - I*d]])/((-c + I*d)*f) - (I*Sqrt[c + I*d]*((b*(b*c - a*d)*
(a^2*(A*c - c*C - B*d) - b^2*(A*c - c*C - B*d) + 2*a*b*(B*c + (A - C)*d)))/
2 - (I/2)*b*(b*c - a*d)*(2*a*b*(A*c - c*C - B*d) - a^2*(B*c + (A - C)*d) +
b^2*(B*c + (A - C)*d)))*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/((
-c - I*d)*f))/(a^2 + b^2) + (2*Sqrt[b*c - a*d]*(-(a^2*(A*b^2 - a*b*B - a^2*
C - 2*b^2*C)*d*(b*c - a*d))/4 + (a*b^2*(b*c - a*d)*(A*b*c - a*B*c - b*c*C -
 a*A*d - b*B*d + a*C*d))/2 + (b^2*(b*c - a*d)*(a^2*C*d + b^2*(2*B*c + A*d)
+ a*b*(2*A*c - 2*c*C - B*d)))/4)*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])
/Sqrt[b*c - a*d]])/(Sqrt[b]*(a^2 + b^2)*(-(b*c) + a*d)*f))/((a^2 + b^2)*(b*
c - a*d))) - (((b^2*(-(A*b*c) + 2*b*c*C - a*C*d))/2 - a*(-(b^2*(B*c + (A -
C)*d))/2 - (a*(b*c*C - b*B*d - a*C*d))/2))*Sqrt[c + d*Tan[e + f*x]])/((a^2
+ b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x]))))/b
```

Maple [B] time = 0.214, size = 5778, normalized size = 18.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^2,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + d\tan\left(e + fx\right)}\left(A + B\tan\left(e + fx\right) + C\tan^{2}\left(e + fx\right)\right)}{\left(a + b\tan\left(e + fx\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**2,x)
```

```
[Out] Integral(sqrt(c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/
(a + b*tan(e + f*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\tan\left(fx+e\right)^{2}+B\tan\left(fx+e\right)+A\right)\sqrt{d\tan\left(fx+e\right)+c}}{\left(b\tan\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*sqrt(d*tan(f*x + e) + c)/
(b*tan(f*x + e) + a)^2, x)
```

3.96 $\int \frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx$

Optimal. Leaf size=543

$$\frac{\left(2a^3b^3\left(20cd(A-C)+B\left(4c^2-13d^2\right)\right)-3a^2b^4\left(8Ac^2-6Ad^2-16Bcd-8c^2C+5Cd^2\right)-3a^4b^2d(5Ad+4Bc-6Cd)+\right.}{4b^{3/2}f\left(a^2+b^2\right.}$$

```
[Out] -(((A - I*B - C)*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*
d]])/((I*a + b)^3*f)) + ((A + I*B - C)*Sqrt[c + I*d]*ArcTanh[Sqrt[c + d*Tan
[e + f*x]]/Sqrt[c + I*d]])/((I*a - b)^3*f) + ((3*a^5*b*B*d^2 + a^6*C*d^2 -
3*a^4*b^2*d*(4*B*c + 5*A*d - 6*C*d) - 3*a^2*b^4*(8*A*c^2 - 8*c^2*C - 16*B*c
*d - 6*A*d^2 + 5*C*d^2) + 2*a^3*b^3*(20*c*(A - C)*d + B*(4*c^2 - 13*d^2)) -
 3*a*b^5*(8*c*(A - C)*d + B*(8*c^2 - d^2)) - b^6*(4*c*(2*c*C + B*d) - A*(8*
c^2 + d^2)))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(
4*b^(3/2)*(a^2 + b^2)^3*(b*c - a*d)^(3/2)*f) - ((A*b^2 - a*(b*B - a*C))*Sqr
t[c + d*Tan[e + f*x]])/(2*b*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^2) - ((3*a^3
*b*B*d + a^4*C*d + b^4*(4*B*c + A*d) + a*b^3*(8*A*c - 8*c*C - 5*B*d) - a^2*
b^2*(4*B*c + 7*A*d - 9*C*d))*Sqrt[c + d*Tan[e + f*x]])/(4*b*(a^2 + b^2)^2*(
b*c - a*d)*f*(a + b*Tan[e + f*x]))
```

Rubi [A] time = 4.03671, antiderivative size = 543, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.17, Rules used = {3645, 3649, 3653, 3539, 3537, 63, 208, 3634}

$$\frac{\left(2a^3b^3\left(20cd(A-C)+B\left(4c^2-13d^2\right)\right)-3a^2b^4\left(8Ac^2-6Ad^2-16Bcd-8c^2C+5Cd^2\right)-3a^4b^2d(5Ad+4Bc-6Cd)+\right.}{4b^{3/2}f\left(a^2+b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a +
 b*Tan[e + f*x])^3,x]
```

```
[Out] -(((A - I*B - C)*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*
d]])/((I*a + b)^3*f)) + ((A + I*B - C)*Sqrt[c + I*d]*ArcTanh[Sqrt[c + d*Tan
[e + f*x]]/Sqrt[c + I*d]])/((I*a - b)^3*f) + ((3*a^5*b*B*d^2 + a^6*C*d^2 -
3*a^4*b^2*d*(4*B*c + 5*A*d - 6*C*d) - 3*a^2*b^4*(8*A*c^2 - 8*c^2*C - 16*B*c
*d - 6*A*d^2 + 5*C*d^2) + 2*a^3*b^3*(20*c*(A - C)*d + B*(4*c^2 - 13*d^2)) -
 3*a*b^5*(8*c*(A - C)*d + B*(8*c^2 - d^2)) - b^6*(4*c*(2*c*C + B*d) - A*(8*
c^2 + d^2)))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(
4*b^(3/2)*(a^2 + b^2)^3*(b*c - a*d)^(3/2)*f) - ((A*b^2 - a*(b*B - a*C))*Sqr
t[c + d*Tan[e + f*x]])/(2*b*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^2) - ((3*a^3
*b*B*d + a^4*C*d + b^4*(4*B*c + A*d) + a*b^3*(8*A*c - 8*c*C - 5*B*d) - a^2*
b^2*(4*B*c + 7*A*d - 9*C*d))*Sqrt[c + d*Tan[e + f*x]])/(4*b*(a^2 + b^2)^2*(
b*c - a*d)*f*(a + b*Tan[e + f*x]))
```

Rule 3645

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*d^2 + c*(c*C - B*d))*(a + b*Tan[e
+ f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
```

```
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
 b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3653

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[e + f*x])^n*(1 + Tan[e
+ f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3634

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}dx = -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2} + \int \frac{\frac{1}{2}(2(\ldots}{\ldots}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2} - \frac{\left(3a^3b\ldots\right.}{\ldots}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2} - \frac{\left(3a^3b\ldots\right.}{\ldots}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2} - \frac{\left(3a^3b\ldots\right.}{\ldots}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2} - \frac{\left(3a^3b\ldots\right.}{\ldots}$$

$$= \frac{\left(3a^5bBd^2+a^6Cd^2-3a^4b^2d(4Bc+5Ad-6Cd)-\ldots\right.}{\ldots}$$

$$= -\frac{(A-iB-C)\sqrt{c-id}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(ia+b)^3f} + \frac{(\ldots}{\ldots}$$

Mathematica [B] time = 6.41253, size = 2819, normalized size = 5.19

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)
)/(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (-2*C*Sqrt[c + d*Tan[e + f*x]])/(3*b*f*(a + b*Tan[e + f*x])^2) - (2*(-(((b^
2*(-3*A*b*c + 4*b*c*C - a*C*d))/2 - a*((-3*b^2*(B*c + (A - C)*d))/2 - (a*(b
*c*C - 3*b*B*d - a*C*d))/2))*Sqrt[c + d*Tan[e + f*x]])/(2*(a^2 + b^2)*(b*c
- a*d)*f*(a + b*Tan[e + f*x])^2) - (-((((I*Sqrt[c - I*d]*(b*(b*c - a*d)*((3
*b*(3*A*b^2 - 3*a*b*B - a^2*C - 4*b^2*C)*d*(b*c - a*d))/4 + 3*a*b*(b*c - a*
d)*(A*b*c - a*B*c - b*c*C - a*A*d - b*B*d + a*C*d) + (3*b*(b*c - a*d)*(a^2*
C*d + b^2*(4*B*c + A*d) + a*b*(4*A*c - 4*c*C - B*d)))/4) + a*((3*(b*c - a*d
)*((b^2*d)/2 - a*(b*c - a*d))*(a^2*C*d + b^2*(4*B*c + A*d) + a*b*(4*A*c - 4
*c*C - B*d)))/4 + (-(b*c) + (a*d)/2)*((3*a*(3*A*b^2 - 3*a*b*B - a^2*C - 4*b
^2*C)*d*(b*c - a*d))/4 - 3*b^2*(b*c - a*d)*(A*b*c - a*B*c - b*c*C - a*A*d -
 b*B*d + a*C*d)) - (d*((3*b^2*(b*c - a*d)*(a^2*C*d + b^2*(4*B*c + A*d) + a*
b*(4*A*c - 4*c*C - B*d)))/4 - a*((3*a*(3*A*b^2 - 3*a*b*B - a^2*C - 4*b^2*C)
*d*(b*c - a*d))/4 - 3*b^2*(b*c - a*d)*(A*b*c - a*B*c - b*c*C - a*A*d - b*B*
```

```
d + a*C*d))))/2) - I*(a*(b*c - a*d)*((3*b*(3*A*b^2 - 3*a*b*B - a^2*C - 4*b^
2*C)*d*(b*c - a*d))/4 + 3*a*b*(b*c - a*d)*(A*b*c - a*B*c - b*c*C - a*A*d -
b*B*d + a*C*d) + (3*b*(b*c - a*d)*(a^2*C*d + b^2*(4*B*c + A*d) + a*b*(4*A*c
 - 4*c*C - B*d)))/4) - b*((3*(b*c - a*d)*((b^2*d)/2 - a*(b*c - a*d))*(a^2*C
*d + b^2*(4*B*c + A*d) + a*b*(4*A*c - 4*c*C - B*d)))/4 + (-(b*c) + (a*d)/2)
*((3*a*(3*A*b^2 - 3*a*b*B - a^2*C - 4*b^2*C)*d*(b*c - a*d))/4 - 3*b^2*(b*c
- a*d)*(A*b*c - a*B*c - b*c*C - a*A*d - b*B*d + a*C*d)) - (d*((3*b^2*(b*c -
 a*d)*(a^2*C*d + b^2*(4*B*c + A*d) + a*b*(4*A*c - 4*c*C - B*d)))/4 - a*((3*
a*(3*A*b^2 - 3*a*b*B - a^2*C - 4*b^2*C)*d*(b*c - a*d))/4 - 3*b^2*(b*c - a*d
)*(A*b*c - a*B*c - b*c*C - a*A*d - b*B*d + a*C*d))))/2)))*ArcTanh[Sqrt[c +
d*Tan[e + f*x]]/Sqrt[c - I*d]])/((-c + I*d)*f) - (I*Sqrt[c + I*d]*(b*(b*c -
 a*d)*((3*b*(3*A*b^2 - 3*a*b*B - a^2*C - 4*b^2*C)*d*(b*c - a*d))/4 + 3*a*b*
(b*c - a*d)*(A*b*c - a*B*c - b*c*C - a*A*d - b*B*d + a*C*d) + (3*b*(b*c - a
*d)*(a^2*C*d + b^2*(4*B*c + A*d) + a*b*(4*A*c - 4*c*C - B*d)))/4) + a*((3*(
b*c - a*d)*((b^2*d)/2 - a*(b*c - a*d))*(a^2*C*d + b^2*(4*B*c + A*d) + a*b*(
4*A*c - 4*c*C - B*d)))/4 + (-(b*c) + (a*d)/2)*((3*a*(3*A*b^2 - 3*a*b*B - a^
2*C - 4*b^2*C)*d*(b*c - a*d))/4 - 3*b^2*(b*c - a*d)*(A*b*c - a*B*c - b*c*C
- a*A*d - b*B*d + a*C*d)) - (d*((3*b^2*(b*c - a*d)*(a^2*C*d + b^2*(4*B*c +
A*d) + a*b*(4*A*c - 4*c*C - B*d)))/4 - a*((3*a*(3*A*b^2 - 3*a*b*B - a^2*C -
 4*b^2*C)*d*(b*c - a*d))/4 - 3*b^2*(b*c - a*d)*(A*b*c - a*B*c - b*c*C - a*A
*d - b*B*d + a*C*d))))/2) + I*(a*(b*c - a*d)*((3*b*(3*A*b^2 - 3*a*b*B - a^2
*C - 4*b^2*C)*d*(b*c - a*d))/4 + 3*a*b*(b*c - a*d)*(A*b*c - a*B*c - b*c*C -
 a*A*d - b*B*d + a*C*d) + (3*b*(b*c - a*d)*(a^2*C*d + b^2*(4*B*c + A*d) + a
*b*(4*A*c - 4*c*C - B*d)))/4) - b*((3*(b*c - a*d)*((b^2*d)/2 - a*(b*c - a*d
))*(a^2*C*d + b^2*(4*B*c + A*d) + a*b*(4*A*c - 4*c*C - B*d)))/4 + (-(b*c) +
 (a*d)/2)*((3*a*(3*A*b^2 - 3*a*b*B - a^2*C - 4*b^2*C)*d*(b*c - a*d))/4 - 3*
b^2*(b*c - a*d)*(A*b*c - a*B*c - b*c*C - a*A*d - b*B*d + a*C*d)) - (d*((3*b
^2*(b*c - a*d)*(a^2*C*d + b^2*(4*B*c + A*d) + a*b*(4*A*c - 4*c*C - B*d)))/4
 - a*((3*a*(3*A*b^2 - 3*a*b*B - a^2*C - 4*b^2*C)*d*(b*c - a*d))/4 - 3*b^2*(
b*c - a*d)*(A*b*c - a*B*c - b*c*C - a*A*d - b*B*d + a*C*d))))/2)))*ArcTanh[
Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/((-c - I*d)*f))/(a^2 + b^2) + (2*S
qrt[b*c - a*d]*(-(a*b*(b*c - a*d)*((3*b*(3*A*b^2 - 3*a*b*B - a^2*C - 4*b^2*
C)*d*(b*c - a*d))/4 + 3*a*b*(b*c - a*d)*(A*b*c - a*B*c - b*c*C - a*A*d - b*
B*d + a*C*d) + (3*b*(b*c - a*d)*(a^2*C*d + b^2*(4*B*c + A*d) + a*b*(4*A*c -
 4*c*C - B*d)))/4)) + (a^2*d*((3*b^2*(b*c - a*d)*(a^2*C*d + b^2*(4*B*c + A*
d) + a*b*(4*A*c - 4*c*C - B*d)))/4 - a*((3*a*(3*A*b^2 - 3*a*b*B - a^2*C - 4
*b^2*C)*d*(b*c - a*d))/4 - 3*b^2*(b*c - a*d)*(A*b*c - a*B*c - b*c*C - a*A*d
 - b*B*d + a*C*d))))/2 + b^2*((3*(b*c - a*d)*((b^2*d)/2 - a*(b*c - a*d))*(a
^2*C*d + b^2*(4*B*c + A*d) + a*b*(4*A*c - 4*c*C - B*d)))/4 + (-(b*c) + (a*d
)/2)*((3*a*(3*A*b^2 - 3*a*b*B - a^2*C - 4*b^2*C)*d*(b*c - a*d))/4 - 3*b^2*(
b*c - a*d)*(A*b*c - a*B*c - b*c*C - a*A*d - b*B*d + a*C*d))))*ArcTanh[(Sqrt
[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(Sqrt[b]*(a^2 + b^2)*(-(b*c
) + a*d)*f))/((a^2 + b^2)*(b*c - a*d))) - (((3*b^2*(b*c - a*d)*(a^2*C*d + b
^2*(4*B*c + A*d) + a*b*(4*A*c - 4*c*C - B*d)))/4 - a*((3*a*(3*A*b^2 - 3*a*b
*B - a^2*C - 4*b^2*C)*d*(b*c - a*d))/4 - 3*b^2*(b*c - a*d)*(A*b*c - a*B*c -
 b*c*C - a*A*d - b*B*d + a*C*d)))*Sqrt[c + d*Tan[e + f*x]])/((a^2 + b^2)*(b
*c - a*d)*f*(a + b*Tan[e + f*x]))))/(2*(a^2 + b^2)*(b*c - a*d))))/(3*b)
```

Maple [B] time = 0.241, size = 9797, normalized size = 18.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + d\tan\left(e + fx\right)}\left(A + B\tan\left(e + fx\right) + C\tan^{2}\left(e + fx\right)\right)}{\left(a + b\tan\left(e + fx\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**3,x)
```

```
[Out] Integral(sqrt(c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/
(a + b*tan(e + f*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\tan\left(fx + e\right)^{2} + B\tan\left(fx + e\right) + A\right)\sqrt{d\tan\left(fx + e\right) + c}}{\left(b\tan\left(fx + e\right) + a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*sqrt(d*tan(f*x + e) + c)/
(b*tan(f*x + e) + a)^3, x)
```

3.97 $\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+\right.$

Optimal. Leaf size=550

$$\frac{2(c+d\tan(e+fx))^{5/2}\left(-2a^2bd^2(192cC-847Bd)+168a^3Cd^3+33ab^2d\left(63d^2(A-C)-18Bcd+8c^2C\right)+b^3\left(-\left(198cd^2(\right.\right.\right.}{3465d^4f}$$

```
[Out] ((I*a + b)^3*(A - I*B - C)*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]
/Sqrt[c - I*d]])/f + ((a + I*b)^3*(I*A - B - I*C)*(c + I*d)^(3/2)*ArcTanh[S
qrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/f + (2*(3*a^2*b*(A*c - c*C - B*d) -
 b^3*(A*c - c*C - B*d) + a^3*(B*c + (A - C)*d) - 3*a*b^2*(B*c + (A - C)*d))
*Sqrt[c + d*Tan[e + f*x]])/f + (2*(a^3*B - 3*a*b^2*B + 3*a^2*b*(A - C) - b^
3*(A - C))*(c + d*Tan[e + f*x])^(3/2))/(3*f) + (2*(168*a^3*C*d^3 - 2*a^2*b*
d^2*(192*c*C - 847*B*d) + 33*a*b^2*d*(8*c^2*C - 18*B*c*d + 63*(A - C)*d^2)
- b^3*(48*c^3*C - 88*B*c^2*d + 198*c*(A - C)*d^2 + 693*B*d^3))*(c + d*Tan[e
 + f*x])^(5/2))/(3465*d^4*f) + (2*b*(99*b*(A*b + a*B - b*C)*d^2 + 4*(b*c -
a*d)*(6*b*c*C - 11*b*B*d - 6*a*C*d))*Tan[e + f*x]*(c + d*Tan[e + f*x])^(5/2
))/(693*d^3*f) - (2*(6*b*c*C - 11*b*B*d - 6*a*C*d)*(a + b*Tan[e + f*x])^2*(
c + d*Tan[e + f*x])^(5/2))/(99*d^2*f) + (2*C*(a + b*Tan[e + f*x])^3*(c + d*
Tan[e + f*x])^(5/2))/(11*d*f)
```

Rubi [A] time = 2.73385, antiderivative size = 550, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.17, Rules used = {3647, 3637, 3630, 3528, 3539, 3537, 63, 208}

$$\frac{2(c+d\tan(e+fx))^{5/2}\left(-2a^2bd^2(192cC-847Bd)+168a^3Cd^3+33ab^2d\left(63d^2(A-C)-18Bcd+8c^2C\right)+b^3\left(-\left(198cd^2(\right.\right.\right.}{3465d^4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] +
 C*Tan[e + f*x]^2),x]
```


```
[Out] ((I*a + b)^3*(A - I*B - C)*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]
/Sqrt[c - I*d]])/f + ((a + I*b)^3*(I*A - B - I*C)*(c + I*d)^(3/2)*ArcTanh[S
qrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/f + (2*(3*a^2*b*(A*c - c*C - B*d) -
 b^3*(A*c - c*C - B*d) + a^3*(B*c + (A - C)*d) - 3*a*b^2*(B*c + (A - C)*d))
*Sqrt[c + d*Tan[e + f*x]])/f + (2*(a^3*B - 3*a*b^2*B + 3*a^2*b*(A - C) - b^
3*(A - C))*(c + d*Tan[e + f*x])^(3/2))/(3*f) + (2*(168*a^3*C*d^3 - 2*a^2*b*
d^2*(192*c*C - 847*B*d) + 33*a*b^2*d*(8*c^2*C - 18*B*c*d + 63*(A - C)*d^2)
- b^3*(48*c^3*C - 88*B*c^2*d + 198*c*(A - C)*d^2 + 693*B*d^3))*(c + d*Tan[e
 + f*x])^(5/2))/(3465*d^4*f) + (2*b*(99*b*(A*b + a*B - b*C)*d^2 + 4*(b*c -
a*d)*(6*b*c*C - 11*b*B*d - 6*a*C*d))*Tan[e + f*x]*(c + d*Tan[e + f*x])^(5/2
))/(693*d^3*f) - (2*(6*b*c*C - 11*b*B*d - 6*a*C*d)*(a + b*Tan[e + f*x])^2*(
c + d*Tan[e + f*x])^(5/2))/(99*d^2*f) + (2*C*(a + b*Tan[e + f*x])^3*(c + d*
Tan[e + f*x])^(5/2))/(11*d*f)
```


Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
```

```
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx = \frac{2C(a+b\tan(e+fx))^3(c+d\,\mathrm{ta}}{11df}$$

$$= -\frac{2(6bcC-11bBd-6aCd)(a+b}{}$$

$$= \frac{2b\left(99b(Ab+aB-bC)d^2+4(b\right.}{}$$

$$= \frac{2\left(168a^3Cd^3-2a^2bd^2(192cC-\right.}{}$$

$$= \frac{2\left(a^3B-3ab^2B+3a^2b(A-C)-\right.}{}$$

$$= \frac{2\left(3a^2b(Ac-cC-Bd)-b^3(Ac\right.}{}$$

$$= \frac{2\left(3a^2b(Ac-cC-Bd)-b^3(Ac\right.}{}$$

$$= \frac{2\left(3a^2b(Ac-cC-Bd)-b^3(Ac\right.}{}$$

$$= \frac{2\left(3a^2b(Ac-cC-Bd)-b^3(Ac\right.}{}$$

$$= -\frac{(a-ib)^3(iA+B-iC)(c-id)^{3}}{f}$$

Mathematica [B] time = 6.4178, size = 1290, normalized size = 2.35

$$\frac{2C(c+d\tan(e+fx))^{5/2}(a+b\tan(e+fx))^3}{11df}+2\left(\frac{(-6bcC+6adC+11bBd)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df}+\frac{2\left(\frac{b\left(99b(Ab-Cb+aB)d^2+4(bc\cdots\right.}{\cdots}\cdots\right)}{\cdots}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e +
f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] (2*C*(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^(5/2))/(11*d*f) + (2*(((-6
*b*c*C + 11*b*B*d + 6*a*C*d)*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(5
/2))/(9*d*f) + (2*((b*(99*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C
- 11*b*B*d - 6*a*C*d))*Tan[e + f*x]*(c + d*Tan[e + f*x])^(5/2))/(14*d*f) -
(2*((2*((-7*a*d*(99*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C - 11*b
*B*d - 6*a*C*d)))/8 + b*((-693*(a^2*B - b^2*B + 2*a*b*(A - C))*d^3)/8 + (c*
(99*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C - 11*b*B*d - 6*a*C*d))
)/4))*(c + d*Tan[e + f*x])^(5/2))/(5*d*f) + ((I/2)*((-7*a*d*(3*a^2*(33*A -
25*C)*d^2 + 4*b^2*c*(6*c*C - 11*B*d) - a*b*d*(48*c*C + 55*B*d)))/8 + (b*c*(
99*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C - 11*b*B*d - 6*a*C*d)))
/4 + (7*a*d*(99*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C - 11*b*B*d
 - 6*a*C*d)))/8 + ((7*I)/2)*d*((99*a*(a^2*B - b^2*B + 2*a*b*(A - C))*d^2)/4
 + (b*(3*a^2*(33*A - 25*C)*d^2 + 4*b^2*c*(6*c*C - 11*B*d) - a*b*d*(48*c*C +
 55*B*d)))/4 - (b*(99*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C - 11
*b*B*d - 6*a*C*d)))/4) - b*((-693*(a^2*B - b^2*B + 2*a*b*(A - C))*d^3)/8 +
(c*(99*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C - 11*b*B*d - 6*a*C*
d)))/4))*((2*(c + d*Tan[e + f*x])^(3/2))/3 + (c - I*d)*((2*(c - I*d)^(3/2)*
ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/(-c + I*d) + 2*Sqrt[c + d*
Tan[e + f*x]])))/f - ((I/2)*((-7*a*d*(3*a^2*(33*A - 25*C)*d^2 + 4*b^2*c*(6*
c*C - 11*B*d) - a*b*d*(48*c*C + 55*B*d)))/8 + (b*c*(99*b*(A*b + a*B - b*C)*
d^2 + 4*(b*c - a*d)*(6*b*c*C - 11*b*B*d - 6*a*C*d)))/4 + (7*a*d*(99*b*(A*b
+ a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C - 11*b*B*d - 6*a*C*d)))/8 - ((7*I
)/2)*d*((99*a*(a^2*B - b^2*B + 2*a*b*(A - C))*d^2)/4 + (b*(3*a^2*(33*A - 25
*C)*d^2 + 4*b^2*c*(6*c*C - 11*B*d) - a*b*d*(48*c*C + 55*B*d)))/4 - (b*(99*b
*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C - 11*b*B*d - 6*a*C*d)))/4)
- b*((-693*(a^2*B - b^2*B + 2*a*b*(A - C))*d^3)/8 + (c*(99*b*(A*b + a*B - b
*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C - 11*b*B*d - 6*a*C*d)))/4))*((2*(c + d*Tan
[e + f*x])^(3/2))/3 + (c + I*d)*((2*(c + I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[
e + f*x]]/Sqrt[c + I*d]])/(-c - I*d) + 2*Sqrt[c + d*Tan[e + f*x]])))/f))/(7
```

```
*d)))/(9*d)))/(11*d)
```

Maple [B] time = 0.198, size = 11056, normalized size = 20.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f
*x+e)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f
*x+e)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b \tan \left(e + fx\right)\right)^{3} \left(c + d \tan \left(e + fx\right)\right)^{\frac{3}{2}} \left(A + B \tan \left(e + fx\right) + C \tan^{2}\left(e + fx\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3*(c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan
(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**3*(c + d*tan(e + f*x))**(3/2)*(A + B*tan(e +
 f*x) + C*tan(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \tan \left(fx+e\right)^{2}+B \tan \left(fx+e\right)+A\right)\left(b \tan \left(fx+e\right)+a\right)^{3}\left(d \tan \left(fx+e\right)+c\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f
*x+e)^2),x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^3*(d
*tan(f*x + e) + c)^(3/2), x)
```

3.98 $\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+\right.$

Optimal. Leaf size=396

$$\frac{2(c+d\tan(e+fx))^{5/2}\left(28a^2Cd^2-18abd(2cC-7Bd)+b^2\left(63d^2(A-C)-18Bcd+8c^2C\right)\right)}{315d^3f}+\frac{2\left(a^2B+2ab(A-C)-b^2\right.}{3f}$$

```
[Out] -(((a - I*b)^2*(B + I*(A - C))*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f
*x]]/Sqrt[c - I*d]])/f) + ((a + I*b)^2*(I*A - B - I*C)*(c + I*d)^(3/2)*ArcT
anh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/f + (2*(2*a*b*(A*c - c*C - B*d
) + a^2*(B*c + (A - C)*d) - b^2*(B*c + (A - C)*d))*Sqrt[c + d*Tan[e + f*x]]
)/f + (2*(a^2*B - b^2*B + 2*a*b*(A - C))*(c + d*Tan[e + f*x])^(3/2))/(3*f)
+ (2*(28*a^2*C*d^2 - 18*a*b*d*(2*c*C - 7*B*d) + b^2*(8*c^2*C - 18*B*c*d + 6
3*(A - C)*d^2))*(c + d*Tan[e + f*x])^(5/2))/(315*d^3*f) - (2*b*(4*b*c*C - 9
*b*B*d - 4*a*C*d)*Tan[e + f*x]*(c + d*Tan[e + f*x])^(5/2))/(63*d^2*f) + (2*
C*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(5/2))/(9*d*f)
```

Rubi [A] time = 1.72651, antiderivative size = 396, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.17, Rules used = {3647, 3637, 3630, 3528, 3539, 3537, 63, 208}

$$\frac{2(c+d\tan(e+fx))^{5/2}\left(28a^2Cd^2-18abd(2cC-7Bd)+b^2\left(63d^2(A-C)-18Bcd+8c^2C\right)\right)}{315d^3f}+\frac{2\left(a^2B+2ab(A-C)-b^2\right.}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] +
 C*Tan[e + f*x]^2),x]
```



```
[Out] -(((a - I*b)^2*(B + I*(A - C))*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f
*x]]/Sqrt[c - I*d]])/f) + ((a + I*b)^2*(I*A - B - I*C)*(c + I*d)^(3/2)*ArcT
anh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/f + (2*(2*a*b*(A*c - c*C - B*d
) + a^2*(B*c + (A - C)*d) - b^2*(B*c + (A - C)*d))*Sqrt[c + d*Tan[e + f*x]]
)/f + (2*(a^2*B - b^2*B + 2*a*b*(A - C))*(c + d*Tan[e + f*x])^(3/2))/(3*f)
+ (2*(28*a^2*C*d^2 - 18*a*b*d*(2*c*C - 7*B*d) + b^2*(8*c^2*C - 18*B*c*d + 6
3*(A - C)*d^2))*(c + d*Tan[e + f*x])^(5/2))/(315*d^3*f) - (2*b*(4*b*c*C - 9
*b*B*d - 4*a*C*d)*Tan[e + f*x]*(c + d*Tan[e + f*x])^(5/2))/(63*d^2*f) + (2*
C*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(5/2))/(9*d*f)
```


Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
```

```
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx = \frac{2C(a+b\tan(e+fx))^2(c+d\,\mathrm{ta}}{9df}$$

$$= -\frac{2b(4bcC-9bBd-4aCd)\tan(}{63d}$$

$$= \frac{2\left(28a^2Cd^2-18abd(2cC-7Bd\right.}{}$$

$$= \frac{2\left(a^2B-b^2B+2ab(A-C)\right)(c+}{3f}$$

$$= \frac{2\left(2ab(Ac-cC-Bd)+a^2(Bc+\right.}{}$$

$$= \frac{2\left(2ab(Ac-cC-Bd)+a^2(Bc+\right.}{}$$

$$= \frac{2\left(2ab(Ac-cC-Bd)+a^2(Bc+\right.}{}$$

$$= \frac{2\left(2ab(Ac-cC-Bd)+a^2(Bc+\right.}{}$$

$$= -\frac{(a-ib)^2(iA+B-iC)(c-id)^{3}}{f}$$

Mathematica [A] time = 6.1604, size = 350, normalized size = 0.88

$$\frac{2\left((c+d\tan(e+fx))^{5/2}\left(28a^2Cd^2+18abd(7Bd-2cC)+b^2\left(63d^2(A-C)-18Bcd+8c^2C\right)\right)+\frac{105}{2}d^3(a-ib)^2(iA+B-i\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e +
f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] (2*((28*a^2*C*d^2 + 18*a*b*d*(-2*c*C + 7*B*d) + b^2*(8*c^2*C - 18*B*c*d + 6
3*(A - C)*d^2))*(c + d*Tan[e + f*x])^(5/2) + 5*b*d*(-4*b*c*C + 9*b*B*d + 4*
a*C*d)*Tan[e + f*x]*(c + d*Tan[e + f*x])^(5/2) + 35*C*d^2*(a + b*Tan[e + f*
x])^2*(c + d*Tan[e + f*x])^(5/2) + (105*(a - I*b)^2*(I*A + B - I*C)*d^3*(-3
*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]] + Sqrt[c +
 d*Tan[e + f*x]]*(4*c - (3*I)*d + d*Tan[e + f*x])))/2 + (105*(a + I*b)^2*((
-I)*A + B + I*C)*d^3*(-3*(c + I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/S
qrt[c + I*d]] + Sqrt[c + d*Tan[e + f*x]]*(4*c + (3*I)*d + d*Tan[e + f*x]))
/2))/(315*d^3*f)
```

Maple [B] time = 0.18, size = 8031, normalized size = 20.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f
*x+e)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f
*x+e)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b \tan \left(e + fx\right)\right)^{2} \left(c + d \tan \left(e + fx\right)\right)^{\frac{3}{2}} \left(A + B \tan \left(e + fx\right) + C \tan^{2}\left(e + fx\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2*(c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan
(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**2*(c + d*tan(e + f*x))**(3/2)*(A + B*tan(e +
 f*x) + C*tan(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \tan \left(fx + e\right)^{2} + B \tan \left(fx + e\right) + A\right)\left(b \tan \left(fx + e\right) + a\right)^{2} \left(d \tan \left(fx + e\right) + c\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f
*x+e)^2),x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^2*(d
*tan(f*x + e) + c)^(3/2), x)
```

3.99 $\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)\right.$

Optimal. Leaf size=273

$$\frac{2(aB+Ab-bC)(c+d\tan(e+fx))^{3/2}}{3f}+\frac{2\sqrt{c+d\tan(e+fx)}(aAd+aBc-aCd+Abc-bBd-bcC)}{f}-\frac{(b+ia)(c-i\ldots}{}$$

```
[Out] -(((I*a + b)*(A - I*B - C)*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]
/Sqrt[c - I*d]])/f) + ((I*a - b)*(A + I*B - C)*(c + I*d)^(3/2)*ArcTanh[Sqrt
[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/f + (2*(A*b*c + a*B*c - b*c*C + a*A*d
- b*B*d - a*C*d)*Sqrt[c + d*Tan[e + f*x]])/f + (2*(A*b + a*B - b*C)*(c + d*
Tan[e + f*x])^(3/2))/(3*f) - (2*(2*b*c*C - 7*b*B*d - 7*a*C*d)*(c + d*Tan[e
+ f*x])^(5/2))/(35*d^2*f) + (2*b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(5/2))
/(7*d*f)
```

Rubi [A] time = 0.879381, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3637, 3630, 3528, 3539, 3537, 63, 208}

$$\frac{2(aB+Ab-bC)(c+d\tan(e+fx))^{3/2}}{3f}+\frac{2\sqrt{c+d\tan(e+fx)}(aAd+aBc-aCd+Abc-bBd-bcC)}{f}-\frac{(b+ia)(c-i\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C
*Tan[e + f*x]^2),x]
```



```
[Out] -(((I*a + b)*(A - I*B - C)*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]
/Sqrt[c - I*d]])/f) + ((I*a - b)*(A + I*B - C)*(c + I*d)^(3/2)*ArcTanh[Sqrt
[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/f + (2*(A*b*c + a*B*c - b*c*C + a*A*d
- b*B*d - a*C*d)*Sqrt[c + d*Tan[e + f*x]])/f + (2*(A*b + a*B - b*C)*(c + d*
Tan[e + f*x])^(3/2))/(3*f) - (2*(2*b*c*C - 7*b*B*d - 7*a*C*d)*(c + d*Tan[e
+ f*x])^(5/2))/(35*d^2*f) + (2*b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(5/2))
/(7*d*f)
```


Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \frac{2bC\tan(e+fx)(c+d\tan(e+\cdots}{7df}$$

$$= -\frac{2(2bcC-7bBd-7aCd)(c+\cdots}{35d^2f}$$

$$= \frac{2(Ab+aB-bC)(c+d\tan(e\cdots}{3f}$$

$$= \frac{2(Abc+aBc-bcC+aAd-\cdots}{\cdots}$$

$$= \frac{2(Abc+aBc-bcC+aAd-\cdots}{\cdots}$$

$$= \frac{2(Abc+aBc-bcC+aAd-\cdots}{\cdots}$$

$$= \frac{2(Abc+aBc-bcC+aAd-\cdots}{\cdots}$$

$$= -\frac{(a-ib)(iA+B-iC)(c-id)\cdots}{f}$$

Mathematica [A] time = 4.51759, size = 260, normalized size = 0.95

$$\frac{\frac{35}{3}d(b+ia)(A-iB-C)\left(\sqrt{c+d\tan(e+fx)}(4c+d\tan(e+fx)-3id)-3(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)\right)+\frac{35}{3}d(b\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*
x] + C*Tan[e + f*x]^2),x]
```

```
[Out] ((2*(-2*b*c*C + 7*b*B*d + 7*a*C*d)*(c + d*Tan[e + f*x])^(5/2))/d + 10*b*C*T
an[e + f*x]*(c + d*Tan[e + f*x])^(5/2) + (35*(I*a + b)*(A - I*B - C)*d*(-3*
(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]] + Sqrt[c +
d*Tan[e + f*x]]*(4*c - (3*I)*d + d*Tan[e + f*x])))/3 + (35*((-I)*a + b)*(A
+ I*B - C)*d*(-3*(c + I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c +
I*d]] + Sqrt[c + d*Tan[e + f*x]]*(4*c + (3*I)*d + d*Tan[e + f*x])))/3)/(35*
d*f)
```

Maple [B] time = 0.15, size = 5149, normalized size = 18.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)
,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b \tan{\left(e + fx \right)}\right) \left(c + d \tan{\left(e + fx \right)}\right)^{\frac{3}{2}} \left(A + B \tan{\left(e + fx \right)} + C \tan^{2}{\left(e + fx \right)}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan(f*
x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))*(c + d*tan(e + f*x))**(3/2)*(A + B*tan(e + f*
x) + C*tan(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \tan\left(fx + e\right)^{2} + B \tan\left(fx + e\right) + A\right)\left(b \tan\left(fx + e\right) + a\right)\left(d \tan\left(fx + e\right) + c\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)*(d*t
an(f*x + e) + c)^(3/2), x)
```

3.100 $\int (c+d\tan(e+fx))^{3/2} \left(A+B\tan(e+fx)+C\tan^2(e+f\right.$ …

Optimal. Leaf size=187

$$\frac{2(d(A-C)+Bc)\sqrt{c+d\tan(e+fx)}}{f}-\frac{(c-id)^{3/2}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}-\frac{(c+id)^{3/2}(B-i(A-C))\tan\ldots}{f}$$

```
[Out] -(((I*A + B - I*C)*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c
- I*d]])/f) - ((B - I*(A - C))*(c + I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f
*x]]/Sqrt[c + I*d]])/f + (2*(B*c + (A - C)*d)*Sqrt[c + d*Tan[e + f*x]])/f +
 (2*B*(c + d*Tan[e + f*x])^(3/2))/(3*f) + (2*C*(c + d*Tan[e + f*x])^(5/2))/
(5*d*f)
```

Rubi [A] time = 0.459659, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3630, 3528, 3539, 3537, 63, 208}

$$\frac{2(d(A-C)+Bc)\sqrt{c+d\tan(e+fx)}}{f}-\frac{(c-id)^{3/2}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}-\frac{(c+id)^{3/2}(B-i(A-C))\tan\ldots}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] -(((I*A + B - I*C)*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c
- I*d]])/f) - ((B - I*(A - C))*(c + I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f
*x]]/Sqrt[c + I*d]])/f + (2*(B*c + (A - C)*d)*Sqrt[c + d*Tan[e + f*x]])/f +
 (2*B*(c + d*Tan[e + f*x])^(3/2))/(3*f) + (2*C*(c + d*Tan[e + f*x])^(5/2))/
(5*d*f)
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int (c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx = \frac{2C(c+d\tan(e+fx))^{5/2}}{5df}+\int (A-C+B\tan(e+f$$

$$= \frac{2B(c+d\tan(e+fx))^{3/2}}{3f}+\frac{2C(c+d\tan(e+fx))^{5/2}}{5df}$$

$$= \frac{2(Bc+(A-C)d)\sqrt{c+d\tan(e+fx)}}{f}+\frac{2B(c+d\tan}{3}$$

$$= \frac{2(Bc+(A-C)d)\sqrt{c+d\tan(e+fx)}}{f}+\frac{2B(c+d\tan}{3}$$

$$= \frac{2(Bc+(A-C)d)\sqrt{c+d\tan(e+fx)}}{f}+\frac{2B(c+d\tan}{3}$$

$$= \frac{2(Bc+(A-C)d)\sqrt{c+d\tan(e+fx)}}{f}+\frac{2B(c+d\tan}{3}$$

$$= -\frac{(iA+B-iC)(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}-$$

Mathematica [A] time = 1.23434, size = 202, normalized size = 1.08

$$\frac{5(iA+B-iC)\left(\sqrt{c+d\tan(e+fx)}(4c+d\tan(e+fx)-3id)-3(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)\right)+5(-iA+B+iC)\left(\right.}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2
),x]
```

```
[Out] ((6*C*(c + d*Tan[e + f*x])^(5/2))/d + 5*(I*A + B - I*C)*(-3*(c - I*d)^(3/2)
*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]] + Sqrt[c + d*Tan[e + f*x]]
*(4*c - (3*I)*d + d*Tan[e + f*x])) + 5*((-I)*A + B + I*C)*(-3*(c + I*d)^(3/
2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]] + Sqrt[c + d*Tan[e + f*x
]]*(4*c + (3*I)*d + d*Tan[e + f*x])))/(15*f)
```

Maple [B] time = 0.113, size = 2517, normalized size = 13.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x)
```

```
[Out] 2/3*B*(c+d*tan(f*x+e))^(3/2)/f-1/4/d/f*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*C*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)*(c^2+d^2)^(1/2)*c+1/4/d/f*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)
*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*C*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*(c^2+d^2)^(1/2)*c-1/4/d/f*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*(c^2+d^2)^(1/2)*c+1/4/d/f*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^
2+d^2)^(1/2)*c+1/4*d/f*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+1/4*d/f
*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^
2+d^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-1/4*d/f*ln((c+d*tan(f*x+e))^(
1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*C*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)+d^2/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2
+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1
/2))*B-d^2/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)
+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*B+1/4/f*ln(d
*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2
)^(1/2))*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)-1/f/(2*(c^2+d^2)^(
1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1
/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*B*c^2-1/4/f*ln((c+d*tan(f*x+e))^(1/2)*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*B*(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)+1/2/f*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*B*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)*c+1/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1
/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*B*c^2-1/2
/f*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(
c^2+d^2)^(1/2))*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c+2/5*C*(c+d*tan(f*x+e))^(5
/2)/f/d-1/4*d/f*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)+(c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+2/f*B*c*(c+d*t
an(f*x+e))^(1/2)+2*d/f*A*(c+d*tan(f*x+e))^(1/2)-2*d/f*C*(c+d*tan(f*x+e))^(1
/2)-d/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(
c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*A*(c^2+d^2)^(1/2)
+d/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2
+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*C*(c^2+d^2)^(1/2)+2*
d/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(
c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*C*c-2*d/f/(2*(c^2+d^2
)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))
^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*A*c+d/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2
)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d
^2)^(1/2)-2*c)^(1/2))*A*(c^2+d^2)^(1/2)-d/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*a
rctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)
^(1/2)-2*c)^(1/2))*C*(c^2+d^2)^(1/2)+2*d/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*ar
ctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^
(1/2)-2*c)^(1/2))*A*c+1/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2
)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))
*B*(c^2+d^2)^(1/2)*c-1/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f
*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*
B*(c^2+d^2)^(1/2)*c-2*d/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(
f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))
```

```
*C*c+1/4/d/f*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)+(c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2-1/4/d/f*ln(d*
tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)
^(1/2))*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2-1/4/d/f*ln((c+d*tan(f*x+e))^(1/
2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*A*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*c^2+1/4/d/f*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*C*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\tan\left(fx+e\right)^2+B\tan\left(fx+e\right)+A\right)\left(d\tan\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(d*tan(f*x + e) + c)^(3/2
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c+d\tan\left(e+fx\right)\right)^{\frac{3}{2}}\left(A+B\tan\left(e+fx\right)+C\tan^2\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2),x)
```

```
[Out] Integral((c + d*tan(e + f*x))**(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)**
2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\tan\left(fx+e\right)^2+B\tan\left(fx+e\right)+A\right)\left(d\tan\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(d*tan(f*x + e) + c)^(3/2
), x)
```

3.101 $\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=271

$$-\frac{2(bc-ad)^{3/2}\left(Ab^2-a(bB-aC)\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{b^{5/2}f\left(a^2+b^2\right)}-\frac{(c-id)^{3/2}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(a-ib)}-\frac{(c+id)^{3}}{}$$

```
[Out] -(((I*A + B - I*C)*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c
- I*d]])/((a - I*b)*f)) - ((A + I*B - C)*(c + I*d)^(3/2)*ArcTanh[Sqrt[c + d
*Tan[e + f*x]]/Sqrt[c + I*d]])/((I*a - b)*f) - (2*(A*b^2 - a*(b*B - a*C))*(
b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]
])/(b^(5/2)*(a^2 + b^2)*f) + (2*(b*c*C + b*B*d - a*C*d)*Sqrt[c + d*Tan[e +
f*x]])/(b^2*f) + (2*C*(c + d*Tan[e + f*x])^(3/2))/(3*b*f)
```

Rubi [A] time = 1.81409, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.149, Rules used = {3647, 3653, 3539, 3537, 63, 208, 3634}

$$-\frac{2(bc-ad)^{3/2}\left(Ab^2-a(bB-aC)\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{b^{5/2}f\left(a^2+b^2\right)}-\frac{(c-id)^{3/2}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(a-ib)}-\frac{(c+id)^{3}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
 + b*Tan[e + f*x]),x]
```

```
[Out] -(((I*A + B - I*C)*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c
- I*d]])/((a - I*b)*f)) - ((A + I*B - C)*(c + I*d)^(3/2)*ArcTanh[Sqrt[c + d
*Tan[e + f*x]]/Sqrt[c + I*d]])/((I*a - b)*f) - (2*(A*b^2 - a*(b*B - a*C))*(
b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]
])/(b^(5/2)*(a^2 + b^2)*f) + (2*(b*c*C + b*B*d - a*C*d)*Sqrt[c + d*Tan[e +
f*x]])/(b^2*f) + (2*C*(c + d*Tan[e + f*x])^(3/2))/(3*b*f)
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3653

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[e + f*x])^n*(1 + Tan[e
```

```
+ f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3634

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}dx = \frac{2C(c+d\tan(e+fx))^{3/2}}{3bf} + \frac{2\int \frac{\sqrt{c+d\tan(e+fx)}\left(\frac{3}{2}(Abc-\right.}{}}{}$$

$$= \frac{2(bcC+bBd-aCd)\sqrt{c+d\tan(e+fx)}}{b^2f} + \frac{2C(c+d}{}$$

$$= \frac{2(bcC+bBd-aCd)\sqrt{c+d\tan(e+fx)}}{b^2f} + \frac{2C(c+d}{}$$

$$= \frac{2(bcC+bBd-aCd)\sqrt{c+d\tan(e+fx)}}{b^2f} + \frac{2C(c+d}{}$$

$$= \frac{2(bcC+bBd-aCd)\sqrt{c+d\tan(e+fx)}}{b^2f} + \frac{2C(c+d}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d}}{\sqrt{bc}}\right.}{b^{5/2}\left(a^2+b^2\right)f}$$

$$= -\frac{(iA+B-iC)(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)f} -$$

Mathematica [A] time = 2.42084, size = 266, normalized size = 0.98

$$\frac{-\frac{6(bc-ad)^{3/2}\left(a(aC-bB)+Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{b^{3/2}\left(a^2+b^2\right)} + \frac{3ib\left((a-ib)(c+id)^{3/2}(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)-(a+ib)(c-id)^{3/2}(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan}}{\sqrt{c-}}\right.\right.}{a^2+b^2}}{3bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/(a + b*Tan[e + f*x]),x]
```

```
[Out] (((3*I)*b*(-((a + I*b)*(A - I*B - C)*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan
[e + f*x]]/Sqrt[c - I*d]]) + (a - I*b)*(A + I*B - C)*(c + I*d)^(3/2)*ArcTan
h[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]]))/(a^2 + b^2) - (6*(A*b^2 + a*(-(
b*B) + a*C))*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/S
qrt[b*c - a*d]])/(b^(3/2)*(a^2 + b^2)) + (6*(b*c*C + b*B*d - a*C*d)*Sqrt[c
+ d*Tan[e + f*x]])/b + 2*C*(c + d*Tan[e + f*x])^(3/2))/(3*b*f)
```

Maple [B] time = 0.192, size = 6055, normalized size = 22.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\tan\left(fx+e\right)^{2}+B\tan\left(fx+e\right)+A\right)\left(d\tan\left(fx+e\right)+c\right)^{\frac{3}{2}}}{b\tan\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e)),x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(d*tan(f*x + e) + c)^(3/2
)/(b*tan(f*x + e) + a), x)
```

3.102 $\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=372

$$-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}+\frac{d\left(3a^2C-abB+Ab^2+2b^2C\right)\sqrt{c+d\tan(e+fx)}}{b^2f\left(a^2+b^2\right)}+\frac{\sqrt{bc-ad}\left(a^2b^2(d(A-\right.}{}$$

```
[Out] -(((I*A + B - I*C)*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c
- I*d]])/((a - I*b)^2*f)) - ((B - I*(A - C))*(c + I*d)^(3/2)*ArcTanh[Sqrt[c
 + d*Tan[e + f*x]]/Sqrt[c + I*d]])/((a + I*b)^2*f) + (Sqrt[b*c - a*d]*(a^3*
b*B*d - 3*a^4*C*d - b^4*(2*B*c + 3*A*d) - a*b^3*(4*A*c - 4*c*C - 5*B*d) + a
^2*b^2*(2*B*c + (A - 7*C)*d))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sq
rt[b*c - a*d]])/(b^(5/2)*(a^2 + b^2)^2*f) + ((A*b^2 - a*b*B + 3*a^2*C + 2*b
^2*C)*d*Sqrt[c + d*Tan[e + f*x]])/(b^2*(a^2 + b^2)*f) - ((A*b^2 - a*(b*B -
a*C))*(c + d*Tan[e + f*x])^(3/2))/(b*(a^2 + b^2)*f*(a + b*Tan[e + f*x]))
```

Rubi [A] time = 2.54731, antiderivative size = 372, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.17, Rules used = {3645, 3647, 3653, 3539, 3537, 63, 208, 3634}

$$-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}+\frac{d\left(3a^2C-abB+Ab^2+2b^2C\right)\sqrt{c+d\tan(e+fx)}}{b^2f\left(a^2+b^2\right)}+\frac{\sqrt{bc-ad}\left(a^2b^2(d(A-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
 + b*Tan[e + f*x])^2,x]
```


```
[Out] -(((I*A + B - I*C)*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c
- I*d]])/((a - I*b)^2*f)) - ((B - I*(A - C))*(c + I*d)^(3/2)*ArcTanh[Sqrt[c
 + d*Tan[e + f*x]]/Sqrt[c + I*d]])/((a + I*b)^2*f) + (Sqrt[b*c - a*d]*(a^3*
b*B*d - 3*a^4*C*d - b^4*(2*B*c + 3*A*d) - a*b^3*(4*A*c - 4*c*C - 5*B*d) + a
^2*b^2*(2*B*c + (A - 7*C)*d))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sq
rt[b*c - a*d]])/(b^(5/2)*(a^2 + b^2)^2*f) + ((A*b^2 - a*b*B + 3*a^2*C + 2*b
^2*C)*d*Sqrt[c + d*Tan[e + f*x]])/(b^2*(a^2 + b^2)*f) - ((A*b^2 - a*(b*B -
a*C))*(c + d*Tan[e + f*x])^(3/2))/(b*(a^2 + b^2)*f*(a + b*Tan[e + f*x]))
```


Rule 3645

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*d^2 + c*(c*C - B*d))*(a + b*Tan[e
+ f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3653

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[e + f*x])^n*(1 + Tan[e
+ f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3634

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}dx = -\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{b\left(a^2+b^2\right)f(a+b\tan(e+fx))} + \int \frac{\sqrt{c}\cdots}{\cdots}$$

$$= \frac{\left(Ab^2-abB+3a^2C+2b^2C\right)d\sqrt{c+d\tan(e+fx)}}{b^2\left(a^2+b^2\right)f} - \cdots$$

$$= \frac{\left(Ab^2-abB+3a^2C+2b^2C\right)d\sqrt{c+d\tan(e+fx)}}{b^2\left(a^2+b^2\right)f} - \cdots$$

$$= \frac{\left(Ab^2-abB+3a^2C+2b^2C\right)d\sqrt{c+d\tan(e+fx)}}{b^2\left(a^2+b^2\right)f} - \cdots$$

$$= \frac{\left(Ab^2-abB+3a^2C+2b^2C\right)d\sqrt{c+d\tan(e+fx)}}{b^2\left(a^2+b^2\right)f} - \cdots$$

$$= \frac{\sqrt{bc-ad}\left(a^3bBd-3a^4Cd-b^4(2Bc+3Ad)-ab^3(4\cdots\right.}{\cdots}$$

$$= -\frac{(iA+B-iC)(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)^2 f} - \cdots$$

Mathematica [B] time = 6.22458, size = 1732, normalized size = 4.66

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (-4*a^2*A*b^3*c*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/
Sqrt[b*c - a*d]] + 2*a^3*b^2*B*c*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c +
d*Tan[e + f*x]])/Sqrt[b*c - a*d]] - 2*a*b^4*B*c*Sqrt[b*c - a*d]*ArcTanh[(Sq
rt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]] + 4*a^2*b^3*c*C*Sqrt[b*c -
 a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]] + a^3*A*b
^2*d*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c -
a*d]] - 3*a*A*b^4*d*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x
]])/Sqrt[b*c - a*d]] + a^4*b*B*d*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c +
d*Tan[e + f*x]])/Sqrt[b*c - a*d]] + 5*a^2*b^3*B*d*Sqrt[b*c - a*d]*ArcTanh[(
Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]] - 3*a^5*C*d*Sqrt[b*c - a
*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]] - 7*a^3*b^2
*C*d*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c -
a*d]] - 4*a*A*b^4*c*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x
]])/Sqrt[b*c - a*d]]*Tan[e + f*x] + 2*a^2*b^3*B*c*Sqrt[b*c - a*d]*ArcTanh[(
Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]]*Tan[e + f*x] - 2*b^5*B*c
*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]
]*Tan[e + f*x] + 4*a*b^4*c*C*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*Ta
n[e + f*x]])/Sqrt[b*c - a*d]]*Tan[e + f*x] + a^2*A*b^3*d*Sqrt[b*c - a*d]*Ar
cTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]]*Tan[e + f*x] - 3*
A*b^5*d*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c
```

```
- a*d]]*Tan[e + f*x] + a^3*b^2*B*d*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*Tan[e + f*x]])/Sqrt[b*c - a*d]]*Tan[e + f*x] + 5*a*b^4*B*d*Sqrt[b*c -
a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]]*Tan[e + f*
x] - 3*a^4*b*C*d*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])
/Sqrt[b*c - a*d]]*Tan[e + f*x] - 7*a^2*b^3*C*d*Sqrt[b*c - a*d]*ArcTanh[(Sqr
t[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]]*Tan[e + f*x] + b^(5/2)*((-I
)*a + b)^2*(I*A + B - I*C)*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]
/Sqrt[c - I*d]]*(a + b*Tan[e + f*x]) + b^(5/2)*(I*a + b)^2*((-I)*A + B + I*
C)*(c + I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]]*(a + b*T
an[e + f*x]) - a^2*A*b^(7/2)*c*Sqrt[c + d*Tan[e + f*x]] - A*b^(11/2)*c*Sqrt
[c + d*Tan[e + f*x]] + a^3*b^(5/2)*B*c*Sqrt[c + d*Tan[e + f*x]] + a*b^(9/2)
*B*c*Sqrt[c + d*Tan[e + f*x]] - a^4*b^(3/2)*c*C*Sqrt[c + d*Tan[e + f*x]] -
a^2*b^(7/2)*c*C*Sqrt[c + d*Tan[e + f*x]] + a^3*A*b^(5/2)*d*Sqrt[c + d*Tan[e
 + f*x]] + a*A*b^(9/2)*d*Sqrt[c + d*Tan[e + f*x]] - a^4*b^(3/2)*B*d*Sqrt[c
+ d*Tan[e + f*x]] - a^2*b^(7/2)*B*d*Sqrt[c + d*Tan[e + f*x]] + 3*a^5*Sqrt[b
]*C*d*Sqrt[c + d*Tan[e + f*x]] + 5*a^3*b^(5/2)*C*d*Sqrt[c + d*Tan[e + f*x]]
 + 2*a*b^(9/2)*C*d*Sqrt[c + d*Tan[e + f*x]] + 2*a^4*b^(3/2)*C*d*Tan[e + f*x
]*Sqrt[c + d*Tan[e + f*x]] + 4*a^2*b^(7/2)*C*d*Tan[e + f*x]*Sqrt[c + d*Tan[
e + f*x]] + 2*b^(11/2)*C*d*Tan[e + f*x]*Sqrt[c + d*Tan[e + f*x]])/(b^(5/2)*
(a^2 + b^2)^2*f*(a + b*Tan[e + f*x]))
```

Maple [B] time = 0.235, size = 9865, normalized size = 26.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^2,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\tan\left(fx+e\right)^{2}+B\tan\left(fx+e\right)+A\right)\left(d\tan\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(b\tan\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(d*tan(f*x + e) + c)^(3/2
)/(b*tan(f*x + e) + a)^2, x)
```

3.103 $\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx$

Optimal. Leaf size=532

$$-\frac{\left(-2a^3b^3\left(12cd(A-C)+B\left(4c^2-9d^2\right)\right)+a^2b^4\left(24Ac^2-26Ad^2-48Bcd-24c^2C+35Cd^2\right)+a^4b^2d(3d(A+2C)+4\cdots}{4b^{5/2}f\left(\cdots\right.}$$

```
[Out] -(((A - I*B - C)*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c -
I*d]])/((I*a + b)^3*f)) + ((A + I*B - C)*(c + I*d)^(3/2)*ArcTanh[Sqrt[c + d
*Tan[e + f*x]]/Sqrt[c + I*d]])/((I*a - b)^3*f) - ((a^5*b*B*d^2 + 3*a^6*C*d^
2 + a^4*b^2*d*(4*B*c + 3*(A + 2*C)*d) - b^6*(8*A*c^2 - 8*c^2*C - 12*B*c*d -
 3*A*d^2) + a^2*b^4*(24*A*c^2 - 24*c^2*C - 48*B*c*d - 26*A*d^2 + 35*C*d^2)
- 2*a^3*b^3*(12*c*(A - C)*d + B*(4*c^2 - 9*d^2)) + a*b^5*(40*c*(A - C)*d +
3*B*(8*c^2 - 5*d^2)))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c -
 a*d]])/(4*b^(5/2)*(a^2 + b^2)^3*Sqrt[b*c - a*d]*f) - ((a^3*b*B*d + 3*a^4*C
*d + b^4*(4*B*c + 3*A*d) + a*b^3*(8*A*c - 8*c*C - 7*B*d) - a^2*b^2*(4*B*c +
 5*A*d - 11*C*d))*Sqrt[c + d*Tan[e + f*x]])/(4*b^2*(a^2 + b^2)^2*f*(a + b*T
an[e + f*x])) - ((A*b^2 - a*(b*B - a*C))*(c + d*Tan[e + f*x])^(3/2))/(2*b*(
a^2 + b^2)*f*(a + b*Tan[e + f*x])^2)
```

Rubi [A] time = 4.08696, antiderivative size = 532, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.149, Rules used = {3645, 3653, 3539, 3537, 63, 208, 3634}

$$-\frac{\left(-2a^3b^3\left(12cd(A-C)+B\left(4c^2-9d^2\right)\right)+a^2b^4\left(24Ac^2-26Ad^2-48Bcd-24c^2C+35Cd^2\right)+a^4b^2d(3d(A+2C)+4\cdots}{4b^{5/2}f\left(\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
 + b*Tan[e + f*x])^3,x]
```


```
[Out] -(((A - I*B - C)*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c -
I*d]])/((I*a + b)^3*f)) + ((A + I*B - C)*(c + I*d)^(3/2)*ArcTanh[Sqrt[c + d
*Tan[e + f*x]]/Sqrt[c + I*d]])/((I*a - b)^3*f) - ((a^5*b*B*d^2 + 3*a^6*C*d^
2 + a^4*b^2*d*(4*B*c + 3*(A + 2*C)*d) - b^6*(8*A*c^2 - 8*c^2*C - 12*B*c*d -
 3*A*d^2) + a^2*b^4*(24*A*c^2 - 24*c^2*C - 48*B*c*d - 26*A*d^2 + 35*C*d^2)
- 2*a^3*b^3*(12*c*(A - C)*d + B*(4*c^2 - 9*d^2)) + a*b^5*(40*c*(A - C)*d +
3*B*(8*c^2 - 5*d^2)))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c -
 a*d]])/(4*b^(5/2)*(a^2 + b^2)^3*Sqrt[b*c - a*d]*f) - ((a^3*b*B*d + 3*a^4*C
*d + b^4*(4*B*c + 3*A*d) + a*b^3*(8*A*c - 8*c*C - 7*B*d) - a^2*b^2*(4*B*c +
 5*A*d - 11*C*d))*Sqrt[c + d*Tan[e + f*x]])/(4*b^2*(a^2 + b^2)^2*f*(a + b*T
an[e + f*x])) - ((A*b^2 - a*(b*B - a*C))*(c + d*Tan[e + f*x])^(3/2))/(2*b*(
a^2 + b^2)*f*(a + b*Tan[e + f*x])^2)
```


Rule 3645

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*d^2 + c*(c*C - B*d))*(a + b*Tan[e
+ f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
```

```
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3653

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[e + f*x])^n*(1 + Tan[e
+ f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3634

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}dx = -\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2} + \int$$

$$= -\frac{\left(a^3bBd+3a^4Cd+b^4(4Bc+3Ad)+ab^3(8Ac-\right.}{4b^2\left(a^2+b\right.}$$

$$= -\frac{\left(a^3bBd+3a^4Cd+b^4(4Bc+3Ad)+ab^3(8Ac-\right.}{4b^2\left(a^2+b\right.}$$

$$= -\frac{\left(a^3bBd+3a^4Cd+b^4(4Bc+3Ad)+ab^3(8Ac-\right.}{4b^2\left(a^2+b\right.}$$

$$= -\frac{\left(a^3bBd+3a^4Cd+b^4(4Bc+3Ad)+ab^3(8Ac-\right.}{4b^2\left(a^2+b\right.}$$

$$= -\frac{\left(a^5bBd^2+3a^6Cd^2+a^4b^2d(4Bc+3(A+2C)d)\right.}{}$$

$$= -\frac{(A-iB-C)(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(ia+b)^3f}$$

Mathematica [B] time = 6.54976, size = 7678, normalized size = 14.43

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/(a + b*Tan[e + f*x])^3,x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.251, size = 14441, normalized size = 27.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\tan\left(fx+e\right)^2+B\tan\left(fx+e\right)+A\right)\left(d\tan\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(b\tan\left(fx+e\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(d*tan(f*x + e) + c)^(3/2
)/(b*tan(f*x + e) + a)^3, x)
```

3.104 $\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+f\right.$

Optimal. Leaf size=503

$$\frac{2(c+d\tan(e+fx))^{7/2}\left(36a^2Cd^2-22abd(2cC-9Bd)+b^2\left(99d^2(A-C)-22Bcd+8c^2C\right)\right)}{693d^3f}-\frac{2\sqrt{c+d\tan(e+fx)}\left(a\right.}{}$$

```
[Out] -(((a - I*b)^2*(I*A + B - I*C)*(c - I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f
*x]]/Sqrt[c - I*d]])/f) + ((a + I*b)^2*(I*A - B - I*C)*(c + I*d)^(5/2)*ArcT
anh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/f - (2*(2*a*b*(c^2*C + 2*B*c*d
 - C*d^2 - A*(c^2 - d^2)) - a^2*(2*c*(A - C)*d + B*(c^2 - d^2)) + b^2*(2*c*
(A - C)*d + B*(c^2 - d^2)))*Sqrt[c + d*Tan[e + f*x]])/f + (2*(2*a*b*(A*c -
c*C - B*d) + a^2*(B*c + (A - C)*d) - b^2*(B*c + (A - C)*d))*(c + d*Tan[e +
f*x])^(3/2))/(3*f) + (2*(a^2*B - b^2*B + 2*a*b*(A - C))*(c + d*Tan[e + f*x]
)^(5/2))/(5*f) + (2*(36*a^2*C*d^2 - 22*a*b*d*(2*c*C - 9*B*d) + b^2*(8*c^2*C
 - 22*B*c*d + 99*(A - C)*d^2))*(c + d*Tan[e + f*x])^(7/2))/(693*d^3*f) - (2
*b*(4*b*c*C - 11*b*B*d - 4*a*C*d)*Tan[e + f*x]*(c + d*Tan[e + f*x])^(7/2))/
(99*d^2*f) + (2*C*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(7/2))/(11*d*
f)
```

Rubi [A] time = 2.31229, antiderivative size = 503, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.17, Rules used = {3647, 3637, 3630, 3528, 3539, 3537, 63, 208}

$$\frac{2(c+d\tan(e+fx))^{7/2}\left(36a^2Cd^2-22abd(2cC-9Bd)+b^2\left(99d^2(A-C)-22Bcd+8c^2C\right)\right)}{693d^3f}-\frac{2\sqrt{c+d\tan(e+fx)}\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] +
 C*Tan[e + f*x]^2),x]
```


```
[Out] -(((a - I*b)^2*(I*A + B - I*C)*(c - I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f
*x]]/Sqrt[c - I*d]])/f) + ((a + I*b)^2*(I*A - B - I*C)*(c + I*d)^(5/2)*ArcT
anh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/f - (2*(2*a*b*(c^2*C + 2*B*c*d
 - C*d^2 - A*(c^2 - d^2)) - a^2*(2*c*(A - C)*d + B*(c^2 - d^2)) + b^2*(2*c*
(A - C)*d + B*(c^2 - d^2)))*Sqrt[c + d*Tan[e + f*x]])/f + (2*(2*a*b*(A*c -
c*C - B*d) + a^2*(B*c + (A - C)*d) - b^2*(B*c + (A - C)*d))*(c + d*Tan[e +
f*x])^(3/2))/(3*f) + (2*(a^2*B - b^2*B + 2*a*b*(A - C))*(c + d*Tan[e + f*x]
)^(5/2))/(5*f) + (2*(36*a^2*C*d^2 - 22*a*b*d*(2*c*C - 9*B*d) + b^2*(8*c^2*C
 - 22*B*c*d + 99*(A - C)*d^2))*(c + d*Tan[e + f*x])^(7/2))/(693*d^3*f) - (2
*b*(4*b*c*C - 11*b*B*d - 4*a*C*d)*Tan[e + f*x]*(c + d*Tan[e + f*x])^(7/2))/
(99*d^2*f) + (2*C*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(7/2))/(11*d*
f)
```


Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
```

```
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx = \frac{2C(a+b\tan(e+fx))^2(c+d\ldots}{11df}$$

$$= -\frac{2b(4bcC-11bBd-4aCd)\ldots}{\ldots}$$

$$= \frac{2\left(36a^2Cd^2-22abd(2cC-9\ldots\right.}{\ldots}$$

$$= \frac{2\left(a^2B-b^2B+2ab(A-C)\right)\ldots}{5f}$$

$$= \frac{2\left(2ab(Ac-cC-Bd)+a^2(B\ldots\right.}{\ldots}$$

$$= -\frac{2\left(2ab\left(c^2C+2Bcd-Cd^2\ldots\right.\right.}{\ldots}$$

$$= -\frac{2\left(2ab\left(c^2C+2Bcd-Cd^2\ldots\right.\right.}{\ldots}$$

$$= -\frac{2\left(2ab\left(c^2C+2Bcd-Cd^2\ldots\right.\right.}{\ldots}$$

$$= -\frac{2\left(2ab\left(c^2C+2Bcd-Cd^2\ldots\right.\right.}{\ldots}$$

$$= -\frac{(a-ib)^2(iA+B-iC)(c-i\ldots}{\ldots}$$

Mathematica [A] time = 6.43647, size = 564, normalized size = 1.12

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{7/2}}{11df} + \frac{2\left(\frac{b\tan(e+fx)(4aCd+11bBd-4bcC)(c+d\tan(e+fx))^{7/2}}{9df} - \frac{2\left(\frac{(c+d\tan(e+fx))^{7/2}\left(-36a^2Cd^2+22\ldots\right.}{\ldots}\ldots\right)}{\ldots}\right)}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e +
f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] (2*C*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(7/2))/(11*d*f) + (2*((b*(
-4*b*c*C + 11*b*B*d + 4*a*C*d)*Tan[e + f*x]*(c + d*Tan[e + f*x])^(7/2))/(9*
d*f) - (2*(((-36*a^2*C*d^2 + 22*a*b*d*(2*c*C - 9*B*d) - b^2*(8*c^2*C - 22*B
*c*d + 99*(A - C)*d^2))*(c + d*Tan[e + f*x])^(7/2))/(14*d*f) + ((I/2)*(((99
*I)/4)*(a^2*B - b^2*B + 2*a*b*(A - C))*d^2 + (99*(2*a*b*B - a^2*(A - C) + b
```

```
^2*(A - C))*d^2)/4)*((2*(c + d*Tan[e + f*x])^(5/2))/5 + (c - I*d)*((2*(c +
d*Tan[e + f*x])^(3/2))/3 + (c - I*d)*((2*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d
*Tan[e + f*x]]/Sqrt[c - I*d]])/(-c + I*d) + 2*Sqrt[c + d*Tan[e + f*x]]))))/
f - ((I/2)*(((-99*I)/4)*(a^2*B - b^2*B + 2*a*b*(A - C))*d^2 + (99*(2*a*b*B
- a^2*(A - C) + b^2*(A - C))*d^2)/4)*((2*(c + d*Tan[e + f*x])^(5/2))/5 + (c
 + I*d)*((2*(c + d*Tan[e + f*x])^(3/2))/3 + (c + I*d)*((2*(c + I*d)^(3/2)*A
rcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/(-c - I*d) + 2*Sqrt[c + d*T
an[e + f*x]]))))/f))/(9*d)))/(11*d)
```

Maple [B] time = 0.192, size = 11478, normalized size = 22.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f
*x+e)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f
*x+e)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2*(c+d*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan
(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\tan\left(fx+e\right)^{2}+B\tan\left(fx+e\right)+A\right)\left(b\tan\left(fx+e\right)+a\right)^{2}\left(d\tan\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f
*x+e)^2),x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^2*(d
*tan(f*x + e) + c)^(5/2), x)
```

3.105 $\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)\right.$

Optimal. Leaf size=353

$$\frac{2\sqrt{c+d\tan(e+fx)}\left(A\left(2acd+b\left(c^2-d^2\right)\right)+a\left(Bc^2-Bd^2-2cCd\right)-b\left(2Bcd+c^2C-Cd^2\right)\right)}{f}+\frac{2(aB+Ab-bC)(c+d\tan}{5f}$$

```
[Out] -(((I*a + b)*(A - I*B - C)*(c - I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]
/Sqrt[c - I*d]])/f) + ((I*a - b)*(A + I*B - C)*(c + I*d)^(5/2)*ArcTanh[Sqrt
[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/f + (2*(a*(B*c^2 - 2*c*C*d - B*d^2) -
b*(c^2*C + 2*B*c*d - C*d^2) + A*(2*a*c*d + b*(c^2 - d^2)))*Sqrt[c + d*Tan[e
 + f*x]])/f + (2*(A*b*c + a*B*c - b*c*C + a*A*d - b*B*d - a*C*d)*(c + d*Tan
[e + f*x])^(3/2))/(3*f) + (2*(A*b + a*B - b*C)*(c + d*Tan[e + f*x])^(5/2))/
(5*f) - (2*(2*b*c*C - 9*b*B*d - 9*a*C*d)*(c + d*Tan[e + f*x])^(7/2))/(63*d^
2*f) + (2*b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(7/2))/(9*d*f)
```

Rubi [A] time = 1.21232, antiderivative size = 351, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3637, 3630, 3528, 3539, 3537, 63, 208}

$$\frac{2\sqrt{c+d\tan(e+fx)}\left(2aAcd+aB\left(c^2-d^2\right)-2acCd+Ab\left(c^2-d^2\right)-b\left(2Bcd+c^2C-Cd^2\right)\right)}{f}+\frac{2(aB+Ab-bC)(c+d\tan}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C
*Tan[e + f*x]^2),x]
```

```
[Out] -(((I*a + b)*(A - I*B - C)*(c - I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]
/Sqrt[c - I*d]])/f) + ((I*a - b)*(A + I*B - C)*(c + I*d)^(5/2)*ArcTanh[Sqrt
[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/f + (2*(2*a*A*c*d - 2*a*c*C*d + A*b*(c
^2 - d^2) + a*B*(c^2 - d^2) - b*(c^2*C + 2*B*c*d - C*d^2))*Sqrt[c + d*Tan[e
 + f*x]])/f + (2*(A*b*c + a*B*c - b*c*C + a*A*d - b*B*d - a*C*d)*(c + d*Tan
[e + f*x])^(3/2))/(3*f) + (2*(A*b + a*B - b*C)*(c + d*Tan[e + f*x])^(5/2))/
(5*f) - (2*(2*b*c*C - 9*b*B*d - 9*a*C*d)*(c + d*Tan[e + f*x])^(7/2))/(63*d^
2*f) + (2*b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(7/2))/(9*d*f)
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \frac{2bC\tan(e+fx)(c+d\tan(e+f}{9df}$$

$$= -\frac{2(2bcC-9bBd-9aCd)(c+d\,\mathrm{t}}{63d^2f}$$

$$= \frac{2(Ab+aB-bC)(c+d\tan(e+f}{5f}$$

$$= \frac{2(Abc+aBc-bcC+aAd-bBd}{3f}$$

$$= \frac{2\left(2aAcd-2acCd+Ab\left(c^2-d^2\right.\right.}{}$$

$$= \frac{2\left(2aAcd-2acCd+Ab\left(c^2-d^2\right.\right.}{}$$

$$= \frac{2\left(2aAcd-2acCd+Ab\left(c^2-d^2\right.\right.}{}$$

$$= \frac{2\left(2aAcd-2acCd+Ab\left(c^2-d^2\right.\right.}{}$$

$$= -\frac{(ia+b)(A-iB-C)(c-id)^{5/2}\,\mathrm{t}}{f}$$

Mathematica [A] time = 4.99884, size = 324, normalized size = 0.92

$$\frac{\frac{63}{2}id(a-ib)(A-iB-C)\left(\frac{2}{5}(c+d\tan(e+fx))^{5/2}+\frac{2}{3}(c-id)\left(\sqrt{c+d\tan(e+fx)}(4c+d\tan(e+fx)-3id)-3(c-id)^{3/2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*
x] + C*Tan[e + f*x]^2),x]
```

```
[Out] ((2*(-2*b*c*C + 9*b*B*d + 9*a*C*d)*(c + d*Tan[e + f*x])^(7/2))/d + 14*b*C*T
an[e + f*x]*(c + d*Tan[e + f*x])^(7/2) + ((63*I)/2)*(a - I*b)*(A - I*B - C)
*d*((2*(c + d*Tan[e + f*x])^(5/2))/5 + (2*(c - I*d)*(-3*(c - I*d)^(3/2)*Arc
Tanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]] + Sqrt[c + d*Tan[e + f*x]]*(4*
c - (3*I)*d + d*Tan[e + f*x])))/3) - ((63*I)/2)*(a + I*b)*(A + I*B - C)*d*(
(2*(c + d*Tan[e + f*x])^(5/2))/5 + (2*(c + I*d)*(-3*(c + I*d)^(3/2)*ArcTanh
[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]] + Sqrt[c + d*Tan[e + f*x]]*(4*c +
(3*I)*d + d*Tan[e + f*x])))/3))/(63*d*f)
```

Maple [B] time = 0.164, size = 7402, normalized size = 21.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)
,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b \tan \left(e + fx\right)\right) \left(c + d \tan \left(e + fx\right)\right)^{\frac{5}{2}} \left(A + B \tan \left(e + fx\right) + C \tan^{2} \left(e + fx\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*
x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))*(c + d*tan(e + f*x))**(5/2)*(A + B*tan(e + f*
x) + C*tan(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \tan \left(fx + e\right)^{2} + B \tan \left(fx + e\right) + A\right)\left(b \tan \left(fx + e\right) + a\right)\left(d \tan \left(fx + e\right) + c\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)*(d*t
an(f*x + e) + c)^(5/2), x)
```

3.106 $\int (c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+f\right.$

Optimal. Leaf size=229

$$\frac{2\left(2cd(A-C)+B\left(c^2-d^2\right)\right)\sqrt{c+d\tan(e+fx)}}{f}+\frac{2(d(A-C)+Bc)(c+d\tan(e+fx))^{3/2}}{3f}-\frac{(c-id)^{5/2}(iA+B-iC)}{f}$$

```
[Out] -(((I*A + B - I*C)*(c - I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c
- I*d]])/f) - ((B - I*(A - C))*(c + I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f
*x]]/Sqrt[c + I*d]])/f + (2*(2*c*(A - C)*d + B*(c^2 - d^2))*Sqrt[c + d*Tan[
e + f*x]])/f + (2*(B*c + (A - C)*d)*(c + d*Tan[e + f*x])^(3/2))/(3*f) + (2*
B*(c + d*Tan[e + f*x])^(5/2))/(5*f) + (2*C*(c + d*Tan[e + f*x])^(7/2))/(7*d
*f)
```

Rubi [A] time = 0.628563, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3630, 3528, 3539, 3537, 63, 208}

$$\frac{2\left(2cd(A-C)+B\left(c^2-d^2\right)\right)\sqrt{c+d\tan(e+fx)}}{f}+\frac{2(d(A-C)+Bc)(c+d\tan(e+fx))^{3/2}}{3f}-\frac{(c-id)^{5/2}(iA+B-iC)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] -(((I*A + B - I*C)*(c - I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c
- I*d]])/f) - ((B - I*(A - C))*(c + I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f
*x]]/Sqrt[c + I*d]])/f + (2*(2*c*(A - C)*d + B*(c^2 - d^2))*Sqrt[c + d*Tan[
e + f*x]])/f + (2*(B*c + (A - C)*d)*(c + d*Tan[e + f*x])^(3/2))/(3*f) + (2*
B*(c + d*Tan[e + f*x])^(5/2))/(5*f) + (2*C*(c + d*Tan[e + f*x])^(7/2))/(7*d
*f)
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
```

```
a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int (c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx = \frac{2C(c+d\tan(e+fx))^{7/2}}{7df}+\int (A-C+B\tan(e+f$$
$$= \frac{2B(c+d\tan(e+fx))^{5/2}}{5f}+\frac{2C(c+d\tan(e+fx))^{7/2}}{}$$
$$= \frac{2(Bc+(A-C)d)(c+d\tan(e+fx))^{3/2}}{3f}+\frac{2B(c+d}{}$$
$$= \frac{2\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\sqrt{c+d\tan(e+fx)}}{f}+$$
$$= \frac{2\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\sqrt{c+d\tan(e+fx)}}{f}+$$
$$= \frac{2\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\sqrt{c+d\tan(e+fx)}}{f}+$$
$$= \frac{2\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\sqrt{c+d\tan(e+fx)}}{f}+$$
$$= -\frac{(iA+B-iC)(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}-$$

Mathematica [A] time = 2.03062, size = 262, normalized size = 1.14

$$7i(A-iB-C)\left(\frac{2}{5}(c+d\tan(e+fx))^{5/2}+\frac{2}{3}(c-id)\left(\sqrt{c+d\tan(e+fx)}(4c+d\tan(e+fx)-3id)-3(c-id)^{3/2}\tanh^{-1}\left(\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2
),x]
```

```
[Out] ((4*C*(c + d*Tan[e + f*x])^(7/2))/d + (7*I)*(A - I*B - C)*((2*(c + d*Tan[e
+ f*x])^(5/2))/5 + (2*(c - I*d)*(-3*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[
e + f*x]]/Sqrt[c - I*d]] + Sqrt[c + d*Tan[e + f*x]]*(4*c - (3*I)*d + d*Tan[
e + f*x])))/3) - (7*I)*(A + I*B - C)*((2*(c + d*Tan[e + f*x])^(5/2))/5 + (2
*(c + I*d)*(-3*(c + I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*
d]] + Sqrt[c + d*Tan[e + f*x]]*(4*c + (3*I)*d + d*Tan[e + f*x])))/3))/(14*f
)
```

Maple [B] time = 0.123, size = 3614, normalized size = 15.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x)
```

```
[Out] 2/5*B*(c+d*tan(f*x+e))^(5/2)/f+1/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2
*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*
c)^(1/2))*B*(c^2+d^2)^(1/2)*c^2+1/4*d/f*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^
(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)*(c^2+d^2)^(1/2)-3/4*d/f*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)*c-1/4*d/f*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)+(c^2+d^2)^(1/2))*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)+
1/4/d/f*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)+(c^2+d^2)^(1/2))*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*c^2+2/
f*B*c^2*(c+d*tan(f*x+e))^(1/2)+2/3/f*B*(c+d*tan(f*x+e))^(3/2)*c+2/3*d/f*A*(
c+d*tan(f*x+e))^(3/2)-2/3*d/f*C*(c+d*tan(f*x+e))^(3/2)-2*d^2/f*B*(c+d*tan(f
*x+e))^(1/2)+2/7*C*(c+d*tan(f*x+e))^(7/2)/f/d+2*d/f/(2*(c^2+d^2)^(1/2)-2*c)
^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(
c^2+d^2)^(1/2)-2*c)^(1/2))*C*(c^2+d^2)^(1/2)*c+2*d/f/(2*(c^2+d^2)^(1/2)-2*c
)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*
(c^2+d^2)^(1/2)-2*c)^(1/2))*A*(c^2+d^2)^(1/2)*c-2*d/f/(2*(c^2+d^2)^(1/2)-2*
c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2
*(c^2+d^2)^(1/2)-2*c)^(1/2))*C*(c^2+d^2)^(1/2)*c+1/4/d/f*ln((c+d*tan(f*x+e)
)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*A*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*c^2-1/4/d/f*ln((c+d*tan(f*x+e))^
(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*C*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*c^2-1/4/d/f*ln(d*tan(f*x+e)+c+(c+d
*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*A*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*c^2-2*d/f/(2*(c^2+d^2)^(1/2)-2*c)^(1
/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2
+d^2)^(1/2)-2*c)^(1/2))*A*(c^2+d^2)^(1/2)*c+1/4/d/f*ln((c+d*tan(f*x+e))^(1/
2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*C*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*c^3+d^3/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c
^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^
(1/2))*A-d^3/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c
)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*C-3/4/f*ln
(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d
^2)^(1/2))*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2+1/f/(2*(c^2+d^2)^(1/2)-2*c)^
(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c
^2+d^2)^(1/2)-2*c)^(1/2))*B*c^3+3/4/f*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*B*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*c^2-1/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*
c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*B*c^3+1/4
*d^2/f*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)+(c^2+d^2)^(1/2))*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-4*d/f*c*C*(c+d*tan(f*x+
```

```
e))^(1/2)-1/4*d^2/f*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+4*d/f*A*c*
(c+d*tan(f*x+e))^(1/2)+d^3/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*t
an(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/
2))*C-d^3/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+
(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*A-1/4/d/f*ln(
(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^
2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3-1/2/f*ln((c+d*tan(f*x+e))^(1/
2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*B*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*c-1/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*a
rctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)
^(1/2)-2*c)^(1/2))*B*(c^2+d^2)^(1/2)*c^2+1/2/f*ln(d*tan(f*x+e)+c+(c+d*tan(f
*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*B*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*c-3*d/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arct
an((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1
/2)-2*c)^(1/2))*C*c^2-3*d^2/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*
tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1
/2))*B*c+3*d/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/
2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*A*c^2-d^2/
f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+
d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*B*(c^2+d^2)^(1/2)+3/4*d
/f*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(
c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c+1/4*d/f*ln((c+d*tan(f*x+e
))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*C*(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)-3/4*d/f*ln((c+d*tan(f*x+e))^(1/
2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*C*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*c+d^2/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*
tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1
/2))*B*(c^2+d^2)^(1/2)+3/4*d/f*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*c+1/4/d/f*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)
^(1/2)+(c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3-1/4*d/f*ln((c+d
*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(
1/2))*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)-1/4/d/f*ln(d*tan(f*x+
e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*
C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3+3*d/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arc
tan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(
1/2)-2*c)^(1/2))*C*c^2-3*d/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+
d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/
2))*A*c^2+3*d^2/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+
2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*B*c
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorit
hm="maxima")

[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c + d\tan\left(e + fx\right)\right)^{\frac{5}{2}} \left(A + B\tan\left(e + fx\right) + C\tan^2\left(e + fx\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2),x)
```

```
[Out] Integral((c + d*tan(e + f*x))**(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)**
2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\tan\left(fx + e\right)^2 + B\tan\left(fx + e\right) + A\right)\left(d\tan\left(fx + e\right) + c\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(d*tan(f*x + e) + c)^(5/2
), x)
```

3.107 $\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=336

$$-\frac{2(bc-ad)^{5/2}\left(Ab^2-a(bB-aC)\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{b^{7/2}f\left(a^2+b^2\right)}+\frac{2\sqrt{c+d\tan(e+fx)}\left((bc-ad)(-aCd+bBd+bcC)+b^2d\right.}{b^3f}$$

```
[Out] -(((I*A + B - I*C)*(c - I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c
- I*d]])/((a - I*b)*f)) + ((I*A - B - I*C)*(c + I*d)^(5/2)*ArcTanh[Sqrt[c +
 d*Tan[e + f*x]]/Sqrt[c + I*d]])/((a + I*b)*f) - (2*(A*b^2 - a*(b*B - a*C))
*(b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*
d]])/(b^(7/2)*(a^2 + b^2)*f) + (2*(b^2*d*(B*c + (A - C)*d) + (b*c - a*d)*(b
*c*C + b*B*d - a*C*d))*Sqrt[c + d*Tan[e + f*x]])/(b^3*f) + (2*(b*c*C + b*B*
d - a*C*d)*(c + d*Tan[e + f*x])^(3/2))/(3*b^2*f) + (2*C*(c + d*Tan[e + f*x]
)^(5/2))/(5*b*f)
```

Rubi [A] time = 2.81173, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.149, Rules used = {3647, 3653, 3539, 3537, 63, 208, 3634}

$$-\frac{2(bc-ad)^{5/2}\left(Ab^2-a(bB-aC)\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{b^{7/2}f\left(a^2+b^2\right)}+\frac{2\sqrt{c+d\tan(e+fx)}\left((bc-ad)(-aCd+bBd+bcC)+b^2d\right.}{b^3f}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
 + b*Tan[e + f*x]),x]
```

```
[Out] -(((I*A + B - I*C)*(c - I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c
- I*d]])/((a - I*b)*f)) + ((I*A - B - I*C)*(c + I*d)^(5/2)*ArcTanh[Sqrt[c +
 d*Tan[e + f*x]]/Sqrt[c + I*d]])/((a + I*b)*f) - (2*(A*b^2 - a*(b*B - a*C))
*(b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*
d]])/(b^(7/2)*(a^2 + b^2)*f) + (2*(b^2*d*(B*c + (A - C)*d) + (b*c - a*d)*(b
*c*C + b*B*d - a*C*d))*Sqrt[c + d*Tan[e + f*x]])/(b^3*f) + (2*(b*c*C + b*B*
d - a*C*d)*(c + d*Tan[e + f*x])^(3/2))/(3*b^2*f) + (2*C*(c + d*Tan[e + f*x]
)^(5/2))/(5*b*f)
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3653

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
```

```
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[e + f*x])^n*(1 + Tan[e
+ f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3634

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}dx = \frac{2C(c+d\tan(e+fx))^{5/2}}{5bf} + \frac{2\int \frac{(c+d\tan(e+fx))^{3/2}\left(\frac{5}{2}(Ab\right.}{}}{}$$

$$= \frac{2(bcC+bBd-aCd)(c+d\tan(e+fx))^{3/2}}{3b^2f} + \frac{2C(c+}{}$$

$$= \frac{2\left(b^2d(Bc+(A-C)d)+(bc-ad)(bcC+bBd-aC\right.}{b^3f}$$

$$= \frac{2\left(b^2d(Bc+(A-C)d)+(bc-ad)(bcC+bBd-aC\right.}{b^3f}$$

$$= \frac{2\left(b^2d(Bc+(A-C)d)+(bc-ad)(bcC+bBd-aC\right.}{b^3f}$$

$$= \frac{2\left(b^2d(Bc+(A-C)d)+(bc-ad)(bcC+bBd-aC\right.}{b^3f}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d}}{\sqrt{b}}\right.}{b^{7/2}\left(a^2+b^2\right)f}$$

$$= -\frac{(iA+B-iC)(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)f} -$$

Mathematica [A] time = 5.32831, size = 322, normalized size = 0.96

$$\frac{\frac{15\left(b^{7/2}(b-ia)(c-id)^{5/2}(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)+b^{7/2}(b+ia)(c+id)^{5/2}(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)-2(bc-ad)^{5/2}\left(a(aC-bB)+Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d}}{\sqrt{b}}\right.\right.}{b^{5/2}\left(a^2+b^2\right)}}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/(a + b*Tan[e + f*x]),x]
```

```
[Out] ((15*(b^(7/2)*((-I)*a + b)*(A - I*B - C)*(c - I*d)^(5/2)*ArcTanh[Sqrt[c + d
*Tan[e + f*x]]/Sqrt[c - I*d]] + b^(7/2)*(I*a + b)*(A + I*B - C)*(c + I*d)^(
5/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]] - 2*(A*b^2 + a*(-(b*B)
 + a*C))*(b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[
b*c - a*d]]))/(b^(5/2)*(a^2 + b^2)) + (30*(b^2*d*(B*c + (A - C)*d) + (b*c -
 a*d)*(b*c*C + b*B*d - a*C*d))*Sqrt[c + d*Tan[e + f*x]])/b^2 + (10*(b*c*C +
 b*B*d - a*C*d)*(c + d*Tan[e + f*x])^(3/2))/b + 6*C*(c + d*Tan[e + f*x])^(5
/2))/(15*b*f)
```

Maple [B] time = 0.214, size = 8698, normalized size = 25.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\tan\left(fx+e\right)^2+B\tan\left(fx+e\right)+A\right)\left(d\tan\left(fx+e\right)+c\right)^{\frac{5}{2}}}{b\tan\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e)),x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(d*tan(f*x + e) + c)^(5/2
)/(b*tan(f*x + e) + a), x)
```

3.108 $\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=473

$$-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}+\frac{d\left(5a^2C-3abB+3Ab^2+2b^2C\right)(c+d\tan(e+fx))^{3/2}}{3b^2f\left(a^2+b^2\right)}-\frac{d\sqrt{c+d\tan(e+}}{}$$

```
[Out] -(((I*A + B - I*C)*(c - I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c
- I*d]])/((a - I*b)^2*f)) - ((B - I*(A - C))*(c + I*d)^(5/2)*ArcTanh[Sqrt[c
 + d*Tan[e + f*x]]/Sqrt[c + I*d]])/((a + I*b)^2*f) + ((b*c - a*d)^(3/2)*(3*
a^3*b*B*d - 5*a^4*C*d - b^4*(2*B*c + 5*A*d) - a*b^3*(4*A*c - 4*c*C - 7*B*d)
 + a^2*b^2*(2*B*c - (A + 9*C)*d))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]]
)/Sqrt[b*c - a*d]])/(b^(7/2)*(a^2 + b^2)^2*f) - (d*(5*a^3*C*d - A*b^2*(b*c
- a*d) - 2*b^3*(2*c*C + B*d) - a^2*b*(5*c*C + 3*B*d) + a*b^2*(B*c + 4*C*d))
*Sqrt[c + d*Tan[e + f*x]])/(b^3*(a^2 + b^2)*f) + ((3*A*b^2 - 3*a*b*B + 5*a^
2*C + 2*b^2*C)*d*(c + d*Tan[e + f*x])^(3/2))/(3*b^2*(a^2 + b^2)*f) - ((A*b^
2 - a*(b*B - a*C))*(c + d*Tan[e + f*x])^(5/2))/(b*(a^2 + b^2)*f*(a + b*Tan[
e + f*x]))
```

Rubi [A] time = 3.89577, antiderivative size = 473, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.17, Rules used = {3645, 3647, 3653, 3539, 3537, 63, 208, 3634}

$$-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}+\frac{d\left(5a^2C-3abB+3Ab^2+2b^2C\right)(c+d\tan(e+fx))^{3/2}}{3b^2f\left(a^2+b^2\right)}-\frac{d\sqrt{c+d\tan(e+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
 + b*Tan[e + f*x])^2,x]
```


```
[Out] -(((I*A + B - I*C)*(c - I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c
- I*d]])/((a - I*b)^2*f)) - ((B - I*(A - C))*(c + I*d)^(5/2)*ArcTanh[Sqrt[c
 + d*Tan[e + f*x]]/Sqrt[c + I*d]])/((a + I*b)^2*f) + ((b*c - a*d)^(3/2)*(3*
a^3*b*B*d - 5*a^4*C*d - b^4*(2*B*c + 5*A*d) - a*b^3*(4*A*c - 4*c*C - 7*B*d)
 + a^2*b^2*(2*B*c - (A + 9*C)*d))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]]
)/Sqrt[b*c - a*d]])/(b^(7/2)*(a^2 + b^2)^2*f) - (d*(5*a^3*C*d - A*b^2*(b*c
- a*d) - 2*b^3*(2*c*C + B*d) - a^2*b*(5*c*C + 3*B*d) + a*b^2*(B*c + 4*C*d))
*Sqrt[c + d*Tan[e + f*x]])/(b^3*(a^2 + b^2)*f) + ((3*A*b^2 - 3*a*b*B + 5*a^
2*C + 2*b^2*C)*d*(c + d*Tan[e + f*x])^(3/2))/(3*b^2*(a^2 + b^2)*f) - ((A*b^
2 - a*(b*B - a*C))*(c + d*Tan[e + f*x])^(5/2))/(b*(a^2 + b^2)*f*(a + b*Tan[
e + f*x]))
```


Rule 3645

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*d^2 + c*(c*C - B*d))*(a + b*Tan[e
+ f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
```

```
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3653

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[e + f*x])^n*(1 + Tan[e
+ f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3634

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{b\left(a^2+b^2\right)f(a+b\tan(e+fx))} + \frac{\int \frac{(c+}{}}{}$$

$$= \frac{\left(3Ab^2-3abB+5a^2C+2b^2C\right)d(c+d\tan(e+fx))}{3b^2\left(a^2+b^2\right)f}$$

$$= -\frac{d\left(5a^3Cd-Ab^2(bc-ad)-2b^3(2cC+Bd)-a^2b(5\right.}{b^3\left(a^2+\right.}$$

$$= -\frac{d\left(5a^3Cd-Ab^2(bc-ad)-2b^3(2cC+Bd)-a^2b(5\right.}{b^3\left(a^2+\right.}$$

$$= -\frac{d\left(5a^3Cd-Ab^2(bc-ad)-2b^3(2cC+Bd)-a^2b(5\right.}{b^3\left(a^2+\right.}$$

$$= -\frac{d\left(5a^3Cd-Ab^2(bc-ad)-2b^3(2cC+Bd)-a^2b(5\right.}{b^3\left(a^2+\right.}$$

$$= \frac{(bc-ad)^{3/2}\left(3a^3bBd-5a^4Cd-b^4(2Bc+5Ad)-ab\right.}{}$$

$$= -\frac{(iA+B-iC)(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)^2f} -$$

Mathematica [B] time = 6.54989, size = 6112, normalized size = 12.92

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.258, size = 14119, normalized size = 29.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^2,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\tan\left(fx+e\right)^{2}+B\tan\left(fx+e\right)+A\right)\left(d\tan\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(b\tan\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(d*tan(f*x + e) + c)^(5/2
)/(b*tan(f*x + e) + a)^2, x)
```

3.109 $\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx$

Optimal. Leaf size=643

$$\frac{\sqrt{bc-ad}\left(2a^3b^3\left(4cd(A-C)+B\left(4c^2+3d^2\right)\right)-3a^2b^4\left(8Ac^2-6Ad^2-16Bcd-8c^2C+21Cd^2\right)+a^4b^2d(d(A-46C)+4\ldots}{4b^7\ldots}$$

```
[Out] -(((A - I*B - C)*(c - I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c -
I*d]])/((I*a + b)^3*f)) + ((A + I*B - C)*(c + I*d)^(5/2)*ArcTanh[Sqrt[c + d
*Tan[e + f*x]]/Sqrt[c + I*d]])/((I*a - b)^3*f) + (Sqrt[b*c - a*d]*(3*a^5*b*
B*d^2 - 15*a^6*C*d^2 + a^4*b^2*d*(4*B*c + (A - 46*C)*d) - 3*a^2*b^4*(8*A*c^
2 - 8*c^2*C - 16*B*c*d - 6*A*d^2 + 21*C*d^2) - a*b^5*(56*c*(A - C)*d + B*(2
4*c^2 - 35*d^2)) - b^6*(4*c*(2*c*C + 5*B*d) - A*(8*c^2 - 15*d^2)) + 2*a^3*b
^3*(4*c*(A - C)*d + B*(4*c^2 + 3*d^2)))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e +
 f*x]])/Sqrt[b*c - a*d]])/(4*b^(7/2)*(a^2 + b^2)^3*f) - (d*(3*a^3*b*B*d - 1
5*a^4*C*d - a*b^3*(8*A*c - 8*c*C - 11*B*d) + a^2*b^2*(4*B*c + (A - 31*C)*d)
 - b^4*(4*B*c + 7*A*d + 8*C*d))*Sqrt[c + d*Tan[e + f*x]])/(4*b^3*(a^2 + b^2
)^2*f) + ((a^3*b*B*d - 5*a^4*C*d - b^4*(4*B*c + 5*A*d) - a*b^3*(8*A*c - 8*c
*C - 9*B*d) + a^2*b^2*(4*B*c + 3*A*d - 13*C*d))*(c + d*Tan[e + f*x])^(3/2))
/(4*b^2*(a^2 + b^2)^2*f*(a + b*Tan[e + f*x])) - ((A*b^2 - a*(b*B - a*C))*(c
 + d*Tan[e + f*x])^(5/2))/(2*b*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^2)
```

Rubi [A] time = 6.06519, antiderivative size = 643, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.17, Rules used = {3645, 3647, 3653, 3539, 3537, 63, 208, 3634}

$$\frac{\sqrt{bc-ad}\left(2a^3b^3\left(4cd(A-C)+B\left(4c^2+3d^2\right)\right)-3a^2b^4\left(8Ac^2-6Ad^2-16Bcd-8c^2C+21Cd^2\right)+a^4b^2d(d(A-46C)+4\ldots}{4b^7\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
 + b*Tan[e + f*x])^3,x]
```

```
[Out] -(((A - I*B - C)*(c - I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c -
I*d]])/((I*a + b)^3*f)) + ((A + I*B - C)*(c + I*d)^(5/2)*ArcTanh[Sqrt[c + d
*Tan[e + f*x]]/Sqrt[c + I*d]])/((I*a - b)^3*f) + (Sqrt[b*c - a*d]*(3*a^5*b*
B*d^2 - 15*a^6*C*d^2 + a^4*b^2*d*(4*B*c + (A - 46*C)*d) - 3*a^2*b^4*(8*A*c^
2 - 8*c^2*C - 16*B*c*d - 6*A*d^2 + 21*C*d^2) - a*b^5*(56*c*(A - C)*d + B*(2
4*c^2 - 35*d^2)) - b^6*(4*c*(2*c*C + 5*B*d) - A*(8*c^2 - 15*d^2)) + 2*a^3*b
^3*(4*c*(A - C)*d + B*(4*c^2 + 3*d^2)))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e +
 f*x]])/Sqrt[b*c - a*d]])/(4*b^(7/2)*(a^2 + b^2)^3*f) - (d*(3*a^3*b*B*d - 1
5*a^4*C*d - a*b^3*(8*A*c - 8*c*C - 11*B*d) + a^2*b^2*(4*B*c + (A - 31*C)*d)
 - b^4*(4*B*c + 7*A*d + 8*C*d))*Sqrt[c + d*Tan[e + f*x]])/(4*b^3*(a^2 + b^2
)^2*f) + ((a^3*b*B*d - 5*a^4*C*d - b^4*(4*B*c + 5*A*d) - a*b^3*(8*A*c - 8*c
*C - 9*B*d) + a^2*b^2*(4*B*c + 3*A*d - 13*C*d))*(c + d*Tan[e + f*x])^(3/2))
/(4*b^2*(a^2 + b^2)^2*f*(a + b*Tan[e + f*x])) - ((A*b^2 - a*(b*B - a*C))*(c
 + d*Tan[e + f*x])^(5/2))/(2*b*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^2)
```

Rule 3645

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*d^2 + c*(c*C - B*d))*(a + b*Tan[e
+ f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3653

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[e + f*x])^n*(1 + Tan[e
+ f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
```

```
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3634

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}+\frac{\int \frac{(c+}{}}{}$$

$$=\frac{\left(a^3bBd-5a^4Cd-b^4(4Bc+5Ad)-ab^3(8Ac-8cC\right.}{4b^2\left(a^2+b^2\right)^2}$$

$$=-\frac{d\left(3a^3bBd-15a^4Cd-ab^3(8Ac-8cC-11Bd)+a\right.}{}$$

$$=-\frac{d\left(3a^3bBd-15a^4Cd-ab^3(8Ac-8cC-11Bd)+a\right.}{}$$

$$=-\frac{d\left(3a^3bBd-15a^4Cd-ab^3(8Ac-8cC-11Bd)+a\right.}{}$$

$$=-\frac{d\left(3a^3bBd-15a^4Cd-ab^3(8Ac-8cC-11Bd)+a\right.}{}$$

$$=\frac{\sqrt{bc-ad}\left(3a^5bBd^2-15a^6Cd^2+a^4b^2d(4Bc+(A-\right.}{}$$

$$=-\frac{(A-iB-C)(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(ia+b)^3f}+$$

Mathematica [B] time = 6.89837, size = 18214, normalized size = 28.33

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/(a + b*Tan[e + f*x])^3,x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.282, size = 20663, normalized size = 32.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\tan\left(fx+e\right)^{2}+B\tan\left(fx+e\right)+A\right)\left(d\tan\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(b\tan\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(d*tan(f*x + e) + c)^(5/2
)/(b*tan(f*x + e) + a)^3, x)
```

3.110 $\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=407

$$\frac{2\sqrt{c+d\tan(e+fx)}\left(-6a^2bd^2(32cC-49Bd)+72a^3Cd^3+21ab^2d\left(15d^2(A-C)-10Bcd+8c^2C\right)+b^3\left(-\left(70cd^2(A-\right.\right.\right.}{105d^4f}$$

```
[Out] ((I*a + b)^3*(A - I*B - C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])
/(Sqrt[c - I*d]*f) - ((I*a - b)^3*(A + I*B - C)*ArcTanh[Sqrt[c + d*Tan[e +
f*x]]/Sqrt[c + I*d]])/(Sqrt[c + I*d]*f) + (2*(72*a^3*C*d^3 - 6*a^2*b*d^2*(3
2*c*C - 49*B*d) + 21*a*b^2*d*(8*c^2*C - 10*B*c*d + 15*(A - C)*d^2) - b^3*(4
8*c^3*C - 56*B*c^2*d + 70*c*(A - C)*d^2 + 105*B*d^3))*Sqrt[c + d*Tan[e + f*
x]])/(105*d^4*f) + (2*b*(35*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*
C - 7*b*B*d - 6*a*C*d))*Tan[e + f*x]*Sqrt[c + d*Tan[e + f*x]])/(105*d^3*f)
- (2*(6*b*c*C - 7*b*B*d - 6*a*C*d)*(a + b*Tan[e + f*x])^2*Sqrt[c + d*Tan[e
+ f*x]])/(35*d^2*f) + (2*C*(a + b*Tan[e + f*x])^3*Sqrt[c + d*Tan[e + f*x]])
/(7*d*f)
```

Rubi [A] time = 1.69893, antiderivative size = 407, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.149, Rules used = {3647, 3637, 3630, 3539, 3537, 63, 208}

$$\frac{2\sqrt{c+d\tan(e+fx)}\left(-6a^2bd^2(32cC-49Bd)+72a^3Cd^3+21ab^2d\left(15d^2(A-C)-10Bcd+8c^2C\right)+b^3\left(-\left(70cd^2(A-\right.\right.\right.}{105d^4f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/Sqrt[c
 + d*Tan[e + f*x]],x]
```


```
[Out] ((I*a + b)^3*(A - I*B - C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])
/(Sqrt[c - I*d]*f) - ((I*a - b)^3*(A + I*B - C)*ArcTanh[Sqrt[c + d*Tan[e +
f*x]]/Sqrt[c + I*d]])/(Sqrt[c + I*d]*f) + (2*(72*a^3*C*d^3 - 6*a^2*b*d^2*(3
2*c*C - 49*B*d) + 21*a*b^2*d*(8*c^2*C - 10*B*c*d + 15*(A - C)*d^2) - b^3*(4
8*c^3*C - 56*B*c^2*d + 70*c*(A - C)*d^2 + 105*B*d^3))*Sqrt[c + d*Tan[e + f*
x]])/(105*d^4*f) + (2*b*(35*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*
C - 7*b*B*d - 6*a*C*d))*Tan[e + f*x]*Sqrt[c + d*Tan[e + f*x]])/(105*d^3*f)
- (2*(6*b*c*C - 7*b*B*d - 6*a*C*d)*(a + b*Tan[e + f*x])^2*Sqrt[c + d*Tan[e
+ f*x]])/(35*d^2*f) + (2*C*(a + b*Tan[e + f*x])^3*Sqrt[c + d*Tan[e + f*x]])
/(7*d*f)
```


Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}dx = \frac{2C(a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}}{7df} + \frac{2\int}{}$$

$$= -\frac{2(6bcC-7bBd-6aCd)(a+b\tan(e+fx))^2\sqrt{c}}{35d^2f}$$

$$= \frac{2b\left(35b(Ab+aB-bC)d^2+4(bc-ad)(6bcC-7b\right.}{105d^3}$$

$$= \frac{2\left(72a^3Cd^3-6a^2bd^2(32cC-49Bd)+21ab^2d\left(8c\right.\right.}{}$$

$$= \frac{2\left(72a^3Cd^3-6a^2bd^2(32cC-49Bd)+21ab^2d\left(8c\right.\right.}{}$$

$$= \frac{2\left(72a^3Cd^3-6a^2bd^2(32cC-49Bd)+21ab^2d\left(8c\right.\right.}{}$$

$$= \frac{2\left(72a^3Cd^3-6a^2bd^2(32cC-49Bd)+21ab^2d\left(8c\right.\right.}{}$$

$$= -\frac{(a-ib)^3(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}f} -$$

<u>Mathematica</u> [B] time = 6.45581, size = 1200, normalized size = 2.95

$$\frac{2C\sqrt{c+d\tan(e+fx)}(a+b\tan(e+fx))^3}{7df} + \frac{2\left(\frac{(-6bcC+6adC+7bBd)\sqrt{c+d\tan(e+fx)}(a+b\tan(e+fx))^2}{5df} + \frac{2\left(\frac{b\left(35b(Ab-Cb+aB)d^2+4(bc-ad)(6b\right.}{}\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] (2*C*(a + b*Tan[e + f*x])^3*Sqrt[c + d*Tan[e + f*x]])/(7*d*f) + (2*(((-6*b*
c*C + 7*b*B*d + 6*a*C*d)*(a + b*Tan[e + f*x])^2*Sqrt[c + d*Tan[e + f*x]])/(
5*d*f) + (2*((b*(35*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C - 7*b*
B*d - 6*a*C*d))*Tan[e + f*x]*Sqrt[c + d*Tan[e + f*x]])/(6*d*f) - (2*((I*Sqr
t[c - I*d]*((b*c*(35*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C - 7*b
*B*d - 6*a*C*d)))/4 + (3*a*d*(35*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6
*b*c*C - 7*b*B*d - 6*a*C*d)))/8 - (3*a*d*(-5*a*d*(6*b*c*C - a*(7*A - C)*d)
+ (4*b*c + a*d)*(6*b*c*C - 7*b*B*d - 6*a*C*d)))/8 - b*((-105*(a^2*B - b^2*B
 + 2*a*b*(A - C))*d^3)/8 + (c*(35*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(
```

```
6*b*c*C - 7*b*B*d - 6*a*C*d)))/4) + ((3*I)/2)*d*((35*a*(a^2*B - b^2*B + 2*a
*b*(A - C))*d^2)/4 - (b*(35*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*
C - 7*b*B*d - 6*a*C*d)))/4 + (b*(-5*a*d*(6*b*c*C - a*(7*A - C)*d) + (4*b*c
+ a*d)*(6*b*c*C - 7*b*B*d - 6*a*C*d)))/4))*ArcTanh[Sqrt[c + d*Tan[e + f*x]]
/Sqrt[c - I*d]])/((-c + I*d)*f) - (I*Sqrt[c + I*d]*((b*c*(35*b*(A*b + a*B -
 b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C - 7*b*B*d - 6*a*C*d)))/4 + (3*a*d*(35*b*
(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C - 7*b*B*d - 6*a*C*d)))/8 - (
3*a*d*(-5*a*d*(6*b*c*C - a*(7*A - C)*d) + (4*b*c + a*d)*(6*b*c*C - 7*b*B*d
- 6*a*C*d)))/8 - b*((-105*(a^2*B - b^2*B + 2*a*b*(A - C))*d^3)/8 + (c*(35*b
*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C - 7*b*B*d - 6*a*C*d)))/4) -
 ((3*I)/2)*d*((35*a*(a^2*B - b^2*B + 2*a*b*(A - C))*d^2)/4 - (b*(35*b*(A*b
+ a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C - 7*b*B*d - 6*a*C*d)))/4 + (b*(-5
*a*d*(6*b*c*C - a*(7*A - C)*d) + (4*b*c + a*d)*(6*b*c*C - 7*b*B*d - 6*a*C*d
)))/4))*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/((-c - I*d)*f) + (
2*((-3*a*d*(35*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C - 7*b*B*d -
 6*a*C*d)))/8 + b*((-105*(a^2*B - b^2*B + 2*a*b*(A - C))*d^3)/8 + (c*(35*b*
(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C - 7*b*B*d - 6*a*C*d)))/4))*S
qrt[c + d*Tan[e + f*x]])/(d*f)))/(3*d)))/(5*d)))/(7*d)
```

Maple [B] time = 0.211, size = 25426, normalized size = 62.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/
2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tan\left(e+fx\right)\right)^{3}\left(A+B\tan\left(e+fx\right)+C\tan^{2}\left(e+fx\right)\right)}{\sqrt{c+d\tan\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x
+e))**(1/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**3*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/s
qrt(c + d*tan(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\tan\left(fx+e\right)^{2}+B\tan\left(fx+e\right)+A\right)\left(b\tan\left(fx+e\right)+a\right)^{3}}{\sqrt{d\tan\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^3/sq
rt(d*tan(f*x + e) + c), x)
```

3.111 $\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=287

$$\frac{2\sqrt{c+d\tan(e+fx)}\left(12a^2Cd^2-10abd(2cC-3Bd)+b^2\left(15d^2(A-C)-10Bcd+8c^2C\right)\right)}{15d^3f}-\frac{(a-ib)^2(B+i(A-C))\tanh}{f\sqrt{c-id}}$$

```
[Out] -(((a - I*b)^2*(B + I*(A - C))*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*
d]])/(Sqrt[c - I*d]*f)) + ((a + I*b)^2*(I*A - B - I*C)*ArcTanh[Sqrt[c + d*T
an[e + f*x]]/Sqrt[c + I*d]])/(Sqrt[c + I*d]*f) + (2*(12*a^2*C*d^2 - 10*a*b*
d*(2*c*C - 3*B*d) + b^2*(8*c^2*C - 10*B*c*d + 15*(A - C)*d^2))*Sqrt[c + d*T
an[e + f*x]])/(15*d^3*f) - (2*b*(4*b*c*C - 5*b*B*d - 4*a*C*d)*Tan[e + f*x]*
Sqrt[c + d*Tan[e + f*x]])/(15*d^2*f) + (2*C*(a + b*Tan[e + f*x])^2*Sqrt[c +
 d*Tan[e + f*x]])/(5*d*f)
```

Rubi [A] time = 1.0014, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.149, Rules used = {3647, 3637, 3630, 3539, 3537, 63, 208}

$$\frac{2\sqrt{c+d\tan(e+fx)}\left(12a^2Cd^2-10abd(2cC-3Bd)+b^2\left(15d^2(A-C)-10Bcd+8c^2C\right)\right)}{15d^3f}-\frac{(a-ib)^2(B+i(A-C))\tanh}{f\sqrt{c-id}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/Sqrt[c
 + d*Tan[e + f*x]],x]
```

```
[Out] -(((a - I*b)^2*(B + I*(A - C))*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*
d]])/(Sqrt[c - I*d]*f)) + ((a + I*b)^2*(I*A - B - I*C)*ArcTanh[Sqrt[c + d*T
an[e + f*x]]/Sqrt[c + I*d]])/(Sqrt[c + I*d]*f) + (2*(12*a^2*C*d^2 - 10*a*b*
d*(2*c*C - 3*B*d) + b^2*(8*c^2*C - 10*B*c*d + 15*(A - C)*d^2))*Sqrt[c + d*T
an[e + f*x]])/(15*d^3*f) - (2*b*(4*b*c*C - 5*b*B*d - 4*a*C*d)*Tan[e + f*x]*
Sqrt[c + d*Tan[e + f*x]])/(15*d^2*f) + (2*C*(a + b*Tan[e + f*x])^2*Sqrt[c +
 d*Tan[e + f*x]])/(5*d*f)
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
```

```
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx = \frac{2C(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df} + \frac{2\int \frac{(a+}{}}{}$$

$$= -\frac{2b(4bcC-5bBd-4aCd)\tan(e+fx)\sqrt{c+d\tan(e}}{15d^2f}$$

$$= \frac{2\left(12a^2Cd^2-10abd(2cC-3Bd)+b^2\left(8c^2C-10Bcd\right.\right.}{15d^3f}$$

$$= \frac{2\left(12a^2Cd^2-10abd(2cC-3Bd)+b^2\left(8c^2C-10Bcd\right.\right.}{15d^3f}$$

$$= \frac{2\left(12a^2Cd^2-10abd(2cC-3Bd)+b^2\left(8c^2C-10Bcd\right.\right.}{15d^3f}$$

$$= \frac{2\left(12a^2Cd^2-10abd(2cC-3Bd)+b^2\left(8c^2C-10Bcd\right.\right.}{15d^3f}$$

$$= -\frac{(a-ib)^2(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}f} - \frac{(a}{}$$

Mathematica [A] time = 5.99704, size = 275, normalized size = 0.96

$$\frac{\frac{2\sqrt{c+d\tan(e+fx)}\left(12a^2Cd^2+10abd(3Bd-2cC)+b^2\left(15d^2(A-C)-10Bcd+8c^2C\right)\right)}{d^2} - \frac{15d(a-ib)^2(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}} + \frac{15id(a+ib)^2(A+iB-C)\tanh}{\sqrt{c+id}}}{15df}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((-15*(a - I*b)^2*(I*A + B - I*C)*d*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c
 - I*d]])/Sqrt[c - I*d] + ((15*I)*(a + I*b)^2*(A + I*B - C)*d*ArcTanh[Sqrt[
c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/Sqrt[c + I*d] + (2*(12*a^2*C*d^2 + 10*a
*b*d*(-2*c*C + 3*B*d) + b^2*(8*c^2*C - 10*B*c*d + 15*(A - C)*d^2))*Sqrt[c +
 d*Tan[e + f*x]])/d^2 + (2*b*(-4*b*c*C + 5*b*B*d + 4*a*C*d)*Tan[e + f*x]*Sq
rt[c + d*Tan[e + f*x]])/d + 6*C*(a + b*Tan[e + f*x])^2*Sqrt[c + d*Tan[e + f
*x]])/(15*d*f)
```

Maple [B] time = 0.183, size = 18289, normalized size = 63.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/
2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tan\left(e+fx\right)\right)^2\left(A+B\tan\left(e+fx\right)+C\tan^2\left(e+fx\right)\right)}{\sqrt{c+d\tan\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x
+e))**(1/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**2*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/s
qrt(c + d*tan(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\tan\left(fx+e\right)^2+B\tan\left(fx+e\right)+A\right)\left(b\tan\left(fx+e\right)+a\right)^2}{\sqrt{d\tan\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^2/sq
rt(d*tan(f*x + e) + c), x)
```

3.112 $\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=194

$$-\frac{(b+ia)(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f\sqrt{c-id}}+\frac{(-b+ia)(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f\sqrt{c+id}}-\frac{2(-3aCd-3bBd+2bc}{3d^2}$$

```
[Out] -(((I*a + b)*(A - I*B - C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])
/(Sqrt[c - I*d]*f)) + ((I*a - b)*(A + I*B - C)*ArcTanh[Sqrt[c + d*Tan[e + f
*x]]/Sqrt[c + I*d]])/(Sqrt[c + I*d]*f) - (2*(2*b*c*C - 3*b*B*d - 3*a*C*d)*S
qrt[c + d*Tan[e + f*x]])/(3*d^2*f) + (2*b*C*Tan[e + f*x]*Sqrt[c + d*Tan[e +
 f*x]])/(3*d*f)
```

Rubi [A] time = 0.498312, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3637, 3630, 3539, 3537, 63, 208}

$$-\frac{(b+ia)(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f\sqrt{c-id}}+\frac{(-b+ia)(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f\sqrt{c+id}}-\frac{2(-3aCd-3bBd+2bc}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/Sqrt[c +
 d*Tan[e + f*x]],x]
```


```
[Out] -(((I*a + b)*(A - I*B - C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])
/(Sqrt[c - I*d]*f)) + ((I*a - b)*(A + I*B - C)*ArcTanh[Sqrt[c + d*Tan[e + f
*x]]/Sqrt[c + I*d]])/(Sqrt[c + I*d]*f) - (2*(2*b*c*C - 3*b*B*d - 3*a*C*d)*S
qrt[c + d*Tan[e + f*x]])/(3*d^2*f) + (2*b*C*Tan[e + f*x]*Sqrt[c + d*Tan[e +
 f*x]])/(3*d*f)
```


Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
```

```
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}dx = \frac{2bC\tan(e+fx)\sqrt{c+d\tan(e+fx)}}{3df} - \frac{2\int \frac{\frac{1}{2}(2bcC-3}{}}{}$$

$$= -\frac{2(2bcC-3bBd-3aCd)\sqrt{c+d\tan(e+fx)}}{3d^2f} + \frac{2b}{}$$

$$= -\frac{2(2bcC-3bBd-3aCd)\sqrt{c+d\tan(e+fx)}}{3d^2f} + \frac{2b}{}$$

$$= -\frac{2(2bcC-3bBd-3aCd)\sqrt{c+d\tan(e+fx)}}{3d^2f} + \frac{2b}{}$$

$$= -\frac{2(2bcC-3bBd-3aCd)\sqrt{c+d\tan(e+fx)}}{3d^2f} + \frac{2b}{}$$

$$= -\frac{(ia+b)(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}f} + \frac{(i}{}$$

Mathematica [A] time = 1.47551, size = 192, normalized size = 0.99

$$\frac{2\left(-\frac{3id(a-ib)(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{2\sqrt{c-id}} + \frac{3id(a+ib)(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{2\sqrt{c+id}} + \frac{(3aCd+3bBd-2bcC)\sqrt{c+d\tan(e+fx)}}{d} + bC\tan(e\right.}{3df}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/Sq
rt[c + d*Tan[e + f*x]],x]
```

```
[Out] (2*((((-3*I)/2)*(a - I*b)*(A - I*B - C)*d*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/
Sqrt[c - I*d]])/Sqrt[c - I*d] + (((3*I)/2)*(a + I*b)*(A + I*B - C)*d*ArcTan
h[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/Sqrt[c + I*d] + ((-2*b*c*C + 3*b
*B*d + 3*a*C*d)*Sqrt[c + d*Tan[e + f*x]])/d + b*C*Tan[e + f*x]*Sqrt[c + d*T
an[e + f*x]]))/(3*d*f)
```

Maple [B] time = 0.154, size = 4132, normalized size = 21.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)
,x)
```

```
[Out] 2/3/f/d^2*C*b*(c+d*tan(f*x+e))^(3/2)+2/f/d*a*C*(c+d*tan(f*x+e))^(1/2)+1/4/f
/d*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(
c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a-1/4/f/d*ln(d*tan(f*x+e)+c
+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*B*(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)*b-2/f/d^2*C*b*c*(c+d*tan(f*x+e))^(1/2)+1/4/f/(c
^2+d^2)^(1/2)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)+(c^2+d^2)^(1/2))*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a-1/4/f/(c^2+d^2
)^(1/2)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)+(c^2+d^2)^(1/2))*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b-1/f/(c^2+d^2)^(1/2)/
(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(c^2+d^2)
^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*B*a*c+1/f/(c^2+d^2)^(1/2)
/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(c^2+d^2
)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*C*b*c-1/f/(c^2+d^2)^(1/2
)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^
2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*B*a*c-1/4/f/d/(c^2+d^2)
^(1/2)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)+(c^2+d^2)^(1/2))*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c+1/f/d/(c^2+d^2)^(1/
2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d
^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*A*a*c^2-1/f/d/(c^2+d^2
)^(1/2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(
c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*C*a*c^2-1/4/f/d/(
c^2+d^2)^(1/2)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+
2*c)^(1/2)+(c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c-1/f/d/(c^2+
d^2)^(1/2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(
2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*B*b*c^2-1/f/d/
(c^2+d^2)^(1/2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1
/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*C*a*c^2+1
/f/d/(c^2+d^2)^(1/2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e
))^(1/2)-(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*A*a*
c^2-1/f/d/(c^2+d^2)^(1/2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(
f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))
*B*b*c^2+1/4/f/d/(c^2+d^2)^(1/2)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*b*c+1/4/f/d/(c^2+d^2)^(1/2)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*a*c-1/4/f/d/(c^2+d^2)^(1/2)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b
*c+1/4/f/d/(c^2+d^2)^(1/2)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c
-1/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(c^2
+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*C*b+1/f/(2*(c^2+d^2)
^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(c^2+d^2)^(1/2)+2*c)^
```

```
(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*A*b+1/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)
*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^
2)^(1/2)-2*c)^(1/2))*B*a+2/f/d*B*b*(c+d*tan(f*x+e))^(1/2)-1/f/(2*(c^2+d^2)^
(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(
1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*C*b+1/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*
arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2
)^(1/2)-2*c)^(1/2))*A*b+1/f/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*ta
n(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2
))*B*a-1/f/(c^2+d^2)^(1/2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan
(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)
)*A*b*c+2/f*d/(c^2+d^2)^(1/2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*
tan(f*x+e))^(1/2)-(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1
/2))*A*a+1/4/f/(c^2+d^2)^(1/2)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*b-1/4/f/(c^2+d^2)^(1/2)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b-1/4
/f/(c^2+d^2)^(1/2)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a+1/4/f/(c^
2+d^2)^(1/2)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)+(c^2+d^2)^(1/2))*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b+1/4/f/d*ln(d*ta
n(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(
1/2))*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a-1/4/f/d*ln(d*tan(f*x+e)+c-(c+d*tan(
f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*A*(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*a-1/4/f/d*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a+
1/4/f/d*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)+(c^2+d^2)^(1/2))*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b-1/f/d*(c^2+d^2)^(1/2
)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^
2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*A*a-2/f*d/(c^2+d^2)^(1/
2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d
^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*C*a+2/f*d/(c^2+d^2)^(1
/2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+
d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*A*a+1/f/(c^2+d^2)^(1/
2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d
^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*C*b*c-1/f/(c^2+d^2)^(1
/2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(c^2+
d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*A*b*c-2/f*d/(c^2+d^2)
^(1/2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c
^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*B*b-1/f/d*(c^2+d^2
)^(1/2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(
c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*A*a+1/f/d*(c^2+d^
2)^(1/2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*
(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*B*b+1/f/d*(c^2+d
^2)^(1/2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2
*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*C*a+1/f/d*(c^2+
d^2)^(1/2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(
2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*B*b+1/f/d*(c^2
+d^2)^(1/2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+
(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*C*a-2/f*d/(c^
2+d^2)^(1/2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)
-(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*B*b-2/f*d/(c
^2+d^2)^(1/2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2
)-(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*C*a
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b \tan \left(e + fx\right)\right) \left(A + B \tan \left(e + fx\right) + C \tan^2 \left(e + fx\right)\right)}{\sqrt{c + d \tan \left(e + fx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e)
)**(1/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/sqrt
(c + d*tan(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C \tan \left(fx + e\right)^2 + B \tan \left(fx + e\right) + A\right)\left(b \tan \left(fx + e\right) + a\right)}{\sqrt{d \tan \left(fx + e\right) + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)/sqrt
(d*tan(f*x + e) + c), x)
```

3.113 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=133

$$-\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f\sqrt{c-id}}-\frac{(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f\sqrt{c+id}}+\frac{2C\sqrt{c+d\tan(e+fx)}}{df}$$

```
[Out] -(((I*A + B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/(Sqrt[c
 - I*d]*f)) - ((B - I*(A - C))*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*
d]])/(Sqrt[c + I*d]*f) + (2*C*Sqrt[c + d*Tan[e + f*x]])/(d*f)
```

Rubi [A] time = 0.215701, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3630, 3539, 3537, 63, 208}

$$-\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f\sqrt{c-id}}-\frac{(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f\sqrt{c+id}}+\frac{2C\sqrt{c+d\tan(e+fx)}}{df}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] -(((I*A + B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/(Sqrt[c
 - I*d]*f)) - ((B - I*(A - C))*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*
d]])/(Sqrt[c + I*d]*f) + (2*C*Sqrt[c + d*Tan[e + f*x]])/(d*f)
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx = \frac{2C\sqrt{c+d\tan(e+fx)}}{df} + \int \frac{A-C+B\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx$$

$$= \frac{2C\sqrt{c+d\tan(e+fx)}}{df} + \frac{1}{2}(A-iB-C)\int \frac{1+i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx + \frac{1}{2}(A+$$

$$= \frac{2C\sqrt{c+d\tan(e+fx)}}{df} + \frac{(iA+B-iC)\,\mathrm{Subst}\left(\int \frac{1}{(-1+x)\sqrt{c-idx}}\,dx, x, i\tan(e\right.}{2f}$$

$$= \frac{2C\sqrt{c+d\tan(e+fx)}}{df} - \frac{(A-iB-C)\,\mathrm{Subst}\left(\int \frac{1}{-1-\frac{ic}{d}+\frac{ix^2}{d}}\,dx, x, \sqrt{c+d\tan\right.}{df}$$

$$= -\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}f} - \frac{(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+}}{\sqrt{c+id}}\right.}{\sqrt{c+id}f}$$

Mathematica [A] time = 0.21263, size = 129, normalized size = 0.97

$$\frac{-\frac{i(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}} + \frac{i(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{\sqrt{c+id}} + \frac{2C\sqrt{c+d\tan(e+fx)}}{d}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/Sqrt[c + d*Tan[e + f*x]],
x]
```

```
[Out] (((-I)*(A - I*B - C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/Sqrt[
c - I*d] + (I*(A + I*B - C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]]
)/Sqrt[c + I*d] + (2*C*Sqrt[c + d*Tan[e + f*x]])/d)/f
```

Maple [B] time = 0.14, size = 5570, normalized size = 41.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \tan(fx+e)^2 + B \tan(fx+e) + A}{\sqrt{d \tan(fx+e) + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)/sqrt(d*tan(f*x + e) + c),
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B \tan(e+fx) + C \tan^2(e+fx)}{\sqrt{c + d \tan(e+fx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/sqrt(c + d*tan(e + f*x)),
 x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \tan(fx+e)^2 + B \tan(fx+e) + A}{\sqrt{d \tan(fx+e) + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)/sqrt(d*tan(f*x + e) + c),
 x)
```

3.114 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=210

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{\sqrt{b}f\left(a^2+b^2\right)\sqrt{bc-ad}}-\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(a-ib)\sqrt{c-id}}-\frac{(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(-b+ia)\sqrt{c+id}}$$

```
[Out] -(((I*A + B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((a - I
*b)*Sqrt[c - I*d]*f)) - ((A + I*B - C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqr
t[c + I*d]])/((I*a - b)*Sqrt[c + I*d]*f) - (2*(A*b^2 - a*(b*B - a*C))*ArcTa
nh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(Sqrt[b]*(a^2 + b^2
)*Sqrt[b*c - a*d]*f)
```

Rubi [A] time = 0.614801, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {3653, 3539, 3537, 63, 208, 3634}

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{\sqrt{b}f\left(a^2+b^2\right)\sqrt{bc-ad}}-\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(a-ib)\sqrt{c-id}}-\frac{(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(-b+ia)\sqrt{c+id}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])*Sqrt[c +
d*Tan[e + f*x]]),x]
```


```
[Out] -(((I*A + B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((a - I
*b)*Sqrt[c - I*d]*f)) - ((A + I*B - C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqr
t[c + I*d]])/((I*a - b)*Sqrt[c + I*d]*f) - (2*(A*b^2 - a*(b*B - a*C))*ArcTa
nh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(Sqrt[b]*(a^2 + b^2
)*Sqrt[b*c - a*d]*f)
```


Rule 3653

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[e + f*x])^n*(1 + Tan[e
+ f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
```

```
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3634

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}\,dx = \frac{\int \frac{bB+a(A-C)-(Ab-aB-bC)\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx}{a^2+b^2} + \frac{\left(Ab^2-abB+a^2C\right)\int \frac{1+\tan\ldots}{(a+b\tan(e+f\ldots}}{a^2+b^2}\ldots$$

$$= \frac{(A-iB-C)\int \frac{1+i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx}{2(a-ib)} + \frac{(A+iB-C)\int \frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx}{2(a+ib)} + \ldots$$

$$= -\frac{(i(A+iB-C))\,\mathrm{Subst}\left(\int \frac{1}{(-1+x)\sqrt{c+idx}}\,dx, x, -i\tan(e+fx)\right)}{2(a+ib)f} + \frac{(iA+\ldots}{\ldots}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\left(a^2+b^2\right)\sqrt{bc-ad}f} - \frac{(A-iB-C)\,\mathrm{Subs}\ldots}{\ldots}$$

$$= -\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)\sqrt{c-id}f} - \frac{(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\,\mathrm{ta}\ldots}}{\sqrt{c}\ldots}\right)}{(ia-b)\sqrt{c+id}f}\ldots$$

Mathematica [A] time = 0.373487, size = 194, normalized size = 0.92

$$\frac{-\frac{2\left(a(aC-bB)+Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}} + \frac{(b-ia)(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}} + \frac{(b+ia)(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{\sqrt{c+id}}}{f\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])*Sqr
t[c + d*Tan[e + f*x]]),x]
```

```
[Out] ((((-I)*a + b)*(A - I*B - C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]
])/Sqrt[c - I*d] + ((I*a + b)*(A + I*B - C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]
]/Sqrt[c + I*d]])/Sqrt[c + I*d] - (2*(A*b^2 + a*(-(b*B) + a*C))*ArcTanh[(Sq
rt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(Sqrt[b]*Sqrt[b*c - a*d])
)/((a^2 + b^2)*f)
```

Maple [B] time = 0.193, size = 13474, normalized size = 64.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))
,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f
*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f
*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\tan\left(e + fx\right) + C\tan^2\left(e + fx\right)}{\left(a + b\tan\left(e + fx\right)\right)\sqrt{c + d\tan\left(e + fx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e))**(1/2)/(a+b*tan
(f*x+e)),x)
```

```
[Out] Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/((a + b*tan(e + f*x))*sqr
t(c + d*tan(e + f*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\tan\left(fx+e\right)^2+B\tan\left(fx+e\right)+A}{\left(b\tan\left(fx+e\right)+a\right)\sqrt{d\tan\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f
*x+e)),x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)/((b*tan(f*x + e) + a)*sqr
t(d*tan(f*x + e) + c)), x)
```

3.115 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=327

$$-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))}-\frac{\left(-a^2b^2(5Ad+2Bc-3Cd)+3a^3bBd+a^4(-C)d+ab^3(4Ac-Bd-4cC)+b^4\right.}{\sqrt{b}f\left(a^2+b^2\right)^2(bc-ad)^{3/2}}$$

```
[Out] -(((I*A + B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((a - I
*b)^2*Sqrt[c - I*d]*f)) - ((B - I*(A - C))*ArcTanh[Sqrt[c + d*Tan[e + f*x]]
/Sqrt[c + I*d]])/((a + I*b)^2*Sqrt[c + I*d]*f) - ((3*a^3*b*B*d - a^4*C*d +
b^4*(2*B*c - A*d) + a*b^3*(4*A*c - 4*c*C - B*d) - a^2*b^2*(2*B*c + 5*A*d -
3*C*d))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(Sqrt[
b]*(a^2 + b^2)^2*(b*c - a*d)^(3/2)*f) - ((A*b^2 - a*(b*B - a*C))*Sqrt[c + d
*Tan[e + f*x]])/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x]))
```

Rubi [A] time = 1.37937, antiderivative size = 327, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.149, Rules used = {3649, 3653, 3539, 3537, 63, 208, 3634}

$$-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))}-\frac{\left(-a^2b^2(5Ad+2Bc-3Cd)+3a^3bBd+a^4(-C)d+ab^3(4Ac-Bd-4cC)+b^4\right.}{\sqrt{b}f\left(a^2+b^2\right)^2(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^2*Sqrt[c
+ d*Tan[e + f*x]]),x]
```


```
[Out] -(((I*A + B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((a - I
*b)^2*Sqrt[c - I*d]*f)) - ((B - I*(A - C))*ArcTanh[Sqrt[c + d*Tan[e + f*x]]
/Sqrt[c + I*d]])/((a + I*b)^2*Sqrt[c + I*d]*f) - ((3*a^3*b*B*d - a^4*C*d +
b^4*(2*B*c - A*d) + a*b^3*(4*A*c - 4*c*C - B*d) - a^2*b^2*(2*B*c + 5*A*d -
3*C*d))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(Sqrt[
b]*(a^2 + b^2)^2*(b*c - a*d)^(3/2)*f) - ((A*b^2 - a*(b*B - a*C))*Sqrt[c + d
*Tan[e + f*x]])/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x]))
```


Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
 b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3653

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
```

```
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[e + f*x])^n*(1 + Tan[e
+ f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3634

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))} - \frac{\int \frac{\frac{1}{2}\left(Ab^2d-2aA(bc-ad)-2(bB-aC)\left(b\right.\right.}{}}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))} - \frac{\int \frac{-\left(2abB+a^2(A-C)-b^2(A-C)\right)(bc-}{\sqrt{c}}}{\left(a^2+\right.}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))} + \frac{(A-iB-C)\int \frac{1+i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}}{2(a-ib)^2}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))} - \frac{(i(A+iB-C))\operatorname{Subst}\left(\int \frac{}{(-}\right.}{2(a}$$

$$= -\frac{\left(3a^3bBd-a^4Cd+b^4(2Bc-Ad)+ab^3(4Ac-4cC-Bd)-a^2b^2(2Bc+\right.}{\sqrt{b}\left(a^2+b^2\right)^2(bc-ad)^{3/2}f}$$

$$= -\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)^2\sqrt{c-id}f} - \frac{(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\tan}}{\sqrt{c}}\right.}{(a+ib)^2\sqrt{c+id}f}$$

Mathematica [A] time = 6.21476, size = 521, normalized size = 1.59

$$-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))} - \frac{\frac{2\sqrt{bc-ad}\left(\frac{1}{2}a^2d\left(Ab^2-a(bB-aC)\right)+\frac{1}{2}b^2\left(-2aA(bc-ad)-2(bB-aC)\left(bc-\frac{ad}{2}\right)+Ab^2d\right)-ab(bc-ad)(-aB+Ab-\right.}{\sqrt{b}f\left(a^2+b^2\right)(ad-bc)}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^2*S
qrt[c + d*Tan[e + f*x]]),x]
```

```
[Out] -((((I*Sqrt[c - I*d]*(I*(a^2*B - b^2*B - 2*a*b*(A - C))*(b*c - a*d) - (2*a*
b*B + a^2*(A - C) - b^2*(A - C))*(b*c - a*d))*ArcTanh[Sqrt[c + d*Tan[e + f*
x]]/Sqrt[c - I*d]])/((-c + I*d)*f) - (I*Sqrt[c + I*d]*((-I)*(a^2*B - b^2*B
- 2*a*b*(A - C))*(b*c - a*d) - (2*a*b*B + a^2*(A - C) - b^2*(A - C))*(b*c -
 a*d))*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/((-c - I*d)*f))/(a^
2 + b^2) + (2*Sqrt[b*c - a*d]*((a^2*(A*b^2 - a*(b*B - a*C))*d)/2 - a*b*(A*b
 - a*B - b*C)*(b*c - a*d) + (b^2*(A*b^2*d - 2*a*A*(b*c - a*d) - 2*(b*B - a*
C)*(b*c - (a*d)/2)))/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c
 - a*d]])/(Sqrt[b]*(a^2 + b^2)*(-(b*c) + a*d)*f))/((a^2 + b^2)*(b*c - a*d))
) - ((A*b^2 - a*(b*B - a*C))*Sqrt[c + d*Tan[e + f*x]])/((a^2 + b^2)*(b*c -
a*d)*f*(a + b*Tan[e + f*x]))
```

Maple [B] time = 0.222, size = 20870, normalized size = 63.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))
^2,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f
*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f
*x+e))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\tan\left(e + fx\right) + C\tan^{2}\left(e + fx\right)}{\left(a + b\tan\left(e + fx\right)\right)^{2}\sqrt{c + d\tan\left(e + fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e))**(1/2)/(a+b*tan
(f*x+e))**2,x)
```

```
[Out] Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/((a + b*tan(e + f*x))**2*
sqrt(c + d*tan(e + f*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\tan\left(fx + e\right)^{2} + B\tan\left(fx + e\right) + A}{\left(b\tan\left(fx + e\right) + a\right)^{2}\sqrt{d\tan\left(fx + e\right) + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f
*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)/((b*tan(f*x + e) + a)^2*s
qrt(d*tan(f*x + e) + c)), x)
```

3.116 $\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=511

$$\frac{2b\sqrt{c+d\tan(e+fx)}\left(6a^2d^2\left(d^2(5A+7C)-5Bcd+12c^2C\right)-15abd\left(cd^2(3A+5C)-6Bc^2d-3Bd^3+8c^3C\right)+b^2\left(6c\right.\right.}{15d^4f\left(c^2+d^2\right)}$$

```
[Out] -(((a - I*b)^3*(I*A + B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*
d]])/((c - I*d)^(3/2)*f)) - ((I*a - b)^3*(A + I*B - C)*ArcTanh[Sqrt[c + d*T
an[e + f*x]]/Sqrt[c + I*d]])/((c + I*d)^(3/2)*f) - (2*(c^2*C - B*c*d + A*d^
2)*(a + b*Tan[e + f*x])^3)/(d*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]]) + (2*
b*(6*a^2*d^2*(12*c^2*C - 5*B*c*d + (5*A + 7*C)*d^2) - 15*a*b*d*(8*c^3*C - 6
*B*c^2*d + c*(3*A + 5*C)*d^2 - 3*B*d^3) + b^2*(48*c^4*C - 40*B*c^3*d + 6*c^
2*(5*A + 3*C)*d^2 - 25*B*c*d^3 + 15*(A - C)*d^4))*Sqrt[c + d*Tan[e + f*x]])
/(15*d^4*(c^2 + d^2)*f) - (2*b^2*(4*(b*c - a*d)*(6*c^2*C - 5*B*c*d + (5*A +
 C)*d^2) - 5*d^2*((A - C)*(b*c - a*d) + B*(a*c + b*d)))*Tan[e + f*x]*Sqrt[c
 + d*Tan[e + f*x]])/(15*d^3*(c^2 + d^2)*f) + (2*b*(6*c^2*C - 5*B*c*d + (5*A
 + C)*d^2)*(a + b*Tan[e + f*x])^2*Sqrt[c + d*Tan[e + f*x]])/(5*d^2*(c^2 + d
^2)*f)
```

Rubi [A] time = 2.46494, antiderivative size = 511, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.17, Rules used = {3645, 3647, 3637, 3630, 3539, 3537, 63, 208}

$$\frac{2b\sqrt{c+d\tan(e+fx)}\left(6a^2d^2\left(d^2(5A+7C)-5Bcd+12c^2C\right)-15abd\left(cd^2(3A+5C)-6Bc^2d-3Bd^3+8c^3C\right)+b^2\left(6c\right.\right.}{15d^4f\left(c^2+d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d
*Tan[e + f*x])^(3/2),x]
```


```
[Out] -(((a - I*b)^3*(I*A + B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*
d]])/((c - I*d)^(3/2)*f)) - ((I*a - b)^3*(A + I*B - C)*ArcTanh[Sqrt[c + d*T
an[e + f*x]]/Sqrt[c + I*d]])/((c + I*d)^(3/2)*f) - (2*(c^2*C - B*c*d + A*d^
2)*(a + b*Tan[e + f*x])^3)/(d*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]]) + (2*
b*(6*a^2*d^2*(12*c^2*C - 5*B*c*d + (5*A + 7*C)*d^2) - 15*a*b*d*(8*c^3*C - 6
*B*c^2*d + c*(3*A + 5*C)*d^2 - 3*B*d^3) + b^2*(48*c^4*C - 40*B*c^3*d + 6*c^
2*(5*A + 3*C)*d^2 - 25*B*c*d^3 + 15*(A - C)*d^4))*Sqrt[c + d*Tan[e + f*x]])
/(15*d^4*(c^2 + d^2)*f) - (2*b^2*(4*(b*c - a*d)*(6*c^2*C - 5*B*c*d + (5*A +
 C)*d^2) - 5*d^2*((A - C)*(b*c - a*d) + B*(a*c + b*d)))*Tan[e + f*x]*Sqrt[c
 + d*Tan[e + f*x]])/(15*d^3*(c^2 + d^2)*f) + (2*b*(6*c^2*C - 5*B*c*d + (5*A
 + C)*d^2)*(a + b*Tan[e + f*x])^2*Sqrt[c + d*Tan[e + f*x]])/(5*d^2*(c^2 + d
^2)*f)
```


Rule 3645

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*d^2 + c*(c*C - B*d))*(a + b*Tan[e
+ f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
```

```
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^3}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{2\int}{}$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^3}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{2b(}{}$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^3}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} - \frac{2b^2}{}$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^3}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{2b(}{}$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^3}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{2b(}{}$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^3}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{2b(}{}$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^3}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{2b(}{}$$

$$= -\frac{(a-ib)^3(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{3/2}f} -$$

Mathematica [C] time = 6.7738, size = 920, normalized size = 1.8

$$\frac{2C(a+b\tan(e+fx))^3}{5df\sqrt{c+d\tan(e+fx)}}+\frac{2\left(\frac{(-6bcC+6adC+5bBd)(a+b\tan(e+fx))^2}{3df\sqrt{c+d\tan(e+fx)}}+\frac{2\left(\frac{\left(15b(Ab-Cb+aB)d^2+4(bc-ad)(6bcC-6adC-5bBd)\right)(a+b\tan(e+fx))}{2df\sqrt{c+d\tan(e+fx)}}+\frac{2\left(\frac{1}{2}\left(-15Ab^3d^3+45\cdots\right.\right.}{\cdots}\right)}{\cdots}\right)}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*C*(a + b*Tan[e + f*x])^3)/(5*d*f*Sqrt[c + d*Tan[e + f*x]]) + (2*(((-6*b*
c*C + 5*b*B*d + 6*a*C*d)*(a + b*Tan[e + f*x])^2)/(3*d*f*Sqrt[c + d*Tan[e +
f*x]]) + (2*(((15*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C - 5*b*B*
d - 6*a*C*d))*(a + b*Tan[e + f*x]))/(2*d*f*Sqrt[c + d*Tan[e + f*x]]) + ((-2
*(-48*b^3*c^3*C + 40*b^3*B*c^2*d + 144*a*b^2*c^2*C*d - 30*A*b^3*c*d^2 - 110
*a*b^2*B*c*d^2 - 144*a^2*b*c*C*d^2 + 30*b^3*c*C*d^2 + 60*a*A*b^2*d^3 + 85*a
^2*b*B*d^3 - 15*b^3*B*d^3 + 48*a^3*C*d^3 - 60*a*b^2*C*d^3))/(d*Sqrt[c + d*T
an[e + f*x]]) + (2*(((45*a^2*A*b*d^3 - 15*A*b^3*d^3 + 15*a^3*B*d^3 - 45*a*b
^2*B*d^3 - 45*a^2*b*C*d^3 + 15*b^3*C*d^3)*(((-I)*ArcTanh[Sqrt[c + d*Tan[e +
 f*x]]/Sqrt[c - I*d]])/Sqrt[c - I*d] + (I*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/
Sqrt[c + I*d]])/Sqrt[c + I*d]))/2 + ((-(c*d*(45*a^2*A*b*d^3 - 15*A*b^3*d^3
+ 15*a^3*B*d^3 - 45*a*b^2*B*d^3 - 45*a^2*b*C*d^3 + 15*b^3*C*d^3))/2 + d^2*(
(-48*b^3*c^3*C + 40*b^3*B*c^2*d + 144*a*b^2*c^2*C*d - 30*A*b^3*c*d^2 - 110*
a*b^2*B*c*d^2 - 144*a^2*b*c*C*d^2 + 30*b^3*c*C*d^2 + 15*a^3*A*d^3 + 15*a*A*
b^2*d^3 + 40*a^2*b*B*d^3 + 33*a^3*C*d^3 - 15*a*b^2*C*d^3)/2 + (48*b^3*c^3*C
 - 40*b^3*B*c^2*d - 144*a*b^2*c^2*C*d + 30*A*b^3*c*d^2 + 110*a*b^2*B*c*d^2
+ 144*a^2*b*c*C*d^2 - 30*b^3*c*C*d^2 - 60*a*A*b^2*d^3 - 85*a^2*b*B*d^3 + 15
*b^3*B*d^3 - 48*a^3*C*d^3 + 60*a*b^2*C*d^3)/2))*(-(Hypergeometric2F1[-1/2,
1, 1/2, (c + d*Tan[e + f*x])/(c - I*d)]/((I*c + d)*Sqrt[c + d*Tan[e + f*x]]
)) + Hypergeometric2F1[-1/2, 1, 1/2, (c + d*Tan[e + f*x])/(c + I*d)]/((I*c
- d)*Sqrt[c + d*Tan[e + f*x]])))/d))/d)/(4*d*f)))/(3*d)))/(5*d)
```

Maple [B] time = 0.244, size = 49725, normalized size = 97.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(3/
2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b\tan\left(e + fx\right)\right)^{3}\left(A + B\tan\left(e + fx\right) + C\tan^{2}\left(e + fx\right)\right)}{\left(c + d\tan\left(e + fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x
+e))**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**3*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/(
c + d*tan(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\tan\left(fx + e\right)^{2} + B\tan\left(fx + e\right) + A\right)\left(b\tan\left(fx + e\right) + a\right)^{3}}{\left(d\tan\left(fx + e\right) + c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^3/(d
*tan(f*x + e) + c)^(3/2), x)
```

3.117 $\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=343

$$\frac{2b\sqrt{c+d\tan(e+fx)}\left(6ad\left(d^2(A+C)-Bcd+2c^2C\right)-b\left(cd^2(3A+5C)-6Bc^2d-3Bd^3+8c^3C\right)\right)}{3d^3f\left(c^2+d^2\right)}-\frac{2\left(Ad^2-Bcd+\right.}{df\left(c^2+d^2\right.}$$

```
[Out] -(((a - I*b)^2*(I*A + B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*
d]])/((c - I*d)^(3/2)*f)) - ((a + I*b)^2*(B - I*(A - C))*ArcTanh[Sqrt[c + d
*Tan[e + f*x]]/Sqrt[c + I*d]])/((c + I*d)^(3/2)*f) - (2*(c^2*C - B*c*d + A*
d^2)*(a + b*Tan[e + f*x])^2)/(d*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]]) + (
2*b*(6*a*d*(2*c^2*C - B*c*d + (A + C)*d^2) - b*(8*c^3*C - 6*B*c^2*d + c*(3*
A + 5*C)*d^2 - 3*B*d^3))*Sqrt[c + d*Tan[e + f*x]])/(3*d^3*(c^2 + d^2)*f) +
(2*b^2*(4*c^2*C - 3*B*c*d + (3*A + C)*d^2)*Tan[e + f*x]*Sqrt[c + d*Tan[e +
f*x]])/(3*d^2*(c^2 + d^2)*f)
```

Rubi [A] time = 1.35366, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.149, Rules used = {3645, 3637, 3630, 3539, 3537, 63, 208}

$$\frac{2b\sqrt{c+d\tan(e+fx)}\left(6ad\left(d^2(A+C)-Bcd+2c^2C\right)-b\left(cd^2(3A+5C)-6Bc^2d-3Bd^3+8c^3C\right)\right)}{3d^3f\left(c^2+d^2\right)}-\frac{2\left(Ad^2-Bcd+\right.}{df\left(c^2+d^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d
*Tan[e + f*x])^(3/2),x]
```

```
[Out] -(((a - I*b)^2*(I*A + B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*
d]])/((c - I*d)^(3/2)*f)) - ((a + I*b)^2*(B - I*(A - C))*ArcTanh[Sqrt[c + d
*Tan[e + f*x]]/Sqrt[c + I*d]])/((c + I*d)^(3/2)*f) - (2*(c^2*C - B*c*d + A*
d^2)*(a + b*Tan[e + f*x])^2)/(d*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]]) + (
2*b*(6*a*d*(2*c^2*C - B*c*d + (A + C)*d^2) - b*(8*c^3*C - 6*B*c^2*d + c*(3*
A + 5*C)*d^2 - 3*B*d^3))*Sqrt[c + d*Tan[e + f*x]])/(3*d^3*(c^2 + d^2)*f) +
(2*b^2*(4*c^2*C - 3*B*c*d + (3*A + C)*d^2)*Tan[e + f*x]*Sqrt[c + d*Tan[e +
f*x]])/(3*d^2*(c^2 + d^2)*f)
```

Rule 3645

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*d^2 + c*(c*C - B*d))*(a + b*Tan[e
+ f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
```

```
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^2}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}}+\frac{2\int}{}$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^2}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}}+\frac{2b^2}{}$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^2}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}}+\frac{2b(}{}$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^2}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}}+\frac{2b(}{}$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^2}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}}+\frac{2b(}{}$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^2}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}}+\frac{2b(}{}$$

$$= -\frac{(a-ib)^2(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{3/2}f}-$$

Mathematica [C] time = 6.57688, size = 476, normalized size = 1.39

$$\frac{2C(a+b\tan(e+fx))^2}{3df\sqrt{c+d\tan(e+fx)}}+\frac{2\left(\frac{(4aCd+3bBd-4bcC)(a+b\tan(e+fx))}{df\sqrt{c+d\tan(e+fx)}}+\frac{-\frac{2\left(8a^2Cd^2+9abBd^2-16abcCd+3Ab^2d^2-6b^2Bcd+8b^2c^2C-3b^2Cd^2\right)}{d\sqrt{c+d\tan(e+fx)}}+\frac{2\left(\left(-\frac{3}{2}cd^3\left(a^2B+2ab\right.\right.\right.}{}}{}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*C*(a + b*Tan[e + f*x])^2)/(3*d*f*Sqrt[c + d*Tan[e + f*x]]) + (2*(((-4*b*
c*C + 3*b*B*d + 4*a*C*d)*(a + b*Tan[e + f*x]))/(d*f*Sqrt[c + d*Tan[e + f*x]
]) + ((-2*(8*b^2*c^2*C - 6*b^2*B*c*d - 16*a*b*c*C*d + 3*A*b^2*d^2 + 9*a*b*B
*d^2 + 8*a^2*C*d^2 - 3*b^2*C*d^2))/(d*Sqrt[c + d*Tan[e + f*x]]) + (2*((3*(a
^2*B - b^2*B + 2*a*b*(A - C))*d^2*(((-I)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/S
qrt[c - I*d]])/Sqrt[c - I*d] + (I*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c +
 I*d]])/Sqrt[c + I*d]))/2 + (((-3*c*(a^2*B - b^2*B + 2*a*b*(A - C))*d^3)/2
- (3*(2*a*b*B - a^2*(A - C) + b^2*(A - C))*d^4)/2)*(-(Hypergeometric2F1[-1/
2, 1, 1/2, (c + d*Tan[e + f*x]])/(c - I*d)]/((I*c + d)*Sqrt[c + d*Tan[e + f*
x]])) + Hypergeometric2F1[-1/2, 1, 1/2, (c + d*Tan[e + f*x])/(c + I*d)]/((I
```

```
*c - d)*Sqrt[c + d*Tan[e + f*x]]))/d))/d)/(2*d*f)))/(3*d)
```

Maple [B] time = 0.198, size = 36710, normalized size = 107.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(3/
2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b \tan \left(e + fx\right)\right)^{2} \left(A + B \tan \left(e + fx\right) + C \tan^{2} \left(e + fx\right)\right)}{\left(c + d \tan \left(e + fx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x
+e))**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**2*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/(
c + d*tan(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\tan\left(fx+e\right)^{2}+B\tan\left(fx+e\right)+A\right)\left(b\tan\left(fx+e\right)+a\right)^{2}}{\left(d\tan\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^2/(d
*tan(f*x + e) + c)^(3/2), x)
```

3.118 $\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=201

$$\frac{2(bc-ad)\left(Ad^2-Bcd+c^2C\right)}{d^2f\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}-\frac{(b+ia)(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(c-id)^{3/2}}+\frac{(-b+ia)(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(c+id)^{3/2}}$$

```
[Out] -(((I*a + b)*(A - I*B - C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])
/((c - I*d)^(3/2)*f)) + ((I*a - b)*(A + I*B - C)*ArcTanh[Sqrt[c + d*Tan[e +
 f*x]]/Sqrt[c + I*d]])/((c + I*d)^(3/2)*f) + (2*(b*c - a*d)*(c^2*C - B*c*d
+ A*d^2))/(d^2*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]]) + (2*b*C*Sqrt[c + d*
Tan[e + f*x]])/(d^2*f)
```

Rubi [A] time = 0.554369, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3635, 3630, 3539, 3537, 63, 208}

$$\frac{2(bc-ad)\left(Ad^2-Bcd+c^2C\right)}{d^2f\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}-\frac{(b+ia)(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(c-id)^{3/2}}+\frac{(-b+ia)(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(c+id)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d*T
an[e + f*x])^(3/2),x]
```

```
[Out] -(((I*a + b)*(A - I*B - C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])
/((c - I*d)^(3/2)*f)) + ((I*a - b)*(A + I*B - C)*ArcTanh[Sqrt[c + d*Tan[e +
 f*x]]/Sqrt[c + I*d]])/((c + I*d)^(3/2)*f) + (2*(b*c - a*d)*(c^2*C - B*c*d
+ A*d^2))/(d^2*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]]) + (2*b*C*Sqrt[c + d*
Tan[e + f*x]])/(d^2*f)
```

Rule 3635

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(c^2*C - B*c*d + A*d^2)*(c +
d*Tan[e + f*x])^(n + 1))/(d^2*f*(n + 1)*(c^2 + d^2)), x] + Dist[1/(d*(c^2 +
 d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(c^
2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*Ta
n[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -
1]
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx = \frac{2(bc-ad)\left(c^2C-Bcd+Ad^2\right)}{d^2\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{\int \frac{ad(Ac-cC+Bd)\cdot}{}}{}$$

$$= \frac{2(bc-ad)\left(c^2C-Bcd+Ad^2\right)}{d^2\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{2bC\sqrt{c+d\tan}}{d^2f}$$

$$= \frac{2(bc-ad)\left(c^2C-Bcd+Ad^2\right)}{d^2\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{2bC\sqrt{c+d\tan}}{d^2f}$$

$$= \frac{2(bc-ad)\left(c^2C-Bcd+Ad^2\right)}{d^2\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{2bC\sqrt{c+d\tan}}{d^2f}$$

$$= \frac{2(bc-ad)\left(c^2C-Bcd+Ad^2\right)}{d^2\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{2bC\sqrt{c+d\tan}}{d^2f}$$

$$= -\frac{(ia+b)(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{3/2}f} + \frac{(i}{}$$

Mathematica [C] time = 2.39151, size = 290, normalized size = 1.44

$$\frac{\frac{(-aAd+aBc+aCd+Abc+bBd-bcC)\left((d-ic)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{c+d\tan(e+fx)}{c-id}\right)+(d+ic)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{c+d\tan(e+fx)}{c+id}\right)\right)}{\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}} + (aB + A}{df}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c
 + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((A*b + a*B - b*C)*(((-I)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/
Sqrt[c - I*d] + (I*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/Sqrt[c
+ I*d]) - (2*(-2*b*c*C + b*B*d + 2*a*C*d))/(d*Sqrt[c + d*Tan[e + f*x]]) + (
(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*(((-I)*c + d)*Hypergeometri
c2F1[-1/2, 1, 1/2, (c + d*Tan[e + f*x])/(c - I*d)] + (I*c + d)*Hypergeometr
ic2F1[-1/2, 1, 1/2, (c + d*Tan[e + f*x])/(c + I*d)]))/((c^2 + d^2)*Sqrt[c +
 d*Tan[e + f*x]]) + (2*C*(a + b*Tan[e + f*x]))/Sqrt[c + d*Tan[e + f*x]])/(d
*f)
```

Maple [B] time = 0.164, size = 23472, normalized size = 116.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(3/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b \tan{\left(e + f x \right)}\right) \left(A + B \tan{\left(e + f x \right)} + C \tan^{2}{\left(e + f x \right)}\right)}{\left(c + d \tan{\left(e + f x \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e)
)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/(c +
 d*tan(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\tan\left(fx+e\right)^2 + B\tan\left(fx+e\right) + A\right)\left(b\tan\left(fx+e\right) + a\right)}{\left(d\tan\left(fx+e\right) + c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)/(d*t
an(f*x + e) + c)^(3/2), x)
```

3.119 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=157

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}-\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(c-id)^{3/2}}-\frac{(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(c+id)^{3/2}}$$

```
[Out] -(((I*A + B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((c - I
*d)^(3/2)*f)) - ((B - I*(A - C))*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c +
I*d]])/((c + I*d)^(3/2)*f) - (2*(c^2*C - B*c*d + A*d^2))/(d*(c^2 + d^2)*f*S
qrt[c + d*Tan[e + f*x]])
```

Rubi [A] time = 0.293992, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3628, 3539, 3537, 63, 208}

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}-\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(c-id)^{3/2}}-\frac{(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(c+id)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] -(((I*A + B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((c - I
*d)^(3/2)*f)) - ((B - I*(A - C))*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c +
I*d]])/((c + I*d)^(3/2)*f) - (2*(c^2*C - B*c*d + A*d^2))/(d*(c^2 + d^2)*f*S
qrt[c + d*Tan[e + f*x]])
```

Rule 3628

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2
- a*b*B + a^2*C)*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 + b^2)), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{2\left(c^2C-Bcd+Ad^2\right)}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}}+\frac{\int\frac{Ac-cC+Bd+(Bc-(A-C)d)\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx}{c^2+d^2}$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}}+\frac{(A-iB-C)\int\frac{1+i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx}{2(c-id)}+ \cdots$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}}+\frac{(iA+B-iC)\,\mathrm{Subst}\left(\int\frac{1}{(-1+x)\sqrt{c-idx}} \cdots\right)}{2(c-id)f}$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}}-\frac{(A-iB-C)\,\mathrm{Subst}\left(\int\frac{1}{-1-\frac{ic}{d}+\frac{ix^2}{d}}\,dx, \cdots\right)}{(c-id)df}$$

$$= -\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{3/2}f}-\frac{(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\tan \cdots}}{\sqrt{c \cdots}}\right)}{(c+id)^{3/2}f}$$

Mathematica [C] time = 0.92493, size = 218, normalized size = 1.39

$$\frac{\frac{(d(C-A)+Bc)\left((d-ic)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{c+d\tan(e+fx)}{c-id}\right)+(d+ic)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{c+d\tan(e+fx)}{c+id}\right)\right)}{\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}-iB\left(\frac{\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}}- \cdots\right)}{df}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(c + d*Tan[e + f*x])^(3/2
),x]
```

```
[Out] ((-I)*B*(ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]]/Sqrt[c - I*d] - Ar
cTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]]/Sqrt[c + I*d]) - (2*C)/Sqrt[c
 + d*Tan[e + f*x]] + ((B*c + (-A + C)*d)*(((-I)*c + d)*Hypergeometric2F1[-1
/2, 1, 1/2, (c + d*Tan[e + f*x])/(c - I*d)] + (I*c + d)*Hypergeometric2F1[-
1/2, 1, 1/2, (c + d*Tan[e + f*x])/(c + I*d)]))/((c^2 + d^2)*Sqrt[c + d*Tan[
e + f*x]]))/(d*f)
```

Maple [B] time = 0.125, size = 11427, normalized size = 72.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(3/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(3/2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\tan\left(e + fx\right) + C\tan^2\left(e + fx\right)}{\left(c + d\tan\left(e + fx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/(c + d*tan(e + f*x))**(3/
2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\tan\left(fx + e\right)^2 + B\tan\left(fx + e\right) + A}{\left(d\tan\left(fx + e\right) + c\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)/(d*tan(f*x + e) + c)^(3/2
), x)
```

3.120 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=262

$$-\frac{2\sqrt{b}\left(Ab^2-a(bB-aC)\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{f\left(a^2+b^2\right)(bc-ad)^{3/2}}+\frac{2\left(Ad^2-Bcd+c^2C\right)}{f\left(c^2+d^2\right)(bc-ad)\sqrt{c+d\tan(e+fx)}}+\frac{(A-iB-C)\tanh^{-1}\left(\cdots\right)}{f(b+ia)(c-i\cdots}$$

```
[Out] ((A - I*B - C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((I*a + b)*
(c - I*d)^(3/2)*f) + ((I*A - B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt
[c + I*d]])/((a + I*b)*(c + I*d)^(3/2)*f) - (2*Sqrt[b]*(A*b^2 - a*(b*B - a*
C))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/((a^2 + b^
2)*(b*c - a*d)^(3/2)*f) + (2*(c^2*C - B*c*d + A*d^2))/((b*c - a*d)*(c^2 + d
^2)*f*Sqrt[c + d*Tan[e + f*x]])
```

Rubi [A] time = 1.27748, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.149, Rules used = {3649, 3653, 3539, 3537, 63, 208, 3634}

$$-\frac{2\sqrt{b}\left(Ab^2-a(bB-aC)\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{f\left(a^2+b^2\right)(bc-ad)^{3/2}}+\frac{2\left(Ad^2-Bcd+c^2C\right)}{f\left(c^2+d^2\right)(bc-ad)\sqrt{c+d\tan(e+fx)}}+\frac{(A-iB-C)\tanh^{-1}\left(\cdots\right)}{f(b+ia)(c-i\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])*(c + d*Ta
n[e + f*x])^(3/2)),x]
```

```
[Out] ((A - I*B - C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((I*a + b)*
(c - I*d)^(3/2)*f) + ((I*A - B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt
[c + I*d]])/((a + I*b)*(c + I*d)^(3/2)*f) - (2*Sqrt[b]*(A*b^2 - a*(b*B - a*
C))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/((a^2 + b^
2)*(b*c - a*d)^(3/2)*f) + (2*(c^2*C - B*c*d + A*d^2))/((b*c - a*d)*(c^2 + d
^2)*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
 b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3653

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
```

```
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[e + f*x])^n*(1 + Tan[e
+ f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3634

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}\,dx = \frac{2\left(c^2C-Bcd+Ad^2\right)}{(bc-ad)\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{2\int \frac{\frac{1}{2}\left(-aAcd+ad(cC-Bd)+Ab\left(c^2+d\right.\right.}{}}{}$$

$$= \frac{2\left(c^2C-Bcd+Ad^2\right)}{(bc-ad)\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{\left(b\left(Ab^2-a(bB-aC)\right)\right)\int \frac{}{(a}}{\left(a^2+b^2\right)(}$$

$$= \frac{2\left(c^2C-Bcd+Ad^2\right)}{(bc-ad)\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{(A-iB-C)\int \frac{1+i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}}{2(a-ib)(c-id)}$$

$$= \frac{2\left(c^2C-Bcd+Ad^2\right)}{(bc-ad)\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{(iA+B-iC)\,\mathrm{Subst}\left(\int \frac{}{(-1+}\right.}{2(a-ib)}$$

$$= -\frac{2\sqrt{b}\left(Ab^2-a(bB-aC)\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{\left(a^2+b^2\right)(bc-ad)^{3/2}f} + \frac{2\left(c^2C\right.}{(bc-ad)\left(c^2+c\right.}$$

$$= -\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)(c-id)^{3/2}f} - \frac{(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan}}{\sqrt{c+}}\right)}{(ia-b)(c+id)^{3/2}f}$$

Mathematica [A] time = 4.73414, size = 296, normalized size = 1.13

$$\frac{\frac{2\sqrt{b}\left(c^2+d^2\right)\left(a(aC-bB)+Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{\left(a^2+b^2\right)\sqrt{bc-ad}} - \frac{i\left(\frac{(a+ib)(c+id)(A-iB-C)(ad-bc)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}} + \frac{(a-ib)(c-id)(A+iB-C)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{\sqrt{c+id}}\right.}{a^2+b^2}}{f\left(c^2+d^2\right)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])*(c
+ d*Tan[e + f*x])^(3/2)),x]
```

```
[Out] (((-I)*(((a + I*b)*(A - I*B - C)*(c + I*d)*(-(b*c) + a*d)*ArcTanh[Sqrt[c +
d*Tan[e + f*x]]/Sqrt[c - I*d]])/Sqrt[c - I*d] + ((a - I*b)*(A + I*B - C)*(c
 - I*d)*(b*c - a*d)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/Sqrt[c
 + I*d]))/(a^2 + b^2) + (2*Sqrt[b]*(A*b^2 + a*(-(b*B) + a*C))*(c^2 + d^2)*A
rcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/((a^2 + b^2)*Sq
rt[b*c - a*d]) - (2*(c^2*C - B*c*d + A*d^2))/Sqrt[c + d*Tan[e + f*x]])/((-(
b*c) + a*d)*(c^2 + d^2)*f)
```

Maple [B] time = 0.216, size = 26343, normalized size = 100.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))^(3/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))
^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))
^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+B\tan\left(e+fx\right)+C\tan^{2}\left(e+fx\right)}{\left(a+b\tan\left(e+fx\right)\right)\left(c+d\tan\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e)
)**(3/2),x)
```

```
[Out] Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/((a + b*tan(e + f*x))*(c
+ d*tan(e + f*x))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\tan\left(fx+e\right)^{2}+B\tan\left(fx+e\right)+A}{\left(b\tan\left(fx+e\right)+a\right)\left(d\tan\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)/((b*tan(f*x + e) + a)*(d*
tan(f*x + e) + c)^(3/2)), x)
```

3.121 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=447

$$-\frac{d\left(A\left(2a^2d^2+b^2\left(c^2+3d^2\right)\right)+a^2\left(-2Bcd+3c^2C+Cd^2\right)-abB\left(c^2+d^2\right)+2b^2c(cC-Bd)\right)}{f\left(a^2+b^2\right)\left(c^2+d^2\right)(bc-ad)^2\sqrt{c+d\tan(e+fx)}}-\frac{Ab^2\cdots}{f\left(a^2+b^2\right)(bc-ad)(a+\cdots}$$

```
[Out] -(((I*A + B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((a - I
*b)^2*(c - I*d)^(3/2)*f)) - ((B - I*(A - C))*ArcTanh[Sqrt[c + d*Tan[e + f*x
]]/Sqrt[c + I*d]])/((a + I*b)^2*(c + I*d)^(3/2)*f) - (Sqrt[b]*(5*a^3*b*B*d
- 3*a^4*C*d + b^4*(2*B*c - 3*A*d) + a*b^3*(4*A*c - 4*c*C + B*d) - a^2*b^2*(
2*B*c + (7*A - C)*d))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c -
 a*d]])/((a^2 + b^2)^2*(b*c - a*d)^(5/2)*f) - (d*(2*b^2*c*(c*C - B*d) - a*b
*B*(c^2 + d^2) + a^2*(3*c^2*C - 2*B*c*d + C*d^2) + A*(2*a^2*d^2 + b^2*(c^2
+ 3*d^2))))/((a^2 + b^2)*(b*c - a*d)^2*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x
]]) - (A*b^2 - a*(b*B - a*C))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x
])*Sqrt[c + d*Tan[e + f*x]])
```

Rubi [A] time = 2.88148, antiderivative size = 446, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.149, Rules used = {3649, 3653, 3539, 3537, 63, 208, 3634}

$$-\frac{d\left(2a^2Ad^2+a^2\left(-2Bcd+3c^2C+Cd^2\right)-abB\left(c^2+d^2\right)+Ab^2\left(c^2+3d^2\right)+2b^2c(cC-Bd)\right)}{f\left(a^2+b^2\right)\left(c^2+d^2\right)(bc-ad)^2\sqrt{c+d\tan(e+fx)}}-\frac{Ab^2\cdots}{f\left(a^2+b^2\right)(bc-ad)(a+\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^2*(c + d*
Tan[e + f*x])^(3/2)),x]
```

```
[Out] -(((I*A + B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((a - I
*b)^2*(c - I*d)^(3/2)*f)) - ((B - I*(A - C))*ArcTanh[Sqrt[c + d*Tan[e + f*x
]]/Sqrt[c + I*d]])/((a + I*b)^2*(c + I*d)^(3/2)*f) - (Sqrt[b]*(5*a^3*b*B*d
- 3*a^4*C*d + b^4*(2*B*c - 3*A*d) + a*b^3*(4*A*c - 4*c*C + B*d) - a^2*b^2*(
2*B*c + (7*A - C)*d))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c -
 a*d]])/((a^2 + b^2)^2*(b*c - a*d)^(5/2)*f) - (d*(2*a^2*A*d^2 + 2*b^2*c*(c*
C - B*d) - a*b*B*(c^2 + d^2) + A*b^2*(c^2 + 3*d^2) + a^2*(3*c^2*C - 2*B*c*d
 + C*d^2)))/((a^2 + b^2)*(b*c - a*d)^2*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x
]]) - (A*b^2 - a*(b*B - a*C))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x
])*Sqrt[c + d*Tan[e + f*x]])
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
 b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
```

```
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3653

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[e + f*x])^n*(1 + Tan[e
+ f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3634

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{Ab^2-a(bB-aC)}{\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}} - \frac{\int \frac{\frac{1}{2}(3A}{}}{}$$

$$= -\frac{d\left(2a^2Ad^2+2b^2c(cC-Bd)-abB\left(c^2+d^2\right)+Ab^2\left(c^2+3d^2\right)+a^2\left(3\right.\right.}{\left(a^2+b^2\right)(bc-ad)^2\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx}}$$

$$= -\frac{d\left(2a^2Ad^2+2b^2c(cC-Bd)-abB\left(c^2+d^2\right)+Ab^2\left(c^2+3d^2\right)+a^2\left(3\right.\right.}{\left(a^2+b^2\right)(bc-ad)^2\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx}}$$

$$= -\frac{d\left(2a^2Ad^2+2b^2c(cC-Bd)-abB\left(c^2+d^2\right)+Ab^2\left(c^2+3d^2\right)+a^2\left(3\right.\right.}{\left(a^2+b^2\right)(bc-ad)^2\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx}}$$

$$= -\frac{d\left(2a^2Ad^2+2b^2c(cC-Bd)-abB\left(c^2+d^2\right)+Ab^2\left(c^2+3d^2\right)+a^2\left(3\right.\right.}{\left(a^2+b^2\right)(bc-ad)^2\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx}}$$

$$= -\frac{\sqrt{b}\left(5a^3bBd-3a^4Cd+b^4(2Bc-3Ad)+ab^3(4Ac-4cC+Bd)-a^2b\right.}{\left(a^2+b^2\right)^2(bc-ad)^{5/2}f}$$

$$= -\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)^2(c-id)^{3/2}f} - \frac{(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d}}{}\right.}{(a+ib)^2(c+id)^{3/2}f}$$

Mathematica [B] time = 6.25202, size = 2078, normalized size = 4.65

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^2*(
c + d*Tan[e + f*x])^(3/2)),x]
```

```
[Out] -((A*b^2 - a*(b*B - a*C))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])*S
qrt[c + d*Tan[e + f*x]])) - ((-2*(((I*Sqrt[c - I*d]*((b*(-(b*c) + a*d)*((-3
*(A*b^2 - a*(b*B - a*C))*d^2)/2 - c*(A*b - a*B - b*C)*(b*c - a*d) + (d*(3*A
*b^2*d - 2*a*A*(b*c - a*d) - (b*B - a*C)*(2*b*c + a*d)))/2))/2 + a*(-(a*d*(
(-3*c*(A*b^2 - a*(b*B - a*C))*d)/2 + (A*b - a*B - b*C)*d*(b*c - a*d)))/2 +
(((b*d^2)/2 - (c*(-(b*c) + a*d))/2)*(3*A*b^2*d - 2*a*A*(b*c - a*d) - (b*B -
 a*C)*(2*b*c + a*d)))/2 - (b*(-(c*((-3*c*(A*b^2 - a*(b*B - a*C))*d)/2 + (A*
b - a*B - b*C)*d*(b*c - a*d))) + (d^2*(3*A*b^2*d - 2*a*A*(b*c - a*d) - (b*B
 - a*C)*(2*b*c + a*d)))/2))/2) - I*((a*(-(b*c) + a*d)*((-3*(A*b^2 - a*(b*B
- a*C))*d^2)/2 - c*(A*b - a*B - b*C)*(b*c - a*d) + (d*(3*A*b^2*d - 2*a*A*(b
*c - a*d) - (b*B - a*C)*(2*b*c + a*d)))/2))/2 - b*(-(a*d*((-3*c*(A*b^2 - a*
(b*B - a*C))*d)/2 + (A*b - a*B - b*C)*d*(b*c - a*d)))/2 + (((b*d^2)/2 - (c*
(-(b*c) + a*d))/2)*(3*A*b^2*d - 2*a*A*(b*c - a*d) - (b*B - a*C)*(2*b*c + a*
d)))/2 - (b*(-(c*((-3*c*(A*b^2 - a*(b*B - a*C))*d)/2 + (A*b - a*B - b*C)*d*
(b*c - a*d))) + (d^2*(3*A*b^2*d - 2*a*A*(b*c - a*d) - (b*B - a*C)*(2*b*c +
a*d)))/2))/2)))*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((-c + I*d
)*f) - (I*Sqrt[c + I*d]*((b*(-(b*c) + a*d)*((-3*(A*b^2 - a*(b*B - a*C))*d^2
)/2 - c*(A*b - a*B - b*C)*(b*c - a*d) + (d*(3*A*b^2*d - 2*a*A*(b*c - a*d) -
 (b*B - a*C)*(2*b*c + a*d)))/2))/2 + a*(-(a*d*((-3*c*(A*b^2 - a*(b*B - a*C)
)*d)/2 + (A*b - a*B - b*C)*d*(b*c - a*d)))/2 + (((b*d^2)/2 - (c*(-(b*c) + a
```

```
*d))/2)*(3*A*b^2*d - 2*a*A*(b*c - a*d) - (b*B - a*C)*(2*b*c + a*d)))/2 - (b
*(-(c*((-3*c*(A*b^2 - a*(b*B - a*C))*d)/2 + (A*b - a*B - b*C)*d*(b*c - a*d)
)) + (d^2*(3*A*b^2*d - 2*a*A*(b*c - a*d) - (b*B - a*C)*(2*b*c + a*d)))/2))/
2) + I*((a*(-(b*c) + a*d)*((-3*(A*b^2 - a*(b*B - a*C))*d^2)/2 - c*(A*b - a*
B - b*C)*(b*c - a*d) + (d*(3*A*b^2*d - 2*a*A*(b*c - a*d) - (b*B - a*C)*(2*b
*c + a*d)))/2))/2 - b*(-(a*d*((-3*c*(A*b^2 - a*(b*B - a*C))*d)/2 + (A*b - a
*B - b*C)*d*(b*c - a*d)))/2 + (((b*d^2)/2 - (c*(-(b*c) + a*d))/2)*(3*A*b^2*
d - 2*a*A*(b*c - a*d) - (b*B - a*C)*(2*b*c + a*d)))/2 - (b*(-(c*((-3*c*(A*b
^2 - a*(b*B - a*C))*d)/2 + (A*b - a*B - b*C)*d*(b*c - a*d))) + (d^2*(3*A*b^
2*d - 2*a*A*(b*c - a*d) - (b*B - a*C)*(2*b*c + a*d)))/2))/2)))*ArcTanh[Sqrt
[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/((-c - I*d)*f))/(a^2 + b^2) + (2*Sqrt[
b*c - a*d]*(-(a*b*(-(b*c) + a*d)*((-3*(A*b^2 - a*(b*B - a*C))*d^2)/2 - c*(A
*b - a*B - b*C)*(b*c - a*d) + (d*(3*A*b^2*d - 2*a*A*(b*c - a*d) - (b*B - a*
C)*(2*b*c + a*d)))/2))/2 + (a^2*b*(-(c*((-3*c*(A*b^2 - a*(b*B - a*C))*d)/2
+ (A*b - a*B - b*C)*d*(b*c - a*d))) + (d^2*(3*A*b^2*d - 2*a*A*(b*c - a*d) -
 (b*B - a*C)*(2*b*c + a*d)))/2))/2 + b^2*(-(a*d*((-3*c*(A*b^2 - a*(b*B - a*
C))*d)/2 + (A*b - a*B - b*C)*d*(b*c - a*d)))/2 + (((b*d^2)/2 - (c*(-(b*c) +
 a*d))/2)*(3*A*b^2*d - 2*a*A*(b*c - a*d) - (b*B - a*C)*(2*b*c + a*d)))/2))*
ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(Sqrt[b]*(a^2
+ b^2)*(-(b*c) + a*d)*f)))/((-(b*c) + a*d)*(c^2 + d^2)) - (2*(-(c*((-3*c*(A
*b^2 - a*(b*B - a*C))*d)/2 + (A*b - a*B - b*C)*d*(b*c - a*d))) + (d^2*(3*A*
b^2*d - 2*a*A*(b*c - a*d) - (b*B - a*C)*(2*b*c + a*d)))/2))/((-(b*c) + a*d)
*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]]))/((a^2 + b^2)*(b*c - a*d))
```

Maple [B] time = 0.263, size = 40619, normalized size = 90.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^(3/
2),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e
))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e
))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**2/(c+d*tan(f*x
+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\tan\left(fx+e\right)^2+B\tan\left(fx+e\right)+A}{\left(b\tan\left(fx+e\right)+a\right)^2\left(d\tan\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e
))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)/((b*tan(f*x + e) + a)^2*(
d*tan(f*x + e) + c)^(3/2)), x)
```

3.122 $\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=585

$$\frac{2b\sqrt{c+d\tan(e+fx)}\left(6a^2d^3\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+3abd\left(-c^2d^2(A-17C)+d^4(5A+3C)-2Bc^3d-8Bcd^3+8\right.\right.}{3d^4f\left(c^2+d^2\right)^2}$$

```
[Out] -(((a - I*b)^3*(I*A + B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*
d]])/((c - I*d)^(5/2)*f)) - ((I*a - b)^3*(A + I*B - C)*ArcTanh[Sqrt[c + d*T
an[e + f*x]]/Sqrt[c + I*d]])/((c + I*d)^(5/2)*f) - (2*(c^2*C - B*c*d + A*d^
2)*(a + b*Tan[e + f*x])^3)/(3*d*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/2)) -
 (2*(b*(2*c^4*C - B*c^3*d + 4*c^2*C*d^2 - 3*B*c*d^3 + 2*A*d^4) + a*d^2*(2*c
*(A - C)*d - B*(c^2 - d^2)))*(a + b*Tan[e + f*x])^2)/(d^2*(c^2 + d^2)^2*f*S
qrt[c + d*Tan[e + f*x]]) + (2*b*(3*a*b*d*(8*c^4*C - 2*B*c^3*d - c^2*(A - 17
*C)*d^2 - 8*B*c*d^3 + (5*A + 3*C)*d^4) - b^2*(16*c^5*C - 8*B*c^4*d + 2*c^3*
(A + 15*C)*d^2 - 17*B*c^2*d^3 + 8*c*(A + C)*d^4 - 3*B*d^5) + 6*a^2*d^3*(2*c
*(A - C)*d - B*(c^2 - d^2)))*Sqrt[c + d*Tan[e + f*x]])/(3*d^4*(c^2 + d^2)^2
*f) + (2*b^2*(b*(8*c^4*C - 4*B*c^3*d + c^2*(A + 15*C)*d^2 - 10*B*c*d^3 + (7
*A + C)*d^4) + 3*a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)))*Tan[e + f*x]*Sqrt[c
 + d*Tan[e + f*x]])/(3*d^3*(c^2 + d^2)^2*f)
```

Rubi [A] time = 2.96752, antiderivative size = 585, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.149, Rules used = {3645, 3637, 3630, 3539, 3537, 63, 208}

$$\frac{2b\sqrt{c+d\tan(e+fx)}\left(6a^2d^3\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+3abd\left(-c^2d^2(A-17C)+d^4(5A+3C)-2Bc^3d-8Bcd^3+8\right.\right.}{3d^4f\left(c^2+d^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d
*Tan[e + f*x])^(5/2),x]
```


```
[Out] -(((a - I*b)^3*(I*A + B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*
d]])/((c - I*d)^(5/2)*f)) - ((I*a - b)^3*(A + I*B - C)*ArcTanh[Sqrt[c + d*T
an[e + f*x]]/Sqrt[c + I*d]])/((c + I*d)^(5/2)*f) - (2*(c^2*C - B*c*d + A*d^
2)*(a + b*Tan[e + f*x])^3)/(3*d*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/2)) -
 (2*(b*(2*c^4*C - B*c^3*d + 4*c^2*C*d^2 - 3*B*c*d^3 + 2*A*d^4) + a*d^2*(2*c
*(A - C)*d - B*(c^2 - d^2)))*(a + b*Tan[e + f*x])^2)/(d^2*(c^2 + d^2)^2*f*S
qrt[c + d*Tan[e + f*x]]) + (2*b*(3*a*b*d*(8*c^4*C - 2*B*c^3*d - c^2*(A - 17
*C)*d^2 - 8*B*c*d^3 + (5*A + 3*C)*d^4) - b^2*(16*c^5*C - 8*B*c^4*d + 2*c^3*
(A + 15*C)*d^2 - 17*B*c^2*d^3 + 8*c*(A + C)*d^4 - 3*B*d^5) + 6*a^2*d^3*(2*c
*(A - C)*d - B*(c^2 - d^2)))*Sqrt[c + d*Tan[e + f*x]])/(3*d^4*(c^2 + d^2)^2
*f) + (2*b^2*(b*(8*c^4*C - 4*B*c^3*d + c^2*(A + 15*C)*d^2 - 10*B*c*d^3 + (7
*A + C)*d^4) + 3*a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)))*Tan[e + f*x]*Sqrt[c
 + d*Tan[e + f*x]])/(3*d^3*(c^2 + d^2)^2*f)
```


Rule 3645

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*d^2 + c*(c*C - B*d))*(a + b*Tan[e
+ f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dis
```

```
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^3}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + 2\int$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^3}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - 2\left(\right.$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^3}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - 2\left(\right.$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^3}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - 2\left(\right.$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^3}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - 2\left(\right.$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^3}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - 2\left(\right.$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^3}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - 2\left(\right.$$

$$= -\frac{(a-ib)^3(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{5/2}f} -$$

Mathematica [C] time = 6.83154, size = 670, normalized size = 1.15

$$\frac{2C(a+b\tan(e+fx))^3}{3df(c+d\tan(e+fx))^{3/2}}+\frac{2\left(-\frac{3(-2aCd-bBd+2bcC)(a+b\tan(e+fx))^2}{df(c+d\tan(e+fx))^{3/2}}+\frac{2\left(-\frac{3(a+b\tan(e+fx))\left(bd^2(aB+Ab-bC)+4(bc-ad)(-2aCd-bBd+2bcC)\right)}{2df(c+d\tan(e+fx))^{3/2}}-\frac{3\left(-\frac{2\left(9a^2bBd\cdots\right.}{\cdots}\cdots\right)}{\cdots}\right)}{\cdots}\right)}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (2*C*(a + b*Tan[e + f*x])^3)/(3*d*f*(c + d*Tan[e + f*x])^(3/2)) + (2*((-3*(
2*b*c*C - b*B*d - 2*a*C*d)*(a + b*Tan[e + f*x])^2)/(d*f*(c + d*Tan[e + f*x]
)^(3/2)) + (2*((-3*(b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(2*b*c*C - b*B*
d - 2*a*C*d))*(a + b*Tan[e + f*x]))/(2*d*f*(c + d*Tan[e + f*x])^(3/2)) - (3
*((-2*(-16*b^3*c^3*C + 8*b^3*B*c^2*d + 48*a*b^2*c^2*C*d - 2*A*b^3*c*d^2 - 1
8*a*b^2*B*c*d^2 - 48*a^2*b*c*C*d^2 + 2*b^3*c*C*d^2 + 9*a^2*b*B*d^3 + b^3*B*
d^3 + 16*a^3*C*d^3))/(3*d*(c + d*Tan[e + f*x])^(3/2)) + (2*((((3*c*(a^3*B -
 3*a*b^2*B + 3*a^2*b*(A - C) - b^3*(A - C))*d^4)/2 + (3*(3*a^2*b*B - b^3*B
- a^3*(A - C) + 3*a*b^2*(A - C))*d^5)/2)*(-Hypergeometric2F1[-3/2, 1, -1/2,
 (c + d*Tan[e + f*x])/(c - I*d)]/(3*(I*c + d)*(c + d*Tan[e + f*x])^(3/2)) +
 Hypergeometric2F1[-3/2, 1, -1/2, (c + d*Tan[e + f*x])/(c + I*d)]/(3*(I*c -
 d)*(c + d*Tan[e + f*x])^(3/2))))/d - (3*(a^3*B - 3*a*b^2*B + 3*a^2*b*(A -
```

```
C) - b^3*(A - C))*d^3*(-(Hypergeometric2F1[-1/2, 1, 1/2, (c + d*Tan[e + f*x
])/(c - I*d)]/((I*c + d)*Sqrt[c + d*Tan[e + f*x]])) + Hypergeometric2F1[-1/
2, 1, 1/2, (c + d*Tan[e + f*x])/(c + I*d)]/((I*c - d)*Sqrt[c + d*Tan[e + f*
x]])))/2))/(3*d)))/(4*d*f)))/d))/(3*d)
```

Maple [B] time = 0.284, size = 85156, normalized size = 145.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(5/
2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x
+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\tan\left(fx+e\right)^2 + B\tan\left(fx+e\right) + A\right)\left(b\tan\left(fx+e\right) + a\right)^3}{\left(d\tan\left(fx+e\right) + c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^3/(d
*tan(f*x + e) + c)^(5/2), x)
```

3.123 $\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=358

$$\frac{2(bc-ad)\left(3ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(-2c^2d^2(A-5C)+4Ad^4-Bc^3d-7Bcd^3+4c^4C\right)\right)}{3d^3f\left(c^2+d^2\right)^2\sqrt{c+d\tan(e+fx)}}-\frac{2\left(Ad^2-Bcd+\right.}{3df\left(c^2+d^2\right.}$$

```
[Out] -(((a - I*b)^2*(I*A + B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*
d]])/((c - I*d)^(5/2)*f)) - ((a + I*b)^2*(B - I*(A - C))*ArcTanh[Sqrt[c + d
*Tan[e + f*x]]/Sqrt[c + I*d]])/((c + I*d)^(5/2)*f) - (2*(c^2*C - B*c*d + A*
d^2)*(a + b*Tan[e + f*x])^2)/(3*d*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/2))
 + (2*(b*c - a*d)*(b*(4*c^4*C - B*c^3*d - 2*c^2*(A - 5*C)*d^2 - 7*B*c*d^3 +
 4*A*d^4) + 3*a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2))))/(3*d^3*(c^2 + d^2)^2*
f*Sqrt[c + d*Tan[e + f*x]]) + (2*b^2*(4*c^2*C - B*c*d + (A + 3*C)*d^2)*Sqrt
[c + d*Tan[e + f*x]])/(3*d^3*(c^2 + d^2)*f)
```

Rubi [A] time = 1.55107, antiderivative size = 358, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.149, Rules used = {3645, 3635, 3630, 3539, 3537, 63, 208}

$$\frac{2(bc-ad)\left(3ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(-2c^2d^2(A-5C)+4Ad^4-Bc^3d-7Bcd^3+4c^4C\right)\right)}{3d^3f\left(c^2+d^2\right)^2\sqrt{c+d\tan(e+fx)}}-\frac{2\left(Ad^2-Bcd+\right.}{3df\left(c^2+d^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d
*Tan[e + f*x])^(5/2),x]
```


```
[Out] -(((a - I*b)^2*(I*A + B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*
d]])/((c - I*d)^(5/2)*f)) - ((a + I*b)^2*(B - I*(A - C))*ArcTanh[Sqrt[c + d
*Tan[e + f*x]]/Sqrt[c + I*d]])/((c + I*d)^(5/2)*f) - (2*(c^2*C - B*c*d + A*
d^2)*(a + b*Tan[e + f*x])^2)/(3*d*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/2))
 + (2*(b*c - a*d)*(b*(4*c^4*C - B*c^3*d - 2*c^2*(A - 5*C)*d^2 - 7*B*c*d^3 +
 4*A*d^4) + 3*a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2))))/(3*d^3*(c^2 + d^2)^2*
f*Sqrt[c + d*Tan[e + f*x]]) + (2*b^2*(4*c^2*C - B*c*d + (A + 3*C)*d^2)*Sqrt
[c + d*Tan[e + f*x]])/(3*d^3*(c^2 + d^2)*f)
```


Rule 3645

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*d^2 + c*(c*C - B*d))*(a + b*Tan[e
+ f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3635

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
```

```
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(c^2*C - B*c*d + A*d^2)*(c +
d*Tan[e + f*x])^(n + 1))/(d^2*f*(n + 1)*(c^2 + d^2)), x] + Dist[1/(d*(c^2 +
 d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(c^
2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*Ta
n[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -
1]
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^2}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{2\int}{}$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^2}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{2(b}{}$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^2}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{2(b}{}$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^2}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{2(b}{}$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^2}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{2(b}{}$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^2}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{2(b}{}$$

$$= -\frac{(a-ib)^2(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{5/2}f} -$$

Mathematica [C] time = 6.47687, size = 502, normalized size = 1.4

$$\frac{2C(a+b\tan(e+fx))^2}{df(c+d\tan(e+fx))^{3/2}} + \frac{2\left(-\frac{(4aCd+bBd-4bcC)(a+b\tan(e+fx))}{df(c+d\tan(e+fx))^{3/2}} - \frac{-\frac{2\left(8a^2Cd^2+abBd^2-16abcCd-Ab^2d^2-2b^2Bcd+8b^2c^2C+b^2Cd^2\right)}{3d(c+d\tan(e+fx))^{3/2}} + \frac{2\left(\frac{3}{2}cd^3\left(a^2B+2ab(A\right.\right.}{}}{}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (2*C*(a + b*Tan[e + f*x])^2)/(d*f*(c + d*Tan[e + f*x])^(3/2)) + (2*(-(((-4*
b*c*C + b*B*d + 4*a*C*d)*(a + b*Tan[e + f*x]))/(d*f*(c + d*Tan[e + f*x])^(3
/2))) - ((-2*(8*b^2*c^2*C - 2*b^2*B*c*d - 16*a*b*c*C*d - A*b^2*d^2 + a*b*B*
d^2 + 8*a^2*C*d^2 + b^2*C*d^2))/(3*d*(c + d*Tan[e + f*x])^(3/2)) + (2*((((3
*c*(a^2*B - b^2*B + 2*a*b*(A - C))*d^3)/2 + (3*(2*a*b*B - a^2*(A - C) + b^2
*(A - C))*d^4)/2)*(-Hypergeometric2F1[-3/2, 1, -1/2, (c + d*Tan[e + f*x])/(
c - I*d)]/(3*(I*c + d)*(c + d*Tan[e + f*x])^(3/2)) + Hypergeometric2F1[-3/2
, 1, -1/2, (c + d*Tan[e + f*x])/(c + I*d)]/(3*(I*c - d)*(c + d*Tan[e + f*x]
)^(3/2))))/d - (3*(a^2*B - b^2*B + 2*a*b*(A - C))*d^2*(-(Hypergeometric2F1[
```

```
-1/2, 1, 1/2, (c + d*Tan[e + f*x])/(c - I*d)]/((I*c + d)*Sqrt[c + d*Tan[e +
 f*x]])) + Hypergeometric2F1[-1/2, 1, 1/2, (c + d*Tan[e + f*x])/(c + I*d)]/
((I*c - d)*Sqrt[c + d*Tan[e + f*x]])))/2))/(3*d))/(2*d*f)))/d
```

Maple [B] time = 0.247, size = 61833, normalized size = 172.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(5/
2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b \tan{\left(e + f x \right)}\right)^{2} \left(A + B \tan{\left(e + f x \right)} + C \tan^{2}{\left(e + f x \right)}\right)}{\left(c + d \tan{\left(e + f x \right)}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x
+e))**(5/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**2*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/(
c + d*tan(e + f*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\tan\left(fx+e\right)^2+B\tan\left(fx+e\right)+A\right)\left(b\tan\left(fx+e\right)+a\right)^2}{\left(d\tan\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^2/(d
*tan(f*x + e) + c)^(5/2), x)
```

3.124 $\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=273

$$\frac{2(bc-ad)\left(Ad^2-Bcd+c^2C\right)}{3d^2f\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}-\frac{2\left(ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(-c^2d^2(A-3C)+Ad^4-2Bcd^3+c^4C\right)\right)}{d^2f\left(c^2+d^2\right)^2\sqrt{c+d\tan(e+fx)}}-\cdots$$

```
[Out] -(((a - I*b)*(I*A + B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]
])/((c - I*d)^(5/2)*f)) + ((I*a - b)*(A + I*B - C)*ArcTanh[Sqrt[c + d*Tan[e
 + f*x]]/Sqrt[c + I*d]])/((c + I*d)^(5/2)*f) + (2*(b*c - a*d)*(c^2*C - B*c*
d + A*d^2))/(3*d^2*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/2)) - (2*(b*(c^4*C
 - c^2*(A - 3*C)*d^2 - 2*B*c*d^3 + A*d^4) + a*d^2*(2*c*(A - C)*d - B*(c^2 -
 d^2))))/(d^2*(c^2 + d^2)^2*f*Sqrt[c + d*Tan[e + f*x]])
```

Rubi [A] time = 0.797563, antiderivative size = 271, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3635, 3628, 3539, 3537, 63, 208}

$$\frac{2(bc-ad)\left(Ad^2-Bcd+c^2C\right)}{3d^2f\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}-\frac{2\left(ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(-c^2d^2(A-3C)+Ad^4-2Bcd^3+c^4C\right)\right)}{d^2f\left(c^2+d^2\right)^2\sqrt{c+d\tan(e+fx)}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d*T
an[e + f*x])^(5/2),x]
```

```
[Out] -(((I*a + b)*(A - I*B - C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])
/((c - I*d)^(5/2)*f)) + ((I*a - b)*(A + I*B - C)*ArcTanh[Sqrt[c + d*Tan[e +
 f*x]]/Sqrt[c + I*d]])/((c + I*d)^(5/2)*f) + (2*(b*c - a*d)*(c^2*C - B*c*d
+ A*d^2))/(3*d^2*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/2)) - (2*(b*(c^4*C -
 c^2*(A - 3*C)*d^2 - 2*B*c*d^3 + A*d^4) + a*d^2*(2*c*(A - C)*d - B*(c^2 - d
^2))))/(d^2*(c^2 + d^2)^2*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 3635

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(c^2*C - B*c*d + A*d^2)*(c +
d*Tan[e + f*x])^(n + 1))/(d^2*f*(n + 1)*(c^2 + d^2)), x] + Dist[1/(d*(c^2 +
 d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(c^
2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*Ta
n[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -
1]
```

Rule 3628

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2
- a*b*B + a^2*C)*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 + b^2)), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx = \frac{2(bc-ad)\left(c^2C-Bcd+Ad^2\right)}{3d^2\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{\int \frac{ad(Ac-cC+}{}}{}$$

$$= \frac{2(bc-ad)\left(c^2C-Bcd+Ad^2\right)}{3d^2\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - \frac{2\left(b\left(c^4C-\right.\right.}{}$$

$$= \frac{2(bc-ad)\left(c^2C-Bcd+Ad^2\right)}{3d^2\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - \frac{2\left(b\left(c^4C-\right.\right.}{}$$

$$= \frac{2(bc-ad)\left(c^2C-Bcd+Ad^2\right)}{3d^2\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - \frac{2\left(b\left(c^4C-\right.\right.}{}$$

$$= \frac{2(bc-ad)\left(c^2C-Bcd+Ad^2\right)}{3d^2\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - \frac{2\left(b\left(c^4C-\right.\right.}{}$$

$$= -\frac{(ia+b)(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{5/2}f} + \frac{(i}{}$$

Mathematica [C] time = 2.7419, size = 300, normalized size = 1.1

$$-\frac{d(-aAd+aBc+aCd+Abc+bBd-bcC)\left(i(c+id)\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{c+d\tan(e+fx)}{c-id}\right)-(d+ic)\text{Hyper}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c
 + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] -(2*(c - I*d)*(c + I*d)*(2*b*c*C + b*B*d - 2*a*C*d) + d*(A*b*c + a*B*c - b*
c*C - a*A*d + b*B*d + a*C*d)*(I*(c + I*d)*Hypergeometric2F1[-3/2, 1, -1/2,
(c + d*Tan[e + f*x])/(c - I*d)] - (I*c + d)*Hypergeometric2F1[-3/2, 1, -1/2
, (c + d*Tan[e + f*x])/(c + I*d)]) + 6*C*(c - I*d)*(c + I*d)*d*(a + b*Tan[e
 + f*x]) - 3*(A*b + a*B - b*C)*d*(I*(c + I*d)*Hypergeometric2F1[-1/2, 1, 1/
2, (c + d*Tan[e + f*x])/(c - I*d)] - (I*c + d)*Hypergeometric2F1[-1/2, 1, 1
/2, (c + d*Tan[e + f*x])/(c + I*d)])*(c + d*Tan[e + f*x]))/(3*d^2*(c^2 + d^
2)*f*(c + d*Tan[e + f*x])^(3/2))
```

Maple [B] time = 0.21, size = 40201, normalized size = 147.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(5/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tan\left(e+fx\right)\right)\left(A+B\tan\left(e+fx\right)+C\tan^{2}\left(e+fx\right)\right)}{\left(c+d\tan\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e)
)**(5/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/(c +
 d*tan(e + f*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\tan\left(fx+e\right)^{2}+B\tan\left(fx+e\right)+A\right)\left(b\tan\left(fx+e\right)+a\right)}{\left(d\tan\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)/(d*t
an(f*x + e) + c)^(5/2), x)
```

3.125 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=209

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}-\frac{2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)}{f\left(c^2+d^2\right)^2\sqrt{c+d\tan(e+fx)}}-\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(c-id)^{5/2}}-\frac{(B-i\ldots}{}$$

```
[Out] -(((I*A + B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((c - I
*d)^(5/2)*f)) - ((B - I*(A - C))*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c +
I*d]])/((c + I*d)^(5/2)*f) - (2*(c^2*C - B*c*d + A*d^2))/(3*d*(c^2 + d^2)*f
*(c + d*Tan[e + f*x])^(3/2)) - (2*(2*c*(A - C)*d - B*(c^2 - d^2)))/((c^2 +
d^2)^2*f*Sqrt[c + d*Tan[e + f*x]])
```

Rubi [A] time = 0.486027, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3628, 3529, 3539, 3537, 63, 208}

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}-\frac{2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)}{f\left(c^2+d^2\right)^2\sqrt{c+d\tan(e+fx)}}-\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(c-id)^{5/2}}-\frac{(B-i\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] -(((I*A + B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((c - I
*d)^(5/2)*f)) - ((B - I*(A - C))*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c +
I*d]])/((c + I*d)^(5/2)*f) - (2*(c^2*C - B*c*d + A*d^2))/(3*d*(c^2 + d^2)*f
*(c + d*Tan[e + f*x])^(3/2)) - (2*(2*c*(A - C)*d - B*(c^2 - d^2)))/((c^2 +
d^2)^2*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 3628

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2
- a*b*B + a^2*C)*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 + b^2)), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
```

```
a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{2\left(c^2C-Bcd+Ad^2\right)}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}}+\frac{\int \frac{Ac-cC+Bd+(Bc-(A-C)d)\tan(e+fx)}{(c+d\tan(e+fx))^{3/2}}\,d}{c^2+d^2}$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}}-\frac{2\left(2c(A-C)d-B\left(c^2-d^2\right)\right)}{\left(c^2+d^2\right)^2f\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}}-\frac{2\left(2c(A-C)d-B\left(c^2-d^2\right)\right)}{\left(c^2+d^2\right)^2f\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}}-\frac{2\left(2c(A-C)d-B\left(c^2-d^2\right)\right)}{\left(c^2+d^2\right)^2f\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}}-\frac{2\left(2c(A-C)d-B\left(c^2-d^2\right)\right)}{\left(c^2+d^2\right)^2f\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{(B+i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{5/2}f}-\frac{(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\,\mathrm{t}}}{\sqrt{}}\right.}{(c+id)^{5/2}f}$$

Mathematica [C] time = 0.853108, size = 223, normalized size = 1.07

$$-\frac{(d(C-A)+Bc)\left(i(c+id)\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{c+d\tan(e+fx)}{c-id}\right)-(d+ic)\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(c + d*Tan[e + f*x])^(5/2
),x]
```

```
[Out] -(2*C*(c^2 + d^2) + (B*c + (-A + C)*d)*(I*(c + I*d)*Hypergeometric2F1[-3/2,
 1, -1/2, (c + d*Tan[e + f*x])/(c - I*d)] - (I*c + d)*Hypergeometric2F1[-3/
2, 1, -1/2, (c + d*Tan[e + f*x])/(c + I*d)]) - 3*B*(I*(c + I*d)*Hypergeomet
ric2F1[-1/2, 1, 1/2, (c + d*Tan[e + f*x])/(c - I*d)] - (I*c + d)*Hypergeome
tric2F1[-1/2, 1, 1/2, (c + d*Tan[e + f*x])/(c + I*d)])*(c + d*Tan[e + f*x])
)/(3*d*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/2))
```

Maple [B] time = 0.149, size = 20647, normalized size = 98.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(5/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(5/2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\tan\left(e + fx\right) + C\tan^2\left(e + fx\right)}{\left(c + d\tan\left(e + fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/(c + d*tan(e + f*x))**(5/
2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\tan\left(fx+e\right)^{2}+B\tan\left(fx+e\right)+A}{\left(d\tan\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)/(d*tan(f*x + e) + c)^(5/2
), x)
```

3.126 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=365

$$-\frac{2b^{3/2}\left(Ab^2-a(bB-aC)\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{f\left(a^2+b^2\right)(bc-ad)^{5/2}}+\frac{2\left(b\left(c^2d^2(3A-C)+Ad^4-2Bc^3d+c^4C\right)-ad^2\left(2cd(A-C)-B\right.\right.}{f\left(c^2+d^2\right)^2(bc-ad)^2\sqrt{c+d\tan(e+fx)}}$$

```
[Out] ((A - I*B - C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((I*a + b)*
(c - I*d)^(5/2)*f) + ((I*A - B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt
[c + I*d]])/((a + I*b)*(c + I*d)^(5/2)*f) - (2*b^(3/2)*(A*b^2 - a*(b*B - a*
C))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/((a^2 + b^
2)*(b*c - a*d)^(5/2)*f) + (2*(c^2*C - B*c*d + A*d^2))/(3*(b*c - a*d)*(c^2 +
 d^2)*f*(c + d*Tan[e + f*x])^(3/2)) + (2*(b*(c^4*C - 2*B*c^3*d + c^2*(3*A -
 C)*d^2 + A*d^4) - a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2))))/((b*c - a*d)^2*(
c^2 + d^2)^2*f*Sqrt[c + d*Tan[e + f*x]])
```

Rubi [A] time = 2.46572, antiderivative size = 365, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.149, Rules used = {3649, 3653, 3539, 3537, 63, 208, 3634}

$$-\frac{2b^{3/2}\left(Ab^2-a(bB-aC)\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{f\left(a^2+b^2\right)(bc-ad)^{5/2}}+\frac{2\left(b\left(c^2d^2(3A-C)+Ad^4-2Bc^3d+c^4C\right)-ad^2\left(2cd(A-C)-B\right.\right.}{f\left(c^2+d^2\right)^2(bc-ad)^2\sqrt{c+d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])*(c + d*Ta
n[e + f*x])^(5/2)),x]
```


```
[Out] ((A - I*B - C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((I*a + b)*
(c - I*d)^(5/2)*f) + ((I*A - B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt
[c + I*d]])/((a + I*b)*(c + I*d)^(5/2)*f) - (2*b^(3/2)*(A*b^2 - a*(b*B - a*
C))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/((a^2 + b^
2)*(b*c - a*d)^(5/2)*f) + (2*(c^2*C - B*c*d + A*d^2))/(3*(b*c - a*d)*(c^2 +
 d^2)*f*(c + d*Tan[e + f*x])^(3/2)) + (2*(b*(c^4*C - 2*B*c^3*d + c^2*(3*A -
 C)*d^2 + A*d^4) - a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2))))/((b*c - a*d)^2*(
c^2 + d^2)^2*f*Sqrt[c + d*Tan[e + f*x]])
```


Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
 b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3653

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[e + f*x])^n*(1 + Tan[e
+ f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3634

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}}\,dx = \frac{2\left(c^2C-Bcd+Ad^2\right)}{3(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{2\int \frac{-\frac{3}{2}\left(aAcd-ad(cC-Bd)-Ab(\right.}{}}{}$$

$$= \frac{2\left(c^2C-Bcd+Ad^2\right)}{3(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{2\left(b\left(c^4C-2Bc^3d+c^2(3\right.\right.}{(bc-a}$$

$$= \frac{2\left(c^2C-Bcd+Ad^2\right)}{3(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{2\left(b\left(c^4C-2Bc^3d+c^2(3\right.\right.}{(bc-a}$$

$$= \frac{2\left(c^2C-Bcd+Ad^2\right)}{3(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{2\left(b\left(c^4C-2Bc^3d+c^2(3\right.\right.}{(bc-a}$$

$$= \frac{2\left(c^2C-Bcd+Ad^2\right)}{3(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{2\left(b\left(c^4C-2Bc^3d+c^2(3\right.\right.}{(bc-a}$$

$$= -\frac{2b^{3/2}\left(Ab^2-a(bB-aC)\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{\left(a^2+b^2\right)(bc-ad)^{5/2}f} + \frac{2\left(c^2\right.}{3(bc-ad)\left(c^2+\right.}$$

$$= \frac{(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(ia+b)(c-id)^{5/2}f} - \frac{(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e}}{\sqrt{c+id}}\right)}{(ia-b)(c+id)^{5/2}f}$$

Mathematica [B] time = 6.26429, size = 1948, normalized size = 5.34

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])*(c
+ d*Tan[e + f*x])^(5/2)),x]
```

```
[Out] (-2*(A*d^2 - c*(-(c*C) + B*d)))/(3*(-(b*c) + a*d)*(c^2 + d^2)*f*(c + d*Tan[
e + f*x])^(3/2)) - (2*((-2*((((I*Sqrt[c - I*d]*((b*(-(b*c) + a*d)*((-3*c*(b*
c - a*d)*(B*c - (A - C)*d))/2 - (3*b*d*(c^2*C - B*c*d + A*d^2))/2 - (3*d*(a
*A*c*d - a*d*(c*C - B*d) - A*b*(c^2 + d^2)))/2))/2 + a*((-3*((b*d^2)/2 - (c
*(-(b*c) + a*d))/2)*(a*A*c*d - a*d*(c*C - B*d) - A*b*(c^2 + d^2)))/2 - (a*d
*((3*d*(b*c - a*d)*(B*c - (A - C)*d))/2 - (3*b*c*(c^2*C - B*c*d + A*d^2))/2
))/2 - (b*((-3*d^2*(a*A*c*d - a*d*(c*C - B*d) - A*b*(c^2 + d^2)))/2 - c*((3
*d*(b*c - a*d)*(B*c - (A - C)*d))/2 - (3*b*c*(c^2*C - B*c*d + A*d^2))/2)))/
2) - I*((a*(-(b*c) + a*d)*((-3*c*(b*c - a*d)*(B*c - (A - C)*d))/2 - (3*b*d*
(c^2*C - B*c*d + A*d^2))/2 - (3*d*(a*A*c*d - a*d*(c*C - B*d) - A*b*(c^2 + d
^2)))/2))/2 - b*((-3*((b*d^2)/2 - (c*(-(b*c) + a*d))/2)*(a*A*c*d - a*d*(c*C
 - B*d) - A*b*(c^2 + d^2)))/2 - (a*d*((3*d*(b*c - a*d)*(B*c - (A - C)*d))/2
 - (3*b*c*(c^2*C - B*c*d + A*d^2))/2))/2 - (b*((-3*d^2*(a*A*c*d - a*d*(c*C
- B*d) - A*b*(c^2 + d^2)))/2 - c*((3*d*(b*c - a*d)*(B*c - (A - C)*d))/2 - (
3*b*c*(c^2*C - B*c*d + A*d^2))/2)))/2)))*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/S
qrt[c - I*d]])/((-c + I*d)*f) - (I*Sqrt[c + I*d]*((b*(-(b*c) + a*d)*((-3*c*
(b*c - a*d)*(B*c - (A - C)*d))/2 - (3*b*d*(c^2*C - B*c*d + A*d^2))/2 - (3*d
*(a*A*c*d - a*d*(c*C - B*d) - A*b*(c^2 + d^2)))/2))/2 + a*((-3*((b*d^2)/2 -
 (c*(-(b*c) + a*d))/2)*(a*A*c*d - a*d*(c*C - B*d) - A*b*(c^2 + d^2)))/2 - (
a*d*((3*d*(b*c - a*d)*(B*c - (A - C)*d))/2 - (3*b*c*(c^2*C - B*c*d + A*d^2)
```

```
)/2))/2 - (b*((-3*d^2*(a*A*c*d - a*d*(c*C - B*d) - A*b*(c^2 + d^2)))/2 - c*
((3*d*(b*c - a*d)*(B*c - (A - C)*d))/2 - (3*b*c*(c^2*C - B*c*d + A*d^2))/2)
))/2) + I*((a*(-(b*c) + a*d)*((-3*c*(b*c - a*d)*(B*c - (A - C)*d))/2 - (3*b
*d*(c^2*C - B*c*d + A*d^2))/2 - (3*d*(a*A*c*d - a*d*(c*C - B*d) - A*b*(c^2
+ d^2)))/2))/2 - b*((-3*((b*d^2)/2 - (c*(-(b*c) + a*d))/2)*(a*A*c*d - a*d*(
c*C - B*d) - A*b*(c^2 + d^2)))/2 - (a*d*((3*d*(b*c - a*d)*(B*c - (A - C)*d)
)/2 - (3*b*c*(c^2*C - B*c*d + A*d^2))/2))/2 - (b*((-3*d^2*(a*A*c*d - a*d*(c
*C - B*d) - A*b*(c^2 + d^2)))/2 - c*((3*d*(b*c - a*d)*(B*c - (A - C)*d))/2
- (3*b*c*(c^2*C - B*c*d + A*d^2))/2)))/2)))*ArcTanh[Sqrt[c + d*Tan[e + f*x]
]/Sqrt[c + I*d]])/((-c - I*d)*f))/(a^2 + b^2) + (2*Sqrt[b*c - a*d]*(-(a*b*(
-(b*c) + a*d)*((-3*c*(b*c - a*d)*(B*c - (A - C)*d))/2 - (3*b*d*(c^2*C - B*c
*d + A*d^2))/2 - (3*d*(a*A*c*d - a*d*(c*C - B*d) - A*b*(c^2 + d^2)))/2))/2
+ (a^2*b*((-3*d^2*(a*A*c*d - a*d*(c*C - B*d) - A*b*(c^2 + d^2)))/2 - c*((3*
d*(b*c - a*d)*(B*c - (A - C)*d))/2 - (3*b*c*(c^2*C - B*c*d + A*d^2))/2)))/2
 + b^2*((-3*((b*d^2)/2 - (c*(-(b*c) + a*d))/2)*(a*A*c*d - a*d*(c*C - B*d) -
 A*b*(c^2 + d^2)))/2 - (a*d*((3*d*(b*c - a*d)*(B*c - (A - C)*d))/2 - (3*b*c
*(c^2*C - B*c*d + A*d^2))/2))/2))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]]
)/Sqrt[b*c - a*d]])/(Sqrt[b]*(a^2 + b^2)*(-(b*c) + a*d)*f)))/((-(b*c) + a*d
)*(c^2 + d^2)) - (2*((-3*d^2*(a*A*c*d - a*d*(c*C - B*d) - A*b*(c^2 + d^2)))
/2 - c*((3*d*(b*c - a*d)*(B*c - (A - C)*d))/2 - (3*b*c*(c^2*C - B*c*d + A*d
^2))/2)))/((-(b*c) + a*d)*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]])))/(3*(-(b
*c) + a*d)*(c^2 + d^2))
```

Maple [B] time = 0.25, size = 45119, normalized size = 123.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))^(5/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))
^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))
^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e)
)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\tan\left(fx+e\right)^{2}+B\tan\left(fx+e\right)+A}{\left(b\tan\left(fx+e\right)+a\right)\left(d\tan\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)/((b*tan(f*x + e) + a)*(d*
tan(f*x + e) + c)^(5/2)), x)
```

3.127 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=679

$$-\frac{d\left(-A\left(-4a^2bd^2\left(2c^2+d^2\right)+4a^3cd^3+4ab^2cd^3+b^3\left(-\left(10c^2d^2+c^4+5d^4\right)\right)\right)+a^2b\left(-6Bc^3d-2Bcd^3+2c^2Cd^2+5c^4\right.}{f\left(a^2+b^2\right)\left(c^2+d^2\right)^2(bc-ad)^3\sqrt{}}$$

```
[Out] -(((I*A + B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((a - I
*b)^2*(c - I*d)^(5/2)*f)) - ((B - I*(A - C))*ArcTanh[Sqrt[c + d*Tan[e + f*x
]]/Sqrt[c + I*d]])/((a + I*b)^2*(c + I*d)^(5/2)*f) - (b^(3/2)*(7*a^3*b*B*d
- 5*a^4*C*d + b^4*(2*B*c - 5*A*d) + a*b^3*(4*A*c - 4*c*C + 3*B*d) - a^2*b^2
*(2*B*c + (9*A + C)*d))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c
 - a*d]])/((a^2 + b^2)^2*(b*c - a*d)^(7/2)*f) - (d*(2*b^2*c*(c*C - B*d) - 3
*a*b*B*(c^2 + d^2) + a^2*(5*c^2*C - 2*B*c*d + 3*C*d^2) + A*(2*a^2*d^2 + b^2
*(3*c^2 + 5*d^2))))/(3*(a^2 + b^2)*(b*c - a*d)^2*(c^2 + d^2)*f*(c + d*Tan[e
 + f*x])^(3/2)) - (A*b^2 - a*(b*B - a*C))/((a^2 + b^2)*(b*c - a*d)*f*(a + b
*Tan[e + f*x])*(c + d*Tan[e + f*x])^(3/2)) - (d*(2*a^3*d^2*(B*c^2 + 2*c*C*d
 - B*d^2) + 2*b^3*c*(2*c^3*C - 3*B*c^2*d - B*d^3) - a*b^2*(B*c^4 - 4*c*C*d^
3 + 3*B*d^4) + a^2*b*(5*c^4*C - 6*B*c^3*d + 2*c^2*C*d^2 - 2*B*c*d^3 + C*d^4
) - A*(4*a^3*c*d^3 + 4*a*b^2*c*d^3 - 4*a^2*b*d^2*(2*c^2 + d^2) - b^3*(c^4 +
 10*c^2*d^2 + 5*d^4))))/((a^2 + b^2)*(b*c - a*d)^3*(c^2 + d^2)^2*f*Sqrt[c +
 d*Tan[e + f*x]])
```

Rubi [A] time = 5.0617, antiderivative size = 678, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.149, Rules used = {3649, 3653, 3539, 3537, 63, 208, 3634}

$$-\frac{d\left(-A\left(-4a^2bd^2\left(2c^2+d^2\right)+4a^3cd^3+4ab^2cd^3+b^3\left(-\left(10c^2d^2+c^4+5d^4\right)\right)\right)+a^2b\left(-6Bc^3d-2Bcd^3+2c^2Cd^2+5c^4\right.}{f\left(a^2+b^2\right)\left(c^2+d^2\right)^2(bc-ad)^3\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^2*(c + d*
Tan[e + f*x])^(5/2)),x]
```

```
[Out] -(((I*A + B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((a - I
*b)^2*(c - I*d)^(5/2)*f)) - ((B - I*(A - C))*ArcTanh[Sqrt[c + d*Tan[e + f*x
]]/Sqrt[c + I*d]])/((a + I*b)^2*(c + I*d)^(5/2)*f) - (b^(3/2)*(7*a^3*b*B*d
- 5*a^4*C*d + b^4*(2*B*c - 5*A*d) + a*b^3*(4*A*c - 4*c*C + 3*B*d) - a^2*b^2
*(2*B*c + (9*A + C)*d))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c
 - a*d]])/((a^2 + b^2)^2*(b*c - a*d)^(7/2)*f) - (d*(2*a^2*A*d^2 + 2*b^2*c*(
c*C - B*d) - 3*a*b*B*(c^2 + d^2) + A*b^2*(3*c^2 + 5*d^2) + a^2*(5*c^2*C - 2
*B*c*d + 3*C*d^2)))/(3*(a^2 + b^2)*(b*c - a*d)^2*(c^2 + d^2)*f*(c + d*Tan[e
 + f*x])^(3/2)) - (A*b^2 - a*(b*B - a*C))/((a^2 + b^2)*(b*c - a*d)*f*(a + b
*Tan[e + f*x])*(c + d*Tan[e + f*x])^(3/2)) - (d*(2*a^3*d^2*(B*c^2 + 2*c*C*d
 - B*d^2) + 2*b^3*c*(2*c^3*C - 3*B*c^2*d - B*d^3) - a*b^2*(B*c^4 - 4*c*C*d^
3 + 3*B*d^4) + a^2*b*(5*c^4*C - 6*B*c^3*d + 2*c^2*C*d^2 - 2*B*c*d^3 + C*d^4
) - A*(4*a^3*c*d^3 + 4*a*b^2*c*d^3 - 4*a^2*b*d^2*(2*c^2 + d^2) - b^3*(c^4 +
 10*c^2*d^2 + 5*d^4))))/((a^2 + b^2)*(b*c - a*d)^3*(c^2 + d^2)^2*f*Sqrt[c +
 d*Tan[e + f*x]])
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
 b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3653

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[e + f*x])^n*(1 + Tan[e
+ f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3634

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{A + B\tan(e+fx) + C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{Ab^2 - a(bB - aC)}{\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}} - \ldots$$

$$= -\frac{d\left(2a^2Ad^2 + 2b^2c(cC - Bd) - 3abB\left(c^2+d^2\right) + Ab^2\left(3c^2+5d^2\right) + \ldots\right.}{3\left(a^2+b^2\right)(bc-ad)^2\left(c^2+d^2\right)f(c+d\tan(e+\ldots}$$

$$= -\frac{d\left(2a^2Ad^2 + 2b^2c(cC - Bd) - 3abB\left(c^2+d^2\right) + Ab^2\left(3c^2+5d^2\right) + \ldots\right.}{3\left(a^2+b^2\right)(bc-ad)^2\left(c^2+d^2\right)f(c+d\tan(e+\ldots}$$

$$= -\frac{d\left(2a^2Ad^2 + 2b^2c(cC - Bd) - 3abB\left(c^2+d^2\right) + Ab^2\left(3c^2+5d^2\right) + \ldots\right.}{3\left(a^2+b^2\right)(bc-ad)^2\left(c^2+d^2\right)f(c+d\tan(e+\ldots}$$

$$= -\frac{d\left(2a^2Ad^2 + 2b^2c(cC - Bd) - 3abB\left(c^2+d^2\right) + Ab^2\left(3c^2+5d^2\right) + \ldots\right.}{3\left(a^2+b^2\right)(bc-ad)^2\left(c^2+d^2\right)f(c+d\tan(e+\ldots}$$

$$= -\frac{d\left(2a^2Ad^2 + 2b^2c(cC - Bd) - 3abB\left(c^2+d^2\right) + Ab^2\left(3c^2+5d^2\right) + \ldots\right.}{3\left(a^2+b^2\right)(bc-ad)^2\left(c^2+d^2\right)f(c+d\tan(e+\ldots}$$

$$= -\frac{b^{3/2}\left(7a^3bBd - 5a^4Cd + b^4(2Bc - 5Ad) + ab^3(4Ac - 4cC + 3Bd) \ldots\right.}{\left(a^2+b^2\right)^2(bc - ad\ldots}$$

$$= -\frac{(iA + B - iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)^2(c-id)^{5/2}f} - \frac{(B - i(A-C))\tanh^{-1}\left(\ldots\right)}{(a+ib)^2(c+id)\ldots}$$

Mathematica [B] time = 6.41783, size = 6052, normalized size = 8.91

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^2*(
c + d*Tan[e + f*x])^(5/2)),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.327, size = 67570, normalized size = 99.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^(5/
2),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e
))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e
))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**2/(c+d*tan(f*x
+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\tan\left(fx+e\right)^{2}+B\tan\left(fx+e\right)+A}{\left(b\tan\left(fx+e\right)+a\right)^{2}\left(d\tan\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e
))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)/((b*tan(f*x + e) + a)^2*(
d*tan(f*x + e) + c)^(5/2)), x)
```

3.128 $\int (a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+\right.$

Optimal. Leaf size=679

$$-\frac{\left(30a^2b^2d^2\left(-8d^2(A-C)-4Bcd+c^2C\right)-20a^3bd^3(2Bd+cC)+5a^4Cd^4-20ab^3d\left(8cd^2(A-C)-2Bc^2d-16Bd^3+c\right.\right.}{64b^{3/2}d^{7/2}f}$$

```
[Out] -(((a - I*b)^(5/2)*(I*A + B - I*C)*Sqrt[c - I*d]*ArcTanh[(Sqrt[c - I*d]*Sqr
t[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/f) - ((a
+ I*b)^(5/2)*(B - I*(A - C))*Sqrt[c + I*d]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a +
b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/f - ((5*a^4*C*d
^4 - 20*a^3*b*d^3*(c*C + 2*B*d) + 30*a^2*b^2*d^2*(c^2*C - 4*B*c*d - 8*(A -
C)*d^2) - 20*a*b^3*d*(c^3*C - 2*B*c^2*d + 8*c*(A - C)*d^2 - 16*B*d^3) + b^4
*(5*c^4*C - 8*B*c^3*d + 16*c^2*(A - C)*d^2 + 64*B*c*d^3 + 128*(A - C)*d^4))
*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x
]])])/(64*b^(3/2)*d^(7/2)*f) + ((64*b*(a^2*B - b^2*B + 2*a*b*(A - C))*d^3 -
 (b*c - a*d)*(16*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(5*b*c*C - 8*b*B*d -
 5*a*C*d)))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(64*b*d^3*f)
 + ((16*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(5*b*c*C - 8*b*B*d - 5*a*C*d)
)*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))/(32*d^3*f) - ((5*b*c
*C - 8*b*B*d - 5*a*C*d)*(a + b*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(3/
2))/(24*d^2*f) + (C*(a + b*Tan[e + f*x])^(5/2)*(c + d*Tan[e + f*x])^(3/2))/
(4*d*f)
```

Rubi [A] time = 9.92627, antiderivative size = 679, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3647, 3655, 6725, 63, 217, 206, 93, 208}

$$-\frac{\left(30a^2b^2d^2\left(-8d^2(A-C)-4Bcd+c^2C\right)-20a^3bd^3(2Bd+cC)+5a^4Cd^4-20ab^3d\left(8cd^2(A-C)-2Bc^2d-16Bd^3+c\right.\right.}{64b^{3/2}d^{7/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^(5/2)*Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x]
 + C*Tan[e + f*x]^2),x]
```

```
[Out] -(((a - I*b)^(5/2)*(I*A + B - I*C)*Sqrt[c - I*d]*ArcTanh[(Sqrt[c - I*d]*Sqr
t[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/f) - ((a
+ I*b)^(5/2)*(B - I*(A - C))*Sqrt[c + I*d]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a +
b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/f - ((5*a^4*C*d
^4 - 20*a^3*b*d^3*(c*C + 2*B*d) + 30*a^2*b^2*d^2*(c^2*C - 4*B*c*d - 8*(A -
C)*d^2) - 20*a*b^3*d*(c^3*C - 2*B*c^2*d + 8*c*(A - C)*d^2 - 16*B*d^3) + b^4
*(5*c^4*C - 8*B*c^3*d + 16*c^2*(A - C)*d^2 + 64*B*c*d^3 + 128*(A - C)*d^4))
*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x
]])])/(64*b^(3/2)*d^(7/2)*f) + ((64*b*(a^2*B - b^2*B + 2*a*b*(A - C))*d^3 -
 (b*c - a*d)*(16*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(5*b*c*C - 8*b*B*d -
 5*a*C*d)))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(64*b*d^3*f)
 + ((16*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(5*b*c*C - 8*b*B*d - 5*a*C*d)
)*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))/(32*d^3*f) - ((5*b*c
*C - 8*b*B*d - 5*a*C*d)*(a + b*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(3/
2))/(24*d^2*f) + (C*(a + b*Tan[e + f*x])^(5/2)*(c + d*Tan[e + f*x])^(3/2))/
(4*d*f)
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[((a + b*ff*x)^m*(c + d*ff*x)^n*(A + B*ff*x + C*ff^2*x^2
))/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int (a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx = \frac{C(a+b\tan(e+fx))^{5/2}(c+d}{4df}$$

$$= -\frac{(5bcC-8bBd-5aCd)(a+}{}$$

$$= \frac{\left(16b(Ab+aB-bC)d^2+(bc-\right.}{}$$

$$= \frac{\left(64b\left(a^2B-b^2B+2ab(A-C\right.\right.}{}$$

$$= \frac{\left(64b\left(a^2B-b^2B+2ab(A-C\right.\right.}{}$$

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$$= \frac{\left(64b\left(a^2B-b^2B+2ab(A-C\right.\right.}{}$$

$$= -\frac{\left(5a^4Cd^4-20a^3bd^3(cC+2B\right.}{}$$

$$= -\frac{(a-ib)^{5/2}(iA+B-iC)\sqrt{c-}}{}$$

Mathematica [A] time = 9.72995, size = 1202, normalized size = 1.77

$$\frac{C(c+d\tan(e+fx))^{3/2}(a+b\tan(e+fx))^{5/2}}{4df}+\frac{\frac{(-5bcC+5adC+8bBd)(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}{6df}+\frac{3\left(16b(Ab-Cb+aB)d^2+(bc-ad)(5b\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Tan[e + f*x])^(5/2)*Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e
+ f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] (C*(a + b*Tan[e + f*x])^(5/2)*(c + d*Tan[e + f*x])^(3/2))/(4*d*f) + (((-5*b
*c*C + 8*b*B*d + 5*a*C*d)*(a + b*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(
3/2))/(6*d*f) + ((3*(16*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(5*b*c*C - 8*
b*B*d - 5*a*C*d))*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))/(8*d
*f) + (((24*b*(a^2*B - b^2*B + 2*a*b*(A - C))*d^3 - (3*(b*c - a*d)*(16*b*(A
*b + a*B - b*C)*d^2 + (b*c - a*d)*(5*b*c*C - 8*b*B*d - 5*a*C*d)))/8)*Sqrt[a
 + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(b*f) + ((24*b*d^3*(b*(3*a^2*b
*(A*c - c*C - B*d) - b^3*(A*c - c*C - B*d) + a^3*(B*c + (A - C)*d) - 3*a*b^
2*(B*c + (A - C)*d)) + Sqrt[-b^2]*(a^3*(A*c - c*C - B*d) - 3*a*b^2*(A*c - c
*C - B*d) - 3*a^2*b*(B*c + (A - C)*d) + b^3*(B*c + (A - C)*d)))*ArcTan[(Sqr
t[c + (b*d)/Sqrt[-b^2]]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sq
rt[c + d*Tan[e + f*x]])])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + (b*d)/Sqrt[-b^2]]
) - (24*b*d^3*(b*(3*a^2*b*(A*c - c*C - B*d) - b^3*(A*c - c*C - B*d) + a^3*(
B*c + (A - C)*d) - 3*a*b^2*(B*c + (A - C)*d)) - Sqrt[-b^2]*(a^3*(A*c - c*C
- B*d) - 3*a*b^2*(A*c - c*C - B*d) - 3*a^2*b*(B*c + (A - C)*d) + b^3*(B*c +
 (A - C)*d)))*ArcTan[(Sqrt[-((b*c + Sqrt[-b^2]*d)/b)]*Sqrt[a + b*Tan[e + f*
x]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + Sqrt[-b^2]
]*Sqrt[-((b*c + Sqrt[-b^2]*d)/b)]) - (3*Sqrt[b]*Sqrt[c - (a*d)/b]*Sqrt[(c/(
c - (a*d)/b) - (a*d)/(b*(c - (a*d)/b)))^(-1)]*Sqrt[c/(c - (a*d)/b) - (a*d)/
(b*(c - (a*d)/b))]*(5*a^4*C*d^4 - 20*a^3*b*d^3*(c*C + 2*B*d) + 30*a^2*b^2*d
^2*(c^2*C - 4*B*c*d - 8*(A - C)*d^2) - 20*a*b^3*d*(c^3*C - 2*B*c^2*d + 8*c*
(A - C)*d^2 - 16*B*d^3) + b^4*(5*c^4*C - 8*B*c^3*d + 16*c^2*(A - C)*d^2 + 6
4*B*c*d^3 + 128*(A - C)*d^4))*ArcSinh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(S
qrt[b]*Sqrt[c - (a*d)/b]*Sqrt[c/(c - (a*d)/b) - (a*d)/(b*(c - (a*d)/b))])]*
Sqrt[(c + d*Tan[e + f*x])/(c - (a*d)/b)])/(8*Sqrt[d]*Sqrt[c + d*Tan[e + f*x
]]))/(b^2*f))/(2*d))/(3*d))/(4*d)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \sqrt{c+d\tan\left(fx+e\right)}\left(a+b\tan\left(fx+e\right)\right)^{\frac{5}{2}}\left(A+B\tan\left(fx+e\right)+C\left(\tan\left(fx+e\right)\right)^{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*t
an(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*t
an(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)*(a+b*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C
*tan(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*t
an(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.129 $\int (a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx\right.$

Optimal. Leaf size=505

$$-\frac{\left(-3a^2bd^2(2Bd+cC)+a^3Cd^3+3ab^2d\left(-8d^2(A-C)-4Bcd+c^2C\right)+b^3\left(-\left(8cd^2(A-C)-2Bc^2d-16Bd^3+c^3C\right)\right)\right)\text{ta}}{8b^{3/2}d^{5/2}f}$$

```
[Out] -(((a - I*b)^(3/2)*(I*A + B - I*C)*Sqrt[c - I*d]*ArcTanh[(Sqrt[c - I*d]*Sqr
t[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/f) + ((a
+ I*b)^(3/2)*(I*A - B - I*C)*Sqrt[c + I*d]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a +
b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/f - ((a^3*C*d^3
 - 3*a^2*b*d^2*(c*C + 2*B*d) + 3*a*b^2*d*(c^2*C - 4*B*c*d - 8*(A - C)*d^2)
- b^3*(c^3*C - 2*B*c^2*d + 8*c*(A - C)*d^2 - 16*B*d^3))*ArcTanh[(Sqrt[d]*Sq
rt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(8*b^(3/2)*d^(
5/2)*f) + ((8*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(b*c*C - 2*b*B*d - a*C*
d))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(8*b*d^2*f) - ((b*c*
C - 2*b*B*d - a*C*d)*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))/(
4*d^2*f) + (C*(a + b*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(3/2))/(3*d*f
)
```

Rubi [A] time = 7.33787, antiderivative size = 505, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3647, 3655, 6725, 63, 217, 206, 93, 208}

$$-\frac{\left(-3a^2bd^2(2Bd+cC)+a^3Cd^3+3ab^2d\left(-8d^2(A-C)-4Bcd+c^2C\right)+b^3\left(-\left(8cd^2(A-C)-2Bc^2d-16Bd^3+c^3C\right)\right)\right)\text{ta}}{8b^{3/2}d^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^(3/2)*Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x]
 + C*Tan[e + f*x]^2),x]
```


```
[Out] -(((a - I*b)^(3/2)*(I*A + B - I*C)*Sqrt[c - I*d]*ArcTanh[(Sqrt[c - I*d]*Sqr
t[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/f) + ((a
+ I*b)^(3/2)*(I*A - B - I*C)*Sqrt[c + I*d]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a +
b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/f - ((a^3*C*d^3
 - 3*a^2*b*d^2*(c*C + 2*B*d) + 3*a*b^2*d*(c^2*C - 4*B*c*d - 8*(A - C)*d^2)
- b^3*(c^3*C - 2*B*c^2*d + 8*c*(A - C)*d^2 - 16*B*d^3))*ArcTanh[(Sqrt[d]*Sq
rt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(8*b^(3/2)*d^(
5/2)*f) + ((8*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(b*c*C - 2*b*B*d - a*C*
d))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(8*b*d^2*f) - ((b*c*
C - 2*b*B*d - a*C*d)*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))/(
4*d^2*f) + (C*(a + b*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(3/2))/(3*d*f
)
```


Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
```

```
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[((a + b*ff*x)^m*(c + d*ff*x)^n*(A + B*ff*x + C*ff^2*x^2
))/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int (a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx = \frac{C(a+b\tan(e+fx))^{3/2}(c+d\tan \ldots}{3df}$$

$$= -\frac{(bcC-2bBd-aCd)\sqrt{a+b\tan \ldots}}{4d^{\ldots}}$$

$$= \frac{\left(8b(Ab+aB-bC)d^2+(bc-ad \ldots\right.}{\ldots}$$

$$= \frac{\left(8b(Ab+aB-bC)d^2+(bc-ad \ldots\right.}{\ldots}$$

$$= \frac{\left(8b(Ab+aB-bC)d^2+(bc-ad \ldots\right.}{\ldots}$$

$$= \frac{\left(8b(Ab+aB-bC)d^2+(bc-ad \ldots\right.}{\ldots}$$

$$= \frac{\left(8b(Ab+aB-bC)d^2+(bc-ad \ldots\right.}{\ldots}$$

$$= \frac{\left(8b(Ab+aB-bC)d^2+(bc-ad \ldots\right.}{\ldots}$$

$$= -\frac{\left(a^3Cd^3-3a^2bd^2(cC+2Bd)+\ldots\right.}{\ldots}$$

$$= -\frac{(a-ib)^{3/2}(iA+B-iC)\sqrt{c-id \ldots}}{f \ldots}$$

Mathematica [A] time = 8.81503, size = 835, normalized size = 1.65

$$\frac{C(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}{3df} + \frac{\frac{3\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(8b(Ab-Cb+aB)d^2+(bc-ad)(bcC-adC-2bBd)\right)}{4bf} + \frac{6b\left(\sqrt{-b^2}\left((Ac-Cc-Bd)a^2-2b(B \ldots\right.\right.}{\ldots}}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Tan[e + f*x])^(3/2)*Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e
+ f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] (C*(a + b*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(3/2))/(3*d*f) + ((-3*(b
*c*C - 2*b*B*d - a*C*d)*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2)
)/(4*d*f) + ((3*(8*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(b*c*C - 2*b*B*d -
 a*C*d))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(4*b*f) + ((6*b
*d^2*(Sqrt[-b^2]*(a^2*(A*c - c*C - B*d) - b^2*(A*c - c*C - B*d) - 2*a*b*(B*
c + (A - C)*d)) + b*(2*a*b*(A*c - c*C - B*d) + a^2*(B*c + (A - C)*d) - b^2*
```

```
(B*c + (A - C)*d)))*ArcTan[(Sqrt[c + (b*d)/Sqrt[-b^2]]*Sqrt[a + b*Tan[e + f
*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[-a + Sqrt[-b
^2]]*Sqrt[c + (b*d)/Sqrt[-b^2]]) + (6*b*d^2*(Sqrt[-b^2]*(a^2*(A*c - c*C - B
*d) - b^2*(A*c - c*C - B*d) - 2*a*b*(B*c + (A - C)*d)) - b*(2*a*b*(A*c - c*
C - B*d) + a^2*(B*c + (A - C)*d) - b^2*(B*c + (A - C)*d)))*ArcTan[(Sqrt[-((
b*c + Sqrt[-b^2]*d)/b)]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + Sqrt[-b^2]]*Sqr
t[c + d*Tan[e + f*x]])])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[-((b*c + Sqrt[-b^2]*d)/
b)]) - (3*Sqrt[b]*Sqrt[c - (a*d)/b]*(a^3*C*d^3 - 3*a^2*b*d^2*(c*C + 2*B*d)
+ 3*a*b^2*d*(c^2*C - 4*B*c*d - 8*(A - C)*d^2) - b^3*(c^3*C - 2*B*c^2*d + 8*
c*(A - C)*d^2 - 16*B*d^3))*ArcSinh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt
[b]*Sqrt[c - (a*d)/b])]*Sqrt[(b*c + b*d*Tan[e + f*x])/(b*c - a*d)])/(4*Sqrt
[d]*Sqrt[c + d*Tan[e + f*x]]))/(b^2*f))/(2*d))/(3*d)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \sqrt{c+d\tan\left(fx+e\right)}\left(a+b\tan\left(fx+e\right)\right)^{\frac{3}{2}}\left(A+B\tan\left(fx+e\right)+C\left(\tan\left(fx+e\right)\right)^{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\tan\left(fx+e\right)^{2}+B\tan\left(fx+e\right)+A\right)\left(b\tan\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{d\tan\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*t
an(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^(3/2
)*sqrt(d*tan(f*x + e) + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*t
an(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)*(a+b*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C
*tan(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*t
an(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.130 $\int \sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+\right.$

Optimal. Leaf size=381

$$-\frac{\left(a^2Cd^2-2abd(2Bd+cC)+b^2\left(-8d^2(A-C)-4Bcd+c^2C\right)\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{4b^{3/2}d^{3/2}f}-\frac{\sqrt{a-ib}\sqrt{c-id}(iA+B-}{}$$

```
[Out] -((Sqrt[a - I*b]*(I*A + B - I*C)*Sqrt[c - I*d]*ArcTanh[(Sqrt[c - I*d]*Sqrt[
a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/f) - (Sqrt[
a + I*b]*(B - I*(A - C))*Sqrt[c + I*d]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Ta
n[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/f - ((a^2*C*d^2 - 2
*a*b*d*(c*C + 2*B*d) + b^2*(c^2*C - 4*B*c*d - 8*(A - C)*d^2))*ArcTanh[(Sqrt
[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(4*b^(3/
2)*d^(3/2)*f) - ((b*c*C - 4*b*B*d - a*C*d)*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c
+ d*Tan[e + f*x]])/(4*b*d*f) + (C*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f
*x])^(3/2))/(2*d*f)
```

Rubi [A] time = 4.97305, antiderivative size = 383, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 8, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3647, 3655, 6725, 63, 217, 206, 93, 208}

$$-\frac{\left(a^2Cd^2-2abd(2Bd+cC)+b^2\left(-8d^2(A-C)-4Bcd+c^2C\right)\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{4b^{3/2}d^{3/2}f}-\frac{\sqrt{a-ib}\sqrt{c-id}(iA+B-}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] +
 C*Tan[e + f*x]^2),x]
```

```
[Out] -((Sqrt[a - I*b]*(I*A + B - I*C)*Sqrt[c - I*d]*ArcTanh[(Sqrt[c - I*d]*Sqrt[
a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/f) + (Sqrt[
a + I*b]*(I*A - B - I*C)*Sqrt[c + I*d]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Ta
n[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/f - ((a^2*C*d^2 - 2
*a*b*d*(c*C + 2*B*d) + b^2*(c^2*C - 4*B*c*d - 8*(A - C)*d^2))*ArcTanh[(Sqrt
[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(4*b^(3/
2)*d^(3/2)*f) - ((b*c*C - 4*b*B*d - a*C*d)*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c
+ d*Tan[e + f*x]])/(4*b*d*f) + (C*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f
*x])^(3/2))/(2*d*f)
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[((a + b*ff*x)^m*(c + d*ff*x)^n*(A + B*ff*x + C*ff^2*x^2
))/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx = \frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan \ldots}{2df}$$

$$= -\frac{(bcC-4bBd-aCd)\sqrt{a+b\,\mathrm{ta}\ldots}}{4b\ldots}$$

$$= -\frac{(bcC-4bBd-aCd)\sqrt{a+b\,\mathrm{ta}\ldots}}{4b\ldots}$$

$$= -\frac{(bcC-4bBd-aCd)\sqrt{a+b\,\mathrm{ta}\ldots}}{4b\ldots}$$

$$= -\frac{(bcC-4bBd-aCd)\sqrt{a+b\,\mathrm{ta}\ldots}}{4b\ldots}$$

$$= -\frac{(bcC-4bBd-aCd)\sqrt{a+b\,\mathrm{ta}\ldots}}{4b\ldots}$$

$$= -\frac{(bcC-4bBd-aCd)\sqrt{a+b\,\mathrm{ta}\ldots}}{4b\ldots}$$

$$= -\frac{\left(a^2Cd^2-2abd(cC+2Bd)+b\ldots\right.}{\ldots}$$

$$= -\frac{\sqrt{a-ib}(iA+B-iC)\sqrt{c-id\ldots}}{f\ldots}$$

Mathematica [A] time = 7.70825, size = 619, normalized size = 1.62

$$\frac{-\frac{\sqrt{b}\sqrt{c-\frac{ad}{b}}\left(a^2Cd^2-2abd(2Bd+cC)+b^2\left(-8d^2(A-C)-4Bcd+c^2C\right)\right)\sqrt{\frac{bc+bd\tan(e+fx)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c-\frac{ad}{b}}}\right)}{2\sqrt{d}\sqrt{c+d\tan(e+fx)}}+\frac{2bd\left(b(aAd+aBc-aCd+Abc-bBd-bcC)-\sqrt{-b^2}(-a(Ac-Bd-cC)+bd(\ldots\right.}{\sqrt{\sqrt{-b^2}-a}\sqrt{\frac{bd}{\sqrt{-b^2}\ldots}}\ldots}}{b^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e +
f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] (C*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))/(2*d*f) + (((-(b*c*
C) + 4*b*B*d + a*C*d)*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(2
*b*f) + ((2*b*d*(b*(A*b*c + a*B*c - b*c*C + a*A*d - b*B*d - a*C*d) - Sqrt[-
b^2]*(b*B*c + b*(A - C)*d - a*(A*c - c*C - B*d)))*ArcTan[(Sqrt[c + (b*d)/Sq
rt[-b^2]]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e
 + f*x]])])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + (b*d)/Sqrt[-b^2]]) - (2*b*d*(b*
(A*b*c + a*B*c - b*c*C + a*A*d - b*B*d - a*C*d) + Sqrt[-b^2]*(b*B*c + b*(A
- C)*d - a*(A*c - c*C - B*d)))*ArcTan[(Sqrt[-((b*c + Sqrt[-b^2]*d)/b)]*Sqrt
[a + b*Tan[e + f*x]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e + f*x]])])/(Sq
rt[a + Sqrt[-b^2]]*Sqrt[-((b*c + Sqrt[-b^2]*d)/b)]) - (Sqrt[b]*Sqrt[c - (a*
```

```
d)/b]*(a^2*C*d^2 - 2*a*b*d*(c*C + 2*B*d) + b^2*(c^2*C - 4*B*c*d - 8*(A - C)
*d^2))*ArcSinh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c - (a*d)/b
])]*Sqrt[(b*c + b*d*Tan[e + f*x])/(b*c - a*d)])/(2*Sqrt[d]*Sqrt[c + d*Tan[e
 + f*x]]))/(b^2*f))/(2*d)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \sqrt{a+b\tan\left(fx+e\right)}\sqrt{c+d\tan\left(fx+e\right)}\left(A+B\tan\left(fx+e\right)+C\left(\tan\left(fx+e\right)\right)^{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\tan\left(fx+e\right)^{2}+B\tan\left(fx+e\right)+A\right)\sqrt{b\tan\left(fx+e\right)+a}\sqrt{d\tan\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*t
an(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*sqrt(b*tan(f*x + e) + a)*
sqrt(d*tan(f*x + e) + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*t
an(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\tan\left(e+fx\right)}\sqrt{c+d\tan\left(e+fx\right)}\left(A+B\tan\left(e+fx\right)+C\tan^{2}\left(e+fx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(1/2)*(c+d*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e)+C
*tan(f*x+e)**2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x))*sqrt(c + d*tan(e + f*x))*(A + B*tan(e + f
*x) + C*tan(e + f*x)**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*t
an(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.131 $\int \frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{a+b\tan(e+fx)}}\,dx$

Optimal. Leaf size=287

$$-\frac{\sqrt{c-id}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a-ib}}-\frac{\sqrt{c+id}(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a+ib}}+\frac{(-aCd+2bB}{}$$

```
[Out] -(((I*A + B - I*C)*Sqrt[c - I*d]*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a - I*b]*f)) - ((B
- I*(A - C))*Sqrt[c + I*d]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])
/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*f) + ((b*c*C + 2
*b*B*d - a*C*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c
+ d*Tan[e + f*x]])])/(b^(3/2)*Sqrt[d]*f) + (C*Sqrt[a + b*Tan[e + f*x]]*Sqrt
[c + d*Tan[e + f*x]])/(b*f)
```

Rubi [A] time = 2.63334, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3647, 3655, 6725, 63, 217, 206, 93, 208}

$$-\frac{\sqrt{c-id}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a-ib}}-\frac{\sqrt{c+id}(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a+ib}}+\frac{(-aCd+2bB}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/Sqrt
[a + b*Tan[e + f*x]],x]
```


```
[Out] -(((I*A + B - I*C)*Sqrt[c - I*d]*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a - I*b]*f)) - ((B
- I*(A - C))*Sqrt[c + I*d]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])
/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*f) + ((b*c*C + 2
*b*B*d - a*C*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c
+ d*Tan[e + f*x]])])/(b^(3/2)*Sqrt[d]*f) + (C*Sqrt[a + b*Tan[e + f*x]]*Sqrt
[c + d*Tan[e + f*x]])/(b*f)
```


Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
```

```
ist[ff/f, Subst[Int[((a + b*ff*x)^m*(c + d*ff*x)^n*(A + B*ff*x + C*ff^2*x^2
))/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{a+b\tan(e+fx)}}dx = \frac{C\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{bf} + \frac{\int \frac{\frac{1}{2}(2Abc-\cdots}{\cdots}}{\cdots}$$

$$= \frac{C\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{bf} + \frac{\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{C\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{bf} + \frac{\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{C\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{bf} + \frac{\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{C\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{bf} + \frac{\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{C\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{bf} + \frac{((A-iB-\cdots}{\cdots}$$

$$= \frac{(bcC+2bBd-aCd)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{b^{3/2}\sqrt{d}f} + \frac{C\sqrt{\cdots}}{\cdots}$$

$$= -\frac{(iA+B-iC)\sqrt{c-id}\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a-ib}f} - \cdots$$

Mathematica [A] time = 4.05468, size = 441, normalized size = 1.54

$$\frac{\frac{b\left(\sqrt{-b^2}(Ac-Bd-cC)+bd(A-C)+bBc\right)\tan^{-1}\left(\frac{\sqrt{\frac{bd}{\sqrt{-b^2}}+c}\sqrt{a+b\tan(e+fx)}}{\sqrt{\sqrt{-b^2}-a}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{\sqrt{-b^2}-a}\sqrt{\frac{bd}{\sqrt{-b^2}}+c}} + \frac{b\left(\sqrt{-b^2}(Ac-Bd-cC)-b(d(A-C)+Bc)\right)\tan^{-1}\left(\frac{\sqrt{-\frac{\sqrt{-b^2}d+bc}{b}}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+\sqrt{-b^2}}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+\sqrt{-b^2}}\sqrt{-\frac{\sqrt{-b^2}d+bc}{b}}} + \frac{\sqrt{b}\sqrt{c-\cdots}}{\cdots}}{b^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)
)/Sqrt[a + b*Tan[e + f*x]],x]
```

```
[Out] ((b*(b*B*c + b*(A - C)*d + Sqrt[-b^2]*(A*c - c*C - B*d))*ArcTan[(Sqrt[c + (
b*d)/Sqrt[-b^2]]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c +
d*Tan[e + f*x]])])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + (b*d)/Sqrt[-b^2]]) + (b*
(Sqrt[-b^2]*(A*c - c*C - B*d) - b*(B*c + (A - C)*d))*ArcTan[(Sqrt[-((b*c +
Sqrt[-b^2]*d)/b)]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c +
d*Tan[e + f*x]])])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[-((b*c + Sqrt[-b^2]*d)/b)]) +
 b*C*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]] + (Sqrt[b]*Sqrt[c -
(a*d)/b]*(b*c*C + 2*b*B*d - a*C*d)*ArcSinh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]
])/(Sqrt[b]*Sqrt[c - (a*d)/b])]*Sqrt[(b*(c + d*Tan[e + f*x]))/(b*c - a*d)])
/(Sqrt[d]*Sqrt[c + d*Tan[e + f*x]]))/(b^2*f)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int (A + B\tan\left(fx+e\right) + C\left(\tan\left(fx+e\right)\right)^{2})\sqrt{c+d\tan\left(fx+e\right)}\frac{1}{\sqrt{a+b\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(1/2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\tan\left(fx+e\right)^{2} + B\tan\left(fx+e\right) + A\right)\sqrt{d\tan\left(fx+e\right)+c}}{\sqrt{b\tan\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*sqrt(d*tan(f*x + e) + c)/
sqrt(b*tan(f*x + e) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+d\tan\left(e+fx\right)}\left(A + B\tan\left(e+fx\right) + C\tan^{2}\left(e+fx\right)\right)}{\sqrt{a+b\tan\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/
sqrt(a + b*tan(e + f*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.132 $\int \frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=300

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{bf\left(a^2+b^2\right)\sqrt{a+b\tan(e+fx)}}-\frac{\sqrt{c-id}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{f(a-ib)^{3/2}}-\frac{\sqrt{c+id}(B-i(A-C$$

```
[Out] -(((I*A + B - I*C)*Sqrt[c - I*d]*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(3/2)*f)) - ((
B - I*(A - C))*Sqrt[c + I*d]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]
])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(3/2)*f) + (2*C*Sq
rt[d]*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e
+ f*x]])])/(b^(3/2)*f) - (2*(A*b^2 - a*(b*B - a*C))*Sqrt[c + d*Tan[e + f*x]
])/(b*(a^2 + b^2)*f*Sqrt[a + b*Tan[e + f*x]])
```

Rubi [A] time = 3.79592, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3645, 3655, 6725, 63, 217, 206, 93, 208}

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{bf\left(a^2+b^2\right)\sqrt{a+b\tan(e+fx)}}-\frac{\sqrt{c-id}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{f(a-ib)^{3/2}}-\frac{\sqrt{c+id}(B-i(A-C$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a +
 b*Tan[e + f*x])^(3/2),x]
```



```
[Out] -(((I*A + B - I*C)*Sqrt[c - I*d]*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(3/2)*f)) - ((
B - I*(A - C))*Sqrt[c + I*d]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]
])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(3/2)*f) + (2*C*Sq
rt[d]*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e
+ f*x]])])/(b^(3/2)*f) - (2*(A*b^2 - a*(b*B - a*C))*Sqrt[c + d*Tan[e + f*x]
])/(b*(a^2 + b^2)*f*Sqrt[a + b*Tan[e + f*x]])
```


Rule 3645

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*d^2 + c*(c*C - B*d))*(a + b*Tan[e
+ f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
```

```
ist[ff/f, Subst[Int[((a + b*ff*x)^m*(c + d*ff*x)^n*(A + B*ff*x + C*ff^2*x^2
))/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{3/2}}\,dx = -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{b\left(a^2+b^2\right)f\sqrt{a+b\tan(e+fx)}} + \frac{2\int \cdots}{\cdots}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{b\left(a^2+b^2\right)f\sqrt{a+b\tan(e+fx)}} + \frac{2\,\mathrm{Su}\cdots}{\cdots}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{b\left(a^2+b^2\right)f\sqrt{a+b\tan(e+fx)}} + \frac{2\,\mathrm{Su}\cdots}{\cdots}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{b\left(a^2+b^2\right)f\sqrt{a+b\tan(e+fx)}} + \frac{\mathrm{Sub}\cdots}{\cdots}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{b\left(a^2+b^2\right)f\sqrt{a+b\tan(e+fx)}} + \frac{\mathrm{Sub}\cdots}{\cdots}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{b\left(a^2+b^2\right)f\sqrt{a+b\tan(e+fx)}} + \frac{((ia\cdots}{\cdots}$$

$$= \frac{2C\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{b^{3/2}f} - \frac{2\left(Ab^2-a(bB\cdots\right.}{b\left(a^2+b^2\right)\cdots}$$

$$= -\frac{(iA+B-iC)\sqrt{c-id}\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{3/2}f}\cdots$$

Mathematica [C] time = 35.7252, size = 621058, normalized size = 2070.19

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)
)/(a + b*Tan[e + f*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \left(A+B\tan\left(fx+e\right)+C\left(\tan\left(fx+e\right)\right)^2\right)\sqrt{c+d\tan\left(fx+e\right)}\left(a+b\tan\left(fx+e\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(3/2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(3/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + d\tan\left(e + fx\right)}\left(A + B\tan\left(e + fx\right) + C\tan^{2}\left(e + fx\right)\right)}{\left(a + b\tan\left(e + fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/
(a + b*tan(e + f*x))**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.133 $\int \frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=370

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{3bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{3/2}}-\frac{2\sqrt{c+d\tan(e+fx)}\left(-a^2b^2(5Ad+3Bc-7Cd)+2a^3bBd+a^4Cd+2ab\right.}{3bf\left(a^2+b^2\right)^2(bc-ad)\sqrt{a+b\tan(e+\right.}$$

```
[Out] -(((I*A + B - I*C)*Sqrt[c - I*d]*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(5/2)*f)) - ((
B - I*(A - C))*Sqrt[c + I*d]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]
])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(5/2)*f) - (2*(A*b
^2 - a*(b*B - a*C))*Sqrt[c + d*Tan[e + f*x]])/(3*b*(a^2 + b^2)*f*(a + b*Tan
[e + f*x])^(3/2)) - (2*(2*a^3*b*B*d + a^4*C*d + b^4*(3*B*c + A*d) + 2*a*b^3
*(3*A*c - 3*c*C - 2*B*d) - a^2*b^2*(3*B*c + 5*A*d - 7*C*d))*Sqrt[c + d*Tan[
e + f*x]])/(3*b*(a^2 + b^2)^2*(b*c - a*d)*f*Sqrt[a + b*Tan[e + f*x]])
```

Rubi [A] time = 2.05186, antiderivative size = 370, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3645, 3649, 3616, 3615, 93, 208}

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{3bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{3/2}}-\frac{2\sqrt{c+d\tan(e+fx)}\left(-a^2b^2(5Ad+3Bc-7Cd)+2a^3bBd+a^4Cd+2ab\right.}{3bf\left(a^2+b^2\right)^2(bc-ad)\sqrt{a+b\tan(e+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a +
 b*Tan[e + f*x])^(5/2),x]
```

```
[Out] -(((I*A + B - I*C)*Sqrt[c - I*d]*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(5/2)*f)) - ((
B - I*(A - C))*Sqrt[c + I*d]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]
])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(5/2)*f) - (2*(A*b
^2 - a*(b*B - a*C))*Sqrt[c + d*Tan[e + f*x]])/(3*b*(a^2 + b^2)*f*(a + b*Tan
[e + f*x])^(3/2)) - (2*(2*a^3*b*B*d + a^4*C*d + b^4*(3*B*c + A*d) + 2*a*b^3
*(3*A*c - 3*c*C - 2*B*d) - a^2*b^2*(3*B*c + 5*A*d - 7*C*d))*Sqrt[c + d*Tan[
e + f*x]])/(3*b*(a^2 + b^2)^2*(b*c - a*d)*f*Sqrt[a + b*Tan[e + f*x]])
```

Rule 3645

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*d^2 + c*(c*C - B*d))*(a + b*Tan[e
+ f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
 b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3616

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3615

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[((a + b*x)^m*(c + d*x)^n)/(A - B*x), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{5/2}}\,dx = -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{3b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{3/2}} + \frac{2\int}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{3b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{3/2}} - \frac{2(2a}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{3b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{3/2}} - \frac{2(2a}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{3b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{3/2}} - \frac{2(2a}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{3b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{3/2}} - \frac{2(2a}{}$$

$$= -\frac{(iA+B-iC)\sqrt{c-id}\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{5/2}f}$$

Mathematica [A] time = 6.95865, size = 603, normalized size = 1.63

$$-\frac{C\sqrt{c+d\tan(e+fx)}}{bf(a+b\tan(e+fx))^{3/2}} - \frac{-\frac{2\sqrt{c+d\tan(e+fx)}\left(\frac{1}{2}b^2(-aCd-2Abc+3bcC)-a\left(b^2(-(d(A-C)+Bc))-\frac{1}{2}a(-aCd-2bBd+bcC)\right)\right)}{3f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))^{3/2}} - \frac{2\left(-\frac{2\sqrt{c+d\tan(e+fx)}\left(\right.}{}\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)
)/(a + b*Tan[e + f*x])^(5/2),x]
```

```
[Out] -((C*Sqrt[c + d*Tan[e + f*x]])/(b*f*(a + b*Tan[e + f*x])^(3/2))) - ((-2*((b
^2*(-2*A*b*c + 3*b*c*C - a*C*d))/2 - a*(-(b^2*(B*c + (A - C)*d)) - (a*(b*c*
C - 2*b*B*d - a*C*d))/2))*Sqrt[c + d*Tan[e + f*x]])/(3*(a^2 + b^2)*(b*c - a
*d)*f*(a + b*Tan[e + f*x])^(3/2)) - (2*((-3*b*(b*c - a*d)*(((a - I*b)^2*(I*
A - B - I*C)*Sqrt[-c - I*d]*ArcTan[(Sqrt[-c - I*d]*Sqrt[a + b*Tan[e + f*x]]
)/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])]])/Sqrt[a + I*b] - ((a + I*b)^2*(
B + I*(A - C))*Sqrt[c - I*d]*ArcTan[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]]
)/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])]])/Sqrt[-a + I*b]))/(2*(a^2 + b^
2)*f) - (2*((b^2*(b*c - a*d)*(a^2*C*d + b^2*(3*B*c + A*d) + a*b*(3*A*c - 3*
c*C - B*d)))/2 - a*((a*(2*A*b^2 - 2*a*b*B - a^2*C - 3*b^2*C)*d*(b*c - a*d))
/2 - (3*b^2*(b*c - a*d)*(A*b*c - a*B*c - b*c*C - a*A*d - b*B*d + a*C*d))/2)
)*Sqrt[c + d*Tan[e + f*x]])/((a^2 + b^2)*(b*c - a*d)*f*Sqrt[a + b*Tan[e + f
*x]])))/(3*(a^2 + b^2)*(b*c - a*d)))/b
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int (A+B\tan(fx+e)+C(\tan(fx+e))^2)\sqrt{c+d\tan(fx+e)}\,(a+b\tan(fx+e))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(5/2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + d\tan\left(e + fx\right)}\left(A + B\tan\left(e + fx\right) + C\tan^{2}\left(e + fx\right)\right)}{\left(a + b\tan\left(e + fx\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**(5/2),x)
```

```
[Out] Integral(sqrt(c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/
(a + b*tan(e + f*x))**(5/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.134 $\int \frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=597

$$\frac{2\sqrt{c+d\tan(e+fx)}\left(a^3b^3\left(80cd(A-C)+B\left(15c^2-49d^2\right)\right)-a^2b^4\left(45Ac^2-29Ad^2-90Bcd-45c^2C+23Cd^2\right)-a^4b^2d(\cdots}{15bf\left(a^2+b^2\right)^3(bc-a\cdots}$$

```
[Out] -(((I*A + B - I*C)*Sqrt[c - I*d]*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(7/2)*f)) - ((
B - I*(A - C))*Sqrt[c + I*d]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]
])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(7/2)*f) - (2*(A*b
^2 - a*(b*B - a*C))*Sqrt[c + d*Tan[e + f*x]])/(5*b*(a^2 + b^2)*f*(a + b*Tan
[e + f*x])^(5/2)) - (2*(4*a^3*b*B*d + a^4*C*d + b^4*(5*B*c + A*d) + 2*a*b^3
*(5*A*c - 5*c*C - 3*B*d) - a^2*b^2*(5*B*c + 9*A*d - 11*C*d))*Sqrt[c + d*Tan
[e + f*x]])/(15*b*(a^2 + b^2)^2*(b*c - a*d)*f*(a + b*Tan[e + f*x])^(3/2)) +
 (2*(8*a^5*b*B*d^2 + 2*a^6*C*d^2 - a^4*b^2*d*(25*B*c + 33*A*d - 39*C*d) - a
^2*b^4*(45*A*c^2 - 45*c^2*C - 90*B*c*d - 29*A*d^2 + 23*C*d^2) + a^3*b^3*(80
*c*(A - C)*d + B*(15*c^2 - 49*d^2)) - a*b^5*(40*c*(A - C)*d + B*(45*c^2 - 3
*d^2)) - b^6*(5*c*(3*c*C + B*d) - A*(15*c^2 + 2*d^2)))*Sqrt[c + d*Tan[e + f
*x]])/(15*b*(a^2 + b^2)^3*(b*c - a*d)^2*f*Sqrt[a + b*Tan[e + f*x]])
```

Rubi [A] time = 3.58853, antiderivative size = 597, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3645, 3649, 3616, 3615, 93, 208}

$$\frac{2\sqrt{c+d\tan(e+fx)}\left(a^3b^3\left(80cd(A-C)+B\left(15c^2-49d^2\right)\right)-a^2b^4\left(45Ac^2-29Ad^2-90Bcd-45c^2C+23Cd^2\right)-a^4b^2d(\cdots}{15bf\left(a^2+b^2\right)^3(bc-a\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a +
 b*Tan[e + f*x])^(7/2),x]
```


```
[Out] -(((I*A + B - I*C)*Sqrt[c - I*d]*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(7/2)*f)) - ((
B - I*(A - C))*Sqrt[c + I*d]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]
])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(7/2)*f) - (2*(A*b
^2 - a*(b*B - a*C))*Sqrt[c + d*Tan[e + f*x]])/(5*b*(a^2 + b^2)*f*(a + b*Tan
[e + f*x])^(5/2)) - (2*(4*a^3*b*B*d + a^4*C*d + b^4*(5*B*c + A*d) + 2*a*b^3
*(5*A*c - 5*c*C - 3*B*d) - a^2*b^2*(5*B*c + 9*A*d - 11*C*d))*Sqrt[c + d*Tan
[e + f*x]])/(15*b*(a^2 + b^2)^2*(b*c - a*d)*f*(a + b*Tan[e + f*x])^(3/2)) +
 (2*(8*a^5*b*B*d^2 + 2*a^6*C*d^2 - a^4*b^2*d*(25*B*c + 33*A*d - 39*C*d) - a
^2*b^4*(45*A*c^2 - 45*c^2*C - 90*B*c*d - 29*A*d^2 + 23*C*d^2) + a^3*b^3*(80
*c*(A - C)*d + B*(15*c^2 - 49*d^2)) - a*b^5*(40*c*(A - C)*d + B*(45*c^2 - 3
*d^2)) - b^6*(5*c*(3*c*C + B*d) - A*(15*c^2 + 2*d^2)))*Sqrt[c + d*Tan[e + f
*x]])/(15*b*(a^2 + b^2)^3*(b*c - a*d)^2*f*Sqrt[a + b*Tan[e + f*x]])
```


Rule 3645

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*d^2 + c*(c*C - B*d))*(a + b*Tan[e
+ f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dis
```

```
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
 b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3616

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3615

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[((a + b*x)^m*(c + d*x)^n)/(A - B*x), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{7/2}}\,dx = -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{5b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{5/2}} + \frac{2\int \frac{\frac{1}{2}((b}{}}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{5b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{5/2}} - \frac{2\left(4a^3b\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{5b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{5/2}} - \frac{2\left(4a^3b\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{5b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{5/2}} - \frac{2\left(4a^3b\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{5b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{5/2}} - \frac{2\left(4a^3b\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{5b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{5/2}} - \frac{2\left(4a^3b\right.}{}$$

$$= -\frac{(iA+B-iC)\sqrt{c-id}\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{7/2}f} -$$

Mathematica [A] time = 7.15435, size = 1108, normalized size = 1.86

$$-\frac{\sqrt{c+d\tan(e+fx)}C}{2bf(a+b\tan(e+fx))^{5/2}} - \frac{-\frac{2\sqrt{c+d\tan(e+fx)}\left(\frac{1}{2}b^2(-4Abc+5bCc-aCd)-a\left(-2(Bc+(A-C)d)b^2-\frac{1}{2}a(bcC-adC-4bBd)\right)\right)}{5\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{5/2}} - \frac{2\left(-\frac{2\sqrt{c+d\tan(e+fx)}\left(b^2(bc-\right.}{}\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)
)/(a + b*Tan[e + f*x])^(7/2),x]
```

```
[Out] -(C*Sqrt[c + d*Tan[e + f*x]])/(2*b*f*(a + b*Tan[e + f*x])^(5/2)) - ((-2*((b
^2*(-4*A*b*c + 5*b*c*C - a*C*d))/2 - a*(-2*b^2*(B*c + (A - C)*d) - (a*(b*c*
C - 4*b*B*d - a*C*d))/2))*Sqrt[c + d*Tan[e + f*x]])/(5*(a^2 + b^2)*(b*c - a
*d)*f*(a + b*Tan[e + f*x])^(5/2)) - (2*((-2*(b^2*(b*c - a*d)*(a^2*C*d + b^2
*(5*B*c + A*d) + a*b*(5*A*c - 5*c*C - B*d)) - a*(a*(4*A*b^2 - 4*a*b*B - a^2
*C - 5*b^2*C)*d*(b*c - a*d) - 5*b^2*(b*c - a*d)*(A*b*c - a*B*c - b*c*C - a*
A*d - b*B*d + a*C*d)))*Sqrt[c + d*Tan[e + f*x]])/(3*(a^2 + b^2)*(b*c - a*d)
*f*(a + b*Tan[e + f*x])^(3/2)) - (2*((-15*b*(b*c - a*d)^2*(((I*a + b)^3*(A
```

```
+ I*B - C)*Sqrt[-c - I*d]*ArcTan[(Sqrt[-c - I*d]*Sqrt[a + b*Tan[e + f*x]])/
(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[a + I*b] + ((a + I*b)^3*(I*
A + B - I*C)*Sqrt[c - I*d]*ArcTan[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/
(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[-a + I*b]))/(2*(a^2 + b^2)
*f) - (2*(b^2*((b*c - a*d)*(b^2*d - (3*a*(b*c - a*d))/2)*(a^2*C*d + b^2*(5*
B*c + A*d) + a*b*(5*A*c - 5*c*C - B*d)) + ((-3*b*c)/2 + (a*d)/2)*(a*(4*A*b^
2 - 4*a*b*B - a^2*C - 5*b^2*C)*d*(b*c - a*d) - 5*b^2*(b*c - a*d)*(A*b*c - a
*B*c - b*c*C - a*A*d - b*B*d + a*C*d))) - a*((3*b*(b*c - a*d)*(b*(4*A*b^2 -
 4*a*b*B - a^2*C - 5*b^2*C)*d*(b*c - a*d) + 5*a*b*(b*c - a*d)*(A*b*c - a*B*
c - b*c*C - a*A*d - b*B*d + a*C*d) + b*(b*c - a*d)*(a^2*C*d + b^2*(5*B*c +
A*d) + a*b*(5*A*c - 5*c*C - B*d))))/2 - a*d*(b^2*(b*c - a*d)*(a^2*C*d + b^2
*(5*B*c + A*d) + a*b*(5*A*c - 5*c*C - B*d)) - a*(a*(4*A*b^2 - 4*a*b*B - a^2
*C - 5*b^2*C)*d*(b*c - a*d) - 5*b^2*(b*c - a*d)*(A*b*c - a*B*c - b*c*C - a*
A*d - b*B*d + a*C*d)))))*Sqrt[c + d*Tan[e + f*x]])/((a^2 + b^2)*(b*c - a*d)
*f*Sqrt[a + b*Tan[e + f*x]])))/(3*(a^2 + b^2)*(b*c - a*d))))/(5*(a^2 + b^2)
*(b*c - a*d)))/(2*b)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int (A+B\tan\left(fx+e\right)+C\left(\tan\left(fx+e\right)\right)^{2})\sqrt{c+d\tan\left(fx+e\right)}\left(a+b\tan\left(fx+e\right)\right)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(7/2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(7/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.135 $\int (a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+\right.$

Optimal. Leaf size=682

$$\frac{\left(6a^2b^2d^2\left(8d^2(A-C)+12Bcd+3c^2C\right)-4a^3bd^3(2Bd+3cC)+3a^4Cd^4-12ab^3d\left(-24cd^2(A-C)-6Bc^2d+16Bd^3+\right.\right.}{64b^{5/2}d^{5/2}f}$$

```
[Out] -(((a - I*b)^(3/2)*(B + I*(A - C))*(c - I*d)^(3/2)*ArcTanh[(Sqrt[c - I*d]*S
qrt[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/f) - ((
a + I*b)^(3/2)*(B - I*(A - C))*(c + I*d)^(3/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[
a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/f + ((3*a^4
*C*d^4 - 4*a^3*b*d^3*(3*c*C + 2*B*d) + 6*a^2*b^2*d^2*(3*c^2*C + 12*B*c*d +
8*(A - C)*d^2) - 12*a*b^3*d*(c^3*C - 6*B*c^2*d - 24*c*(A - C)*d^2 + 16*B*d^
3) + b^4*(3*c^4*C - 8*B*c^3*d + 48*c^2*(A - C)*d^2 - 192*B*c*d^3 - 128*(A -
 C)*d^4))*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Ta
n[e + f*x]])])/(64*b^(5/2)*d^(5/2)*f) + ((64*b*(a^2*B - b^2*B + 2*a*b*(A -
C))*d^3 + (b*c - a*d)*(48*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(3*b*c*C -
8*b*B*d - 3*a*C*d)))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(64
*b^2*d^2*f) + ((48*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(3*b*c*C - 8*b*B*d
 - 3*a*C*d))*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))/(96*b*d^2
*f) - ((3*b*c*C - 8*b*B*d - 3*a*C*d)*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e
+ f*x])^(5/2))/(24*d^2*f) + (C*(a + b*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*
x])^(5/2))/(4*d*f)
```

Rubi [A] time = 11.8965, antiderivative size = 682, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3647, 3655, 6725, 63, 217, 206, 93, 208}

$$\frac{\left(6a^2b^2d^2\left(8d^2(A-C)+12Bcd+3c^2C\right)-4a^3bd^3(2Bd+3cC)+3a^4Cd^4-12ab^3d\left(-24cd^2(A-C)-6Bc^2d+16Bd^3+\right.\right.}{64b^{5/2}d^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*
x] + C*Tan[e + f*x]^2),x]
```


```
[Out] -(((a - I*b)^(3/2)*(B + I*(A - C))*(c - I*d)^(3/2)*ArcTanh[(Sqrt[c - I*d]*S
qrt[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/f) - ((
a + I*b)^(3/2)*(B - I*(A - C))*(c + I*d)^(3/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[
a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/f + ((3*a^4
*C*d^4 - 4*a^3*b*d^3*(3*c*C + 2*B*d) + 6*a^2*b^2*d^2*(3*c^2*C + 12*B*c*d +
8*(A - C)*d^2) - 12*a*b^3*d*(c^3*C - 6*B*c^2*d - 24*c*(A - C)*d^2 + 16*B*d^
3) + b^4*(3*c^4*C - 8*B*c^3*d + 48*c^2*(A - C)*d^2 - 192*B*c*d^3 - 128*(A -
 C)*d^4))*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Ta
n[e + f*x]])])/(64*b^(5/2)*d^(5/2)*f) + ((64*b*(a^2*B - b^2*B + 2*a*b*(A -
C))*d^3 + (b*c - a*d)*(48*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(3*b*c*C -
8*b*B*d - 3*a*C*d)))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(64
*b^2*d^2*f) + ((48*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(3*b*c*C - 8*b*B*d
 - 3*a*C*d))*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))/(96*b*d^2
*f) - ((3*b*c*C - 8*b*B*d - 3*a*C*d)*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e
+ f*x])^(5/2))/(24*d^2*f) + (C*(a + b*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*
x])^(5/2))/(4*d*f)
```


Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[((a + b*ff*x)^m*(c + d*ff*x)^n*(A + B*ff*x + C*ff^2*x^2
))/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int (a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx=\frac{C(a+b\tan(e+fx))^{3/2}(c+\ldots}{4df}$$

$$=-\frac{(3bcC-8bBd-3aCd)\sqrt{a\ldots}}{\ldots}$$

$$=\frac{\left(48b(Ab+aB-bC)d^2+(b\ldots\right.}{\ldots}$$

$$=\frac{\left(64b\left(a^2B-b^2B+2ab(A-\ldots\right.\right.}{\ldots}$$

$$=\frac{\left(64b\left(a^2B-b^2B+2ab(A-\ldots\right.\right.}{\ldots}$$

$$=\frac{\left(64b\left(a^2B-b^2B+2ab(A-\ldots\right.\right.}{\ldots}$$

$$=\frac{\left(64b\left(a^2B-b^2B+2ab(A-\ldots\right.\right.}{\ldots}$$

$$=\frac{\left(64b\left(a^2B-b^2B+2ab(A-\ldots\right.\right.}{\ldots}$$

$$=\frac{\left(64b\left(a^2B-b^2B+2ab(A-\ldots\right.\right.}{\ldots}$$

$$=\frac{\left(3a^4Cd^4-4a^3bd^3(3cC+2\ldots\right.}{\ldots}$$

$$=-\frac{(a-ib)^{3/2}(B+i(A-C))(\ldots}{\ldots}$$

Mathematica [A] time = 8.17153, size = 1316, normalized size = 1.93

$$\frac{C(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}{4df}+\frac{\frac{(-3bcC+3adC+8bBd)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{6df}+\frac{\left(48b(Ab-Cb+aB)d^2+(bc-ad)(3bcC-\ldots\right.}{\ldots}}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[
e + f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] (C*(a + b*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(5/2))/(4*d*f) + (((-3*b
*c*C + 8*b*B*d + 3*a*C*d)*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(5/
2))/(6*d*f) + (((48*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(3*b*c*C - 8*b*B*
d - 3*a*C*d))*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))/(8*b*f)
+ (((24*b*(a^2*B - b^2*B + 2*a*b*(A - C))*d^3 - (3*(-(b*c) + a*d)*(48*b*(A*
b + a*B - b*C)*d^2 + (b*c - a*d)*(3*b*c*C - 8*b*B*d - 3*a*C*d)))/8)*Sqrt[a
+ b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(b*f) + ((24*(-(b^4*Sqrt[-b^2]*
d^2*(a^2*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - b^2*(c^2*C + 2*B*c*d -
 C*d^2 - A*(c^2 - d^2)) + 2*a*b*(2*c*(A - C)*d + B*(c^2 - d^2)))) + b^5*d^2
*(2*a*b*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - a^2*(2*c*(A - C)*d + B*
(c^2 - d^2)) + b^2*(2*c*(A - C)*d + B*(c^2 - d^2))))*ArcTan[(Sqrt[-c - (Sqr
t[-b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + d*Ta
n[e + f*x]])])/(b^2*Sqrt[a + Sqrt[-b^2]]*Sqrt[-c - (Sqrt[-b^2]*d)/b]) + (24
*(-(b^4*Sqrt[-b^2]*d^2*(a^2*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - b^2
*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) + 2*a*b*(2*c*(A - C)*d + B*(c^2
- d^2)))) - b^5*d^2*(2*a*b*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - a^2*
(2*c*(A - C)*d + B*(c^2 - d^2)) + b^2*(2*c*(A - C)*d + B*(c^2 - d^2))))*Arc
Tan[(Sqrt[c - (Sqrt[-b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + Sqrt[-
b^2]]*Sqrt[c + d*Tan[e + f*x]])])/(b^2*Sqrt[-a + Sqrt[-b^2]]*Sqrt[c - (Sqrt
[-b^2]*d)/b]) + (3*Sqrt[b]*Sqrt[c - (a*d)/b]*Sqrt[(c/(c - (a*d)/b) - (a*d)/
(b*(c - (a*d)/b)))^(-1)]*Sqrt[c/(c - (a*d)/b) - (a*d)/(b*(c - (a*d)/b))]*(3
*a^4*C*d^4 - 4*a^3*b*d^3*(3*c*C + 2*B*d) + 6*a^2*b^2*d^2*(3*c^2*C + 12*B*c*
d + 8*(A - C)*d^2) - 12*a*b^3*d*(c^3*C - 6*B*c^2*d - 24*c*(A - C)*d^2 + 16*
B*d^3) + b^4*(3*c^4*C - 8*B*c^3*d + 48*c^2*(A - C)*d^2 - 192*B*c*d^3 - 128*
(A - C)*d^4))*ArcSinh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c -
(a*d)/b]*Sqrt[c/(c - (a*d)/b) - (a*d)/(b*(c - (a*d)/b))])]*Sqrt[(c + d*Tan[
e + f*x])/(c - (a*d)/b)])/(8*Sqrt[d]*Sqrt[c + d*Tan[e + f*x]]))/(b^2*f))/(2
*b))/(3*d))/(4*d)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \left(a+b\tan\left(fx+e\right)\right)^{\frac{3}{2}}\left(c+d\tan\left(fx+e\right)\right)^{\frac{3}{2}}\left(A+B\tan\left(fx+e\right)+C\left(\tan\left(fx+e\right)\right)^{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(3/2)*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(3/2)*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\tan\left(fx+e\right)^{2}+B\tan\left(fx+e\right)+A\right)\left(b\tan\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\tan\left(fx+e\right)+c\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*t
an(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^(3/2
)*(d*tan(f*x + e) + c)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*t
an(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(3/2)*(c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C
*tan(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*t
an(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.136 $\int \sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+f\right.$

Optimal. Leaf size=508

$$\frac{\left(-a^2bd^2(2Bd+3cC)+a^3Cd^3+ab^2d\left(8d^2(A-C)+12Bcd+3c^2C\right)+b^3\left(-\left(-24cd^2(A-C)-6Bc^2d+16Bd^3+c^3C\right)\right)\right)\mathrm{ta}}{8b^{5/2}d^{3/2}f}$$

```
[Out] -((Sqrt[a - I*b]*(I*A + B - I*C)*(c - I*d)^(3/2)*ArcTanh[(Sqrt[c - I*d]*Sqr
t[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/f) - (Sqr
t[a + I*b]*(B - I*(A - C))*(c + I*d)^(3/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a +
b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/f + ((a^3*C*d^3
 - a^2*b*d^2*(3*c*C + 2*B*d) + a*b^2*d*(3*c^2*C + 12*B*c*d + 8*(A - C)*d^2)
 - b^3*(c^3*C - 6*B*c^2*d - 24*c*(A - C)*d^2 + 16*B*d^3))*ArcTanh[(Sqrt[d]*
Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(8*b^(5/2)*d
^(3/2)*f) + ((8*b*(A*b + a*B - b*C)*d^2 - (b*c - a*d)*(b*c*C - 6*b*B*d - a*
C*d))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(8*b^2*d*f) - ((b*
c*C - 6*b*B*d - a*C*d)*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))
/(12*b*d*f) + (C*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(5/2))/(3*d*
f)
```

Rubi [A] time = 7.48825, antiderivative size = 508, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3647, 3655, 6725, 63, 217, 206, 93, 208}

$$\frac{\left(-a^2bd^2(2Bd+3cC)+a^3Cd^3+ab^2d\left(8d^2(A-C)+12Bcd+3c^2C\right)+b^3\left(-\left(-24cd^2(A-C)-6Bc^2d+16Bd^3+c^3C\right)\right)\right)\mathrm{ta}}{8b^{5/2}d^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x]
 + C*Tan[e + f*x]^2),x]
```

```
[Out] -((Sqrt[a - I*b]*(I*A + B - I*C)*(c - I*d)^(3/2)*ArcTanh[(Sqrt[c - I*d]*Sqr
t[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/f) - (Sqr
t[a + I*b]*(B - I*(A - C))*(c + I*d)^(3/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a +
b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/f + ((a^3*C*d^3
 - a^2*b*d^2*(3*c*C + 2*B*d) + a*b^2*d*(3*c^2*C + 12*B*c*d + 8*(A - C)*d^2)
 - b^3*(c^3*C - 6*B*c^2*d - 24*c*(A - C)*d^2 + 16*B*d^3))*ArcTanh[(Sqrt[d]*
Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(8*b^(5/2)*d
^(3/2)*f) + ((8*b*(A*b + a*B - b*C)*d^2 - (b*c - a*d)*(b*c*C - 6*b*B*d - a*
C*d))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(8*b^2*d*f) - ((b*
c*C - 6*b*B*d - a*C*d)*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))
/(12*b*d*f) + (C*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(5/2))/(3*d*
f)
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
```

```
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[((a + b*ff*x)^m*(c + d*ff*x)^n*(A + B*ff*x + C*ff^2*x^2
))/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx = \frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(}{3df}$$

$$= -\frac{(bcC-6bBd-aCd)\sqrt{a+b\tan}}{12b}$$

$$= \frac{\left(8b(Ab+aB-bC)d^2-(bc-ad\right.}{}$$

$$= \frac{\left(8b(Ab+aB-bC)d^2-(bc-ad\right.}{}$$

$$= \frac{\left(8b(Ab+aB-bC)d^2-(bc-ad\right.}{}$$

$$= \frac{\left(8b(Ab+aB-bC)d^2-(bc-ad\right.}{}$$

$$= \frac{\left(8b(Ab+aB-bC)d^2-(bc-ad\right.}{}$$

$$= \frac{\left(8b(Ab+aB-bC)d^2-(bc-ad\right.}{}$$

$$= \frac{\left(a^3Cd^3-a^2bd^2(3cC+2Bd)+a\right.}{}$$

$$= -\frac{\sqrt{a-ib}(iA+B-iC)(c-id)^{3/2}}{f}$$

Mathematica [A] time = 8.79454, size = 867, normalized size = 1.71

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3df}+\frac{\frac{(-bcC+adC+6bBd)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{4bf}+\frac{\frac{3\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(8b(Ab-Cb+\right.}{4bf}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e
+ f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] (C*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(5/2))/(3*d*f) + (((-(b*c*
C) + 6*b*B*d + a*C*d)*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))/
(4*b*f) + ((3*(8*b*(A*b + a*B - b*C)*d^2 - (b*c - a*d)*(b*c*C - 6*b*B*d - a
*C*d))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(4*b*f) + ((6*b^2
*d*(b*(2*a*A*c*d - 2*a*c*C*d + A*b*(c^2 - d^2) + a*B*(c^2 - d^2) - b*(c^2*C
 + 2*B*c*d - C*d^2)) - Sqrt[-b^2]*(a*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^
```

```
2)) + b*(2*c*(A - C)*d + B*(c^2 - d^2))))*ArcTan[(Sqrt[c + (b*d)/Sqrt[-b^2]
]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e + f*x]]
)])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + (b*d)/Sqrt[-b^2]]) - (6*b^2*d*(b*(2*a*A
*c*d - 2*a*c*C*d + A*b*(c^2 - d^2) + a*B*(c^2 - d^2) - b*(c^2*C + 2*B*c*d -
 C*d^2)) + Sqrt[-b^2]*(a*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) + b*(2*c
*(A - C)*d + B*(c^2 - d^2))))*ArcTan[(Sqrt[-((b*c + Sqrt[-b^2]*d)/b)]*Sqrt[
a + b*Tan[e + f*x]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e + f*x]])])/(Sqr
t[a + Sqrt[-b^2]]*Sqrt[-((b*c + Sqrt[-b^2]*d)/b)]) + (3*Sqrt[b]*Sqrt[c - (a
*d)/b]*(a^3*C*d^3 - a^2*b*d^2*(3*c*C + 2*B*d) + a*b^2*d*(3*c^2*C + 12*B*c*d
 + 8*(A - C)*d^2) - b^3*(c^3*C - 6*B*c^2*d - 24*c*(A - C)*d^2 + 16*B*d^3))*
ArcSinh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c - (a*d)/b])]*Sqr
t[(b*c + b*d*Tan[e + f*x])/(b*c - a*d)])/(4*Sqrt[d]*Sqrt[c + d*Tan[e + f*x]
]))/(b^2*f))/(2*b))/(3*d)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \sqrt{a+b\tan\left(fx+e\right)}\left(c+d\tan\left(fx+e\right)\right)^{\frac{3}{2}}\left(A+B\tan\left(fx+e\right)+C\left(\tan\left(fx+e\right)\right)^{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\tan\left(fx+e\right)^{2}+B\tan\left(fx+e\right)+A\right)\sqrt{b\tan\left(fx+e\right)+a}\left(d\tan\left(fx+e\right)+c\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*t
an(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*sqrt(b*tan(f*x + e) + a)*
(d*tan(f*x + e) + c)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*t
an(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(1/2)*(c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C
*tan(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*t
an(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.137 $\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{a+b\tan(e+fx)}}\,dx$

Optimal. Leaf size=384

$$\frac{\left(3a^2Cd^2-2abd(2Bd+3cC)+b^2\left(8d^2(A-C)+12Bcd+3c^2C\right)\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{4b^{5/2}\sqrt{d}f}-\frac{(c-id)^{3/2}(iA+B-iC)}{f\sqrt{}}$$

```
[Out] -(((I*A + B - I*C)*(c - I*d)^(3/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a - I*b]*f)) + ((
I*A - B - I*C)*(c + I*d)^(3/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*
x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*f) + ((3*a^2
*C*d^2 - 2*a*b*d*(3*c*C + 2*B*d) + b^2*(3*c^2*C + 12*B*c*d + 8*(A - C)*d^2)
)*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*
x]])])/(4*b^(5/2)*Sqrt[d]*f) + ((3*b*c*C + 4*b*B*d - 3*a*C*d)*Sqrt[a + b*Ta
n[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(4*b^2*f) + (C*Sqrt[a + b*Tan[e + f*x
]]*(c + d*Tan[e + f*x])^(3/2))/(2*b*f)
```

Rubi [A] time = 4.31065, antiderivative size = 384, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3647, 3655, 6725, 63, 217, 206, 93, 208}

$$\frac{\left(3a^2Cd^2-2abd(2Bd+3cC)+b^2\left(8d^2(A-C)+12Bcd+3c^2C\right)\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{4b^{5/2}\sqrt{d}f}-\frac{(c-id)^{3/2}(iA+B-iC)}{f\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/Sq
rt[a + b*Tan[e + f*x]],x]
```


```
[Out] -(((I*A + B - I*C)*(c - I*d)^(3/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a - I*b]*f)) + ((
I*A - B - I*C)*(c + I*d)^(3/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*
x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*f) + ((3*a^2
*C*d^2 - 2*a*b*d*(3*c*C + 2*B*d) + b^2*(3*c^2*C + 12*B*c*d + 8*(A - C)*d^2)
)*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*
x]])])/(4*b^(5/2)*Sqrt[d]*f) + ((3*b*c*C + 4*b*B*d - 3*a*C*d)*Sqrt[a + b*Ta
n[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(4*b^2*f) + (C*Sqrt[a + b*Tan[e + f*x
]]*(c + d*Tan[e + f*x])^(3/2))/(2*b*f)
```


Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[((a + b*ff*x)^m*(c + d*ff*x)^n*(A + B*ff*x + C*ff^2*x^2
))/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{a+b\tan(e+fx)}}\,dx = \frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2bf} + \int$$

$$= \frac{(3bcC+4bBd-3aCd)\sqrt{a+b\tan(e+fx)}\sqrt{c+}}{4b^2f}$$

$$= \frac{(3bcC+4bBd-3aCd)\sqrt{a+b\tan(e+fx)}\sqrt{c+}}{4b^2f}$$

$$= \frac{(3bcC+4bBd-3aCd)\sqrt{a+b\tan(e+fx)}\sqrt{c+}}{4b^2f}$$

$$= \frac{(3bcC+4bBd-3aCd)\sqrt{a+b\tan(e+fx)}\sqrt{c+}}{4b^2f}$$

$$= \frac{(3bcC+4bBd-3aCd)\sqrt{a+b\tan(e+fx)}\sqrt{c+}}{4b^2f}$$

$$= \frac{(3bcC+4bBd-3aCd)\sqrt{a+b\tan(e+fx)}\sqrt{c+}}{4b^2f}$$

$$= \frac{\left(3a^2Cd^2-2abd(3cC+2Bd)+b^2\left(3c^2C+12Bc\right.\right.}{4b^{5/2}\sqrt{}}$$

$$= -\frac{(iA+B-iC)(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e}}{\sqrt{a-ib}\sqrt{c+d\tan(e}}\right.}{\sqrt{a-ib}f}$$

Mathematica [A] time = 7.57049, size = 613, normalized size = 1.6

$$\frac{\frac{\sqrt{b}\sqrt{c-\frac{ad}{b}}\left(3a^2Cd^2-2abd(2Bd+3cC)+b^2\left(8d^2(A-C)+12Bcd+3c^2C\right)\right)\sqrt{\frac{bc+bd\tan(e+fx)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c-\frac{ad}{b}}}\right)}{2\sqrt{d}\sqrt{c+d\tan(e+fx)}}-\frac{2b^2\left(\sqrt{-b^2}\left(-A\left(c^2-d^2\right)+2Bcd+c^2C-Cd^2\right)-b\left(2cd(A-C)+B\left(c^2-d^2\right.\right.\right.}{\sqrt{\sqrt{-b^2}-a}\sqrt{\frac{bd}{\sqrt{-b^2}}+c}}}{b^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/Sqrt[a + b*Tan[e + f*x]],x]
```

```
[Out] (C*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))/(2*b*f) + (((3*b*c*
C + 4*b*B*d - 3*a*C*d)*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(
2*b*f) + ((-2*b^2*(Sqrt[-b^2]*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - b
*(2*c*(A - C)*d + B*(c^2 - d^2)))*ArcTan[(Sqrt[c + (b*d)/Sqrt[-b^2]]*Sqrt[a
 + b*Tan[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e + f*x]])])/(Sqr
t[-a + Sqrt[-b^2]]*Sqrt[c + (b*d)/Sqrt[-b^2]]) - (2*b^2*(Sqrt[-b^2]*(c^2*C
+ 2*B*c*d - C*d^2 - A*(c^2 - d^2)) + b*(2*c*(A - C)*d + B*(c^2 - d^2)))*Arc
Tan[(Sqrt[-((b*c + Sqrt[-b^2]*d)/b)]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + Sq
rt[-b^2]]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[-((b*c + S
```

```
qrt[-b^2]*d)/b)]) + (Sqrt[b]*Sqrt[c - (a*d)/b]*(3*a^2*C*d^2 - 2*a*b*d*(3*c*
C + 2*B*d) + b^2*(3*c^2*C + 12*B*c*d + 8*(A - C)*d^2))*ArcSinh[(Sqrt[d]*Sqr
t[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c - (a*d)/b])]*Sqrt[(b*c + b*d*Tan[e +
 f*x])/(b*c - a*d)])/(2*Sqrt[d]*Sqrt[c + d*Tan[e + f*x]]))/(b^2*f))/(2*b)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \left(A + B\tan\left(fx+e\right) + C\left(\tan\left(fx+e\right)\right)^{2}\right)\left(c + d\tan\left(fx+e\right)\right)^{\frac{3}{2}}\frac{1}{\sqrt{a+b\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(1/2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(1/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.138 $\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=382

$$-\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{bf\left(a^2+b^2\right)\sqrt{a+b\tan(e+fx)}}+\frac{d\left(3a^2C-2abB+2Ab^2+b^2C\right)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{b^2f\left(a^2+b^2\right)}-$$

```
[Out] -(((I*A + B - I*C)*(c - I*d)^(3/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(3/2)*f)) -
((B - I*(A - C))*(c + I*d)^(3/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(3/2)*f) + (Sq
rt[d]*(3*b*c*C + 2*b*B*d - 3*a*C*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x
]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(b^(5/2)*f) + ((2*A*b^2 - 2*a*b*B
+ 3*a^2*C + b^2*C)*d*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(b^
2*(a^2 + b^2)*f) - (2*(A*b^2 - a*(b*B - a*C))*(c + d*Tan[e + f*x])^(3/2))/(
b*(a^2 + b^2)*f*Sqrt[a + b*Tan[e + f*x]])
```

Rubi [A] time = 5.73866, antiderivative size = 382, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {3645, 3647, 3655, 6725, 63, 217, 206, 93, 208}

$$-\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{bf\left(a^2+b^2\right)\sqrt{a+b\tan(e+fx)}}+\frac{d\left(3a^2C-2abB+2Ab^2+b^2C\right)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{b^2f\left(a^2+b^2\right)}-$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
 + b*Tan[e + f*x])^(3/2),x]
```


```
[Out] -(((I*A + B - I*C)*(c - I*d)^(3/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(3/2)*f)) -
((B - I*(A - C))*(c + I*d)^(3/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(3/2)*f) + (Sq
rt[d]*(3*b*c*C + 2*b*B*d - 3*a*C*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x
]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(b^(5/2)*f) + ((2*A*b^2 - 2*a*b*B
+ 3*a^2*C + b^2*C)*d*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(b^
2*(a^2 + b^2)*f) - (2*(A*b^2 - a*(b*B - a*C))*(c + d*Tan[e + f*x])^(3/2))/(
b*(a^2 + b^2)*f*Sqrt[a + b*Tan[e + f*x]])
```


Rule 3645

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*d^2 + c*(c*C - B*d))*(a + b*Tan[e
+ f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[((a + b*ff*x)^m*(c + d*ff*x)^n*(A + B*ff*x + C*ff^2*x^2
))/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{3/2}}\,dx = -\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{b\left(a^2+b^2\right)f\sqrt{a+b\tan(e+fx)}}+\frac{2\int}{}$$

$$=\frac{\left(2Ab^2-2abB+3a^2C+b^2C\right)d\sqrt{a+b\tan(e+fx)}}{b^2\left(a^2+b^2\right)f}$$

$$=\frac{\left(2Ab^2-2abB+3a^2C+b^2C\right)d\sqrt{a+b\tan(e+fx)}}{b^2\left(a^2+b^2\right)f}$$

$$=\frac{\left(2Ab^2-2abB+3a^2C+b^2C\right)d\sqrt{a+b\tan(e+fx)}}{b^2\left(a^2+b^2\right)f}$$

$$=\frac{\left(2Ab^2-2abB+3a^2C+b^2C\right)d\sqrt{a+b\tan(e+fx)}}{b^2\left(a^2+b^2\right)f}$$

$$=\frac{\left(2Ab^2-2abB+3a^2C+b^2C\right)d\sqrt{a+b\tan(e+fx)}}{b^2\left(a^2+b^2\right)f}$$

$$=\frac{\left(2Ab^2-2abB+3a^2C+b^2C\right)d\sqrt{a+b\tan(e+fx)}}{b^2\left(a^2+b^2\right)f}$$

$$=\frac{\sqrt{d}(3bcC+2bBd-3aCd)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{b^{5/2}f}$$

$$=-\frac{(iA+B-iC)(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{3/2}f}$$

Mathematica [C] time = 39.4796, size = 1073629, normalized size = 2810.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/(a + b*Tan[e + f*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int (A+B\tan(fx+e)+C(\tan(fx+e))^2)(c+d\tan(fx+e))^{\frac{3}{2}}(a+b\tan(fx+e))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(3/2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(3/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c + d \tan{\left(e + f x \right)}\right)^{\frac{3}{2}} \left(A + B \tan{\left(e + f x \right)} + C \tan^{2}{\left(e + f x \right)}\right)}{\left(a + b \tan{\left(e + f x \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**(3/2),x)
```

```
[Out] Integral((c + d*tan(e + f*x))**(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)**
2)/(a + b*tan(e + f*x))**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.139 $\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=402

$$-\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{3bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{3/2}}-\frac{2\sqrt{c+d\tan(e+fx)}\left(-a^2b^2(d(A-3C)+Bc)+a^4Cd+2ab^3(Ac-Bd\right.}{b^2f\left(a^2+b^2\right)^2\sqrt{a+b\tan(e+fx)}}$$

```
[Out] -(((I*A + B - I*C)*(c - I*d)^(3/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(5/2)*f)) -
((B - I*(A - C))*(c + I*d)^(3/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(5/2)*f) + (2*
C*d^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Ta
n[e + f*x]])])/(b^(5/2)*f) - (2*(a^4*C*d + b^4*(B*c + A*d) + 2*a*b^3*(A*c -
 c*C - B*d) - a^2*b^2*(B*c + (A - 3*C)*d))*Sqrt[c + d*Tan[e + f*x]])/(b^2*(
a^2 + b^2)^2*f*Sqrt[a + b*Tan[e + f*x]]) - (2*(A*b^2 - a*(b*B - a*C))*(c +
d*Tan[e + f*x])^(3/2))/(3*b*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^(3/2))
```

Rubi [A] time = 7.12732, antiderivative size = 402, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3645, 3655, 6725, 63, 217, 206, 93, 208}

$$-\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{3bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{3/2}}-\frac{2\sqrt{c+d\tan(e+fx)}\left(-a^2b^2(d(A-3C)+Bc)+a^4Cd+2ab^3(Ac-Bd\right.}{b^2f\left(a^2+b^2\right)^2\sqrt{a+b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
 + b*Tan[e + f*x])^(5/2),x]
```

```
[Out] -(((I*A + B - I*C)*(c - I*d)^(3/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(5/2)*f)) -
((B - I*(A - C))*(c + I*d)^(3/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(5/2)*f) + (2*
C*d^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Ta
n[e + f*x]])])/(b^(5/2)*f) - (2*(a^4*C*d + b^4*(B*c + A*d) + 2*a*b^3*(A*c -
 c*C - B*d) - a^2*b^2*(B*c + (A - 3*C)*d))*Sqrt[c + d*Tan[e + f*x]])/(b^2*(
a^2 + b^2)^2*f*Sqrt[a + b*Tan[e + f*x]]) - (2*(A*b^2 - a*(b*B - a*C))*(c +
d*Tan[e + f*x])^(3/2))/(3*b*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^(3/2))
```

Rule 3645

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*d^2 + c*(c*C - B*d))*(a + b*Tan[e
+ f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[((a + b*ff*x)^m*(c + d*ff*x)^n*(A + B*ff*x + C*ff^2*x^2
))/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{5/2}}\,dx = -\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{3b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{3/2}} +$$

$$= -\frac{2\left(a^4Cd+b^4(Bc+Ad)+2ab^3(Ac-cC-Bd)\right.}{b^2\left(a^2+b^2\right)^2 f\sqrt{a}}$$

$$= -\frac{2\left(a^4Cd+b^4(Bc+Ad)+2ab^3(Ac-cC-Bd)\right.}{b^2\left(a^2+b^2\right)^2 f\sqrt{a}}$$

$$= -\frac{2\left(a^4Cd+b^4(Bc+Ad)+2ab^3(Ac-cC-Bd)\right.}{b^2\left(a^2+b^2\right)^2 f\sqrt{a}}$$

$$= -\frac{2\left(a^4Cd+b^4(Bc+Ad)+2ab^3(Ac-cC-Bd)\right.}{b^2\left(a^2+b^2\right)^2 f\sqrt{a}}$$

$$= -\frac{2\left(a^4Cd+b^4(Bc+Ad)+2ab^3(Ac-cC-Bd)\right.}{b^2\left(a^2+b^2\right)^2 f\sqrt{a}}$$

$$= -\frac{2\left(a^4Cd+b^4(Bc+Ad)+2ab^3(Ac-cC-Bd)\right.}{b^2\left(a^2+b^2\right)^2 f\sqrt{a}}$$

$$= \frac{2Cd^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{b^{5/2}f} - \frac{2\left(a^4Cd+b^4\right.}{}$$

$$= -\frac{(iA+B-iC)(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e}}{\sqrt{a-ib}\sqrt{c+d\tan(e}}\right.}{(a-ib)^{5/2}f}$$

Mathematica [C] time = 40.5942, size = 1347065, normalized size = 3350.91

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/(a + b*Tan[e + f*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \left(A+B\tan\left(fx+e\right)+C\left(\tan\left(fx+e\right)\right)^2\right)\left(c+d\tan\left(fx+e\right)\right)^{\frac{3}{2}}\left(a+b\tan\left(fx+e\right)\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(5/2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(5/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.140 $\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=586

$$-\frac{2\sqrt{c+d\tan(e+fx)}\left(-a^3b^3\left(50cd(A-C)+B\left(15c^2-39d^2\right)\right)+a^2b^4\left(45Ac^2-49Ad^2-90Bcd-45c^2C+58Cd^2\right)+\right.}{15b^2f\left(a^2+b^2\right)^3(}$$

```
[Out] -(((I*A + B - I*C)*(c - I*d)^(3/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(7/2)*f)) -
((B - I*(A - C))*(c + I*d)^(3/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(7/2)*f) - (2*
(2*a^3*b*B*d + 3*a^4*C*d + b^4*(5*B*c + 3*A*d) + 2*a*b^3*(5*A*c - 5*c*C - 4
*B*d) - a^2*b^2*(5*B*c + 7*A*d - 13*C*d))*Sqrt[c + d*Tan[e + f*x]])/(15*b^2
*(a^2 + b^2)^2*f*(a + b*Tan[e + f*x])^(3/2)) - (2*(2*a^5*b*B*d^2 + 3*a^6*C*
d^2 + a^4*b^2*d*(10*B*c + (8*A + C)*d) + a^2*b^4*(45*A*c^2 - 45*c^2*C - 90*
B*c*d - 49*A*d^2 + 58*C*d^2) - a^3*b^3*(50*c*(A - C)*d + B*(15*c^2 - 39*d^2
)) + a*b^5*(70*c*(A - C)*d + B*(45*c^2 - 23*d^2)) + b^6*(5*c*(3*c*C + 4*B*d
) - 3*A*(5*c^2 - d^2)))*Sqrt[c + d*Tan[e + f*x]])/(15*b^2*(a^2 + b^2)^3*(b*
c - a*d)*f*Sqrt[a + b*Tan[e + f*x]]) - (2*(A*b^2 - a*(b*B - a*C))*(c + d*Ta
n[e + f*x])^(3/2))/(5*b*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^(5/2))
```

Rubi [A] time = 3.66806, antiderivative size = 586, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3645, 3649, 3616, 3615, 93, 208}

$$-\frac{2\sqrt{c+d\tan(e+fx)}\left(-a^3b^3\left(50cd(A-C)+B\left(15c^2-39d^2\right)\right)+a^2b^4\left(45Ac^2-49Ad^2-90Bcd-45c^2C+58Cd^2\right)+\right.}{15b^2f\left(a^2+b^2\right)^3(}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
 + b*Tan[e + f*x])^(7/2),x]
```



```
[Out] -(((I*A + B - I*C)*(c - I*d)^(3/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(7/2)*f)) -
((B - I*(A - C))*(c + I*d)^(3/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(7/2)*f) - (2*
(2*a^3*b*B*d + 3*a^4*C*d + b^4*(5*B*c + 3*A*d) + 2*a*b^3*(5*A*c - 5*c*C - 4
*B*d) - a^2*b^2*(5*B*c + 7*A*d - 13*C*d))*Sqrt[c + d*Tan[e + f*x]])/(15*b^2
*(a^2 + b^2)^2*f*(a + b*Tan[e + f*x])^(3/2)) - (2*(2*a^5*b*B*d^2 + 3*a^6*C*
d^2 + a^4*b^2*d*(10*B*c + (8*A + C)*d) + a^2*b^4*(45*A*c^2 - 45*c^2*C - 90*
B*c*d - 49*A*d^2 + 58*C*d^2) - a^3*b^3*(50*c*(A - C)*d + B*(15*c^2 - 39*d^2
)) + a*b^5*(70*c*(A - C)*d + B*(45*c^2 - 23*d^2)) + b^6*(5*c*(3*c*C + 4*B*d
) - 3*A*(5*c^2 - d^2)))*Sqrt[c + d*Tan[e + f*x]])/(15*b^2*(a^2 + b^2)^3*(b*
c - a*d)*f*Sqrt[a + b*Tan[e + f*x]]) - (2*(A*b^2 - a*(b*B - a*C))*(c + d*Ta
n[e + f*x])^(3/2))/(5*b*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^(5/2))
```


Rule 3645

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*d^2 + c*(c*C - B*d))*(a + b*Tan[e
+ f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dis
```

```
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
 b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3616

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3615

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[((a + b*x)^m*(c + d*x)^n)/(A - B*x), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{7/2}}dx = -\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{5b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{5/2}} +$$

$$= -\frac{2\left(2a^3bBd+3a^4Cd+b^4(5Bc+3Ad)+2ab^3(5\right.}{15b^2\left(a^2+\right.}$$

$$= -\frac{2\left(2a^3bBd+3a^4Cd+b^4(5Bc+3Ad)+2ab^3(5\right.}{15b^2\left(a^2+\right.}$$

$$= -\frac{2\left(2a^3bBd+3a^4Cd+b^4(5Bc+3Ad)+2ab^3(5\right.}{15b^2\left(a^2+\right.}$$

$$= -\frac{2\left(2a^3bBd+3a^4Cd+b^4(5Bc+3Ad)+2ab^3(5\right.}{15b^2\left(a^2+\right.}$$

$$= -\frac{2\left(2a^3bBd+3a^4Cd+b^4(5Bc+3Ad)+2ab^3(5\right.}{15b^2\left(a^2+\right.}$$

$$= -\frac{(iA+B-iC)(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e}}{\sqrt{a-ib}\sqrt{c+d\tan(e}}\right.}{(a-ib)^{7/2}f}$$

Mathematica [B] time = 9.00577, size = 3134, normalized size = 5.35

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/(a + b*Tan[e + f*x])^(7/2),x]
```

```
[Out] -((C*(c + d*Tan[e + f*x])^(3/2))/(b*f*(a + b*Tan[e + f*x])^(5/2))) - (-((3*
b*c*C - 2*b*B*d - 3*a*C*d)*Sqrt[c + d*Tan[e + f*x]])/(4*b*f*(a + b*Tan[e +
f*x])^(5/2)) - ((-2*((b^2*(8*A*b^2*c^2 + 3*a^2*C*d^2 - 2*a*b*d*(3*c*C - B*d
) - 5*b^2*c*(c*C + 2*B*d)))/4 - a*(-(a*(8*b^2*d*(B*c + (A - C)*d) + (b*c -
a*d)*(3*b*c*C - 2*b*B*d - 3*a*C*d)))/4 + 2*b^3*(2*c*(A - C)*d + B*(c^2 - d^
2))))*Sqrt[c + d*Tan[e + f*x]])/(5*(a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e +
 f*x])^(5/2)) - (2*((-2*(b^2*(((2*b^2*d - (5*a*(b*c - a*d))/2)*(8*A*b^2*c^2
 + 3*a^2*C*d^2 - 2*a*b*d*(3*c*C - B*d) - 5*b^2*c*(c*C + 2*B*d)))/4 + ((-5*b
*c)/2 + (a*d)/2)*(-(a*(8*b^2*d*(B*c + (A - C)*d) + (b*c - a*d)*(3*b*c*C - 2
*b*B*d - 3*a*C*d)))/4 + 2*b^3*(2*c*(A - C)*d + B*(c^2 - d^2)))) - a*((5*b*(
b*c - a*d)*((b*(8*A*b^2*c^2 + 3*a^2*C*d^2 - 2*a*b*d*(3*c*C - B*d) - 5*b^2*c
*(c*C + 2*B*d)))/4 - (b*(8*b^2*d*(B*c + (A - C)*d) + (b*c - a*d)*(3*b*c*C -
 2*b*B*d - 3*a*C*d)))/4 - 2*a*b^2*(2*c*(A - C)*d + B*(c^2 - d^2))))/2 - 2*a
*d*((b^2*(8*A*b^2*c^2 + 3*a^2*C*d^2 - 2*a*b*d*(3*c*C - B*d) - 5*b^2*c*(c*C
+ 2*B*d)))/4 - a*(-(a*(8*b^2*d*(B*c + (A - C)*d) + (b*c - a*d)*(3*b*c*C - 2
*b*B*d - 3*a*C*d)))/4 + 2*b^3*(2*c*(A - C)*d + B*(c^2 - d^2))))))*Sqrt[c +
d*Tan[e + f*x]])/(3*(a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])^(3/2)) -
 (2*((-15*b^2*(b*c - a*d)^2*(((-3*a^2*A*b*c^2 + A*b^3*c^2 + a^3*B*c^2 - 3*a
*b^2*B*c^2 + 3*a^2*b*c^2*C - b^3*c^2*C + 2*a^3*A*c*d - 6*a*A*b^2*c*d + 6*a^
2*b*B*c*d - 2*b^3*B*c*d - 2*a^3*c*C*d + 6*a*b^2*c*C*d + 3*a^2*A*b*d^2 - A*b
^3*d^2 - a^3*B*d^2 + 3*a*b^2*B*d^2 - 3*a^2*b*C*d^2 + b^3*C*d^2 + I*(-(a^3*A
```

```
*c^2) + 3*a*A*b^2*c^2 - 3*a^2*b*B*c^2 + b^3*B*c^2 + a^3*c^2*C - 3*a*b^2*c^2
*C - 6*a^2*A*b*c*d + 2*A*b^3*c*d + 2*a^3*B*c*d - 6*a*b^2*B*c*d + 6*a^2*b*c*
C*d - 2*b^3*c*C*d + a^3*A*d^2 - 3*a*A*b^2*d^2 + 3*a^2*b*B*d^2 - b^3*B*d^2 -
 a^3*C*d^2 + 3*a*b^2*C*d^2))*ArcTan[(Sqrt[-c - I*d]*Sqrt[a + b*Tan[e + f*x]
])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*Sqrt[-c - I*d]
) + ((3*a^2*A*b*c^2 - A*b^3*c^2 - a^3*B*c^2 + 3*a*b^2*B*c^2 - 3*a^2*b*c^2*C
 + b^3*c^2*C - 2*a^3*A*c*d + 6*a*A*b^2*c*d - 6*a^2*b*B*c*d + 2*b^3*B*c*d +
2*a^3*c*C*d - 6*a*b^2*c*C*d - 3*a^2*A*b*d^2 + A*b^3*d^2 + a^3*B*d^2 - 3*a*b
^2*B*d^2 + 3*a^2*b*C*d^2 - b^3*C*d^2 + I*(-(a^3*A*c^2) + 3*a*A*b^2*c^2 - 3*
a^2*b*B*c^2 + b^3*B*c^2 + a^3*c^2*C - 3*a*b^2*c^2*C - 6*a^2*A*b*c*d + 2*A*b
^3*c*d + 2*a^3*B*c*d - 6*a*b^2*B*c*d + 6*a^2*b*c*C*d - 2*b^3*c*C*d + a^3*A*
d^2 - 3*a*A*b^2*d^2 + 3*a^2*b*B*d^2 - b^3*B*d^2 - a^3*C*d^2 + 3*a*b^2*C*d^2
))*ArcTan[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + I*b]*Sqrt[c +
 d*Tan[e + f*x]])])/(Sqrt[-a + I*b]*Sqrt[c - I*d])))/(2*(a^2 + b^2)*f) - (2
*(b^2*((b^2*d - (3*a*(b*c - a*d))/2)*(((2*b^2*d - (5*a*(b*c - a*d))/2)*(8*A
*b^2*c^2 + 3*a^2*C*d^2 - 2*a*b*d*(3*c*C - B*d) - 5*b^2*c*(c*C + 2*B*d)))/4
+ ((-5*b*c)/2 + (a*d)/2)*(-(a*(8*b^2*d*(B*c + (A - C)*d) + (b*c - a*d)*(3*b
*c*C - 2*b*B*d - 3*a*C*d)))/4 + 2*b^3*(2*c*(A - C)*d + B*(c^2 - d^2)))) + (
(-3*b*c)/2 + (a*d)/2)*((5*b*(b*c - a*d)*((b*(8*A*b^2*c^2 + 3*a^2*C*d^2 - 2*
a*b*d*(3*c*C - B*d) - 5*b^2*c*(c*C + 2*B*d)))/4 - (b*(8*b^2*d*(B*c + (A - C
)*d) + (b*c - a*d)*(3*b*c*C - 2*b*B*d - 3*a*C*d)))/4 - 2*a*b^2*(2*c*(A - C)
*d + B*(c^2 - d^2))))/2 - 2*a*d*((b^2*(8*A*b^2*c^2 + 3*a^2*C*d^2 - 2*a*b*d*
(3*c*C - B*d) - 5*b^2*c*(c*C + 2*B*d)))/4 - a*(-(a*(8*b^2*d*(B*c + (A - C)*
d) + (b*c - a*d)*(3*b*c*C - 2*b*B*d - 3*a*C*d)))/4 + 2*b^3*(2*c*(A - C)*d +
 B*(c^2 - d^2)))))) - a*((3*b*(b*c - a*d)*((-5*a*(b*c - a*d)*((b*(8*A*b^2*c
^2 + 3*a^2*C*d^2 - 2*a*b*d*(3*c*C - B*d) - 5*b^2*c*(c*C + 2*B*d)))/4 - (b*(
8*b^2*d*(B*c + (A - C)*d) + (b*c - a*d)*(3*b*c*C - 2*b*B*d - 3*a*C*d)))/4 -
 2*a*b^2*(2*c*(A - C)*d + B*(c^2 - d^2))))/2 - 2*b*d*((b^2*(8*A*b^2*c^2 + 3
*a^2*C*d^2 - 2*a*b*d*(3*c*C - B*d) - 5*b^2*c*(c*C + 2*B*d)))/4 - a*(-(a*(8*
b^2*d*(B*c + (A - C)*d) + (b*c - a*d)*(3*b*c*C - 2*b*B*d - 3*a*C*d)))/4 + 2
*b^3*(2*c*(A - C)*d + B*(c^2 - d^2)))) + b*(((2*b^2*d - (5*a*(b*c - a*d))/2
)*(8*A*b^2*c^2 + 3*a^2*C*d^2 - 2*a*b*d*(3*c*C - B*d) - 5*b^2*c*(c*C + 2*B*d
)))/4 + ((-5*b*c)/2 + (a*d)/2)*(-(a*(8*b^2*d*(B*c + (A - C)*d) + (b*c - a*d
)*(3*b*c*C - 2*b*B*d - 3*a*C*d)))/4 + 2*b^3*(2*c*(A - C)*d + B*(c^2 - d^2))
))))/2 - a*d*(b^2*(((2*b^2*d - (5*a*(b*c - a*d))/2)*(8*A*b^2*c^2 + 3*a^2*C*
d^2 - 2*a*b*d*(3*c*C - B*d) - 5*b^2*c*(c*C + 2*B*d)))/4 + ((-5*b*c)/2 + (a*
d)/2)*(-(a*(8*b^2*d*(B*c + (A - C)*d) + (b*c - a*d)*(3*b*c*C - 2*b*B*d - 3*
a*C*d)))/4 + 2*b^3*(2*c*(A - C)*d + B*(c^2 - d^2)))) - a*((5*b*(b*c - a*d)*
((b*(8*A*b^2*c^2 + 3*a^2*C*d^2 - 2*a*b*d*(3*c*C - B*d) - 5*b^2*c*(c*C + 2*B
*d)))/4 - (b*(8*b^2*d*(B*c + (A - C)*d) + (b*c - a*d)*(3*b*c*C - 2*b*B*d -
3*a*C*d)))/4 - 2*a*b^2*(2*c*(A - C)*d + B*(c^2 - d^2))))/2 - 2*a*d*((b^2*(8
*A*b^2*c^2 + 3*a^2*C*d^2 - 2*a*b*d*(3*c*C - B*d) - 5*b^2*c*(c*C + 2*B*d)))/
4 - a*(-(a*(8*b^2*d*(B*c + (A - C)*d) + (b*c - a*d)*(3*b*c*C - 2*b*B*d - 3*
a*C*d)))/4 + 2*b^3*(2*c*(A - C)*d + B*(c^2 - d^2))))))))*Sqrt[c + d*Tan[e +
 f*x]])/((a^2 + b^2)*(b*c - a*d)*f*Sqrt[a + b*Tan[e + f*x]])))/(3*(a^2 + b^
2)*(b*c - a*d))))/(5*(a^2 + b^2)*(b*c - a*d)))/(2*b))/b
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \left(A + B\tan\left(fx + e\right) + C\left(\tan\left(fx + e\right)\right)^2\right)\left(c + d\tan\left(fx + e\right)\right)^{\frac{3}{2}}\left(a + b\tan\left(fx + e\right)\right)^{-\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(7/2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(7/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.141 $\int \sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+f\right.$

Optimal. Leaf size=697

$$-\frac{\left(2a^2b^2d^2\left(8d^2(A-C)+20Bcd+15c^2C\right)-4a^3bd^3(2Bd+5cC)+5a^4Cd^4-4ab^3d\left(40cd^2(A-C)+30Bc^2d-16Bd^3+5\right.\right.}{64b^{7/2}d^{3/2}f}$$

```
[Out] -((Sqrt[a - I*b]*(I*A + B - I*C)*(c - I*d)^(5/2)*ArcTanh[(Sqrt[c - I*d]*Sqr
t[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/f) + (Sqr
t[a + I*b]*(I*A - B - I*C)*(c + I*d)^(5/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a +
b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/f - ((5*a^4*C*d
^4 - 4*a^3*b*d^3*(5*c*C + 2*B*d) + 2*a^2*b^2*d^2*(15*c^2*C + 20*B*c*d + 8*(
A - C)*d^2) - 4*a*b^3*d*(5*c^3*C + 30*B*c^2*d + 40*c*(A - C)*d^2 - 16*B*d^3
) + b^4*(5*c^4*C - 40*B*c^3*d - 240*c^2*(A - C)*d^2 + 320*B*c*d^3 + 128*(A
- C)*d^4))*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*T
an[e + f*x]])])/(64*b^(7/2)*d^(3/2)*f) + ((64*b^2*d^2*(A*b*c + a*B*c - b*c*
C + a*A*d - b*B*d - a*C*d) + (b*c - a*d)*(48*b*(A*b + a*B - b*C)*d^2 - 5*(b
*c - a*d)*(b*c*C - 8*b*B*d - a*C*d)))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*T
an[e + f*x]])/(64*b^3*d*f) + ((48*b*(A*b + a*B - b*C)*d^2 - 5*(b*c - a*d)*(
b*c*C - 8*b*B*d - a*C*d))*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/
2))/(96*b^2*d*f) - ((b*c*C - 8*b*B*d - a*C*d)*Sqrt[a + b*Tan[e + f*x]]*(c +
 d*Tan[e + f*x])^(5/2))/(24*b*d*f) + (C*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan
[e + f*x])^(7/2))/(4*d*f)
```

Rubi [A] time = 10.4159, antiderivative size = 697, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3647, 3655, 6725, 63, 217, 206, 93, 208}

$$-\frac{\left(2a^2b^2d^2\left(8d^2(A-C)+20Bcd+15c^2C\right)-4a^3bd^3(2Bd+5cC)+5a^4Cd^4-4ab^3d\left(40cd^2(A-C)+30Bc^2d-16Bd^3+5\right.\right.}{64b^{7/2}d^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x]
 + C*Tan[e + f*x]^2),x]
```


```
[Out] -((Sqrt[a - I*b]*(I*A + B - I*C)*(c - I*d)^(5/2)*ArcTanh[(Sqrt[c - I*d]*Sqr
t[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/f) + (Sqr
t[a + I*b]*(I*A - B - I*C)*(c + I*d)^(5/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a +
b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/f - ((5*a^4*C*d
^4 - 4*a^3*b*d^3*(5*c*C + 2*B*d) + 2*a^2*b^2*d^2*(15*c^2*C + 20*B*c*d + 8*(
A - C)*d^2) - 4*a*b^3*d*(5*c^3*C + 30*B*c^2*d + 40*c*(A - C)*d^2 - 16*B*d^3
) + b^4*(5*c^4*C - 40*B*c^3*d - 240*c^2*(A - C)*d^2 + 320*B*c*d^3 + 128*(A
- C)*d^4))*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*T
an[e + f*x]])])/(64*b^(7/2)*d^(3/2)*f) + ((64*b^2*d^2*(A*b*c + a*B*c - b*c*
C + a*A*d - b*B*d - a*C*d) + (b*c - a*d)*(48*b*(A*b + a*B - b*C)*d^2 - 5*(b
*c - a*d)*(b*c*C - 8*b*B*d - a*C*d)))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*T
an[e + f*x]])/(64*b^3*d*f) + ((48*b*(A*b + a*B - b*C)*d^2 - 5*(b*c - a*d)*(
b*c*C - 8*b*B*d - a*C*d))*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/
2))/(96*b^2*d*f) - ((b*c*C - 8*b*B*d - a*C*d)*Sqrt[a + b*Tan[e + f*x]]*(c +
 d*Tan[e + f*x])^(5/2))/(24*b*d*f) + (C*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan
[e + f*x])^(7/2))/(4*d*f)
```


Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[((a + b*ff*x)^m*(c + d*ff*x)^n*(A + B*ff*x + C*ff^2*x^2
))/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx = \frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(}{4df}$$

$$= -\frac{(bcC-8bBd-aCd)\sqrt{a+b\tan}}{24b}$$

$$= \frac{\left(48b(Ab+aB-bC)d^2-5(bc-\right.}{}$$

$$= \frac{\left(64b^2d^2(Abc+aBc-bcC+aA\right.}{}$$

$$= \frac{\left(64b^2d^2(Abc+aBc-bcC+aA\right.}{}$$

$$= \frac{\left(64b^2d^2(Abc+aBc-bcC+aA\right.}{}$$

$$= \frac{\left(64b^2d^2(Abc+aBc-bcC+aA\right.}{}$$

$$= \frac{\left(64b^2d^2(Abc+aBc-bcC+aA\right.}{}$$

$$= \frac{\left(64b^2d^2(Abc+aBc-bcC+aA\right.}{}$$

$$= -\frac{\left(5a^4Cd^4-4a^3bd^3(5cC+2Bd)\right.}{}$$

$$= -\frac{\sqrt{a-ib}(iA+B-iC)(c-id)^{5/2}}{f}$$

Mathematica [A] time = 9.26618, size = 1261, normalized size = 1.81

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{7/2}}{4df}+\frac{\frac{(-bcC+adC+8bBd)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{6bf}+\frac{\left(48b(Ab-Cb+aB)d^2-5(bc-ad)(bcC-adC-8bBd)\right)}{8bf}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e
+ f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] (C*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(7/2))/(4*d*f) + (((-(b*c*
C) + 8*b*B*d + a*C*d)*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(5/2))/
(6*b*f) + (((48*b*(A*b + a*B - b*C)*d^2 - 5*(b*c - a*d)*(b*c*C - 8*b*B*d -
a*C*d))*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))/(8*b*f) + (((2
4*b^2*d^2*(A*b*c + a*B*c - b*c*C + a*A*d - b*B*d - a*C*d) - (3*(-(b*c) + a*
d)*(48*b*(A*b + a*B - b*C)*d^2 - 5*(b*c - a*d)*(b*c*C - 8*b*B*d - a*C*d)))/
8)*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(b*f) + ((-24*b^3*d*(
Sqrt[-b^2]*(b*(A - C)*d*(3*c^2 - d^2) + b*B*(c^3 - 3*c*d^2) - a*(A*c^3 - c^
3*C - 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 + B*d^3)) - b*(A*(b*c^3 + 3*a*c^2*d
 - 3*b*c*d^2 - a*d^3) - b*(c^3*C + 3*B*c^2*d - 3*c*C*d^2 - B*d^3) + a*(B*c^
3 - 3*c^2*C*d - 3*B*c*d^2 + C*d^3)))*ArcTan[(Sqrt[c + (b*d)/Sqrt[-b^2]]*Sqr
t[a + b*Tan[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e + f*x]])]])/(
Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + (b*d)/Sqrt[-b^2]]) - (24*b^3*d*(Sqrt[-b^2]*(
b*(A - C)*d*(3*c^2 - d^2) + b*B*(c^3 - 3*c*d^2) - a*(A*c^3 - c^3*C - 3*B*c^
2*d - 3*A*c*d^2 + 3*c*C*d^2 + B*d^3)) + b*(A*(b*c^3 + 3*a*c^2*d - 3*b*c*d^2
 - a*d^3) - b*(c^3*C + 3*B*c^2*d - 3*c*C*d^2 - B*d^3) + a*(B*c^3 - 3*c^2*C*
d - 3*B*c*d^2 + C*d^3)))*ArcTan[(Sqrt[-((b*c + Sqrt[-b^2]*d)/b)]*Sqrt[a + b
*Tan[e + f*x]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e + f*x]])]])/(Sqrt[a +
 Sqrt[-b^2]]*Sqrt[-((b*c + Sqrt[-b^2]*d)/b)]) - (3*Sqrt[b]*Sqrt[c - (a*d)/b
]*Sqrt[(c/(c - (a*d)/b) - (a*d)/(b*(c - (a*d)/b)))^(-1)]*Sqrt[c/(c - (a*d)/
b) - (a*d)/(b*(c - (a*d)/b))]*(5*a^4*C*d^4 - 4*a^3*b*d^3*(5*c*C + 2*B*d) +
2*a^2*b^2*d^2*(15*c^2*C + 20*B*c*d + 8*(A - C)*d^2) - 4*a*b^3*d*(5*c^3*C +
30*B*c^2*d + 40*c*(A - C)*d^2 - 16*B*d^3) + b^4*(5*c^4*C - 40*B*c^3*d - 240
*c^2*(A - C)*d^2 + 320*B*c*d^3 + 128*(A - C)*d^4))*ArcSinh[(Sqrt[d]*Sqrt[a
+ b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c - (a*d)/b]*Sqrt[c/(c - (a*d)/b) - (a*d)/
(b*(c - (a*d)/b))])]*Sqrt[(c + d*Tan[e + f*x])/(c - (a*d)/b)])/(8*Sqrt[d]*S
qrt[c + d*Tan[e + f*x]]))/(b^2*f))/(2*b))/(3*b))/(4*d)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \sqrt{a + b\tan\left(fx + e\right)}\left(c + d\tan\left(fx + e\right)\right)^{\frac{5}{2}}\left(A + B\tan\left(fx + e\right) + C\left(\tan\left(fx + e\right)\right)^{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*t
an(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*t
an(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(1/2)*(c+d*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C
*tan(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*t
an(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.142 $\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{a+b\tan(e+fx)}}\,dx$

Optimal. Leaf size=505

$$-\frac{\left(-3a^2bd^2(2Bd+5cC)+5a^3Cd^3+ab^2d\left(8d^2(A-C)+20Bcd+15c^2C\right)+b^3\left(-\left(40cd^2(A-C)+30Bc^2d-16Bd^3+5\right.\right.\right.}{8b^{7/2}\sqrt{d}f}$$

```
[Out] -(((I*A + B - I*C)*(c - I*d)^(5/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a - I*b]*f)) - ((
B - I*(A - C))*(c + I*d)^(5/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*
x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*f) - ((5*a^3
*C*d^3 - 3*a^2*b*d^2*(5*c*C + 2*B*d) + a*b^2*d*(15*c^2*C + 20*B*c*d + 8*(A
- C)*d^2) - b^3*(5*c^3*C + 30*B*c^2*d + 40*c*(A - C)*d^2 - 16*B*d^3))*ArcTa
nh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/
(8*b^(7/2)*Sqrt[d]*f) + ((8*b^2*d*(B*c + (A - C)*d) + (b*c - a*d)*(5*b*c*C
+ 6*b*B*d - 5*a*C*d))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(8
*b^3*f) + ((5*b*c*C + 6*b*B*d - 5*a*C*d)*Sqrt[a + b*Tan[e + f*x]]*(c + d*Ta
n[e + f*x])^(3/2))/(12*b^2*f) + (C*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e +
f*x])^(5/2))/(3*b*f)
```

Rubi [A] time = 6.23032, antiderivative size = 505, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3647, 3655, 6725, 63, 217, 206, 93, 208}

$$-\frac{\left(-3a^2bd^2(2Bd+5cC)+5a^3Cd^3+ab^2d\left(8d^2(A-C)+20Bcd+15c^2C\right)+b^3\left(-\left(40cd^2(A-C)+30Bc^2d-16Bd^3+5\right.\right.\right.}{8b^{7/2}\sqrt{d}f}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/Sq
rt[a + b*Tan[e + f*x]],x]
```


```
[Out] -(((I*A + B - I*C)*(c - I*d)^(5/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a - I*b]*f)) - ((
B - I*(A - C))*(c + I*d)^(5/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*
x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*f) - ((5*a^3
*C*d^3 - 3*a^2*b*d^2*(5*c*C + 2*B*d) + a*b^2*d*(15*c^2*C + 20*B*c*d + 8*(A
- C)*d^2) - b^3*(5*c^3*C + 30*B*c^2*d + 40*c*(A - C)*d^2 - 16*B*d^3))*ArcTa
nh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/
(8*b^(7/2)*Sqrt[d]*f) + ((8*b^2*d*(B*c + (A - C)*d) + (b*c - a*d)*(5*b*c*C
+ 6*b*B*d - 5*a*C*d))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(8
*b^3*f) + ((5*b*c*C + 6*b*B*d - 5*a*C*d)*Sqrt[a + b*Tan[e + f*x]]*(c + d*Ta
n[e + f*x])^(3/2))/(12*b^2*f) + (C*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e +
f*x])^(5/2))/(3*b*f)
```


Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
```

```
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[((a + b*ff*x)^m*(c + d*ff*x)^n*(A + B*ff*x + C*ff^2*x^2
))/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{a+b\tan(e+fx)}}\,dx = \frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3bf} + \frac{\int}{}$$

$$= \frac{(5bcC+6bBd-5aCd)\sqrt{a+b\tan(e+fx)}(c+d}{12b^2f}$$

$$= \frac{\left(8b^2d(Bc+(A-C)d)+(bc-ad)(5bcC+6bBd\right.}{8b}$$

$$= \frac{\left(8b^2d(Bc+(A-C)d)+(bc-ad)(5bcC+6bBd\right.}{8b}$$

$$= \frac{\left(8b^2d(Bc+(A-C)d)+(bc-ad)(5bcC+6bBd\right.}{8b}$$

$$= \frac{\left(8b^2d(Bc+(A-C)d)+(bc-ad)(5bcC+6bBd\right.}{8b}$$

$$= \frac{\left(8b^2d(Bc+(A-C)d)+(bc-ad)(5bcC+6bBd\right.}{8b}$$

$$= \frac{\left(8b^2d(Bc+(A-C)d)+(bc-ad)(5bcC+6bBd\right.}{8b}$$

$$= -\frac{\left(5a^3Cd^3-3a^2bd^2(5cC+2Bd)+ab^2d\left(15c^2C\right.\right.}{}$$

$$= -\frac{(iA+B-iC)(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e}}{\sqrt{a-ib}\sqrt{c+d\tan(e}}\right.}{\sqrt{a-ib}f}$$

Mathematica [A] time = 8.55599, size = 780, normalized size = 1.54

$$-\frac{3\sqrt{b}\sqrt{c-\frac{ad}{b}}\left(-3a^2bd^2(2Bd+5cC)+5a^3Cd^3+ab^2d\left(8d^2(A-C)+20Bcd+15c^2C\right)+b^3\left(-\left(40cd^2(A-C)+30Bc^2d-16Bd^3+5c^3C\right)\right)\right)\sqrt{\frac{bc+bd\tan(e+fx)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c-\frac{ad}{b}}}\right)}{4\sqrt{d}\sqrt{c+d\tan(e+fx)}} + \frac{6b^3\left(\sqrt{-b}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/Sqrt[a + b*Tan[e + f*x]],x]
```

```
[Out] (C*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(5/2))/(3*b*f) + (((5*b*c*
C + 6*b*B*d - 5*a*C*d)*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))
/(4*b*f) + ((3*(8*b^2*d*(B*c + (A - C)*d) + (b*c - a*d)*(5*b*c*C + 6*b*B*d
- 5*a*C*d))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(4*b*f) + ((
6*b^3*(b*(A - C)*d*(3*c^2 - d^2) + b*B*(c^3 - 3*c*d^2) + Sqrt[-b^2]*(A*c^3
```

```
- c^3*C - 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 + B*d^3))*ArcTan[(Sqrt[c + (b*d
)/Sqrt[-b^2]]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + d*T
an[e + f*x]])])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + (b*d)/Sqrt[-b^2]]) - (6*b^3
*(b*(A - C)*d*(3*c^2 - d^2) + b*B*(c^3 - 3*c*d^2) - Sqrt[-b^2]*(A*c^3 - c^3
*C - 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 + B*d^3))*ArcTan[(Sqrt[-((b*c + Sqrt
[-b^2]*d)/b)]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + d*Ta
n[e + f*x]])])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[-((b*c + Sqrt[-b^2]*d)/b)]) - (3*
Sqrt[b]*Sqrt[c - (a*d)/b]*(5*a^3*C*d^3 - 3*a^2*b*d^2*(5*c*C + 2*B*d) + a*b^
2*d*(15*c^2*C + 20*B*c*d + 8*(A - C)*d^2) - b^3*(5*c^3*C + 30*B*c^2*d + 40*
c*(A - C)*d^2 - 16*B*d^3))*ArcSinh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt
[b]*Sqrt[c - (a*d)/b])]*Sqrt[(b*c + b*d*Tan[e + f*x])/(b*c - a*d)])/(4*Sqrt
[d]*Sqrt[c + d*Tan[e + f*x]]))/(b^2*f))/(2*b))/(3*b)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int (A + B\tan(fx+e) + C(\tan(fx+e))^2)(c + d\tan(fx+e))^{\frac{5}{2}}\frac{1}{\sqrt{a+b\tan(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(1/2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(1/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.143 $\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=535

$$\frac{\sqrt{d}\left(15a^2Cd^2-6abd(2Bd+5cC)+b^2\left(8d^2(A-C)+20Bcd+15c^2C\right)\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{4b^{7/2}f}-\frac{2\left(Ab^2-a(bB-aC)\right)}{bf\left(a^2+b^2\right)\sqrt{a}}$$

```
[Out] -(((I*A + B - I*C)*(c - I*d)^(5/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(3/2)*f)) -
((B - I*(A - C))*(c + I*d)^(5/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(3/2)*f) + (Sq
rt[d]*(15*a^2*C*d^2 - 6*a*b*d*(5*c*C + 2*B*d) + b^2*(15*c^2*C + 20*B*c*d +
8*(A - C)*d^2))*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c
+ d*Tan[e + f*x]])])/(4*b^(7/2)*f) - (d*(15*a^3*C*d - 8*A*b^2*(b*c - a*d) -
 3*a^2*b*(5*c*C + 4*B*d) - b^3*(7*c*C + 4*B*d) + a*b^2*(8*B*c + 7*C*d))*Sqr
t[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(4*b^3*(a^2 + b^2)*f) + ((4
*A*b^2 - 4*a*b*B + 5*a^2*C + b^2*C)*d*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e
 + f*x])^(3/2))/(2*b^2*(a^2 + b^2)*f) - (2*(A*b^2 - a*(b*B - a*C))*(c + d*T
an[e + f*x])^(5/2))/(b*(a^2 + b^2)*f*Sqrt[a + b*Tan[e + f*x]])
```

Rubi [A] time = 8.31384, antiderivative size = 535, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {3645, 3647, 3655, 6725, 63, 217, 206, 93, 208}

$$\frac{\sqrt{d}\left(15a^2Cd^2-6abd(2Bd+5cC)+b^2\left(8d^2(A-C)+20Bcd+15c^2C\right)\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{4b^{7/2}f}-\frac{2\left(Ab^2-a(bB-aC)\right)}{bf\left(a^2+b^2\right)\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
 + b*Tan[e + f*x])^(3/2),x]
```


```
[Out] -(((I*A + B - I*C)*(c - I*d)^(5/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(3/2)*f)) -
((B - I*(A - C))*(c + I*d)^(5/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(3/2)*f) + (Sq
rt[d]*(15*a^2*C*d^2 - 6*a*b*d*(5*c*C + 2*B*d) + b^2*(15*c^2*C + 20*B*c*d +
8*(A - C)*d^2))*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c
+ d*Tan[e + f*x]])])/(4*b^(7/2)*f) - (d*(15*a^3*C*d - 8*A*b^2*(b*c - a*d) -
 3*a^2*b*(5*c*C + 4*B*d) - b^3*(7*c*C + 4*B*d) + a*b^2*(8*B*c + 7*C*d))*Sqr
t[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(4*b^3*(a^2 + b^2)*f) + ((4
*A*b^2 - 4*a*b*B + 5*a^2*C + b^2*C)*d*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e
 + f*x])^(3/2))/(2*b^2*(a^2 + b^2)*f) - (2*(A*b^2 - a*(b*B - a*C))*(c + d*T
an[e + f*x])^(5/2))/(b*(a^2 + b^2)*f*Sqrt[a + b*Tan[e + f*x]])
```


Rule 3645

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*d^2 + c*(c*C - B*d))*(a + b*Tan[e
+ f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
```

```
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[((a + b*ff*x)^m*(c + d*ff*x)^n*(A + B*ff*x + C*ff^2*x^2
))/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{3/2}}\,dx = -\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{b\left(a^2+b^2\right)f\sqrt{a+b\tan(e+fx)}} + \frac{2\int}{}$$

$$= \frac{\left(4Ab^2-4abB+5a^2C+b^2C\right)d\sqrt{a+b\tan(e+fx)}(}{2b^2\left(a^2+b^2\right)f}$$

$$= -\frac{d\left(15a^3Cd-8Ab^2(bc-ad)-3a^2b(5cC+4Bd)-b\right.}{}$$

$$= -\frac{d\left(15a^3Cd-8Ab^2(bc-ad)-3a^2b(5cC+4Bd)-b\right.}{}$$

$$= -\frac{d\left(15a^3Cd-8Ab^2(bc-ad)-3a^2b(5cC+4Bd)-b\right.}{}$$

$$= -\frac{d\left(15a^3Cd-8Ab^2(bc-ad)-3a^2b(5cC+4Bd)-b\right.}{}$$

$$= -\frac{d\left(15a^3Cd-8Ab^2(bc-ad)-3a^2b(5cC+4Bd)-b\right.}{}$$

$$= -\frac{d\left(15a^3Cd-8Ab^2(bc-ad)-3a^2b(5cC+4Bd)-b\right.}{}$$

$$= \frac{\sqrt{d}\left(15a^2Cd^2-6abd(5cC+2Bd)+b^2\left(15c^2C+20b\right.\right.}{4b^{7/2}f}$$

$$= -\frac{(iA+B-iC)(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right.}{(a-ib)^{3/2}f}$$

Mathematica [C] time = 44.0091, size = 1654245, normalized size = 3092.05

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/(a + b*Tan[e + f*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \left(A + B\tan\left(fx+e\right) + C\left(\tan\left(fx+e\right)\right)^{2}\right)\left(c + d\tan\left(fx+e\right)\right)^{\frac{5}{2}}\left(a + b\tan\left(fx+e\right)\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(3/2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(3/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.144 $\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=545

$$-\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{3bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{3/2}}+\frac{2(c+d\tan(e+fx))^{3/2}\left(a^2b^2(d(A-11C)+3Bc)+2a^3bBd-5a^4Cd-\right.}{3b^2f\left(a^2+b^2\right)^2\sqrt{a+b\tan(e+}}$$

```
[Out] -(((I*A + B - I*C)*(c - I*d)^(5/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(5/2)*f)) -
((B - I*(A - C))*(c + I*d)^(5/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(5/2)*f) + (d^
(3/2)*(5*b*c*C + 2*b*B*d - 5*a*C*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x
]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(b^(7/2)*f) - (d*(2*a^3*b*B*d - 5*
a^4*C*d - 2*a*b^3*(2*A*c - 2*c*C - 3*B*d) + 2*a^2*b^2*(B*c - 5*C*d) - b^4*(
2*B*c + (4*A + C)*d))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(b
^3*(a^2 + b^2)^2*f) + (2*(2*a^3*b*B*d - 5*a^4*C*d - b^4*(3*B*c + 5*A*d) - 2
*a*b^3*(3*A*c - 3*c*C - 4*B*d) + a^2*b^2*(3*B*c + (A - 11*C)*d))*(c + d*Tan
[e + f*x])^(3/2))/(3*b^2*(a^2 + b^2)^2*f*Sqrt[a + b*Tan[e + f*x]]) - (2*(A*
b^2 - a*(b*B - a*C))*(c + d*Tan[e + f*x])^(5/2))/(3*b*(a^2 + b^2)*f*(a + b*
Tan[e + f*x])^(3/2))
```

Rubi [A] time = 11.0662, antiderivative size = 545, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {3645, 3647, 3655, 6725, 63, 217, 206, 93, 208}

$$-\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{3bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{3/2}}+\frac{2(c+d\tan(e+fx))^{3/2}\left(a^2b^2(d(A-11C)+3Bc)+2a^3bBd-5a^4Cd-\right.}{3b^2f\left(a^2+b^2\right)^2\sqrt{a+b\tan(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
 + b*Tan[e + f*x])^(5/2),x]
```

```
[Out] -(((I*A + B - I*C)*(c - I*d)^(5/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(5/2)*f)) -
((B - I*(A - C))*(c + I*d)^(5/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(5/2)*f) + (d^
(3/2)*(5*b*c*C + 2*b*B*d - 5*a*C*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x
]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(b^(7/2)*f) - (d*(2*a^3*b*B*d - 5*
a^4*C*d - 2*a*b^3*(2*A*c - 2*c*C - 3*B*d) + 2*a^2*b^2*(B*c - 5*C*d) - b^4*(
2*B*c + (4*A + C)*d))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(b
^3*(a^2 + b^2)^2*f) + (2*(2*a^3*b*B*d - 5*a^4*C*d - b^4*(3*B*c + 5*A*d) - 2
*a*b^3*(3*A*c - 3*c*C - 4*B*d) + a^2*b^2*(3*B*c + (A - 11*C)*d))*(c + d*Tan
[e + f*x])^(3/2))/(3*b^2*(a^2 + b^2)^2*f*Sqrt[a + b*Tan[e + f*x]]) - (2*(A*
b^2 - a*(b*B - a*C))*(c + d*Tan[e + f*x])^(5/2))/(3*b*(a^2 + b^2)*f*(a + b*
Tan[e + f*x])^(3/2))
```

Rule 3645

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*d^2 + c*(c*C - B*d))*(a + b*Tan[e
+ f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dis
```

```
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[((a + b*ff*x)^m*(c + d*ff*x)^n*(A + B*ff*x + C*ff^2*x^2
))/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{5/2}}\,dx = -\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{3b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{3/2}} +$$

$$= \frac{2\left(2a^3bBd-5a^4Cd-b^4(3Bc+5Ad)-2ab^3(3A\right.}{3b^2\left(a^2+\right.}$$

$$= -\frac{d\left(2a^3bBd-5a^4Cd-2ab^3(2Ac-2cC-3Bd)\right.}{}$$

$$= -\frac{d\left(2a^3bBd-5a^4Cd-2ab^3(2Ac-2cC-3Bd)\right.}{}$$

$$= -\frac{d\left(2a^3bBd-5a^4Cd-2ab^3(2Ac-2cC-3Bd)\right.}{}$$

$$= -\frac{d\left(2a^3bBd-5a^4Cd-2ab^3(2Ac-2cC-3Bd)\right.}{}$$

$$= -\frac{d\left(2a^3bBd-5a^4Cd-2ab^3(2Ac-2cC-3Bd)\right.}{}$$

$$= -\frac{d\left(2a^3bBd-5a^4Cd-2ab^3(2Ac-2cC-3Bd)\right.}{}$$

$$= \frac{d^{3/2}(5bcC+2bBd-5aCd)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+}}{\sqrt{b}\sqrt{c+d\tan(e+}}\right.}{b^{7/2}f}$$

$$= -\frac{(iA+B-iC)(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e}}{\sqrt{a-ib}\sqrt{c+d\tan(e}}\right.}{(a-ib)^{5/2}f}$$

Mathematica [C] time = 46.4635, size = 2018669, normalized size = 3703.98

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/(a + b*Tan[e + f*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int (A + B\tan (fx + e) + C(\tan (fx + e))^2)(c + d\tan (fx + e))^{\frac{5}{2}}(a + b\tan (fx + e))^{-\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(5/2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(5/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.145 $\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=590

$$-\frac{2\sqrt{c+d\tan(e+fx)}\left(-a^3b^3\left(2cd(A-C)+B\left(c^2-d^2\right)\right)-3a^2b^4\left(-A\left(c^2-d^2\right)+2Bcd+c^2C-2Cd^2\right)+3a^4b^2Cd^2+a^6C\right.}{b^3f\left(a^2+b^2\right)^3\sqrt{a+b\tan(e+fx)}}$$

```
[Out] -((((I*A + B - I*C)*(c - I*d)^(5/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(7/2)*f)) -
((B - I*(A - C))*(c + I*d)^(5/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(7/2)*f) + (2*
C*d^(5/2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Ta
n[e + f*x]])])/(b^(7/2)*f) - (2*(a^6*C*d^2 + 3*a^4*b^2*C*d^2 - 3*a^2*b^4*(c
^2*C + 2*B*c*d - 2*C*d^2 - A*(c^2 - d^2)) + b^6*(c*(c*C + 2*B*d) - A*(c^2 -
 d^2)) - a^3*b^3*(2*c*(A - C)*d + B*(c^2 - d^2)) + 3*a*b^5*(2*c*(A - C)*d +
 B*(c^2 - d^2)))*Sqrt[c + d*Tan[e + f*x]])/(b^3*(a^2 + b^2)^3*f*Sqrt[a + b*
Tan[e + f*x]]) - (2*(a^4*C*d + b^4*(B*c + A*d) + 2*a*b^3*(A*c - c*C - B*d)
- a^2*b^2*(B*c + (A - 3*C)*d))*(c + d*Tan[e + f*x])^(3/2))/(3*b^2*(a^2 + b^
2)^2*f*(a + b*Tan[e + f*x])^(3/2)) - (2*(A*b^2 - a*(b*B - a*C))*(c + d*Tan[
e + f*x])^(5/2))/(5*b*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^(5/2))
```

Rubi [A] time = 14.0201, antiderivative size = 590, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3645, 3655, 6725, 63, 217, 206, 93, 208}

$$-\frac{2\sqrt{c+d\tan(e+fx)}\left(-a^3b^3\left(2cd(A-C)+B\left(c^2-d^2\right)\right)-3a^2b^4\left(-A\left(c^2-d^2\right)+2Bcd+c^2C-2Cd^2\right)+3a^4b^2Cd^2+a^6C\right.}{b^3f\left(a^2+b^2\right)^3\sqrt{a+b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
 + b*Tan[e + f*x])^(7/2),x]
```


```
[Out] -((((I*A + B - I*C)*(c - I*d)^(5/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(7/2)*f)) -
((B - I*(A - C))*(c + I*d)^(5/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(7/2)*f) + (2*
C*d^(5/2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Ta
n[e + f*x]])])/(b^(7/2)*f) - (2*(a^6*C*d^2 + 3*a^4*b^2*C*d^2 - 3*a^2*b^4*(c
^2*C + 2*B*c*d - 2*C*d^2 - A*(c^2 - d^2)) + b^6*(c*(c*C + 2*B*d) - A*(c^2 -
 d^2)) - a^3*b^3*(2*c*(A - C)*d + B*(c^2 - d^2)) + 3*a*b^5*(2*c*(A - C)*d +
 B*(c^2 - d^2)))*Sqrt[c + d*Tan[e + f*x]])/(b^3*(a^2 + b^2)^3*f*Sqrt[a + b*
Tan[e + f*x]]) - (2*(a^4*C*d + b^4*(B*c + A*d) + 2*a*b^3*(A*c - c*C - B*d)
- a^2*b^2*(B*c + (A - 3*C)*d))*(c + d*Tan[e + f*x])^(3/2))/(3*b^2*(a^2 + b^
2)^2*f*(a + b*Tan[e + f*x])^(3/2)) - (2*(A*b^2 - a*(b*B - a*C))*(c + d*Tan[
e + f*x])^(5/2))/(5*b*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^(5/2))
```


Rule 3645

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*d^2 + c*(c*C - B*d))*(a + b*Tan[e
+ f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dis
```

```
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[((a + b*ff*x)^m*(c + d*ff*x)^n*(A + B*ff*x + C*ff^2*x^2
))/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{7/2}}\,dx = -\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{5b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{5/2}} + \frac{2\int \cdots}{\cdots}$$

$$= -\frac{2\left(a^4Cd+b^4(Bc+Ad)+2ab^3(Ac-cC-Bd)-a^{\cdots}\right.}{3b^2\left(a^2+b^2\right)^2 f(a+b\cdots}$$

$$= -\frac{2\left(a^6Cd^2+3a^4b^2Cd^2-3a^2b^4\left(c^2C+2Bcd-2Cd^{\cdots}\right.\right.}{\cdots}$$

$$= -\frac{2\left(a^6Cd^2+3a^4b^2Cd^2-3a^2b^4\left(c^2C+2Bcd-2Cd^{\cdots}\right.\right.}{\cdots}$$

$$= -\frac{2\left(a^6Cd^2+3a^4b^2Cd^2-3a^2b^4\left(c^2C+2Bcd-2Cd^{\cdots}\right.\right.}{\cdots}$$

$$= -\frac{2\left(a^6Cd^2+3a^4b^2Cd^2-3a^2b^4\left(c^2C+2Bcd-2Cd^{\cdots}\right.\right.}{\cdots}$$

$$= -\frac{2\left(a^6Cd^2+3a^4b^2Cd^2-3a^2b^4\left(c^2C+2Bcd-2Cd^{\cdots}\right.\right.}{\cdots}$$

$$= -\frac{2\left(a^6Cd^2+3a^4b^2Cd^2-3a^2b^4\left(c^2C+2Bcd-2Cd^{\cdots}\right.\right.}{\cdots}$$

$$= \frac{2Cd^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{b^{7/2}f} - \frac{2\left(a^6Cd^2+3a^4b^{\cdots}\right.}{\cdots}$$

$$= -\frac{(iA+B-iC)(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx\cdots}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx\cdots}}\right.}{(a-ib)^{7/2}f}$$

Mathematica [C] time = 48.3229, size = 2345519, normalized size = 3975.46

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/(a + b*Tan[e + f*x])^(7/2),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \left(A+B\tan\left(fx+e\right)+C\left(\tan\left(fx+e\right)\right)^{2}\right)\left(c+d\tan\left(fx+e\right)\right)^{\frac{5}{2}}\left(a+b\tan\left(fx+e\right)\right)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(7/2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(7/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.146 $\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=946

$$-\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)(c-id)^{5/2}}{(a-ib)^{9/2}f}-\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{7b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{7/2}}-\frac{2\left(5Cda^4+2bBda\right.}{}$$

```
[Out] -(((I*A + B - I*C)*(c - I*d)^(5/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(9/2)*f)) -
((B - I*(A - C))*(c + I*d)^(5/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(9/2)*f) - (2*
(6*a^5*b*B*d^2 + 15*a^6*C*d^2 + a^4*b^2*d*(14*B*c + 8*A*d + 37*C*d) + 3*a^2
*b^4*(35*A*c^2 - 35*c^2*C - 70*B*c*d - 39*A*d^2 + 54*C*d^2) - a^3*b^3*(98*c
*(A - C)*d + B*(35*c^2 - 75*d^2)) + a*b^5*(182*c*(A - C)*d + B*(105*c^2 - 7
1*d^2)) + b^6*(7*c*(5*c*C + 8*B*d) - 5*A*(7*c^2 - 3*d^2)))*Sqrt[c + d*Tan[e
 + f*x]])/(105*b^3*(a^2 + b^2)^3*f*(a + b*Tan[e + f*x])^(3/2)) - (2*(6*a^7*
b*B*d^3 + 15*a^8*C*d^3 + 2*a^6*b^2*d^2*(7*B*c + 4*A*d + 26*C*d) - 2*a*b^7*(
210*A*c^3 - 210*c^3*C - 525*B*c^2*d - 406*A*c*d^2 + 406*c*C*d^2 + 88*B*d^3)
 - a^4*b^4*(105*B*c^3 + 525*A*c^2*d - 525*c^2*C*d - 749*B*c*d^2 - 311*A*d^3
 + 221*C*d^3) + 2*a^2*b^6*(315*B*c^3 + 875*A*c^2*d - 875*c^2*C*d - 812*B*c*
d^2 - 261*A*d^3 + 291*C*d^3) + 2*a^5*b^3*d*(56*c*(A - C)*d + B*(35*c^2 - 12
*d^2)) - b^8*(5*d*(49*A*c^2 - 49*c^2*C - 3*A*d^2) + 7*B*(15*c^3 - 23*c*d^2)
) - 2*a^3*b^5*(210*c^3*C + 700*B*c^2*d - 798*c*C*d^2 - 317*B*d^3 - 42*A*(5*
c^3 - 19*c*d^2)))*Sqrt[c + d*Tan[e + f*x]])/(105*b^3*(a^2 + b^2)^4*(b*c - a
*d)*f*Sqrt[a + b*Tan[e + f*x]]) - (2*(2*a^3*b*B*d + 5*a^4*C*d + b^4*(7*B*c
+ 5*A*d) + 2*a*b^3*(7*A*c - 7*c*C - 6*B*d) - a^2*b^2*(7*B*c + 9*A*d - 19*C*
d))*(c + d*Tan[e + f*x])^(3/2))/(35*b^2*(a^2 + b^2)^2*f*(a + b*Tan[e + f*x]
)^(5/2)) - (2*(A*b^2 - a*(b*B - a*C))*(c + d*Tan[e + f*x])^(5/2))/(7*b*(a^2
 + b^2)*f*(a + b*Tan[e + f*x])^(7/2))
```

Rubi [A] time = 6.46419, antiderivative size = 946, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3645, 3649, 3616, 3615, 93, 208}

$$-\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)(c-id)^{5/2}}{(a-ib)^{9/2}f}-\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{7b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{7/2}}-\frac{2\left(5Cda^4+2bBda\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
 + b*Tan[e + f*x])^(9/2),x]
```

```
[Out] -(((I*A + B - I*C)*(c - I*d)^(5/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(9/2)*f)) -
((B - I*(A - C))*(c + I*d)^(5/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(9/2)*f) - (2*
(6*a^5*b*B*d^2 + 15*a^6*C*d^2 + a^4*b^2*d*(14*B*c + 8*A*d + 37*C*d) + 3*a^2
*b^4*(35*A*c^2 - 35*c^2*C - 70*B*c*d - 39*A*d^2 + 54*C*d^2) - a^3*b^3*(98*c
*(A - C)*d + B*(35*c^2 - 75*d^2)) + a*b^5*(182*c*(A - C)*d + B*(105*c^2 - 7
1*d^2)) + b^6*(7*c*(5*c*C + 8*B*d) - 5*A*(7*c^2 - 3*d^2)))*Sqrt[c + d*Tan[e
 + f*x]])/(105*b^3*(a^2 + b^2)^3*f*(a + b*Tan[e + f*x])^(3/2)) - (2*(6*a^7*
b*B*d^3 + 15*a^8*C*d^3 + 2*a^6*b^2*d^2*(7*B*c + 4*A*d + 26*C*d) - 2*a*b^7*(
210*A*c^3 - 210*c^3*C - 525*B*c^2*d - 406*A*c*d^2 + 406*c*C*d^2 + 88*B*d^3)
```

```
 - a^4*b^4*(105*B*c^3 + 525*A*c^2*d - 525*c^2*C*d - 749*B*c*d^2 - 311*A*d^3
 + 221*C*d^3) + 2*a^2*b^6*(315*B*c^3 + 875*A*c^2*d - 875*c^2*C*d - 812*B*c*
d^2 - 261*A*d^3 + 291*C*d^3) + 2*a^5*b^3*d*(56*c*(A - C)*d + B*(35*c^2 - 12
*d^2)) - b^8*(5*d*(49*A*c^2 - 49*c^2*C - 3*A*d^2) + 7*B*(15*c^3 - 23*c*d^2)
) - 2*a^3*b^5*(210*c^3*C + 700*B*c^2*d - 798*c*C*d^2 - 317*B*d^3 - 42*A*(5*
c^3 - 19*c*d^2)))*Sqrt[c + d*Tan[e + f*x]])/(105*b^3*(a^2 + b^2)^4*(b*c - a
*d)*f*Sqrt[a + b*Tan[e + f*x]]) - (2*(2*a^3*b*B*d + 5*a^4*C*d + b^4*(7*B*c
+ 5*A*d) + 2*a*b^3*(7*A*c - 7*c*C - 6*B*d) - a^2*b^2*(7*B*c + 9*A*d - 19*C*
d))*(c + d*Tan[e + f*x])^(3/2))/(35*b^2*(a^2 + b^2)^2*f*(a + b*Tan[e + f*x]
)^(5/2)) - (2*(A*b^2 - a*(b*B - a*C))*(c + d*Tan[e + f*x])^(5/2))/(7*b*(a^2
 + b^2)*f*(a + b*Tan[e + f*x])^(7/2))
```

Rule 3645

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*d^2 + c*(c*C - B*d))*(a + b*Tan[e
+ f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
 b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3616

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3615

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[((a + b*x)^m*(c + d*x)^n)/(A - B*x), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
```

```
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{9/2}}\,dx = -\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{7b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{7/2}} + \frac{2\int$$

$$= -\frac{2\left(2a^3bBd+5a^4Cd+b^4(7Bc+5Ad)+2ab^3(7Ac}{35b^2\left(a^2+b\right.}$$

$$= -\frac{2\left(6a^5bBd^2+15a^6Cd^2+a^4b^2d(14Bc+8Ad+37C\right.}{}$$

$$= -\frac{2\left(6a^5bBd^2+15a^6Cd^2+a^4b^2d(14Bc+8Ad+37C\right.}{}$$

$$= -\frac{2\left(6a^5bBd^2+15a^6Cd^2+a^4b^2d(14Bc+8Ad+37C\right.}{}$$

$$= -\frac{2\left(6a^5bBd^2+15a^6Cd^2+a^4b^2d(14Bc+8Ad+37C\right.}{}$$

$$= -\frac{2\left(6a^5bBd^2+15a^6Cd^2+a^4b^2d(14Bc+8Ad+37C\right.}{}$$

$$= -\frac{(iA+B-iC)(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right.}{(a-ib)^{9/2}f}$$

Mathematica [C] time = 52.8871, size = 2719441, normalized size = 2874.67

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/(a + b*Tan[e + f*x])^(9/2),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int (A + B\tan(fx+e) + C(\tan(fx+e))^2)(c + d\tan(fx+e))^{\frac{5}{2}}(a + b\tan(fx+e))^{-\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(9/2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(9/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(9/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(9/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.147 $\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=505

$$\frac{\left(-15a^2bd^2(cC-2Bd)+5a^3Cd^3+5ab^2d\left(8d^2(A-C)-4Bcd+3c^2C\right)+b^3\left(-\left(8cd^2(A-C)-6Bc^2d+16Bd^3+5c^3C\right)\right.\right.}{8\sqrt{b}d^{7/2}f}$$

```
[Out] -(((a - I*b)^(5/2)*(I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[c - I*d]*f)) - ((
a + I*b)^(5/2)*(B - I*(A - C))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*
x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[c + I*d]*f) + ((5*a^3
*C*d^3 - 15*a^2*b*d^2*(c*C - 2*B*d) + 5*a*b^2*d*(3*c^2*C - 4*B*c*d + 8*(A -
 C)*d^2) - b^3*(5*c^3*C - 6*B*c^2*d + 8*c*(A - C)*d^2 + 16*B*d^3))*ArcTanh[
(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(8*
Sqrt[b]*d^(7/2)*f) + ((8*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(5*b*c*C - 6
*b*B*d - 5*a*C*d))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(8*d^
3*f) - ((5*b*c*C - 6*b*B*d - 5*a*C*d)*(a + b*Tan[e + f*x])^(3/2)*Sqrt[c + d
*Tan[e + f*x]])/(12*d^2*f) + (C*(a + b*Tan[e + f*x])^(5/2)*Sqrt[c + d*Tan[e
 + f*x]])/(3*d*f)
```

Rubi [A] time = 5.95301, antiderivative size = 505, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3647, 3655, 6725, 63, 217, 206, 93, 208}

$$\frac{\left(-15a^2bd^2(cC-2Bd)+5a^3Cd^3+5ab^2d\left(8d^2(A-C)-4Bcd+3c^2C\right)+b^3\left(-\left(8cd^2(A-C)-6Bc^2d+16Bd^3+5c^3C\right)\right.\right.}{8\sqrt{b}d^{7/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/Sq
rt[c + d*Tan[e + f*x]],x]
```

```
[Out] -(((a - I*b)^(5/2)*(I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[c - I*d]*f)) - ((
a + I*b)^(5/2)*(B - I*(A - C))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*
x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[c + I*d]*f) + ((5*a^3
*C*d^3 - 15*a^2*b*d^2*(c*C - 2*B*d) + 5*a*b^2*d*(3*c^2*C - 4*B*c*d + 8*(A -
 C)*d^2) - b^3*(5*c^3*C - 6*B*c^2*d + 8*c*(A - C)*d^2 + 16*B*d^3))*ArcTanh[
(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(8*
Sqrt[b]*d^(7/2)*f) + ((8*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(5*b*c*C - 6
*b*B*d - 5*a*C*d))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(8*d^
3*f) - ((5*b*c*C - 6*b*B*d - 5*a*C*d)*(a + b*Tan[e + f*x])^(3/2)*Sqrt[c + d
*Tan[e + f*x]])/(12*d^2*f) + (C*(a + b*Tan[e + f*x])^(5/2)*Sqrt[c + d*Tan[e
 + f*x]])/(3*d*f)
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
```

```
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[((a + b*ff*x)^m*(c + d*ff*x)^n*(A + B*ff*x + C*ff^2*x^2
))/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx = \frac{C(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}{3df} + \frac{\int}{}$$

$$= -\frac{(5bcC-6bBd-5aCd)(a+b\tan(e+fx))^{3/2}\sqrt{}}{12d^2f}$$

$$= \frac{\left(8b(Ab+aB-bC)d^2+(bc-ad)(5bcC-6bBd\right.}{8d}$$

$$= \frac{\left(8b(Ab+aB-bC)d^2+(bc-ad)(5bcC-6bBd\right.}{8d}$$

$$= \frac{\left(8b(Ab+aB-bC)d^2+(bc-ad)(5bcC-6bBd\right.}{8d}$$

$$= \frac{\left(8b(Ab+aB-bC)d^2+(bc-ad)(5bcC-6bBd\right.}{8d}$$

$$= \frac{\left(8b(Ab+aB-bC)d^2+(bc-ad)(5bcC-6bBd\right.}{8d}$$

$$= \frac{\left(8b(Ab+aB-bC)d^2+(bc-ad)(5bcC-6bBd\right.}{8d}$$

$$= \frac{\left(5a^3Cd^3-15a^2bd^2(cC-2Bd)+5ab^2d\left(3c^2C-\right.\right.}{}$$

$$= -\frac{(a-ib)^{5/2}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e}}{\sqrt{a-ib}\sqrt{c+d\tan(e}}\right.}{\sqrt{c-id}f}$$

Mathematica [A] time = 8.34208, size = 785, normalized size = 1.55

$$\frac{3\sqrt{b}\sqrt{c-\frac{ad}{b}}\left(-15a^2bd^2(cC-2Bd)+5a^3Cd^3+5ab^2d\left(8d^2(A-C)-4Bcd+3c^2C\right)+b^3\left(-\left(8cd^2(A-C)-6Bc^2d+16Bd^3+5c^3C\right)\right)\right)\sqrt{\frac{bc+bd\tan(e+fx)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c-\frac{ad}{b}}}\right)}{4\sqrt{d}\sqrt{c+d\tan(e+fx)}} - \frac{6d^3\left(\sqrt{-b^2}\left(a^3\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] (C*(a + b*Tan[e + f*x])^(5/2)*Sqrt[c + d*Tan[e + f*x]])/(3*d*f) + (((-5*b*c
*C + 6*b*B*d + 5*a*C*d)*(a + b*Tan[e + f*x])^(3/2)*Sqrt[c + d*Tan[e + f*x]]
)/(4*d*f) + ((3*(8*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(5*b*c*C - 6*b*B*d
 - 5*a*C*d))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(4*d*f) + (
(-6*(Sqrt[-b^2]*(3*a^2*b*B - b^3*B - a^3*(A - C) + 3*a*b^2*(A - C)) - b*(a^
```

```
3*B - 3*a*b^2*B + 3*a^2*b*(A - C) - b^3*(A - C)))*d^3*ArcTan[(Sqrt[c + (b*d
)/Sqrt[-b^2]]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + d*T
an[e + f*x]])])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + (b*d)/Sqrt[-b^2]]) - (6*(Sq
rt[-b^2]*(3*a^2*b*B - b^3*B - a^3*(A - C) + 3*a*b^2*(A - C)) + b*(a^3*B - 3
*a*b^2*B + 3*a^2*b*(A - C) - b^3*(A - C)))*d^3*ArcTan[(Sqrt[-((b*c + Sqrt[-
b^2]*d)/b)]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + d*Tan[
e + f*x]])])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[-((b*c + Sqrt[-b^2]*d)/b)]) + (3*Sq
rt[b]*Sqrt[c - (a*d)/b]*(5*a^3*C*d^3 - 15*a^2*b*d^2*(c*C - 2*B*d) + 5*a*b^2
*d*(3*c^2*C - 4*B*c*d + 8*(A - C)*d^2) - b^3*(5*c^3*C - 6*B*c^2*d + 8*c*(A
- C)*d^2 + 16*B*d^3))*ArcSinh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*S
qrt[c - (a*d)/b])]*Sqrt[(b*c + b*d*Tan[e + f*x])/(b*c - a*d)])/(4*Sqrt[d]*S
qrt[c + d*Tan[e + f*x]]))/(b*d*f))/(2*d))/(3*d)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int (A + B\tan(fx+e) + C(\tan(fx+e))^2)(a + b\tan(fx+e))^{\frac{5}{2}} \frac{1}{\sqrt{c + d\tan(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(1/2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(1/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan
(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.148 $\int \frac{(a+b\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=383

$$\frac{\left(3a^2Cd^2-6abd(cC-2Bd)+b^2\left(8d^2(A-C)-4Bcd+3c^2C\right)\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{4\sqrt{b}d^{5/2}f}-\frac{(a-ib)^{3/2}(iA+B-iC)\tanh^{-1}}{f\sqrt{c-id}}$$

```
[Out] -(((a - I*b)^(3/2)*(I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[c - I*d]*f)) + ((
a + I*b)^(3/2)*(I*A - B - I*C)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*
x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[c + I*d]*f) + ((3*a^2
*C*d^2 - 6*a*b*d*(c*C - 2*B*d) + b^2*(3*c^2*C - 4*B*c*d + 8*(A - C)*d^2))*A
rcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]]
)])/(4*Sqrt[b]*d^(5/2)*f) - ((3*b*c*C - 4*b*B*d - 3*a*C*d)*Sqrt[a + b*Tan[e
 + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(4*d^2*f) + (C*(a + b*Tan[e + f*x])^(3/2
)*Sqrt[c + d*Tan[e + f*x]])/(2*d*f)
```

Rubi [A] time = 4.07667, antiderivative size = 383, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3647, 3655, 6725, 63, 217, 206, 93, 208}

$$\frac{\left(3a^2Cd^2-6abd(cC-2Bd)+b^2\left(8d^2(A-C)-4Bcd+3c^2C\right)\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{4\sqrt{b}d^{5/2}f}-\frac{(a-ib)^{3/2}(iA+B-iC)\tanh^{-1}}{f\sqrt{c-id}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/Sq
rt[c + d*Tan[e + f*x]],x]
```


```
[Out] -(((a - I*b)^(3/2)*(I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[c - I*d]*f)) + ((
a + I*b)^(3/2)*(I*A - B - I*C)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*
x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[c + I*d]*f) + ((3*a^2
*C*d^2 - 6*a*b*d*(c*C - 2*B*d) + b^2*(3*c^2*C - 4*B*c*d + 8*(A - C)*d^2))*A
rcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]]
)])/(4*Sqrt[b]*d^(5/2)*f) - ((3*b*c*C - 4*b*B*d - 3*a*C*d)*Sqrt[a + b*Tan[e
 + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(4*d^2*f) + (C*(a + b*Tan[e + f*x])^(3/2
)*Sqrt[c + d*Tan[e + f*x]])/(2*d*f)
```


Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[((a + b*ff*x)^m*(c + d*ff*x)^n*(A + B*ff*x + C*ff^2*x^2
))/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx = \frac{C(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{2df} + \frac{\int \frac{\sqrt{a+ \cdots}}{\cdots}}{\cdots}$$

$$= -\frac{(3bcC-4bBd-3aCd)\sqrt{a+b\tan(e+fx)}\sqrt{c+d \cdots}}{4d^2f}$$

$$= -\frac{(3bcC-4bBd-3aCd)\sqrt{a+b\tan(e+fx)}\sqrt{c+d \cdots}}{4d^2f}$$

$$= -\frac{(3bcC-4bBd-3aCd)\sqrt{a+b\tan(e+fx)}\sqrt{c+d \cdots}}{4d^2f}$$

$$= -\frac{(3bcC-4bBd-3aCd)\sqrt{a+b\tan(e+fx)}\sqrt{c+d \cdots}}{4d^2f}$$

$$= -\frac{(3bcC-4bBd-3aCd)\sqrt{a+b\tan(e+fx)}\sqrt{c+d \cdots}}{4d^2f}$$

$$= -\frac{(3bcC-4bBd-3aCd)\sqrt{a+b\tan(e+fx)}\sqrt{c+d \cdots}}{4d^2f}$$

$$= \frac{\left(3a^2Cd^2-6abd(cC-2Bd)+b^2\left(3c^2C-4Bcd+8(\cdots\right.\right.}{4\sqrt{b}d^{5/2}f}$$

$$= -\frac{(a-ib)^{3/2}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx \cdots}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx \cdots}}\right.}{\sqrt{c-id}f}$$

Mathematica [A] time = 7.45282, size = 607, normalized size = 1.58

$$\frac{\frac{\sqrt{b}\sqrt{c-\frac{ad}{b}}\left(3a^2Cd^2-6abd(cC-2Bd)+b^2\left(8d^2(A-C)-4Bcd+3c^2C\right)\right)\sqrt{\frac{bc+bd\tan(e+fx)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c-\frac{ad}{b}}}\right)}{2\sqrt{d}\sqrt{c+d\tan(e+fx)}}+\frac{2d^2\left(b\left(a^2B+2ab(A-C)-b^2B\right)-\sqrt{-b^2}\left(a^2(-(A-C))+2abB+b^2(A-C)\right)\right)\tan \cdots}{\sqrt{\sqrt{-b^2}-a}\sqrt{\frac{bd}{\sqrt{-b^2}}+c}}}{bdf}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] (C*(a + b*Tan[e + f*x])^(3/2)*Sqrt[c + d*Tan[e + f*x]])/(2*d*f) + (((-3*b*c
*C + 4*b*B*d + 3*a*C*d)*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/
(2*d*f) + ((2*(b*(a^2*B - b^2*B + 2*a*b*(A - C)) - Sqrt[-b^2]*(2*a*b*B - a^
2*(A - C) + b^2*(A - C)))*d^2*ArcTan[(Sqrt[c + (b*d)/Sqrt[-b^2]]*Sqrt[a + b
*Tan[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[-a
 + Sqrt[-b^2]]*Sqrt[c + (b*d)/Sqrt[-b^2]]) - (2*(b*(a^2*B - b^2*B + 2*a*b*(
A - C)) + Sqrt[-b^2]*(2*a*b*B - a^2*(A - C) + b^2*(A - C)))*d^2*ArcTan[(Sqr
t[-((b*c + Sqrt[-b^2]*d)/b)]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + Sqrt[-b^2]
]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[-((b*c + Sqrt[-b^2
```

```
]*d)/b)]) + (Sqrt[b]*Sqrt[c - (a*d)/b]*(3*a^2*C*d^2 - 6*a*b*d*(c*C - 2*B*d)
 + b^2*(3*c^2*C - 4*B*c*d + 8*(A - C)*d^2))*ArcSinh[(Sqrt[d]*Sqrt[a + b*Tan
[e + f*x]])/(Sqrt[b]*Sqrt[c - (a*d)/b])]*Sqrt[(b*c + b*d*Tan[e + f*x])/(b*c
 - a*d)])/(2*Sqrt[d]*Sqrt[c + d*Tan[e + f*x]]))/(b*d*f))/(2*d)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int (A + B\tan(fx+e) + C(\tan(fx+e))^2)(a + b\tan(fx+e))^{\frac{3}{2}}\frac{1}{\sqrt{c+d\tan(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(1/2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(1/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\tan(e+fx))^{\frac{3}{2}}(A + B\tan(e+fx) + C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan
(f*x+e))**(1/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)**
2)/sqrt(c + d*tan(e + f*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.149 $\int \frac{\sqrt{a+b\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=290

$$-\frac{\sqrt{a-ib}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{c-id}}+\frac{\sqrt{a+ib}(iA-B-iC)\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{c+id}}-\frac{(-aCd-2i}{}$$

```
[Out] -((Sqrt[a - I*b]*(I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[c - I*d]*f)) + (Sqr
t[a + I*b]*(I*A - B - I*C)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])
/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[c + I*d]*f) - ((b*c*C - 2
*b*B*d - a*C*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c
+ d*Tan[e + f*x]])])/(Sqrt[b]*d^(3/2)*f) + (C*Sqrt[a + b*Tan[e + f*x]]*Sqrt
[c + d*Tan[e + f*x]])/(d*f)
```

Rubi [A] time = 2.55491, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3647, 3655, 6725, 63, 217, 206, 93, 208}

$$-\frac{\sqrt{a-ib}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{c-id}}+\frac{\sqrt{a+ib}(iA-B-iC)\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{c+id}}-\frac{(-aCd-2i}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/Sqrt
[c + d*Tan[e + f*x]],x]
```

```
[Out] -((Sqrt[a - I*b]*(I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[c - I*d]*f)) + (Sqr
t[a + I*b]*(I*A - B - I*C)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])
/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[c + I*d]*f) - ((b*c*C - 2
*b*B*d - a*C*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c
+ d*Tan[e + f*x]])])/(Sqrt[b]*d^(3/2)*f) + (C*Sqrt[a + b*Tan[e + f*x]]*Sqrt
[c + d*Tan[e + f*x]])/(d*f)
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
```

```
ist[ff/f, Subst[Int[((a + b*ff*x)^m*(c + d*ff*x)^n*(A + B*ff*x + C*ff^2*x^2
))/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+b\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx = \frac{C\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{df} + \frac{\int \frac{1}{2}(-b\cdots}{\cdots}$$

$$= \frac{C\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{df} + \frac{\text{Subst}\cdots}{\cdots}$$

$$= \frac{C\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{df} + \frac{\text{Subst}\cdots}{\cdots}$$

$$= \frac{C\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{df} + \frac{\text{Subst}\cdots}{\cdots}$$

$$= \frac{C\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{df} + \frac{\text{Subst}\cdots}{\cdots}$$

$$= \frac{C\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{df} + \frac{((ia+\cdots}{\cdots}$$

$$= -\frac{(bcC-2bBd-aCd)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{b}d^{3/2}f} + \cdots$$

$$= -\frac{\sqrt{a-ib}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{c-id}f}\cdots$$

Mathematica [A] time = 6.85748, size = 456, normalized size = 1.57

$$\frac{-\frac{d\left(\sqrt{-b^2}(bB-a(A-C))-b(aB+Ab-bC)\right)\tan^{-1}\left(\frac{\sqrt{\frac{bd}{\sqrt{-b^2}}+c}\sqrt{a+b\tan(e+fx)}}{\sqrt{\sqrt{-b^2}-a}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{\sqrt{-b^2}-a}\sqrt{\frac{bd}{\sqrt{-b^2}}+c}} - \frac{d\left(\sqrt{-b^2}(bB-a(A-C))+b(aB+Ab-bC)\right)\tan^{-1}\left(\frac{\sqrt{-\frac{\sqrt{-b^2}d+bc}{b}}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+\sqrt{-b^2}}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+\sqrt{-b^2}}\sqrt{-\frac{\sqrt{-b^2}d+bc}{b}}} - \cdots}{bdf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)
)/Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] (C*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(d*f) + (-(((Sqrt[-b^
2]*(b*B - a*(A - C)) - b*(A*b + a*B - b*C))*d*ArcTan[(Sqrt[c + (b*d)/Sqrt[-
b^2]]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e + f
*x]])])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + (b*d)/Sqrt[-b^2]])) - ((Sqrt[-b^2]*
(b*B - a*(A - C)) + b*(A*b + a*B - b*C))*d*ArcTan[(Sqrt[-((b*c + Sqrt[-b^2]
*d)/b)]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e +
f*x]])])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[-((b*c + Sqrt[-b^2]*d)/b)]) - (Sqrt[b]*
Sqrt[c - (a*d)/b]*(b*c*C - 2*b*B*d - a*C*d)*ArcSinh[(Sqrt[d]*Sqrt[a + b*Tan
[e + f*x]])/(Sqrt[b]*Sqrt[c - (a*d)/b])]*Sqrt[(b*c + b*d*Tan[e + f*x])/(b*c
 - a*d)])/(Sqrt[d]*Sqrt[c + d*Tan[e + f*x]]))/(b*d*f)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \left(A+B\tan \left(fx+e\right)+C\left(\tan \left(fx+e\right)\right)^{2}\right)\sqrt{a+b\tan \left(fx+e\right)}\frac{1}{\sqrt{c+d\tan \left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(1/2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\tan \left(fx+e\right)^{2}+B\tan \left(fx+e\right)+A\right)\sqrt{b\tan \left(fx+e\right)+a}}{\sqrt{d\tan \left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*sqrt(b*tan(f*x + e) + a)/
sqrt(d*tan(f*x + e) + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\tan \left(e+fx\right)}\left(A+B\tan \left(e+fx\right)+C\tan^{2}\left(e+fx\right)\right)}{\sqrt{c+d\tan \left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan
(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/
sqrt(c + d*tan(e + f*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.150 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=239

$$-\frac{(B+i(A-C))\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a-ib}\sqrt{c-id}}+\frac{(iA-B-iC)\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a+ib}\sqrt{c+id}}+\frac{2C\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{b}\sqrt{d}f}$$

```
[Out] -(((B + I*(A - C))*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a
 - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a - I*b]*Sqrt[c - I*d]*f)) + ((I*
A - B - I*C)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b
]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*Sqrt[c + I*d]*f) + (2*C*ArcTan
h[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(
Sqrt[b]*Sqrt[d]*f)
```

Rubi [A] time = 1.45688, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3655, 6725, 63, 217, 206, 93, 208}

$$-\frac{(B+i(A-C))\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a-ib}\sqrt{c-id}}+\frac{(iA-B-iC)\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a+ib}\sqrt{c+id}}+\frac{2C\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{b}\sqrt{d}f}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(Sqrt[a + b*Tan[e + f*x]]*Sqrt[
c + d*Tan[e + f*x]]),x]
```

```
[Out] -(((B + I*(A - C))*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a
 - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a - I*b]*Sqrt[c - I*d]*f)) + ((I*
A - B - I*C)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b
]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*Sqrt[c + I*d]*f) + (2*C*ArcTan
h[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(
Sqrt[b]*Sqrt[d]*f)
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[((a + b*ff*x)^m*(c + d*ff*x)^n*(A + B*ff*x + C*ff^2*x^2
))/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{C}{\sqrt{a+bx}\sqrt{c+dx}}+\frac{A-C+Bx}{\sqrt{a+bx}\sqrt{c+dx}(1+x^2)}\right) dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{A-C+Bx}{\sqrt{a+bx}\sqrt{c+dx}(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f} + \frac{C\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+\cdots}}\right.}{\cdots}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{-B+i(A-C)}{2(i-x)\sqrt{a+bx}\sqrt{c+dx}}+\frac{B+i(A-C)}{2(i+x)\sqrt{a+bx}\sqrt{c+dx}}\right) dx, x, \tan(e+fx)\right)}{f} + \cdots$$

$$= \frac{(-B+i(A-C))\,\text{Subst}\left(\int \frac{1}{(i-x)\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, \tan(e+fx)\right)}{2f} + \frac{(B+i\cdots}{\cdots}$$

$$= \frac{2C\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{b}\sqrt{d}f} + \frac{(-B+i(A-C))\,\text{Subst}\left(\int \frac{1}{a+ib-(c+id)x^2}\right.}{f}$$

$$= -\frac{(B+i(A-C))\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a-ib}\sqrt{c-id}f} - \frac{(B-i(A-C))\tanh^{-1}\cdots}{\sqrt{a+ib}\sqrt{\cdots}}$$

Mathematica [A] time = 2.31499, size = 362, normalized size = 1.51

$$\frac{\frac{\left(\sqrt{-b^2}(A-C)+bB\right)\tan^{-1}\left(\frac{\sqrt{\frac{bd}{\sqrt{-b^2}}+c}\sqrt{a+b\tan(e+fx)}}{\sqrt{\sqrt{-b^2}-a}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{\sqrt{-b^2}-a}\sqrt{\frac{bd}{\sqrt{-b^2}}+c}}-\frac{\left(\sqrt{-b^2}(C-A)+bB\right)\tan^{-1}\left(\frac{\sqrt{-\frac{\sqrt{-b^2}d+bc}{b}}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+\sqrt{-b^2}}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+\sqrt{-b^2}}\sqrt{-\frac{\sqrt{-b^2}d+bc}{b}}}+\frac{2\sqrt{b}C\sqrt{c-\frac{ad}{b}}\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c-\frac{ad}{b}}}\right)}{\sqrt{d}\sqrt{c+d\tan(e+fx)}}}{bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(Sqrt[a + b*Tan[e + f*x]]
*Sqrt[c + d*Tan[e + f*x]]),x]
```

```
[Out] (((b*B + Sqrt[-b^2]*(A - C))*ArcTan[(Sqrt[c + (b*d)/Sqrt[-b^2]]*Sqrt[a + b*
Tan[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e + f*x]])]])/(Sqrt[-a
+ Sqrt[-b^2]]*Sqrt[c + (b*d)/Sqrt[-b^2]]) - ((b*B + Sqrt[-b^2]*(-A + C))*Ar
cTan[(Sqrt[-((b*c + Sqrt[-b^2]*d)/b)]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + S
qrt[-b^2]]*Sqrt[c + d*Tan[e + f*x]])]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[-((b*c +
Sqrt[-b^2]*d)/b)]) + (2*Sqrt[b]*C*Sqrt[c - (a*d)/b]*ArcSinh[(Sqrt[d]*Sqrt[a
 + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c - (a*d)/b])]*Sqrt[(b*(c + d*Tan[e + f*x
]))/(b*c - a*d)])/(Sqrt[d]*Sqrt[c + d*Tan[e + f*x]]))/(b*f)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int (A+B\tan\left(fx+e\right)+C\left(\tan\left(fx+e\right)\right)^2)\frac{1}{\sqrt{a+b\tan\left(fx+e\right)}}\frac{1}{\sqrt{c+d\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))
^(1/2),x)
```

```
[Out] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))
^(1/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f
*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f
*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\tan\left(e + fx\right) + C\tan^2\left(e + fx\right)}{\sqrt{a + b\tan\left(e + fx\right)}\sqrt{c + d\tan\left(e + fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**(1/2)/(c+d*tan
(f*x+e))**(1/2),x)
```

```
[Out] Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/(sqrt(a + b*tan(e + f*x))
*sqrt(c + d*tan(e + f*x))), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f
*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.151 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=251

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{f\left(a^2+b^2\right)(bc-ad)\sqrt{a+b\tan(e+fx)}}-\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{f(a-ib)^{3/2}\sqrt{c-id}}-\frac{(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{}}{\sqrt{a+ib}\sqrt{}}\right)}{f(a+ib)^{3/2}\sqrt{c+ic}}$$

```
[Out] -(((I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a
 - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(3/2)*Sqrt[c - I*d]*f)) - ((
B - I*(A - C))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I
*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(3/2)*Sqrt[c + I*d]*f) - (2*(A*b
^2 - a*(b*B - a*C))*Sqrt[c + d*Tan[e + f*x]])/((a^2 + b^2)*(b*c - a*d)*f*Sq
rt[a + b*Tan[e + f*x]])
```

Rubi [A] time = 0.96859, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.102, Rules used = {3649, 3616, 3615, 93, 208}

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{f\left(a^2+b^2\right)(bc-ad)\sqrt{a+b\tan(e+fx)}}-\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{f(a-ib)^{3/2}\sqrt{c-id}}-\frac{(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{}}{\sqrt{a+ib}\sqrt{}}\right)}{f(a+ib)^{3/2}\sqrt{c+ic}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^(3/2)*Sqr
t[c + d*Tan[e + f*x]]),x]
```


```
[Out] -(((I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a
 - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(3/2)*Sqrt[c - I*d]*f)) - ((
B - I*(A - C))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I
*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(3/2)*Sqrt[c + I*d]*f) - (2*(A*b
^2 - a*(b*B - a*C))*Sqrt[c + d*Tan[e + f*x]])/((a^2 + b^2)*(b*c - a*d)*f*Sq
rt[a + b*Tan[e + f*x]])
```


Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
 b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3616

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
```

```
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3615

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[((a + b*x)^m*(c + d*x)^n)/(A - B*x), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}\,dx &= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{\left(a^2+b^2\right)(bc-ad)f\sqrt{a+b\tan(e+fx)}} - \frac{2\int \frac{-\frac{1}{2}(bB+a(A-C))(bc-ad\cdots}{\sqrt{a+b\tan(\cdots}}}{\left(a^2+\cdots\right.} \\
&= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{\left(a^2+b^2\right)(bc-ad)f\sqrt{a+b\tan(e+fx)}} + \frac{(A-iB-C)\int \frac{\cdots}{\sqrt{a+b\tan\cdots}}}{2(a\cdots} \\
&= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{\left(a^2+b^2\right)(bc-ad)f\sqrt{a+b\tan(e+fx)}} + \frac{(A-iB-C)\,\mathrm{Subst}\left(\cdots\right.}{\cdots} \\
&= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{\left(a^2+b^2\right)(bc-ad)f\sqrt{a+b\tan(e+fx)}} + \frac{(A-iB-C)\,\mathrm{Subst}\left(\cdots\right.}{\cdots} \\
&= -\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{3/2}\sqrt{c-id}f} - \frac{(B-i(A-C))\tanh^{\cdots}}{(a+ib)^{3/\cdots}}
\end{aligned}
$$

Mathematica [A] time = 2.48187, size = 264, normalized size = 1.05

$$
\frac{\frac{2\left(a(aC-bB)+Ab^2\right)\sqrt{c+d\tan(e+fx)}}{(ad-bc)\sqrt{a+b\tan(e+fx)}}+\frac{(b+ia)(A+iB-C)\tan^{-1}\left(\frac{\sqrt{-c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+ib}\sqrt{-c-id}}+\frac{(a+ib)(iA+B-iC)\tan^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-a+ib}\sqrt{c-id}}}{f\left(a^2+b^2\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^(3/
2)*Sqrt[c + d*Tan[e + f*x]]),x]
```

```
[Out] (((I*a + b)*(A + I*B - C)*ArcTan[(Sqrt[-c - I*d]*Sqrt[a + b*Tan[e + f*x]])/
(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*Sqrt[-c - I*d]) +
 ((a + I*b)*(I*A + B - I*C)*ArcTan[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])
/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[-a + I*b]*Sqrt[c - I*d])
 + (2*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[c + d*Tan[e + f*x]])/((-(b*c) + a*d)*
Sqrt[a + b*Tan[e + f*x]]))/((a^2 + b^2)*f)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int (A + B\tan(fx + e) + C(\tan(fx + e))^2)\frac{1}{\sqrt{c + d\tan(fx + e)}}(a + b\tan(fx + e))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))
^(3/2),x)
```

```
[Out] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))
^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\tan(fx + e)^2 + B\tan(fx + e) + A}{(b\tan(fx + e) + a)^{\frac{3}{2}}\sqrt{d\tan(fx + e) + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f
*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)/((b*tan(f*x + e) + a)^(3/
2)*sqrt(d*tan(f*x + e) + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f
*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))^{\frac{3}{2}}\sqrt{c + d\tan(e + fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e))**(1/2)/(a+b*tan
(f*x+e))**(3/2),x)
```

```
[Out] Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/((a + b*tan(e + f*x))**(3
/2)*sqrt(c + d*tan(e + f*x))), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f
*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.152 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=375

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{3f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))^{3/2}}-\frac{2\sqrt{c+d\tan(e+fx)}\left(-a^2b^2(8Ad+3Bc-4Cd)+5a^3bBd-2a^4Cd+ab^3(\right.}{3f\left(a^2+b^2\right)^2(bc-ad)^2\sqrt{a+b\tan(e+}}$$

```
[Out] -(((I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a
 - I*b]*Sqrt[c + d*Tan[e + f*x]])]])/((a - I*b)^(5/2)*Sqrt[c - I*d]*f)) - ((
B - I*(A - C))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I
*b]*Sqrt[c + d*Tan[e + f*x]])]])/((a + I*b)^(5/2)*Sqrt[c + I*d]*f) - (2*(A*b
^2 - a*(b*B - a*C))*Sqrt[c + d*Tan[e + f*x]])/(3*(a^2 + b^2)*(b*c - a*d)*f*
(a + b*Tan[e + f*x])^(3/2)) - (2*(5*a^3*b*B*d - 2*a^4*C*d + b^4*(3*B*c - 2*
A*d) + a*b^3*(6*A*c - 6*c*C - B*d) - a^2*b^2*(3*B*c + 8*A*d - 4*C*d))*Sqrt[
c + d*Tan[e + f*x]])/(3*(a^2 + b^2)^2*(b*c - a*d)^2*f*Sqrt[a + b*Tan[e + f*
x]])
```

Rubi [A] time = 1.7653, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.102, Rules used = {3649, 3616, 3615, 93, 208}

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{3f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))^{3/2}}-\frac{2\sqrt{c+d\tan(e+fx)}\left(-a^2b^2(8Ad+3Bc-4Cd)+5a^3bBd-2a^4Cd+ab^3(\right.}{3f\left(a^2+b^2\right)^2(bc-ad)^2\sqrt{a+b\tan(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^(5/2)*Sqr
t[c + d*Tan[e + f*x]]),x]
```

```
[Out] -(((I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a
 - I*b]*Sqrt[c + d*Tan[e + f*x]])]])/((a - I*b)^(5/2)*Sqrt[c - I*d]*f)) - ((
B - I*(A - C))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I
*b]*Sqrt[c + d*Tan[e + f*x]])]])/((a + I*b)^(5/2)*Sqrt[c + I*d]*f) - (2*(A*b
^2 - a*(b*B - a*C))*Sqrt[c + d*Tan[e + f*x]])/(3*(a^2 + b^2)*(b*c - a*d)*f*
(a + b*Tan[e + f*x])^(3/2)) - (2*(5*a^3*b*B*d - 2*a^4*C*d + b^4*(3*B*c - 2*
A*d) + a*b^3*(6*A*c - 6*c*C - B*d) - a^2*b^2*(3*B*c + 8*A*d - 4*C*d))*Sqrt[
c + d*Tan[e + f*x]])/(3*(a^2 + b^2)^2*(b*c - a*d)^2*f*Sqrt[a + b*Tan[e + f*
x]])
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
 b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3616

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3615

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[((a + b*x)^m*(c + d*x)^n)/(A - B*x), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}\,dx = -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}\left(2Ab^2d-3aA(bc-a\ldots\right.}{\ldots}}{\ldots}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}} - \frac{2\left(5a^3bBd-2a^4Cd\ldots\right.}{\ldots}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}} - \frac{2\left(5a^3bBd-2a^4Cd\ldots\right.}{\ldots}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}} - \frac{2\left(5a^3bBd-2a^4Cd\ldots\right.}{\ldots}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}} - \frac{2\left(5a^3bBd-2a^4Cd\ldots\right.}{\ldots}$$

$$= -\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{5/2}\sqrt{c-id}f} - \frac{(B-i(A-C))\tanh^{-1}\ldots}{(a+ib)^{5/\ldots}}$$

Mathematica [A] time = 6.00611, size = 388, normalized size = 1.03

$$\frac{\frac{2\sqrt{c+d\tan(e+fx)}\left(a^2b^2(8Ad+3Bc-4Cd)-5a^3bBd+2a^4Cd+ab^3(-6Ac+Bd+6cC)+b^4(2Ad-3Bc)\right)}{(bc-ad)^2\sqrt{a+b\tan(e+fx)}}+\frac{2\left(a^2+b^2\right)\left(a(aC-bB)+Ab^2\right)\sqrt{c+d\tan(e+fx)}}{(ad-bc)(a+b\tan(e+fx))^{3/2}}+\frac{3i(a-ib)^2(A+i}{}}{3f\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^(5/
2)*Sqrt[c + d*Tan[e + f*x]]),x]
```

```
[Out] (((3*I)*(a - I*b)^2*(A + I*B - C)*ArcTan[(Sqrt[-c - I*d]*Sqrt[a + b*Tan[e +
 f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*Sqrt[-c -
 I*d]) + (3*(a + I*b)^2*(I*A + B - I*C)*ArcTan[(Sqrt[c - I*d]*Sqrt[a + b*Ta
n[e + f*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[-a + I*b]*Sq
rt[c - I*d]) + (2*(a^2 + b^2)*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[c + d*Tan[e +
 f*x]])/((-(b*c) + a*d)*(a + b*Tan[e + f*x])^(3/2)) + (2*(-5*a^3*b*B*d + 2*
a^4*C*d + b^4*(-3*B*c + 2*A*d) + a*b^3*(-6*A*c + 6*c*C + B*d) + a^2*b^2*(3*
B*c + 8*A*d - 4*C*d))*Sqrt[c + d*Tan[e + f*x]])/((b*c - a*d)^2*Sqrt[a + b*T
an[e + f*x]]))/(3*(a^2 + b^2)^2*f)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int (A+B\tan(fx+e)+C(\tan(fx+e))^2)\frac{1}{\sqrt{c+d\tan(fx+e)}}(a+b\tan(fx+e))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))
^(5/2),x)
```

```
[Out] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))
^(5/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f
*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f
*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e))**(1/2)/(a+b*tan
(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f
*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.153 $\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=528

$$\frac{\sqrt{b}\left(15a^2Cd^2-10abd(3cC-2Bd)+b^2\left(8d^2(A-C)-12Bcd+15c^2C\right)\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{4d^{7/2}f}+\frac{b\left(d^2(4A+C)-4Bc\right.}{}$$

```
[Out] -(((a - I*b)^(5/2)*(I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c - I*d)^(3/2)*f)) -
((a + I*b)^(5/2)*(B - I*(A - C))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c + I*d)^(3/2)*f) + (Sq
rt[b]*(15*a^2*C*d^2 - 10*a*b*d*(3*c*C - 2*B*d) + b^2*(15*c^2*C - 12*B*c*d +
 8*(A - C)*d^2))*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c
 + d*Tan[e + f*x]])])/(4*d^(7/2)*f) - (2*(c^2*C - B*c*d + A*d^2)*(a + b*Tan
[e + f*x])^(5/2))/(d*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]]) - (b*(3*(b*c -
 a*d)*(5*c^2*C - 4*B*c*d + (4*A + C)*d^2) - 4*d^2*((A - C)*(b*c - a*d) + B*
(a*c + b*d)))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(4*d^3*(c^
2 + d^2)*f) + (b*(5*c^2*C - 4*B*c*d + (4*A + C)*d^2)*(a + b*Tan[e + f*x])^(
3/2)*Sqrt[c + d*Tan[e + f*x]])/(2*d^2*(c^2 + d^2)*f)
```

Rubi [A] time = 8.18832, antiderivative size = 528, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {3645, 3647, 3655, 6725, 63, 217, 206, 93, 208}

$$\frac{\sqrt{b}\left(15a^2Cd^2-10abd(3cC-2Bd)+b^2\left(8d^2(A-C)-12Bcd+15c^2C\right)\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{4d^{7/2}f}+\frac{b\left(d^2(4A+C)-4Bc\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c
 + d*Tan[e + f*x])^(3/2),x]
```


```
[Out] -(((a - I*b)^(5/2)*(I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c - I*d)^(3/2)*f)) -
((a + I*b)^(5/2)*(B - I*(A - C))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c + I*d)^(3/2)*f) + (Sq
rt[b]*(15*a^2*C*d^2 - 10*a*b*d*(3*c*C - 2*B*d) + b^2*(15*c^2*C - 12*B*c*d +
 8*(A - C)*d^2))*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c
 + d*Tan[e + f*x]])])/(4*d^(7/2)*f) - (2*(c^2*C - B*c*d + A*d^2)*(a + b*Tan
[e + f*x])^(5/2))/(d*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]]) - (b*(3*(b*c -
 a*d)*(5*c^2*C - 4*B*c*d + (4*A + C)*d^2) - 4*d^2*((A - C)*(b*c - a*d) + B*
(a*c + b*d)))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(4*d^3*(c^
2 + d^2)*f) + (b*(5*c^2*C - 4*B*c*d + (4*A + C)*d^2)*(a + b*Tan[e + f*x])^(
3/2)*Sqrt[c + d*Tan[e + f*x]])/(2*d^2*(c^2 + d^2)*f)
```


Rule 3645

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*d^2 + c*(c*C - B*d))*(a + b*Tan[e
+ f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
```

```
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[((a + b*ff*x)^m*(c + d*ff*x)^n*(A + B*ff*x + C*ff^2*x^2
))/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx=-\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{5/2}}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}}+\frac{2\int}{}$$

$$=-\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{5/2}}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}}+\frac{b(5}{}$$

$$=-\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{5/2}}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}}-\frac{b(3}{}$$

$$=-\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{5/2}}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}}-\frac{b(3}{}$$

$$=-\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{5/2}}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}}-\frac{b(3}{}$$

$$=-\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{5/2}}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}}-\frac{b(3}{}$$

$$=-\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{5/2}}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}}-\frac{b(3}{}$$

$$=-\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{5/2}}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}}-\frac{b(3}{}$$

$$=\frac{\sqrt{b}\left(15a^2Cd^2-10abd(3cC-2Bd)+b^2\left(15c^2C-12\right.\right.}{4d^{7/2}f}$$

$$=-\frac{(a-ib)^{5/2}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx}}\right.}{(c-id)^{3/2}f}$$

Mathematica [C] time = 44.2267, size = 1653959, normalized size = 3132.5

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int (A + B\tan(fx + e) + C(\tan(fx + e))^2)(a + b\tan(fx + e))^{\frac{5}{2}}(c + d\tan(fx + e))^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(3/2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(3/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan
(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

<u>**Giac**</u> **[F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.154 $\int \frac{(a+b\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=380

$$\frac{b\left(d^2(2A+C)-2Bcd+3c^2C\right)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{d^2f\left(c^2+d^2\right)}-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{3/2}}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}-$$

```
[Out] -(((a - I*b)^(3/2)*(I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c - I*d)^(3/2)*f)) -
((a + I*b)^(3/2)*(B - I*(A - C))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c + I*d)^(3/2)*f) - (Sq
rt[b]*(3*b*c*C - 2*b*B*d - 3*a*C*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x
]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(d^(5/2)*f) - (2*(c^2*C - B*c*d +
A*d^2)*(a + b*Tan[e + f*x])^(3/2))/(d*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]
]) + (b*(3*c^2*C - 2*B*c*d + (2*A + C)*d^2)*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c
 + d*Tan[e + f*x]])/(d^2*(c^2 + d^2)*f)
```

Rubi [A] time = 5.62733, antiderivative size = 380, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {3645, 3647, 3655, 6725, 63, 217, 206, 93, 208}

$$\frac{b\left(d^2(2A+C)-2Bcd+3c^2C\right)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{d^2f\left(c^2+d^2\right)}-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{3/2}}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}-$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c
 + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] -(((a - I*b)^(3/2)*(I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c - I*d)^(3/2)*f)) -
((a + I*b)^(3/2)*(B - I*(A - C))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c + I*d)^(3/2)*f) - (Sq
rt[b]*(3*b*c*C - 2*b*B*d - 3*a*C*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x
]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(d^(5/2)*f) - (2*(c^2*C - B*c*d +
A*d^2)*(a + b*Tan[e + f*x])^(3/2))/(d*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]
]) + (b*(3*c^2*C - 2*B*c*d + (2*A + C)*d^2)*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c
 + d*Tan[e + f*x]])/(d^2*(c^2 + d^2)*f)
```

Rule 3645

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*d^2 + c*(c*C - B*d))*(a + b*Tan[e
+ f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[((a + b*ff*x)^m*(c + d*ff*x)^n*(A + B*ff*x + C*ff^2*x^2
))/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{3/2}}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \cdots$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{3/2}}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \cdots$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{3/2}}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \cdots$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{3/2}}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \cdots$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{3/2}}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \cdots$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{3/2}}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \cdots$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{3/2}}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \cdots$$

$$= -\frac{\sqrt{b}(3bcC-2bBd-3aCd)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e\cdots}}{\sqrt{b}\sqrt{c+d\tan(e\cdots}}\cdots\right.}{d^{5/2}f}\cdots$$

$$= -\frac{(a-ib)^{3/2}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e\cdots}}{\sqrt{a-ib}\sqrt{c+d\tan(e\cdots}}\cdots\right.}{(c-id)^{3/2}f}\cdots$$

Mathematica [C] time = 39.3691, size = 1073499, normalized size = 2825.

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \left(A+B\tan\left(fx+e\right)+C\left(\tan\left(fx+e\right)\right)^{2}\right)\left(a+b\tan\left(fx+e\right)\right)^{\frac{3}{2}}\left(c+d\tan\left(fx+e\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(3/2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(3/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b\tan\left(e + fx\right)\right)^{\frac{3}{2}} \left(A + B\tan\left(e + fx\right) + C\tan^{2}\left(e + fx\right)\right)}{\left(c + d\tan\left(e + fx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan
(f*x+e))**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)**
2)/(c + d*tan(e + f*x))**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.155 $\int \frac{\sqrt{a+b\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=299

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)\sqrt{a+b\tan(e+fx)}}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}-\frac{\sqrt{a-ib}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{f(c-id)^{3/2}}-\frac{\sqrt{a+ib}(B-i(A-C))\,t}{f(c}$$

```
[Out] -((Sqrt[a - I*b]*(I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c - I*d)^(3/2)*f)) - (S
qrt[a + I*b]*(B - I*(A - C))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]
])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c + I*d)^(3/2)*f) + (2*Sqrt
[b]*C*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e
+ f*x]])])/(d^(3/2)*f) - (2*(c^2*C - B*c*d + A*d^2)*Sqrt[a + b*Tan[e + f*x]
])/(d*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]])
```

Rubi [A] time = 3.33311, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3645, 3655, 6725, 63, 217, 206, 93, 208}

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)\sqrt{a+b\tan(e+fx)}}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}-\frac{\sqrt{a-ib}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{f(c-id)^{3/2}}-\frac{\sqrt{a+ib}(B-i(A-C))\,t}{f(c}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c +
 d*Tan[e + f*x])^(3/2),x]
```


```
[Out] -((Sqrt[a - I*b]*(I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c - I*d)^(3/2)*f)) - (S
qrt[a + I*b]*(B - I*(A - C))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]
])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c + I*d)^(3/2)*f) + (2*Sqrt
[b]*C*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e
+ f*x]])])/(d^(3/2)*f) - (2*(c^2*C - B*c*d + A*d^2)*Sqrt[a + b*Tan[e + f*x]
])/(d*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]])
```


Rule 3645

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*d^2 + c*(c*C - B*d))*(a + b*Tan[e
+ f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
```

```
ist[ff/f, Subst[Int[((a + b*ff*x)^m*(c + d*ff*x)^n*(A + B*ff*x + C*ff^2*x^2
))/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+b\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{2\left(c^2C-Bcd+Ad^2\right)\sqrt{a+b\tan(e+fx)}}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{2\int \frac{1}{2}(Aa}{\quad}$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)\sqrt{a+b\tan(e+fx)}}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{2\,\mathrm{Subst}}{\quad}$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)\sqrt{a+b\tan(e+fx)}}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{2\,\mathrm{Subst}}{\quad}$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)\sqrt{a+b\tan(e+fx)}}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{(bC)\,\mathrm{Su}}{\quad}$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)\sqrt{a+b\tan(e+fx)}}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{(2C)\,\mathrm{Su}}{\quad}$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)\sqrt{a+b\tan(e+fx)}}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{((ia+b)}{\quad}$$

$$= \frac{2\sqrt{b}C\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{d^{3/2}f} - \frac{2\left(c^2C-Bcd+Ad\right.}{d\left(c^2+d^2\right)f\sqrt{\quad}}$$

$$= -\frac{\sqrt{a-ib}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(c-id)^{3/2}f} -$$

Mathematica [C] time = 35.4382, size = 621084, normalized size = 2077.2

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)
)/(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \left(A+B\tan\left(fx+e\right)+C\left(\tan\left(fx+e\right)\right)^2\right)\sqrt{a+b\tan\left(fx+e\right)}\left(c+d\tan\left(fx+e\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(3/2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(3/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + b\tan\left(e + fx\right)}\left(A + B\tan\left(e + fx\right) + C\tan^2\left(e + fx\right)\right)}{\left(c + d\tan\left(e + fx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan
(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/
(c + d*tan(e + f*x))**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.156 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=251

$$\frac{2\left(Ad^2-Bcd+c^2C\right)\sqrt{a+b\tan(e+fx)}}{f\left(c^2+d^2\right)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{(B+i(A-C))\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a-ib}(c-id)^{3/2}}+\frac{(iA-B-iC)\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a+ib}(c+id)^{3/2}}$$

```
[Out] -(((B + I*(A - C))*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a
 - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a - I*b]*(c - I*d)^(3/2)*f)) + ((
I*A - B - I*C)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I
*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*(c + I*d)^(3/2)*f) + (2*(c^2
*C - B*c*d + A*d^2)*Sqrt[a + b*Tan[e + f*x]])/((b*c - a*d)*(c^2 + d^2)*f*Sq
rt[c + d*Tan[e + f*x]])
```

Rubi [A] time = 1.00079, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.102, Rules used = {3649, 3616, 3615, 93, 208}

$$\frac{2\left(Ad^2-Bcd+c^2C\right)\sqrt{a+b\tan(e+fx)}}{f\left(c^2+d^2\right)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{(B+i(A-C))\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a-ib}(c-id)^{3/2}}+\frac{(iA-B-iC)\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a+ib}(c+id)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(Sqrt[a + b*Tan[e + f*x]]*(c +
d*Tan[e + f*x])^(3/2)),x]
```

```
[Out] -(((B + I*(A - C))*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a
 - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a - I*b]*(c - I*d)^(3/2)*f)) + ((
I*A - B - I*C)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I
*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*(c + I*d)^(3/2)*f) + (2*(c^2
*C - B*c*d + A*d^2)*Sqrt[a + b*Tan[e + f*x]])/((b*c - a*d)*(c^2 + d^2)*f*Sq
rt[c + d*Tan[e + f*x]])
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
 b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3616

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
```

```
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3615

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[((a + b*x)^m*(c + d*x)^n)/(A - B*x), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}\,dx &= \frac{2\left(c^2C-Bcd+Ad^2\right)\sqrt{a+b\tan(e+fx)}}{(bc-ad)\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}}+\frac{2\int\frac{\frac{1}{2}(bc-ad)(Ac-cC+Bd)+\frac{1}{2}\cdots}{\sqrt{a+b\tan(e+\cdots}}}{(bc-ad\cdots}\\
&= \frac{2\left(c^2C-Bcd+Ad^2\right)\sqrt{a+b\tan(e+fx)}}{(bc-ad)\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}}+\frac{(A-iB-C)\int\frac{1\cdots}{\sqrt{a+b\tan(\cdots}}}{2(c-\cdots}\\
&= \frac{2\left(c^2C-Bcd+Ad^2\right)\sqrt{a+b\tan(e+fx)}}{(bc-ad)\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}}+\frac{(A-iB-C)\,\text{Subst}\left(\int\cdots\right.}{\cdots}\\
&= \frac{2\left(c^2C-Bcd+Ad^2\right)\sqrt{a+b\tan(e+fx)}}{(bc-ad)\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}}+\frac{(A-iB-C)\,\text{Subst}\left(\int\cdots\right.}{(\cdots}\\
&= -\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a-ib}(c-id)^{3/2}f}-\frac{(B-i(A-C))\tanh^{-1}\cdots}{\sqrt{a+ib}(\cdots}
\end{aligned}
$$

Mathematica [A] time = 3.11554, size = 275, normalized size = 1.1

$$
-\frac{\frac{2\left(Ad^2-Bcd+c^2C\right)\sqrt{a+b\tan(e+fx)}}{\sqrt{c+d\tan(e+fx)}}+(bc-ad)\left(\frac{(d+ic)(A+iB-C)\tan^{-1}\left(\frac{\sqrt{-c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+ib}\sqrt{-c-id}}+\frac{(c+id)(iA+B-iC)\tan^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-a+ib}\sqrt{c-id}}\right)}{f\left(c^2+d^2\right)(ad-bc)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(Sqrt[a + b*Tan[e + f*x]]
*(c + d*Tan[e + f*x])^(3/2)),x]
```

```
[Out] -(((b*c - a*d)*(((A + I*B - C)*(I*c + d)*ArcTan[(Sqrt[-c - I*d]*Sqrt[a + b*
Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*Sq
rt[-c - I*d]) + ((I*A + B - I*C)*(c + I*d)*ArcTan[(Sqrt[c - I*d]*Sqrt[a + b
*Tan[e + f*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[-a + I*b]
*Sqrt[c - I*d])) + (2*(c^2*C - B*c*d + A*d^2)*Sqrt[a + b*Tan[e + f*x]])/Sqr
t[c + d*Tan[e + f*x]])/((-(b*c) + a*d)*(c^2 + d^2)*f))
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int (A + B\tan(fx+e) + C(\tan(fx+e))^2)\frac{1}{\sqrt{a+b\tan(fx+e)}}(c+d\tan(fx+e))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))
^(3/2),x)
```

```
[Out] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))
^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\tan(fx+e)^2 + B\tan(fx+e) + A}{\sqrt{b\tan(fx+e)+a}(d\tan(fx+e)+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f
*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)/(sqrt(b*tan(f*x + e) + a)
*(d*tan(f*x + e) + c)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f
*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\tan(e+fx) + C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**(1/2)/(c+d*tan
(f*x+e))**(3/2),x)
```

```
[Out] Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/(sqrt(a + b*tan(e + f*x))
*(c + d*tan(e + f*x))**(3/2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f
*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.157 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=383

$$-\frac{2d\sqrt{a+b\tan(e+fx)}\left(A\left(a^2d^2+b^2\left(c^2+2d^2\right)\right)+a^2\left(-Bcd+2c^2C+Cd^2\right)-abB\left(c^2+d^2\right)+b^2c(cC-Bd)\right)}{f\left(a^2+b^2\right)\left(c^2+d^2\right)(bc-ad)^2\sqrt{c+d\tan(e+fx)}}-\frac{}{f\left(a^2+b\right.}$$

```
[Out] -(((I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a
 - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(3/2)*(c - I*d)^(3/2)*f)) -
((B - I*(A - C))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a +
 I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(3/2)*(c + I*d)^(3/2)*f) - (2*
(A*b^2 - a*(b*B - a*C)))/((a^2 + b^2)*(b*c - a*d)*f*Sqrt[a + b*Tan[e + f*x]
]*Sqrt[c + d*Tan[e + f*x]]) - (2*d*(b^2*c*(c*C - B*d) - a*b*B*(c^2 + d^2) +
 a^2*(2*c^2*C - B*c*d + C*d^2) + A*(a^2*d^2 + b^2*(c^2 + 2*d^2)))*Sqrt[a +
b*Tan[e + f*x]])/((a^2 + b^2)*(b*c - a*d)^2*(c^2 + d^2)*f*Sqrt[c + d*Tan[e
+ f*x]])
```

Rubi [A] time = 1.87757, antiderivative size = 382, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.102, Rules used = {3649, 3616, 3615, 93, 208}

$$-\frac{2d\sqrt{a+b\tan(e+fx)}\left(a^2Ad^2+a^2\left(-Bcd+2c^2C+Cd^2\right)-abB\left(c^2+d^2\right)+Ab^2\left(c^2+2d^2\right)+b^2c(cC-Bd)\right)}{f\left(a^2+b^2\right)\left(c^2+d^2\right)(bc-ad)^2\sqrt{c+d\tan(e+fx)}}-\frac{}{f\left(a^2+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^(3/2)*(c
+ d*Tan[e + f*x])^(3/2)),x]
```

```
[Out] -(((I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a
 - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(3/2)*(c - I*d)^(3/2)*f)) -
((B - I*(A - C))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a +
 I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(3/2)*(c + I*d)^(3/2)*f) - (2*
(A*b^2 - a*(b*B - a*C)))/((a^2 + b^2)*(b*c - a*d)*f*Sqrt[a + b*Tan[e + f*x]
]*Sqrt[c + d*Tan[e + f*x]]) - (2*d*(a^2*A*d^2 + b^2*c*(c*C - B*d) - a*b*B*(
c^2 + d^2) + A*b^2*(c^2 + 2*d^2) + a^2*(2*c^2*C - B*c*d + C*d^2))*Sqrt[a +
b*Tan[e + f*x]])/((a^2 + b^2)*(b*c - a*d)^2*(c^2 + d^2)*f*Sqrt[c + d*Tan[e
+ f*x]])
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
 b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3616

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3615

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[((a + b*x)^m*(c + d*x)^n)/(A - B*x), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{2\left(Ab^2-a(bB-aC)\right)}{\left(a^2+b^2\right)(bc-ad)f\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}} - \frac{2\ldots}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)}{\left(a^2+b^2\right)(bc-ad)f\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}} - \frac{2a\ldots}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)}{\left(a^2+b^2\right)(bc-ad)f\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}} - \frac{2a\ldots}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)}{\left(a^2+b^2\right)(bc-ad)f\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}} - \frac{2a\ldots}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)}{\left(a^2+b^2\right)(bc-ad)f\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}} - \frac{2a\ldots}{}$$

$$= -\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{3/2}(c-id)^{3/2}f} - \frac{(B-i(A-C))\tan\ldots}{(a+ib)^{3}\ldots}$$

Mathematica [A] time = 6.67112, size = 484, normalized size = 1.26

$$-\frac{2\left(Ab^2-a(bB-aC)\right)}{f\left(a^2+b^2\right)(bc-ad)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}-\frac{2\left(-\frac{2\sqrt{a+b\tan(e+fx)}\left(\frac{1}{2}d^2\left(-aA(bc-ad)-(bB-aC)(ad+bc)+2Ab^2d\right)-c\left(\frac{1}{2}d(\right.\right.}{f\left(c^2+d^2\right)(ad-bc)\sqrt{c+d\tan(e+}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^(3/
2)*(c + d*Tan[e + f*x])^(3/2)),x]
```

```
[Out] (-2*(A*b^2 - a*(b*B - a*C)))/((a^2 + b^2)*(b*c - a*d)*f*Sqrt[a + b*Tan[e +
f*x]]*Sqrt[c + d*Tan[e + f*x]]) - (2*(((b*c - a*d)^2*(((I*a + b)*(A + I*B -
 C)*(c - I*d)*ArcTan[(Sqrt[-c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*
b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*Sqrt[-c - I*d]) + ((a + I*b)*
(I*A + B - I*C)*(c + I*d)*ArcTan[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(
Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[-a + I*b]*Sqrt[c - I*d])))
/(2*(-(b*c) + a*d)*(c^2 + d^2)*f) - (2*(-(c*(-(c*(A*b^2 - a*(b*B - a*C))*d)
 + ((A*b - a*B - b*C)*d*(b*c - a*d))/2)) + (d^2*(2*A*b^2*d - a*A*(b*c - a*d
) - (b*B - a*C)*(b*c + a*d)))/2)*Sqrt[a + b*Tan[e + f*x]])/((-(b*c) + a*d)*
(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]])))/((a^2 + b^2)*(b*c - a*d))
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int (A+B\tan(fx+e)+C(\tan(fx+e))^2)(a+b\tan(fx+e))^{-\frac{3}{2}}(c+d\tan(fx+e))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))
^(3/2),x)
```

```
[Out] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))
^(3/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(f
*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(f
*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+B\tan\left(e+fx\right)+C\tan^{2}\left(e+fx\right)}{\left(a+b\tan\left(e+fx\right)\right)^{\frac{3}{2}}\left(c+d\tan\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**(3/2)/(c+d*tan
(f*x+e))**(3/2),x)
```

```
[Out] Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/((a + b*tan(e + f*x))**(3
/2)*(c + d*tan(e + f*x))**(3/2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(f
*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.158 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=598

$$-\frac{2d\sqrt{a+b\tan(e+fx)}\left(-a^2b^2\left(11Ac^2d+17Ad^3+3Bc^3-3Bcd^2+5c^2Cd-Cd^3\right)+a^4(-d)\left(d^2(3A+5C)-3Bcd+8c^2C\right.}{3f\left(a^2+b^2\right)^2\left(c^2+d^2\right)(bc-ad)^3}$$

```
[Out] -(((I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a
 - I*b]*Sqrt[c + d*Tan[e + f*x]])]])/((a - I*b)^(5/2)*(c - I*d)^(3/2)*f)) -
((B - I*(A - C))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a +
 I*b]*Sqrt[c + d*Tan[e + f*x]])]])/((a + I*b)^(5/2)*(c + I*d)^(3/2)*f) - (2*
(A*b^2 - a*(b*B - a*C)))/(3*(a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])^
(3/2)*Sqrt[c + d*Tan[e + f*x]]) - (2*(7*a^3*b*B*d - 4*a^4*C*d + b^4*(3*B*c
- 4*A*d) + a*b^3*(6*A*c - 6*c*C + B*d) - a^2*b^2*(3*B*c + 2*(5*A - C)*d)))/
(3*(a^2 + b^2)^2*(b*c - a*d)^2*f*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e
+ f*x]]) - (2*d*(8*a^3*b*B*d*(c^2 + d^2) + 2*a*b^3*(3*A*c - 3*c*C + B*d)*(c
^2 + d^2) - a^4*d*(8*c^2*C - 3*B*c*d + (3*A + 5*C)*d^2) - a^2*b^2*(3*B*c^3
+ 11*A*c^2*d + 5*c^2*C*d - 3*B*c*d^2 + 17*A*d^3 - C*d^3) - b^4*(d*(5*A*c^2
+ 3*c^2*C + 8*A*d^2) - 3*B*(c^3 + 2*c*d^2)))*Sqrt[a + b*Tan[e + f*x]])/(3*(
a^2 + b^2)^2*(b*c - a*d)^3*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]])
```

Rubi [A] time = 3.43465, antiderivative size = 598, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.102, Rules used = {3649, 3616, 3615, 93, 208}

$$-\frac{2d\sqrt{a+b\tan(e+fx)}\left(-a^2b^2\left(11Ac^2d+17Ad^3+3Bc^3-3Bcd^2+5c^2Cd-Cd^3\right)+a^4(-d)\left(d^2(3A+5C)-3Bcd+8c^2C\right.}{3f\left(a^2+b^2\right)^2\left(c^2+d^2\right)(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^(5/2)*(c
+ d*Tan[e + f*x])^(3/2)),x]
```


```
[Out] -(((I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a
 - I*b]*Sqrt[c + d*Tan[e + f*x]])]])/((a - I*b)^(5/2)*(c - I*d)^(3/2)*f)) -
((B - I*(A - C))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a +
 I*b]*Sqrt[c + d*Tan[e + f*x]])]])/((a + I*b)^(5/2)*(c + I*d)^(3/2)*f) - (2*
(A*b^2 - a*(b*B - a*C)))/(3*(a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])^
(3/2)*Sqrt[c + d*Tan[e + f*x]]) - (2*(7*a^3*b*B*d - 4*a^4*C*d + b^4*(3*B*c
- 4*A*d) + a*b^3*(6*A*c - 6*c*C + B*d) - a^2*b^2*(3*B*c + 2*(5*A - C)*d)))/
(3*(a^2 + b^2)^2*(b*c - a*d)^2*f*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e
+ f*x]]) - (2*d*(8*a^3*b*B*d*(c^2 + d^2) + 2*a*b^3*(3*A*c - 3*c*C + B*d)*(c
^2 + d^2) - a^4*d*(8*c^2*C - 3*B*c*d + (3*A + 5*C)*d^2) - a^2*b^2*(3*B*c^3
+ 11*A*c^2*d + 5*c^2*C*d - 3*B*c*d^2 + 17*A*d^3 - C*d^3) - b^4*(d*(5*A*c^2
+ 3*c^2*C + 8*A*d^2) - 3*B*(c^3 + 2*c*d^2)))*Sqrt[a + b*Tan[e + f*x]])/(3*(
a^2 + b^2)^2*(b*c - a*d)^3*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]])
```


Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
```

```
 b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3616

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3615

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[((a + b*x)^m*(c + d*x)^n)/(A - B*x), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{2\left(Ab^2-a(bB-aC)\right)}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}} - \frac{2}{\;}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}} - \frac{2}{\;}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}} - \frac{2}{\;}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}} - \frac{2}{\;}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}} - \frac{2}{\;}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}} - \frac{2}{\;}$$

$$= -\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{5/2}(c-id)^{3/2}f} - \frac{(B-i(A-C))\tanh^{-1}}{(a+ib)^{5/2}(}$$

Mathematica [A] time = 6.84906, size = 902, normalized size = 1.51

$$-\frac{2\left(Ab^2-a(bB-aC)\right)}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}} - \frac{2\left(-\frac{2\left(\frac{1}{2}b^2\left(4Adb^2-3aA(bc-ad)-(bB-aC)(3bc+ad)\right)-a\left(\frac{3}{2}b(Ab-Cb-aB\right.\right.}{\left(a^2+b^2\right)(bc-ad)f\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan}}\right.}{\;}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^(5/
2)*(c + d*Tan[e + f*x])^(3/2)),x]
```

```
[Out] (-2*(A*b^2 - a*(b*B - a*C)))/(3*(a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*
x])^(3/2)*Sqrt[c + d*Tan[e + f*x]]) - (2*((-2*(-(a*(-2*a*(A*b^2 - a*(b*B -
a*C))*d + (3*b*(A*b - a*B - b*C)*(b*c - a*d))/2)) + (b^2*(4*A*b^2*d - 3*a*A
*(b*c - a*d) - (b*B - a*C)*(3*b*c + a*d)))/2))/((a^2 + b^2)*(b*c - a*d)*f*S
qrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]]) - (2*((-3*(b*c - a*d)^3*(
((a - I*b)^2*(A + I*B - C)*(I*c + d)*ArcTan[(Sqrt[-c - I*d]*Sqrt[a + b*Tan[
e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*Sqrt[-
c - I*d]) + ((a + I*b)^2*(I*A + B - I*C)*(c + I*d)*ArcTan[(Sqrt[c - I*d]*Sq
rt[a + b*Tan[e + f*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[-
```

```
a + I*b]*Sqrt[c - I*d])))/(4*(-(b*c) + a*d)*(c^2 + d^2)*f) - (2*(d^2*((-(b*
c)/2 - (a*d)/2)*(-2*a*(A*b^2 - a*(b*B - a*C))*d + (3*b*(A*b - a*B - b*C)*(b
*c - a*d))/2) + ((b^2*d - (a*(b*c - a*d))/2)*(4*A*b^2*d - 3*a*A*(b*c - a*d)
 - (b*B - a*C)*(3*b*c + a*d)))/2) - c*((d*(b*c - a*d)*(-2*b*(A*b^2 - a*(b*B
 - a*C))*d - (3*a*(A*b - a*B - b*C)*(b*c - a*d))/2 + (b*(4*A*b^2*d - 3*a*A*
(b*c - a*d) - (b*B - a*C)*(3*b*c + a*d)))/2))/2 - c*d*(-(a*(-2*a*(A*b^2 - a
*(b*B - a*C))*d + (3*b*(A*b - a*B - b*C)*(b*c - a*d))/2)) + (b^2*(4*A*b^2*d
 - 3*a*A*(b*c - a*d) - (b*B - a*C)*(3*b*c + a*d)))/2)))*Sqrt[a + b*Tan[e +
f*x]])/((-(b*c) + a*d)*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]])))/((a^2 + b^
2)*(b*c - a*d))))/(3*(a^2 + b^2)*(b*c - a*d))
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \left(A+B\tan\left(fx+e\right)+C\left(\tan\left(fx+e\right)\right)^{2}\right)\left(a+b\tan\left(fx+e\right)\right)^{-\frac{5}{2}}\left(c+d\tan\left(fx+e\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))
^(3/2),x)
```

```
[Out] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))
^(3/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(5/2)/(c+d*tan(f
*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(5/2)/(c+d*tan(f
*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**(5/2)/(c+d*tan
(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(5/2)/(c+d*tan(f
*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.159 $\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=549

$$\frac{b\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(2ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(d^4(4A+C)-2Bc^3d-6Bcd^3+10c^2Cd\right.\right.}{d^3f\left(c^2+d^2\right)^2}$$

```
[Out] -(((a - I*b)^(5/2)*(I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c - I*d)^(5/2)*f)) -
((a + I*b)^(5/2)*(B - I*(A - C))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c + I*d)^(5/2)*f) - (b^
(3/2)*(5*b*c*C - 2*b*B*d - 5*a*C*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x
]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(d^(7/2)*f) - (2*(c^2*C - B*c*d +
A*d^2)*(a + b*Tan[e + f*x])^(5/2))/(3*d*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^
(3/2)) - (2*(b*(5*c^4*C - 2*B*c^3*d - c^2*(A - 11*C)*d^2 - 8*B*c*d^3 + 5*A*
d^4) + 3*a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)))*(a + b*Tan[e + f*x])^(3/2))
/(3*d^2*(c^2 + d^2)^2*f*Sqrt[c + d*Tan[e + f*x]]) + (b*(b*(5*c^4*C - 2*B*c^
3*d + 10*c^2*C*d^2 - 6*B*c*d^3 + (4*A + C)*d^4) + 2*a*d^2*(2*c*(A - C)*d -
B*(c^2 - d^2)))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(d^3*(c^
2 + d^2)^2*f)
```

Rubi [A] time = 10.4997, antiderivative size = 549, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {3645, 3647, 3655, 6725, 63, 217, 206, 93, 208}

$$\frac{b\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(2ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(d^4(4A+C)-2Bc^3d-6Bcd^3+10c^2Cd\right.\right.}{d^3f\left(c^2+d^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c
 + d*Tan[e + f*x])^(5/2),x]
```


```
[Out] -(((a - I*b)^(5/2)*(I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c - I*d)^(5/2)*f)) -
((a + I*b)^(5/2)*(B - I*(A - C))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c + I*d)^(5/2)*f) - (b^
(3/2)*(5*b*c*C - 2*b*B*d - 5*a*C*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x
]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(d^(7/2)*f) - (2*(c^2*C - B*c*d +
A*d^2)*(a + b*Tan[e + f*x])^(5/2))/(3*d*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^
(3/2)) - (2*(b*(5*c^4*C - 2*B*c^3*d - c^2*(A - 11*C)*d^2 - 8*B*c*d^3 + 5*A*
d^4) + 3*a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)))*(a + b*Tan[e + f*x])^(3/2))
/(3*d^2*(c^2 + d^2)^2*f*Sqrt[c + d*Tan[e + f*x]]) + (b*(b*(5*c^4*C - 2*B*c^
3*d + 10*c^2*C*d^2 - 6*B*c*d^3 + (4*A + C)*d^4) + 2*a*d^2*(2*c*(A - C)*d -
B*(c^2 - d^2)))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(d^3*(c^
2 + d^2)^2*f)
```


Rule 3645

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*d^2 + c*(c*C - B*d))*(a + b*Tan[e
+ f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dis
```

```
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[((a + b*ff*x)^m*(c + d*ff*x)^n*(A + B*ff*x + C*ff^2*x^2
))/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{5/2}}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \cdots$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{5/2}}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - \cdots$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{5/2}}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - \cdots$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{5/2}}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - \cdots$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{5/2}}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - \cdots$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{5/2}}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - \cdots$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{5/2}}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - \cdots$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{5/2}}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - \cdots$$

$$= -\frac{b^{3/2}(5bcC-2bBd-5aCd)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(\cdots}}{\sqrt{b}\sqrt{c+d\tan(\cdots}}\right.}{d^{7/2}f}$$

$$= -\frac{(a-ib)^{5/2}(B+i(A-C))\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(\cdots}}{\sqrt{a-ib}\sqrt{c+d\tan(\cdots}}\right.}{(c-id)^{5/2}f}$$

Mathematica [C] time = 46.6346, size = 2018643, normalized size = 3676.95

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \left(A+B\tan\left(fx+e\right)+C\left(\tan\left(fx+e\right)\right)^{2}\right)\left(a+b\tan\left(fx+e\right)\right)^{\frac{5}{2}}\left(c+d\tan\left(fx+e\right)\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(5/2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(5/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan
(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.160 $\int \frac{(a+b\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=407

$$-\frac{2\sqrt{a+b\tan(e+fx)}\left(ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(-c^2d^2(A-3C)+Ad^4-2Bcd^3+c^4C\right)\right)}{d^2f\left(c^2+d^2\right)^2\sqrt{c+d\tan(e+fx)}}-\frac{2\left(Ad^2-Bcd+c^2C\right.}{3df\left(c^2+d^2\right)(c}$$

```
[Out] -(((a - I*b)^(3/2)*(I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c - I*d)^(5/2)*f)) -
((a + I*b)^(3/2)*(B - I*(A - C))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c + I*d)^(5/2)*f) + (2*
b^(3/2)*C*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Ta
n[e + f*x]])])/(d^(5/2)*f) - (2*(c^2*C - B*c*d + A*d^2)*(a + b*Tan[e + f*x]
)^(3/2))/(3*d*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/2)) - (2*(b*(c^4*C - c^
2*(A - 3*C)*d^2 - 2*B*c*d^3 + A*d^4) + a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)
))*Sqrt[a + b*Tan[e + f*x]])/(d^2*(c^2 + d^2)^2*f*Sqrt[c + d*Tan[e + f*x]])
```

Rubi [A] time = 7.16258, antiderivative size = 407, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3645, 3655, 6725, 63, 217, 206, 93, 208}

$$-\frac{2\sqrt{a+b\tan(e+fx)}\left(ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(-c^2d^2(A-3C)+Ad^4-2Bcd^3+c^4C\right)\right)}{d^2f\left(c^2+d^2\right)^2\sqrt{c+d\tan(e+fx)}}-\frac{2\left(Ad^2-Bcd+c^2C\right.}{3df\left(c^2+d^2\right)(c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c
 + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] -(((a - I*b)^(3/2)*(I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c - I*d)^(5/2)*f)) -
((a + I*b)^(3/2)*(B - I*(A - C))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c + I*d)^(5/2)*f) + (2*
b^(3/2)*C*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Ta
n[e + f*x]])])/(d^(5/2)*f) - (2*(c^2*C - B*c*d + A*d^2)*(a + b*Tan[e + f*x]
)^(3/2))/(3*d*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/2)) - (2*(b*(c^4*C - c^
2*(A - 3*C)*d^2 - 2*B*c*d^3 + A*d^4) + a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)
))*Sqrt[a + b*Tan[e + f*x]])/(d^2*(c^2 + d^2)^2*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 3645

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*d^2 + c*(c*C - B*d))*(a + b*Tan[e
+ f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[((a + b*ff*x)^m*(c + d*ff*x)^n*(A + B*ff*x + C*ff^2*x^2
))/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{3/2}}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{2\int}{}$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{3/2}}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - \frac{2(b}{}$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{3/2}}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - \frac{2(b}{}$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{3/2}}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - \frac{2(b}{}$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{3/2}}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - \frac{2(b}{}$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{3/2}}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - \frac{2(b}{}$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{3/2}}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} - \frac{2(b}{}$$

$$= \frac{2b^{3/2}C\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{d^{5/2}f} - \frac{2\left(c^2C-Bcd+\right.}{3d\left(c^2+d^2\right)}$$

$$= -\frac{(a-ib)^{3/2}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx}}\right.}{(c-id)^{5/2}f}$$

Mathematica [C] time = 40.7888, size = 1347117, normalized size = 3309.87

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int (A+B\tan(fx+e)+C(\tan(fx+e))^2)(a+b\tan(fx+e))^{\frac{3}{2}}(c+d\tan(fx+e))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(5/2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(5/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan
(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.161 $\int \frac{\sqrt{a+b\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=373

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)\sqrt{a+b\tan(e+fx)}}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}+\frac{2\sqrt{a+b\tan(e+fx)}\left(3ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(-c^2d^2(5A-7C)\right.\right.}{3df\left(c^2+d^2\right)^2(bc-ad)\sqrt{c+d\tan(e+fx)}}$$

```
[Out] -((Sqrt[a - I*b]*(I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c - I*d)^(5/2)*f)) - (S
qrt[a + I*b]*(B - I*(A - C))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]
])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c + I*d)^(5/2)*f) - (2*(c^2
*C - B*c*d + A*d^2)*Sqrt[a + b*Tan[e + f*x]])/(3*d*(c^2 + d^2)*f*(c + d*Tan
[e + f*x])^(3/2)) + (2*(b*(c^4*C + 2*B*c^3*d - c^2*(5*A - 7*C)*d^2 - 4*B*c*
d^3 + A*d^4) + 3*a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)))*Sqrt[a + b*Tan[e +
f*x]])/(3*d*(b*c - a*d)*(c^2 + d^2)^2*f*Sqrt[c + d*Tan[e + f*x]])
```

Rubi [A] time = 1.92182, antiderivative size = 373, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3645, 3649, 3616, 3615, 93, 208}

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)\sqrt{a+b\tan(e+fx)}}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}+\frac{2\sqrt{a+b\tan(e+fx)}\left(3ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(-c^2d^2(5A-7C)\right.\right.}{3df\left(c^2+d^2\right)^2(bc-ad)\sqrt{c+d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c +
 d*Tan[e + f*x])^(5/2),x]
```

```
[Out] -((Sqrt[a - I*b]*(I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c - I*d)^(5/2)*f)) - (S
qrt[a + I*b]*(B - I*(A - C))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]
])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c + I*d)^(5/2)*f) - (2*(c^2
*C - B*c*d + A*d^2)*Sqrt[a + b*Tan[e + f*x]])/(3*d*(c^2 + d^2)*f*(c + d*Tan
[e + f*x])^(3/2)) + (2*(b*(c^4*C + 2*B*c^3*d - c^2*(5*A - 7*C)*d^2 - 4*B*c*
d^3 + A*d^4) + 3*a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)))*Sqrt[a + b*Tan[e +
f*x]])/(3*d*(b*c - a*d)*(c^2 + d^2)^2*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 3645

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*d^2 + c*(c*C - B*d))*(a + b*Tan[e
+ f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
 b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3616

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3615

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[((a + b*x)^m*(c + d*x)^n)/(A - B*x), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+b\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{2\left(c^2C-Bcd+Ad^2\right)\sqrt{a+b\tan(e+fx)}}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{2\int \frac{\frac{1}{2}(Ac}{}}{}$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)\sqrt{a+b\tan(e+fx)}}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{2\left(b\left(c^4C\right.\right.}{}$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)\sqrt{a+b\tan(e+fx)}}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{2\left(b\left(c^4C\right.\right.}{}$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)\sqrt{a+b\tan(e+fx)}}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{2\left(b\left(c^4C\right.\right.}{}$$

$$= -\frac{2\left(c^2C-Bcd+Ad^2\right)\sqrt{a+b\tan(e+fx)}}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} + \frac{2\left(b\left(c^4C\right.\right.}{}$$

$$= -\frac{\sqrt{a-ib}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(c-id)^{5/2}f} -$$

Mathematica [A] time = 6.91028, size = 609, normalized size = 1.63

$$-\frac{C\sqrt{a+b\tan(e+fx)}}{df(c+d\tan(e+fx))^{3/2}} - \frac{-\frac{2\sqrt{a+b\tan(e+fx)}\left(\frac{1}{2}d^2(-ad(2A-3C)-bcC)-c\left(d^2(-(aB+Ab-bC))-\frac{1}{2}c(aCd-2bBd-bcC)\right)\right)}{3f\left(c^2+d^2\right)(ad-bc)(c+d\tan(e+fx))^{3/2}} - \frac{2\left(-\frac{2\sqrt{a+b\tan(e+fx)}\left(-\frac{1}{2}d^2(bc\right.}{}\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)
)/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] -((C*Sqrt[a + b*Tan[e + f*x]])/(d*f*(c + d*Tan[e + f*x])^(3/2))) - ((-2*((d
^2*(-(b*c*C) - a*(2*A - 3*C)*d))/2 - c*(-((A*b + a*B - b*C)*d^2) - (c*(-(b*
c*C) - 2*b*B*d + a*C*d))/2))*Sqrt[a + b*Tan[e + f*x]])/(3*(-(b*c) + a*d)*(c
^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/2)) - (2*((-3*d*(b*c - a*d)^2*((Sqrt[a
+ I*b]*(B - I*(A - C))*(c - I*d)^2*ArcTan[(Sqrt[-c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])]])/Sqrt[-c - I*d] + (Sqrt[
-a + I*b]*(I*A + B - I*C)*(c + I*d)^2*ArcTan[(Sqrt[c - I*d]*Sqrt[a + b*Tan[
e + f*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])]])/Sqrt[c - I*d]))/(2*(
-(b*c) + a*d)*(c^2 + d^2)*f) - (2*(-(d^2*(b*c - a*d)*(3*a*d*(A*c - c*C + B*
d) + b*(c^2*C - B*c*d + A*d^2)))/2 - c*((-3*d^2*(b*c - a*d)*(A*b*c + a*B*c
- b*c*C - a*A*d + b*B*d + a*C*d))/2 + (b*c*(b*c - a*d)*(c^2*C + 2*B*c*d - (
2*A - 3*C)*d^2))/2))*Sqrt[a + b*Tan[e + f*x]])/((-(b*c) + a*d)*(c^2 + d^2)*
f*Sqrt[c + d*Tan[e + f*x]])))/(3*(-(b*c) + a*d)*(c^2 + d^2)))/d
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int (A+B\tan(fx+e)+C(\tan(fx+e))^2)\sqrt{a+b\tan(fx+e)}\left(c+d\tan(fx+e)\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(5/2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan
(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.162 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=379

$$\frac{2\left(Ad^2-Bcd+c^2C\right)\sqrt{a+b\tan(e+fx)}}{3f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^{3/2}}+\frac{2\sqrt{a+b\tan(e+fx)}\left(b\left(4c^2d^2(2A-C)+2Ad^4-5Bc^3d+Bcd^3+2c^4C\right)-\right.}{3f\left(c^2+d^2\right)^2(bc-ad)^2\sqrt{c+d\tan(e+fx}}$$

```
[Out] -(((B + I*(A - C))*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a
 - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a - I*b]*(c - I*d)^(5/2)*f)) + ((
I*A - B - I*C)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I
*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*(c + I*d)^(5/2)*f) + (2*(c^2
*C - B*c*d + A*d^2)*Sqrt[a + b*Tan[e + f*x]])/(3*(b*c - a*d)*(c^2 + d^2)*f*
(c + d*Tan[e + f*x])^(3/2)) + (2*(b*(2*c^4*C - 5*B*c^3*d + 4*c^2*(2*A - C)*
d^2 + B*c*d^3 + 2*A*d^4) - 3*a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)))*Sqrt[a
+ b*Tan[e + f*x]])/(3*(b*c - a*d)^2*(c^2 + d^2)^2*f*Sqrt[c + d*Tan[e + f*x]
])
```

Rubi [A] time = 1.81413, antiderivative size = 379, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.102, Rules used = {3649, 3616, 3615, 93, 208}

$$\frac{2\left(Ad^2-Bcd+c^2C\right)\sqrt{a+b\tan(e+fx)}}{3f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^{3/2}}+\frac{2\sqrt{a+b\tan(e+fx)}\left(b\left(4c^2d^2(2A-C)+2Ad^4-5Bc^3d+Bcd^3+2c^4C\right)-\right.}{3f\left(c^2+d^2\right)^2(bc-ad)^2\sqrt{c+d\tan(e+fx}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(Sqrt[a + b*Tan[e + f*x]]*(c +
d*Tan[e + f*x])^(5/2)),x]
```

```
[Out] -(((B + I*(A - C))*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a
 - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a - I*b]*(c - I*d)^(5/2)*f)) + ((
I*A - B - I*C)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I
*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*(c + I*d)^(5/2)*f) + (2*(c^2
*C - B*c*d + A*d^2)*Sqrt[a + b*Tan[e + f*x]])/(3*(b*c - a*d)*(c^2 + d^2)*f*
(c + d*Tan[e + f*x])^(3/2)) + (2*(b*(2*c^4*C - 5*B*c^3*d + 4*c^2*(2*A - C)*
d^2 + B*c*d^3 + 2*A*d^4) - 3*a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)))*Sqrt[a
+ b*Tan[e + f*x]])/(3*(b*c - a*d)^2*(c^2 + d^2)^2*f*Sqrt[c + d*Tan[e + f*x]
])
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
 b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3616

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3615

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[((a + b*x)^m*(c + d*x)^n)/(A - B*x), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}\,dx = \frac{2\left(c^2C-Bcd+Ad^2\right)\sqrt{a+b\tan(e+fx)}}{3(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}}+\frac{2\int \frac{1}{2}\left(2Abd^2+3Ac(bc-ad)\right.}{}$$

$$= \frac{2\left(c^2C-Bcd+Ad^2\right)\sqrt{a+b\tan(e+fx)}}{3(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}}+\frac{2\left(b\left(2c^4C-5Bc^3d+\right.\right.}{}$$

$$= \frac{2\left(c^2C-Bcd+Ad^2\right)\sqrt{a+b\tan(e+fx)}}{3(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}}+\frac{2\left(b\left(2c^4C-5Bc^3d+\right.\right.}{}$$

$$= \frac{2\left(c^2C-Bcd+Ad^2\right)\sqrt{a+b\tan(e+fx)}}{3(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}}+\frac{2\left(b\left(2c^4C-5Bc^3d+\right.\right.}{}$$

$$= \frac{2\left(c^2C-Bcd+Ad^2\right)\sqrt{a+b\tan(e+fx)}}{3(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}}+\frac{2\left(b\left(2c^4C-5Bc^3d+\right.\right.}{}$$

$$= -\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a-ib}(c-id)^{5/2}f}-\frac{(B-i(A-C))\tanh^{-1}}{\sqrt{a+ib}(}$$

Mathematica [A] time = 5.36485, size = 403, normalized size = 1.06

$$\frac{\frac{2(c^2+d^2)(bc-ad)\left(Ad^2-Bcd+c^2C\right)\sqrt{a+b\tan(e+fx)}}{(c+d\tan(e+fx))^{3/2}}+\frac{2\sqrt{a+b\tan(e+fx)}\left(3ad^2\left(2cd(C-A)+B\left(c^2-d^2\right)\right)+b\left(4c^2d^2(2A-C)+2Ad^4-5Bc^3d+Bcd^3+2c^4C\right)\right)}{\sqrt{c+d\tan(e+fx)}}+3(bc-}{3f\left(c^2+d^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(Sqrt[a + b*Tan[e + f*x]]
*(c + d*Tan[e + f*x])^(5/2)),x]
```

```
[Out] (3*(b*c - a*d)^2*((I*(A + I*B - C)*(c - I*d)^2*ArcTan[(Sqrt[-c - I*d]*Sqrt[
a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])]])/(Sqrt[a + I
*b]*Sqrt[-c - I*d]) + ((I*A + B - I*C)*(c + I*d)^2*ArcTan[(Sqrt[c - I*d]*Sq
rt[a + b*Tan[e + f*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])]])/(Sqrt[-
a + I*b]*Sqrt[c - I*d])) + (2*(b*c - a*d)*(c^2 + d^2)*(c^2*C - B*c*d + A*d^
2)*Sqrt[a + b*Tan[e + f*x]])/(c + d*Tan[e + f*x])^(3/2) + (2*(b*(2*c^4*C -
5*B*c^3*d + 4*c^2*(2*A - C)*d^2 + B*c*d^3 + 2*A*d^4) + 3*a*d^2*(2*c*(-A + C
)*d + B*(c^2 - d^2)))*Sqrt[a + b*Tan[e + f*x]])/Sqrt[c + d*Tan[e + f*x]])/(
3*(b*c - a*d)^2*(c^2 + d^2)^2*f)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int (A+B\tan(fx+e)+C(\tan(fx+e))^2)\frac{1}{\sqrt{a+b\tan(fx+e)}}(c+d\tan(fx+e))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))
^(5/2),x)
```

```
[Out] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))
^(5/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f
*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f
*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**(1/2)/(c+d*tan
(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f
*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.163 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=651

$$-\frac{2d\sqrt{a+b\tan(e+fx)}\left(-A\left(-a^2bd^2\left(11c^2+5d^2\right)+6a^3cd^3+6ab^2cd^3+b^3\left(-\left(17c^2d^2+3c^4+8d^4\right)\right)\right)+a^2b\left(-8Bc^3d-2B\right.\right.}{3f\left(a^2+b^2\right)\left(c^2\right.}$$

```
[Out] -(((I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a
 - I*b]*Sqrt[c + d*Tan[e + f*x]])]])/((a - I*b)^(3/2)*(c - I*d)^(5/2)*f)) -
((B - I*(A - C))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a +
 I*b]*Sqrt[c + d*Tan[e + f*x]])]])/((a + I*b)^(3/2)*(c + I*d)^(5/2)*f) - (2*
(A*b^2 - a*(b*B - a*C)))/((a^2 + b^2)*(b*c - a*d)*f*Sqrt[a + b*Tan[e + f*x]
]*(c + d*Tan[e + f*x])^(3/2)) - (2*d*(b^2*c*(c*C - B*d) - 3*a*b*B*(c^2 + d^
2) + a^2*(4*c^2*C - B*c*d + 3*C*d^2) + A*(a^2*d^2 + b^2*(3*c^2 + 4*d^2)))*S
qrt[a + b*Tan[e + f*x]])/(3*(a^2 + b^2)*(b*c - a*d)^2*(c^2 + d^2)*f*(c + d*
Tan[e + f*x])^(3/2)) - (2*d*(b^3*c*(5*c^3*C - 8*B*c^2*d - c*C*d^2 - 2*B*d^3
) + a^2*b*(8*c^4*C - 8*B*c^3*d + 5*c^2*C*d^2 - 2*B*c*d^3 + 3*C*d^4) + 3*a^3
*d^2*(2*c*C*d + B*(c^2 - d^2)) + 3*a*b^2*(2*c*C*d^3 - B*(c^4 + c^2*d^2 + 2*
d^4)) - A*(6*a^3*c*d^3 + 6*a*b^2*c*d^3 - a^2*b*d^2*(11*c^2 + 5*d^2) - b^3*(
3*c^4 + 17*c^2*d^2 + 8*d^4)))*Sqrt[a + b*Tan[e + f*x]])/(3*(a^2 + b^2)*(b*c
 - a*d)^3*(c^2 + d^2)^2*f*Sqrt[c + d*Tan[e + f*x]])
```

Rubi [A] time = 3.43109, antiderivative size = 650, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.102, Rules used = {3649, 3616, 3615, 93, 208}

$$-\frac{2d\sqrt{a+b\tan(e+fx)}\left(-A\left(-a^2bd^2\left(11c^2+5d^2\right)+6a^3cd^3+6ab^2cd^3+b^3\left(-\left(17c^2d^2+3c^4+8d^4\right)\right)\right)+a^2b\left(-8Bc^3d-2B\right.\right.}{3f\left(a^2+b^2\right)\left(c^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^(3/2)*(c
+ d*Tan[e + f*x])^(5/2)),x]
```

```
[Out] -(((I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a
 - I*b]*Sqrt[c + d*Tan[e + f*x]])]])/((a - I*b)^(3/2)*(c - I*d)^(5/2)*f)) -
((B - I*(A - C))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a +
 I*b]*Sqrt[c + d*Tan[e + f*x]])]])/((a + I*b)^(3/2)*(c + I*d)^(5/2)*f) - (2*
(A*b^2 - a*(b*B - a*C)))/((a^2 + b^2)*(b*c - a*d)*f*Sqrt[a + b*Tan[e + f*x]
]*(c + d*Tan[e + f*x])^(3/2)) - (2*d*(a^2*A*d^2 + b^2*c*(c*C - B*d) - 3*a*b
*B*(c^2 + d^2) + A*b^2*(3*c^2 + 4*d^2) + a^2*(4*c^2*C - B*c*d + 3*C*d^2))*S
qrt[a + b*Tan[e + f*x]])/(3*(a^2 + b^2)*(b*c - a*d)^2*(c^2 + d^2)*f*(c + d*
Tan[e + f*x])^(3/2)) - (2*d*(b^3*c*(5*c^3*C - 8*B*c^2*d - c*C*d^2 - 2*B*d^3
) + a^2*b*(8*c^4*C - 8*B*c^3*d + 5*c^2*C*d^2 - 2*B*c*d^3 + 3*C*d^4) + 3*a^3
*d^2*(2*c*C*d + B*(c^2 - d^2)) + 3*a*b^2*(2*c*C*d^3 - B*(c^4 + c^2*d^2 + 2*
d^4)) - A*(6*a^3*c*d^3 + 6*a*b^2*c*d^3 - a^2*b*d^2*(11*c^2 + 5*d^2) - b^3*(
3*c^4 + 17*c^2*d^2 + 8*d^4)))*Sqrt[a + b*Tan[e + f*x]])/(3*(a^2 + b^2)*(b*c
 - a*d)^3*(c^2 + d^2)^2*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
 b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3616

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3615

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[((a + b*x)^m*(c + d*x)^n)/(A - B*x), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{2\left(Ab^2-a(bB-aC)\right)}{\left(a^2+b^2\right)(bc-ad)f\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}} - \frac{2\int}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)}{\left(a^2+b^2\right)(bc-ad)f\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}} - \frac{2d}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)}{\left(a^2+b^2\right)(bc-ad)f\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}} - \frac{2d}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)}{\left(a^2+b^2\right)(bc-ad)f\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}} - \frac{2d}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)}{\left(a^2+b^2\right)(bc-ad)f\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}} - \frac{2d}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)}{\left(a^2+b^2\right)(bc-ad)f\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}} - \frac{2d}{}$$

$$= -\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{3/2}(c-id)^{5/2}f} - \frac{(B-i(A-C))\tanh^{-1}}{(a+ib)^{3/2}(}$$

Mathematica [A] time = 6.87487, size = 903, normalized size = 1.39

$$-\frac{2\left(Ab^2-a(bB-aC)\right)}{\left(a^2+b^2\right)(bc-ad)f\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}} - \frac{2\left(-\frac{2\sqrt{a+b\tan(e+fx)}\left(\frac{1}{2}d^2\left(4Adb^2-aA(bc-ad)-(bB-aC)(bc+3ad)\right)-c\left(\frac{1}{2}\right.\right.}{3(ad-bc)\left(c^2+d^2\right)f(c+d\tan(e+}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^(3/
2)*(c + d*Tan[e + f*x])^(5/2)),x]
```

```
[Out] (-2*(A*b^2 - a*(b*B - a*C)))/((a^2 + b^2)*(b*c - a*d)*f*Sqrt[a + b*Tan[e +
f*x]]*(c + d*Tan[e + f*x])^(3/2)) - (2*((-2*(-(c*(-2*c*(A*b^2 - a*(b*B - a*
C))*d + ((A*b - a*B - b*C)*d*(b*c - a*d))/2)) + (d^2*(4*A*b^2*d - a*A*(b*c
- a*d) - (b*B - a*C)*(b*c + 3*a*d)))/2)*Sqrt[a + b*Tan[e + f*x]])/(3*(-(b*c
) + a*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/2)) - (2*((3*(b*c - a*d)^3*(
((I*a + b)*(A + I*B - C)*(c - I*d)^2*ArcTan[(Sqrt[-c - I*d]*Sqrt[a + b*Tan[
e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*Sqrt[-
c - I*d]) + ((a + I*b)*(I*A + B - I*C)*(c + I*d)^2*ArcTan[(Sqrt[c - I*d]*Sq
rt[a + b*Tan[e + f*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[-
```

```
a + I*b]*Sqrt[c - I*d]))))/(4*(-(b*c) + a*d)*(c^2 + d^2)*f) - (2*(d^2*(((b*c
)/2 - (3*a*d)/2)*(-2*c*(A*b^2 - a*(b*B - a*C))*d + ((A*b - a*B - b*C)*d*(b*
c - a*d))/2) + ((b*d^2 - (3*c*(-(b*c) + a*d))/2)*(4*A*b^2*d - a*A*(b*c - a*
d) - (b*B - a*C)*(b*c + 3*a*d)))/2) - c*((3*d*(-(b*c) + a*d)*(-2*(A*b^2 - a
*(b*B - a*C))*d^2 - (c*(A*b - a*B - b*C)*(b*c - a*d))/2 + (d*(4*A*b^2*d - a
*A*(b*c - a*d) - (b*B - a*C)*(b*c + 3*a*d)))/2))/2 - b*c*(-(c*(-2*c*(A*b^2
- a*(b*B - a*C))*d + ((A*b - a*B - b*C)*d*(b*c - a*d))/2)) + (d^2*(4*A*b^2*
d - a*A*(b*c - a*d) - (b*B - a*C)*(b*c + 3*a*d)))/2)))*Sqrt[a + b*Tan[e + f
*x]])/((-(b*c) + a*d)*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]]))))/(3*(-(b*c)
+ a*d)*(c^2 + d^2))))/((a^2 + b^2)*(b*c - a*d))
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int (A + B\tan(fx+e) + C(\tan(fx+e))^2)(a + b\tan(fx+e))^{-\frac{3}{2}}(c + d\tan(fx+e))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))
^(5/2),x)
```

```
[Out] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))
^(5/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(f
*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(f
*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**(3/2)/(c+d*tan
(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(f
*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.164 $\int (a+b\tan(e+fx))^m (c+d\tan(e+fx))^n \left(A + B\tan(e + fx\right.$

Optimal. Leaf size=376

$$\frac{C(a+b\tan(e+fx))^{m+1}(c+d\tan(e+fx))^n\left(\frac{b(c+d\tan(e+fx))}{bc-ad}\right)^{-n}\text{Hypergeometric2F1}\left(m+1,-n,m+2,-\frac{d(a+b\tan(e+}{bc-ad}\right.}{bf(m+1)}$$

```
[Out] -((B + I*(A - C))*AppellF1[1 + m, -n, 1, 2 + m, -((d*(a + b*Tan[e + f*x]))/
(b*c - a*d)), (a + b*Tan[e + f*x])/(a - I*b)]*(a + b*Tan[e + f*x])^(1 + m)*
(c + d*Tan[e + f*x])^n)/(2*(a - I*b)*f*(1 + m)*((b*(c + d*Tan[e + f*x]))/(b
*c - a*d))^n) - ((A + I*B - C)*AppellF1[1 + m, -n, 1, 2 + m, -((d*(a + b*Ta
n[e + f*x]))/(b*c - a*d)), (a + b*Tan[e + f*x])/(a + I*b)]*(a + b*Tan[e + f
*x])^(1 + m)*(c + d*Tan[e + f*x])^n)/(2*(I*a - b)*f*(1 + m)*((b*(c + d*Tan[
e + f*x]))/(b*c - a*d))^n) + (C*Hypergeometric2F1[1 + m, -n, 2 + m, -((d*(a
 + b*Tan[e + f*x]))/(b*c - a*d))]*(a + b*Tan[e + f*x])^(1 + m)*(c + d*Tan[e
 + f*x])^n)/(b*f*(1 + m)*((b*(c + d*Tan[e + f*x]))/(b*c - a*d))^n)
```

Rubi [A] time = 0.899859, antiderivative size = 376, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3655, 6725, 70, 69, 137, 136}

$$-\frac{(B+i(A-C))(a+b\tan(e+fx))^{m+1}(c+d\tan(e+fx))^n\left(\frac{b(c+d\tan(e+fx))}{bc-ad}\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{d(a+b\tan(e+fx))}{bc-ad},\right.}{2f(m+1)(a-ib)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(A + B*Tan[e + f*x] + C*T
an[e + f*x]^2),x]
```

```
[Out] -((B + I*(A - C))*AppellF1[1 + m, -n, 1, 2 + m, -((d*(a + b*Tan[e + f*x]))/
(b*c - a*d)), (a + b*Tan[e + f*x])/(a - I*b)]*(a + b*Tan[e + f*x])^(1 + m)*
(c + d*Tan[e + f*x])^n)/(2*(a - I*b)*f*(1 + m)*((b*(c + d*Tan[e + f*x]))/(b
*c - a*d))^n) - ((A + I*B - C)*AppellF1[1 + m, -n, 1, 2 + m, -((d*(a + b*Ta
n[e + f*x]))/(b*c - a*d)), (a + b*Tan[e + f*x])/(a + I*b)]*(a + b*Tan[e + f
*x])^(1 + m)*(c + d*Tan[e + f*x])^n)/(2*(I*a - b)*f*(1 + m)*((b*(c + d*Tan[
e + f*x]))/(b*c - a*d))^n) + (C*Hypergeometric2F1[1 + m, -n, 2 + m, -((d*(a
 + b*Tan[e + f*x]))/(b*c - a*d))]*(a + b*Tan[e + f*x])^(1 + m)*(c + d*Tan[e
 + f*x])^n)/(b*f*(1 + m)*((b*(c + d*Tan[e + f*x]))/(b*c - a*d))^n)
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[((a + b*ff*x)^m*(c + d*ff*x)^n*(A + B*ff*x + C*ff^2*x^2
))/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^n\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx = \frac{\text{Subst}\left(\int \frac{(a+bx)^m(c+dx)^n\left(A+Bx+Cx^2\right)}{1+x^2} \cdots\right.}{f}$$

$$= \frac{\text{Subst}\left(\int \left(C(a+bx)^m(c+dx)^n \cdots\right.\right.}{\cdots}$$

$$= \frac{(-B+i(A-C))\,\text{Subst}\left(\int \frac{(a+bx)^m \cdots}{i \cdots}\right.}{2f}$$

$$= \frac{\left((-B+i(A-C))(c+d\tan(e+\cdots\right.}{\cdots}$$

$$= -\frac{(B+i(A-C))F_1\left(1+m;-n,1;\cdots\right.}{\cdots}$$

Mathematica [F] time = 22.7369, size = 0, normalized size = 0.

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^n\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(A + B*Tan[e + f*x]
 + C*Tan[e + f*x]^2),x]
```

```
[Out] Integrate[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(A + B*Tan[e + f*x]
 + C*Tan[e + f*x]^2), x]
```

Maple [F] time = 0.649, size = 0, normalized size = 0.

$$\int \left(a+b\tan\left(fx+e\right)\right)^{m}\left(c+d\tan\left(fx+e\right)\right)^{n}\left(A+B\tan\left(fx+e\right)+C\left(\tan\left(fx+e\right)\right)^{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^n*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x
)
```

```
[Out] int((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^n*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\tan\left(fx+e\right)^{2}+B\tan\left(fx+e\right)+A\right)\left(b\tan\left(fx+e\right)+a\right)^{m}\left(d\tan\left(fx+e\right)+c\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^n*(A+B*tan(f*x+e)+C*tan(f*x+e
)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m*(d
*tan(f*x + e) + c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(C\tan\left(fx+e\right)^{2}+B\tan\left(fx+e\right)+A\right)\left(b\tan\left(fx+e\right)+a\right)^{m}\left(d\tan\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^n*(A+B*tan(f*x+e)+C*tan(f*x+e
)^2),x, algorithm="fricas")
```

```
[Out] integral((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m*(d*
tan(f*x + e) + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**m*(c+d*tan(f*x+e))**n*(A+B*tan(f*x+e)+C*tan(f*x
+e)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\tan\left(fx+e\right)^2+B\tan\left(fx+e\right)+A\right)\left(b\tan\left(fx+e\right)+a\right)^m\left(d\tan\left(fx+e\right)+c\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^n*(A+B*tan(f*x+e)+C*tan(f*x+e
)^2),x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m*(d
*tan(f*x + e) + c)^n, x)
```

3.165 $\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^3\left(A+B\tan(e+fx\right.$

Optimal. Leaf size=560

$$\frac{(c-id)^3(A-iB-C)(a+b\tan(e+fx))^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{a+b\tan(e+fx)}{a-ib}\right)}{2f(m+1)(b+ia)}-\frac{(c+id)^3(A+iB-}{}$$

```
[Out] ((b*c*(2 + m)*(b^2*d*(B*c + (A - C)*d)*(3 + m)*(4 + m) - 2*(b*c - a*d)*(3*a
*C*d - b*(3*c*C + B*d*(4 + m)))) + d*(b^3*(2*c*(A - C)*d + B*(c^2 - d^2))*(
2 + m)*(3 + m)*(4 + m) - a*(b^2*d*(B*c + (A - C)*d)*(3 + m)*(4 + m) - 2*(b*
c - a*d)*(3*a*C*d - b*(3*c*C + B*d*(4 + m))))))*(a + b*Tan[e + f*x])^(1 + m
))/(b^4*f*(1 + m)*(2 + m)*(3 + m)*(4 + m)) + ((A - I*B - C)*(c - I*d)^3*Hyp
ergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a - I*b)]*(a + b*Tan[
e + f*x])^(1 + m))/(2*(I*a + b)*f*(1 + m)) - ((A + I*B - C)*(c + I*d)^3*Hyp
ergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a + I*b)]*(a + b*Tan[
e + f*x])^(1 + m))/(2*(I*a - b)*f*(1 + m)) + (d*(b^2*d*(B*c + (A - C)*d)*(3
 + m)*(4 + m) - 2*(b*c - a*d)*(3*a*C*d - b*(3*c*C + B*d*(4 + m))))*Tan[e +
f*x]*(a + b*Tan[e + f*x])^(1 + m))/(b^3*f*(2 + m)*(3 + m)*(4 + m)) - ((3*a*
C*d - b*(3*c*C + B*d*(4 + m)))*(a + b*Tan[e + f*x])^(1 + m)*(c + d*Tan[e +
f*x])^2)/(b^2*f*(3 + m)*(4 + m)) + (C*(a + b*Tan[e + f*x])^(1 + m)*(c + d*T
an[e + f*x])^3)/(b*f*(4 + m))
```

Rubi [A] time = 2.37664, antiderivative size = 551, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3647, 3637, 3630, 3539, 3537, 68}

$$\frac{(a+b\tan(e+fx))^{m+1}\left(d\left(b^3(m+2)(m+3)(m+4)\left(2cd(A-C)+B\left(c^2-d^2\right)\right)-a\left(2(bc-ad)(-3aCd+bBd(m+4)\right.\right.\right.}{b^4f(}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*T
an[e + f*x]^2),x]
```

```
[Out] ((b*c*(2 + m)*(b^2*d*(B*c + (A - C)*d)*(3 + m)*(4 + m) + 2*(b*c - a*d)*(3*b
*c*C - 3*a*C*d + b*B*d*(4 + m))) + d*(b^3*(2*c*(A - C)*d + B*(c^2 - d^2))*(
2 + m)*(3 + m)*(4 + m) - a*(b^2*d*(B*c + (A - C)*d)*(3 + m)*(4 + m) + 2*(b*
c - a*d)*(3*b*c*C - 3*a*C*d + b*B*d*(4 + m)))))*(a + b*Tan[e + f*x])^(1 + m
))/(b^4*f*(1 + m)*(2 + m)*(3 + m)*(4 + m)) + ((A - I*B - C)*(c - I*d)^3*Hyp
ergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a - I*b)]*(a + b*Tan[
e + f*x])^(1 + m))/(2*(I*a + b)*f*(1 + m)) - ((A + I*B - C)*(c + I*d)^3*Hyp
ergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a + I*b)]*(a + b*Tan[
e + f*x])^(1 + m))/(2*(I*a - b)*f*(1 + m)) + (d*(b^2*d*(B*c + (A - C)*d)*(3
 + m)*(4 + m) + 2*(b*c - a*d)*(3*b*c*C - 3*a*C*d + b*B*d*(4 + m)))*Tan[e +
f*x]*(a + b*Tan[e + f*x])^(1 + m))/(b^3*f*(2 + m)*(3 + m)*(4 + m)) + ((3*b*
c*C - 3*a*C*d + b*B*d*(4 + m))*(a + b*Tan[e + f*x])^(1 + m)*(c + d*Tan[e +
f*x])^2)/(b^2*f*(3 + m)*(4 + m)) + (C*(a + b*Tan[e + f*x])^(1 + m)*(c + d*T
an[e + f*x])^3)/(b*f*(4 + m))
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
```

```
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx = \frac{C(a+b\tan(e+fx))^{1+m}(c+\cdots}{bf(4+m)}$$

$$= \frac{(3bcC-3aCd+bBd(4+m))\cdots}{b^2\cdots}$$

$$= \frac{d\left(b^2d(Bc+(A-C)d)(3+m\cdots\right.}{\cdots}$$

$$= \frac{\left(bc(2+m)\left(b^2d(Bc+(A-C)\cdots\right.\right.}{\cdots}$$

$$= \frac{\left(bc(2+m)\left(b^2d(Bc+(A-C)\cdots\right.\right.}{\cdots}$$

$$= \frac{\left(bc(2+m)\left(b^2d(Bc+(A-C)\cdots\right.\right.}{\cdots}$$

$$= \frac{\left(bc(2+m)\left(b^2d(Bc+(A-C)\cdots\right.\right.}{\cdots}$$

Mathematica [B] time = 6.38738, size = 1390, normalized size = 2.48

$$\frac{C(c+d\tan(e+fx))^3(a+b\tan(e+fx))^{m+1}}{bf(m+4)} + \frac{\frac{(3bcC-3adC+bBd(m+4))(c+d\tan(e+fx))^2(a+b\tan(e+fx))^{m+1}}{bf(m+3)} + \frac{d\left(d(Bc+(A-C)d)(m+3)(m+4)\cdots\right.}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x]
 + C*Tan[e + f*x]^2),x]
```

```
[Out] (C*(a + b*Tan[e + f*x])^(1 + m)*(c + d*Tan[e + f*x])^3)/(b*f*(4 + m)) + (((
3*b*c*C - 3*a*C*d + b*B*d*(4 + m))*(a + b*Tan[e + f*x])^(1 + m)*(c + d*Tan[
e + f*x])^2)/(b*f*(3 + m)) + ((d*(b^2*d*(B*c + (A - C)*d)*(3 + m)*(4 + m) +
 2*(b*c - a*d)*(3*b*c*C - 3*a*C*d + b*B*d*(4 + m)))*Tan[e + f*x]*(a + b*Tan
[e + f*x])^(1 + m))/(b*f*(2 + m)) - (((-(b*c*(2 + m)*(b^2*d*(B*c + (A - C)*
d)*(3 + m)*(4 + m) + 2*(b*c - a*d)*(3*b*c*C - 3*a*C*d + b*B*d*(4 + m)))) +
d*(-(b^3*(2*c*(A - C)*d + B*(c^2 - d^2))*(2 + m)*(3 + m)*(4 + m)) + a*(b^2*
d*(B*c + (A - C)*d)*(3 + m)*(4 + m) + 2*(b*c - a*d)*(3*b*c*C - 3*a*C*d + b*
B*d*(4 + m)))))*(a + b*Tan[e + f*x])^(1 + m))/(b*f*(1 + m)) + ((I/2)*(a*d*(
b^2*d*(B*c + (A - C)*d)*(3 + m)*(4 + m) + 2*(b*c - a*d)*(3*b*c*C - 3*a*C*d
+ b*B*d*(4 + m))) + b*c*(2 + m)*(b^2*d*(B*c + (A - C)*d)*(3 + m)*(4 + m) +
2*(b*c - a*d)*(3*b*c*C - 3*a*C*d + b*B*d*(4 + m))) - b*c*(2 + m)*(-((2*a*d
+ b*c*(1 + m))*(3*b*c*C - 3*a*C*d + b*B*d*(4 + m))) + b*c*(3 + m)*(A*b*c*(4
 + m) - C*(3*a*d + b*c*(1 + m)))) - d*(-(b^3*(2*c*(A - C)*d + B*(c^2 - d^2)
)*(2 + m)*(3 + m)*(4 + m)) + a*(b^2*d*(B*c + (A - C)*d)*(3 + m)*(4 + m) + 2
*(b*c - a*d)*(3*b*c*C - 3*a*C*d + b*B*d*(4 + m)))) - I*b*(2 + m)*(b^2*c*(2*
c*(A - C)*d + B*(c^2 - d^2))*(3 + m)*(4 + m) - d*(b^2*d*(B*c + (A - C)*d)*(
3 + m)*(4 + m) + 2*(b*c - a*d)*(3*b*c*C - 3*a*C*d + b*B*d*(4 + m))) + d*(-(
(2*a*d + b*c*(1 + m))*(3*b*c*C - 3*a*C*d + b*B*d*(4 + m))) + b*c*(3 + m)*(A
*b*c*(4 + m) - C*(3*a*d + b*c*(1 + m))))))*Hypergeometric2F1[1, 1 + m, 2 +
m, ((-I)*a - I*b*Tan[e + f*x])/((-I)*a + b)]*(a + b*Tan[e + f*x])^(1 + m))/
((a + I*b)*f*(1 + m)) - ((I/2)*(a*d*(b^2*d*(B*c + (A - C)*d)*(3 + m)*(4 + m
) + 2*(b*c - a*d)*(3*b*c*C - 3*a*C*d + b*B*d*(4 + m))) + b*c*(2 + m)*(b^2*d
*(B*c + (A - C)*d)*(3 + m)*(4 + m) + 2*(b*c - a*d)*(3*b*c*C - 3*a*C*d + b*B
*d*(4 + m))) - b*c*(2 + m)*(-((2*a*d + b*c*(1 + m))*(3*b*c*C - 3*a*C*d + b*
```

```
B*d*(4 + m))) + b*c*(3 + m)*(A*b*c*(4 + m) - C*(3*a*d + b*c*(1 + m)))) - d*
(-(b^3*(2*c*(A - C)*d + B*(c^2 - d^2))*(2 + m)*(3 + m)*(4 + m)) + a*(b^2*d*
(B*c + (A - C)*d)*(3 + m)*(4 + m) + 2*(b*c - a*d)*(3*b*c*C - 3*a*C*d + b*B*
d*(4 + m)))) + I*b*(2 + m)*(b^2*c*(2*c*(A - C)*d + B*(c^2 - d^2))*(3 + m)*(
4 + m) - d*(b^2*d*(B*c + (A - C)*d)*(3 + m)*(4 + m) + 2*(b*c - a*d)*(3*b*c*
C - 3*a*C*d + b*B*d*(4 + m))) + d*(-((2*a*d + b*c*(1 + m))*(3*b*c*C - 3*a*C
*d + b*B*d*(4 + m))) + b*c*(3 + m)*(A*b*c*(4 + m) - C*(3*a*d + b*c*(1 + m))
))))*Hypergeometric2F1[1, 1 + m, 2 + m, -((I*a + I*b*Tan[e + f*x])/((-I)*a
- b))]*(a + b*Tan[e + f*x])^(1 + m))/((a - I*b)*f*(1 + m)))/(b*(2 + m)))/(b
*(3 + m)))/(b*(4 + m))
```

Maple [F] time = 0.839, size = 0, normalized size = 0.

$$\int \left(a+b\tan\left(fx+e\right)\right)^{m}\left(c+d\tan\left(fx+e\right)\right)^{3}\left(A+B\tan\left(fx+e\right)+C\left(\tan\left(fx+e\right)\right)^{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x
)
```

```
[Out] int((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x
)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e
)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cd^3\tan\left(fx+e\right)^5+\left(3\,Ccd^2+Bd^3\right)\tan\left(fx+e\right)^4+Ac^3+\left(3\,Cc^2d+3\,Bcd^2+Ad^3\right)\tan\left(fx+e\right)^3+\left(Cc^3+3\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e
)^2),x, algorithm="fricas")
```

```
[Out] integral((C*d^3*tan(f*x + e)^5 + (3*C*c*d^2 + B*d^3)*tan(f*x + e)^4 + A*c^3
 + (3*C*c^2*d + 3*B*c*d^2 + A*d^3)*tan(f*x + e)^3 + (C*c^3 + 3*B*c^2*d + 3*
A*c*d^2)*tan(f*x + e)^2 + (B*c^3 + 3*A*c^2*d)*tan(f*x + e))*(b*tan(f*x + e)
 + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**m*(c+d*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*x
+e)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\tan\left(fx+e\right)^{2}+B\tan\left(fx+e\right)+A\right)\left(d\tan\left(fx+e\right)+c\right)^{3}\left(b\tan\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e
)^2),x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(d*tan(f*x + e) + c)^3*(b
*tan(f*x + e) + a)^m, x)
```

3.166 $\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^2 \left(A+B\tan(e+fx)\right.$

Optimal. Leaf size=363

$$\frac{(c-id)^2(A-iB-C)(a+b\tan(e+fx))^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{a+b\tan(e+fx)}{a-ib}\right)}{2f(m+1)(b+ia)}+\frac{(c+id)^2(iA-B-iC}{}$$

```
[Out] ((2*a^2*C*d^2 - a*b*d*(2*c*C + B*d)*(3 + m) + b^2*(2 + m)*(2*c^2*C + 2*B*c*
d*(3 + m) + (A - C)*d^2*(3 + m)))*(a + b*Tan[e + f*x])^(1 + m))/(b^3*f*(1 +
 m)*(2 + m)*(3 + m)) + ((A - I*B - C)*(c - I*d)^2*Hypergeometric2F1[1, 1 +
m, 2 + m, (a + b*Tan[e + f*x])/(a - I*b)]*(a + b*Tan[e + f*x])^(1 + m))/(2*
(I*a + b)*f*(1 + m)) + ((I*A - B - I*C)*(c + I*d)^2*Hypergeometric2F1[1, 1
+ m, 2 + m, (a + b*Tan[e + f*x])/(a + I*b)]*(a + b*Tan[e + f*x])^(1 + m))/(
2*(a + I*b)*f*(1 + m)) - (d*(2*a*C*d - b*(2*c*C + B*d*(3 + m)))*Tan[e + f*x
]*(a + b*Tan[e + f*x])^(1 + m))/(b^2*f*(2 + m)*(3 + m)) + (C*(a + b*Tan[e +
 f*x])^(1 + m)*(c + d*Tan[e + f*x])^2)/(b*f*(3 + m))
```

Rubi [A] time = 1.15153, antiderivative size = 360, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3647, 3637, 3630, 3539, 3537, 68}

$$\frac{(a+b\tan(e+fx))^{m+1}\left(2a^2Cd^2-abd(m+3)(Bd+2cC)+b^2(m+2)\left(d^2(m+3)(A-C)+2Bcd(m+3)+2c^2C\right)\right)}{b^3f(m+1)(m+2)(m+3)}+\frac{(c-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*T
an[e + f*x]^2),x]
```

```
[Out] ((2*a^2*C*d^2 - a*b*d*(2*c*C + B*d)*(3 + m) + b^2*(2 + m)*(2*c^2*C + 2*B*c*
d*(3 + m) + (A - C)*d^2*(3 + m)))*(a + b*Tan[e + f*x])^(1 + m))/(b^3*f*(1 +
 m)*(2 + m)*(3 + m)) + ((A - I*B - C)*(c - I*d)^2*Hypergeometric2F1[1, 1 +
m, 2 + m, (a + b*Tan[e + f*x])/(a - I*b)]*(a + b*Tan[e + f*x])^(1 + m))/(2*
(I*a + b)*f*(1 + m)) + ((I*A - B - I*C)*(c + I*d)^2*Hypergeometric2F1[1, 1
+ m, 2 + m, (a + b*Tan[e + f*x])/(a + I*b)]*(a + b*Tan[e + f*x])^(1 + m))/(
2*(a + I*b)*f*(1 + m)) + (d*(2*b*c*C - 2*a*C*d + b*B*d*(3 + m))*Tan[e + f*x
]*(a + b*Tan[e + f*x])^(1 + m))/(b^2*f*(2 + m)*(3 + m)) + (C*(a + b*Tan[e +
 f*x])^(1 + m)*(c + d*Tan[e + f*x])^2)/(b*f*(3 + m))
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
```

```
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx=\frac{C(a+b\tan(e+fx))^{1+m}(c+\ldots}{bf(3+m)}$$

$$=\frac{d(2bcC-2aCd+bBd(3+m)\ldots}{b^2f(2\ldots}$$

$$=\frac{\left(2a^2Cd^2-abd(2cC+Bd)(3+\ldots\right.}{\ldots}$$

$$=\frac{\left(2a^2Cd^2-abd(2cC+Bd)(3+\ldots\right.}{\ldots}$$

$$=\frac{\left(2a^2Cd^2-abd(2cC+Bd)(3+\ldots\right.}{\ldots}$$

$$=\frac{\left(2a^2Cd^2-abd(2cC+Bd)(3+\ldots\right.}{\ldots}$$

Mathematica [A] time = 6.33159, size = 505, normalized size = 1.39

$$\frac{C(c+d\tan(e+fx))^2(a+b\tan(e+fx))^{m+1}}{bf(m+3)}+\frac{\frac{d\tan(e+fx)(-2aCd+bBd(m+3)+2bcC)(a+b\tan(e+fx))^{m+1}}{bf(m+2)}-\frac{i(a+b\tan(e+fx))^{m+1}\left(b^2(m+2)(m+3)(\cdots\right.}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x]
 + C*Tan[e + f*x]^2),x]
```

```
[Out] (C*(a + b*Tan[e + f*x])^(1 + m)*(c + d*Tan[e + f*x])^2)/(b*f*(3 + m)) + ((d
*(2*b*c*C - 2*a*C*d + b*B*d*(3 + m))*Tan[e + f*x]*(a + b*Tan[e + f*x])^(1 +
 m))/(b*f*(2 + m)) - (((-(b*c*(2 + m)*(2*b*c*C - 2*a*C*d + b*B*d*(3 + m)))
- d*(b^2*(B*c + (A - C)*d)*(2 + m)*(3 + m) - a*(2*b*c*C - 2*a*C*d + b*B*d*(
3 + m))))*(a + b*Tan[e + f*x])^(1 + m))/(b*f*(1 + m)) + ((I/2)*(-(b^2*(A*c^
2 - c^2*C - 2*B*c*d - A*d^2 + C*d^2)*(2 + m)*(3 + m)) - I*b^2*(2*c*(A - C)*
d + B*(c^2 - d^2))*(2 + m)*(3 + m))*Hypergeometric2F1[1, 1 + m, 2 + m, ((-I
)*a - I*b*Tan[e + f*x])/((-I)*a + b)]*(a + b*Tan[e + f*x])^(1 + m))/((a + I
*b)*f*(1 + m)) - ((I/2)*(-(b^2*(A*c^2 - c^2*C - 2*B*c*d - A*d^2 + C*d^2)*(2
 + m)*(3 + m)) + I*b^2*(2*c*(A - C)*d + B*(c^2 - d^2))*(2 + m)*(3 + m))*Hyp
ergeometric2F1[1, 1 + m, 2 + m, -((I*a + I*b*Tan[e + f*x])/((-I)*a - b))]*(
a + b*Tan[e + f*x])^(1 + m))/((a - I*b)*f*(1 + m)))/(b*(2 + m)))/(b*(3 + m)
)
```

Maple [F] time = 0.591, size = 0, normalized size = 0.

$$\int \left(a+b\tan\left(fx+e\right)\right)^{m}\left(c+d\tan\left(fx+e\right)\right)^{2}\left(A+B\tan\left(fx+e\right)+C\left(\tan\left(fx+e\right)\right)^{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x
)
```

```
[Out] int((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x
)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e
)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cd^2\tan\left(fx+e\right)^4+\left(2\,Ccd+Bd^2\right)\tan\left(fx+e\right)^3+Ac^2+\left(Cc^2+2\,Bcd+Ad^2\right)\tan\left(fx+e\right)^2+\left(Bc^2+2\,Acd\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e
)^2),x, algorithm="fricas")
```

```
[Out] integral((C*d^2*tan(f*x + e)^4 + (2*C*c*d + B*d^2)*tan(f*x + e)^3 + A*c^2 +
 (C*c^2 + 2*B*c*d + A*d^2)*tan(f*x + e)^2 + (B*c^2 + 2*A*c*d)*tan(f*x + e))
*(b*tan(f*x + e) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**m*(c+d*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x
+e)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\tan\left(fx+e\right)^2 + B\tan\left(fx+e\right) + A\right)\left(d\tan\left(fx+e\right) + c\right)^2\left(b\tan\left(fx+e\right) + a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e
)^2),x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(d*tan(f*x + e) + c)^2*(b
*tan(f*x + e) + a)^m, x)
```

3.167 $\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))\left(A+B\tan(e+fx)+\right.$

Optimal. Leaf size=247

$$\frac{(c-id)(A-iB-C)(a+b\tan(e+fx))^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{a+b\tan(e+fx)}{a-ib}\right)}{2f(m+1)(b+ia)}-\frac{(c+id)(A+iB-C)(a}{}$$

```
[Out] -(((a*C*d - b*(c*C + B*d)*(2 + m))*(a + b*Tan[e + f*x])^(1 + m))/(b^2*f*(1
+ m)*(2 + m))) + ((A - I*B - C)*(c - I*d)*Hypergeometric2F1[1, 1 + m, 2 + m
, (a + b*Tan[e + f*x])/(a - I*b)]*(a + b*Tan[e + f*x])^(1 + m))/(2*(I*a + b
)*f*(1 + m)) - ((A + I*B - C)*(c + I*d)*Hypergeometric2F1[1, 1 + m, 2 + m,
(a + b*Tan[e + f*x])/(a + I*b)]*(a + b*Tan[e + f*x])^(1 + m))/(2*(I*a - b)*
f*(1 + m)) + (C*d*Tan[e + f*x]*(a + b*Tan[e + f*x])^(1 + m))/(b*f*(2 + m))
```

Rubi [A] time = 0.528326, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {3637, 3630, 3539, 3537, 68}

$$\frac{(c-id)(A-iB-C)(a+b\tan(e+fx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\tan(e+fx)}{a-ib}\right)}{2f(m+1)(b+ia)}-\frac{(c+id)(A+iB-C)(a+b\tan(e+fx))^{m}}{2f(m+1)(-}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan
[e + f*x]^2),x]
```

```
[Out] -(((a*C*d - b*(c*C + B*d)*(2 + m))*(a + b*Tan[e + f*x])^(1 + m))/(b^2*f*(1
+ m)*(2 + m))) + ((A - I*B - C)*(c - I*d)*Hypergeometric2F1[1, 1 + m, 2 + m
, (a + b*Tan[e + f*x])/(a - I*b)]*(a + b*Tan[e + f*x])^(1 + m))/(2*(I*a + b
)*f*(1 + m)) - ((A + I*B - C)*(c + I*d)*Hypergeometric2F1[1, 1 + m, 2 + m,
(a + b*Tan[e + f*x])/(a + I*b)]*(a + b*Tan[e + f*x])^(1 + m))/(2*(I*a - b)*
f*(1 + m)) + (C*d*Tan[e + f*x]*(a + b*Tan[e + f*x])^(1 + m))/(b*f*(2 + m))
```

Rule 3637

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(n +
 1))/(d*f*(n + 2)), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
```

```
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx = \frac{Cd\tan(e+fx)(a+b\tan(e+\ldots}{bf(2+m)}$$

$$= -\frac{(aCd-b(cC+Bd)(2+m))(a\ldots}{b^2f(1+m)(2\ldots}$$

$$= -\frac{(aCd-b(cC+Bd)(2+m))(a\ldots}{b^2f(1+m)(2\ldots}$$

$$= -\frac{(aCd-b(cC+Bd)(2+m))(a\ldots}{b^2f(1+m)(2\ldots}$$

$$= -\frac{(aCd-b(cC+Bd)(2+m))(a\ldots}{b^2f(1+m)(2\ldots}$$

Mathematica [A] time = 2.71089, size = 202, normalized size = 0.82

$$\frac{(a+b\tan(e+fx))^{m+1}\left(-\frac{ib(m+2)(c-id)(A-iB-C)\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{a+b\tan(e+fx)}{a-ib}\right)}{(m+1)(a-ib)}+\frac{ib(m+2)(c+id)(A+iB-C)\text{Hypergeometric2}\ldots}{(m+1)(a+ib)}\ldots\right.}{2bf(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] +
 C*Tan[e + f*x]^2),x]
```

```
[Out] ((a + b*Tan[e + f*x])^(1 + m)*((-2*a*C*d + 2*b*(c*C + B*d)*(2 + m))/(b*(1 +
 m)) - (I*b*(A - I*B - C)*(c - I*d)*(2 + m)*Hypergeometric2F1[1, 1 + m, 2 +
 m, (a + b*Tan[e + f*x])/(a - I*b)])/((a - I*b)*(1 + m)) + (I*b*(A + I*B -
C)*(c + I*d)*(2 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x]
)/(a + I*b)])/((a + I*b)*(1 + m)) + 2*C*d*Tan[e + f*x]))/(2*b*f*(2 + m))
```

Maple [F] time = 0.492, size = 0, normalized size = 0.

$$\int \left(a+b\tan\left(fx+e\right)\right)^m\left(c+d\tan\left(fx+e\right)\right)\left(A+B\tan\left(fx+e\right)+C\left(\tan\left(fx+e\right)\right)^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x)
```

```
[Out] int((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cd\tan\left(fx+e\right)^{3}+(Cc+Bd)\tan\left(fx+e\right)^{2}+Ac+(Bc+Ad)\tan\left(fx+e\right)\right)\left(b\tan\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*d*tan(f*x + e)^3 + (C*c + B*d)*tan(f*x + e)^2 + A*c + (B*c + A*
d)*tan(f*x + e))*(b*tan(f*x + e) + a)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tan\left(e+fx\right)\right)^{m}\left(c+d\tan\left(e+fx\right)\right)\left(A+B\tan\left(e+fx\right)+C\tan^{2}\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**m*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)
**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**m*(c + d*tan(e + f*x))*(A + B*tan(e + f*x) +
 C*tan(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\tan\left(fx+e\right)^{2}+B\tan\left(fx+e\right)+A\right)\left(d\tan\left(fx+e\right)+c\right)\left(b\tan\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(d*tan(f*x + e) + c)*(b*t
an(f*x + e) + a)^m, x)
```

3.168 $\int (a+b\tan(e+fx))^m \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)$

Optimal. Leaf size=178

$$\frac{(A-iB-C)(a+b\tan(e+fx))^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{a+b\tan(e+fx)}{a-ib}\right)}{2f(m+1)(b+ia)}+\frac{(iA-B-iC)(a+b\tan(e+f$$

```
[Out] (C*(a + b*Tan[e + f*x])^(1 + m))/(b*f*(1 + m)) + ((A - I*B - C)*Hypergeomet
ric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a - I*b)]*(a + b*Tan[e + f*x]
)^(1 + m))/(2*(I*a + b)*f*(1 + m)) + ((I*A - B - I*C)*Hypergeometric2F1[1,
1 + m, 2 + m, (a + b*Tan[e + f*x])/(a + I*b)]*(a + b*Tan[e + f*x])^(1 + m))
/(2*(a + I*b)*f*(1 + m))
```

Rubi [A] time = 0.184451, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {3630, 3539, 3537, 68}

$$\frac{(A-iB-C)(a+b\tan(e+fx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\tan(e+fx)}{a-ib}\right)}{2f(m+1)(b+ia)}+\frac{(iA-B-iC)(a+b\tan(e+fx))^{m+1}\,{}_2F_1\left(1,m+\right.}{2f(m+1)(a+ib)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^m*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] (C*(a + b*Tan[e + f*x])^(1 + m))/(b*f*(1 + m)) + ((A - I*B - C)*Hypergeomet
ric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a - I*b)]*(a + b*Tan[e + f*x]
)^(1 + m))/(2*(I*a + b)*f*(1 + m)) + ((I*A - B - I*C)*Hypergeometric2F1[1,
1 + m, 2 + m, (a + b*Tan[e + f*x])/(a + I*b)]*(a + b*Tan[e + f*x])^(1 + m))
/(2*(a + I*b)*f*(1 + m))
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
 b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int (a+b\tan(e+fx))^m \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx = \frac{C(a+b\tan(e+fx))^{1+m}}{bf(1+m)} + \int (a+b\tan(e+fx))$$

$$= \frac{C(a+b\tan(e+fx))^{1+m}}{bf(1+m)} + \frac{1}{2}(A-iB-C)\int (1+$$

$$= \frac{C(a+b\tan(e+fx))^{1+m}}{bf(1+m)} + \frac{(iA+B-iC)\,\mathrm{Subst}\,(}{}$$

$$= \frac{C(a+b\tan(e+fx))^{1+m}}{bf(1+m)} - \frac{(iA+B-iC)\,{}_2F_1\,(1,1}{}$$

Mathematica [A] time = 0.209102, size = 135, normalized size = 0.76

$$\frac{(a+b\tan(e+fx))^{m+1}\left(-\frac{i(A-iB-C)\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{a+b\tan(e+fx)}{a-ib}\right)}{a-ib}+\frac{i(A+iB-C)\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{a+b\tan(e+}{a+ib}\right.}{a+ib}\right.}{2f(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^m*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] (((2*C)/b - (I*(A - I*B - C)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[
e + f*x])/(a - I*b)])/(a - I*b) + (I*(A + I*B - C)*Hypergeometric2F1[1, 1 +
 m, 2 + m, (a + b*Tan[e + f*x])/(a + I*b)])/(a + I*b))*(a + b*Tan[e + f*x])
^(1 + m))/(2*f*(1 + m))
```

Maple [F] time = 0.39, size = 0, normalized size = 0.

$$\int \left(a+b\tan\left(fx+e\right)\right)^m \left(A+B\tan\left(fx+e\right)+C\left(\tan\left(fx+e\right)\right)^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x)
```

```
[Out] int((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\tan\left(fx+e\right)^2+B\tan\left(fx+e\right)+A\right)\left(b\tan\left(fx+e\right)+a\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm="
maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\tan\left(fx+e\right)^2+B\tan\left(fx+e\right)+A\right)\left(b\tan\left(fx+e\right)+a\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm="
fricas")
```

```
[Out] integral((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tan\left(e+fx\right)\right)^m\left(A+B\tan\left(e+fx\right)+C\tan^2\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**m*(A+B*tan(f*x+e)+C*tan(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**m*(A + B*tan(e + f*x) + C*tan(e + f*x)**2),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\tan\left(fx+e\right)^2+B\tan\left(fx+e\right)+A\right)\left(b\tan\left(fx+e\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm="
giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m, x
)
```

3.169 $\int \frac{(a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=258

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{d(a+b\tan(e+fx))}{bc-ad}\right)}{f(m+1)\left(c^2+d^2\right)(bc-ad)}-\frac{(iA+B-iC)(a$$

```
[Out] -((I*A + B - I*C)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(
a - I*b)]*(a + b*Tan[e + f*x])^(1 + m))/(2*(a - I*b)*(c - I*d)*f*(1 + m)) -
 ((A + I*B - C)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a
+ I*b)]*(a + b*Tan[e + f*x])^(1 + m))/(2*(I*a - b)*(c + I*d)*f*(1 + m)) + (
(c^2*C - B*c*d + A*d^2)*Hypergeometric2F1[1, 1 + m, 2 + m, -((d*(a + b*Tan[
e + f*x]))/(b*c - a*d))]*(a + b*Tan[e + f*x])^(1 + m))/((b*c - a*d)*(c^2 +
d^2)*f*(1 + m))
```

Rubi [A] time = 0.482221, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3653, 3539, 3537, 68, 3634}

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{d(a+b\tan(e+fx))}{bc-ad}\right)}{f(m+1)\left(c^2+d^2\right)(bc-ad)}-\frac{(iA+B-iC)(a+b\tan(e+fx))^{m}}{2f(m+1)(a}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^m*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d
*Tan[e + f*x]),x]
```

```
[Out] -((I*A + B - I*C)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(
a - I*b)]*(a + b*Tan[e + f*x])^(1 + m))/(2*(a - I*b)*(c - I*d)*f*(1 + m)) -
 ((A + I*B - C)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a
+ I*b)]*(a + b*Tan[e + f*x])^(1 + m))/(2*(I*a - b)*(c + I*d)*f*(1 + m)) + (
(c^2*C - B*c*d + A*d^2)*Hypergeometric2F1[1, 1 + m, 2 + m, -((d*(a + b*Tan[
e + f*x]))/(b*c - a*d))]*(a + b*Tan[e + f*x])^(1 + m))/((b*c - a*d)*(c^2 +
d^2)*f*(1 + m))
```

Rule 3653

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[e + f*x])^n*(1 + Tan[e
+ f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3634

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^m \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{c+d\tan(e+fx)}\,dx = \frac{\int (a+b\tan(e+fx))^m(Ac-cC+Bd+(Bc-(A-C$$

$$= \frac{(A-iB-C)\int (1+i\tan(e+fx))(a+b\tan(e+fx)}{2(c-id)}$$

$$= \frac{\left(c^2C-Bcd+Ad^2\right)\,{}_2F_1\left(1,1+m;2+m;-\frac{d(a+b\tan(e}{bc-ad}\right.}{(bc-ad)\left(c^2+d^2\right)f(1+}$$

$$= -\frac{(iA+B-iC)\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a-ib}\right)(a}{2(a-ib)(c-id)f(1+m)}$$

Mathematica [A] time = 0.998107, size = 204, normalized size = 0.79

$$\frac{(a+b\tan(e+fx))^{m+1}\left(\frac{2\left(Ad^2-Bcd+c^2C\right)\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{d(a+b\tan(e+fx))}{ad-bc}\right)}{bc-ad}+\frac{(d-ic)(A-iB-C)\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{a+}{}\right.}{a-ib}\right.}{2f(m+1)\left(c^2+d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])^m*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(c + d*Tan[e + f*x]),x]
```

```
[Out] ((((A - I*B - C)*((-I)*c + d)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan
[e + f*x])/(a - I*b)])/(a - I*b) + ((A + I*B - C)*(I*c + d)*Hypergeometric2
F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a + I*b)])/(a + I*b) + (2*(c^2*C
- B*c*d + A*d^2)*Hypergeometric2F1[1, 1 + m, 2 + m, (d*(a + b*Tan[e + f*x])
)/(-(b*c) + a*d)])/(b*c - a*d))*(a + b*Tan[e + f*x])^(1 + m))/(2*(c^2 + d^2
)*f*(1 + m))
```

Maple [F] time = 0.543, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tan\left(fx+e\right)\right)^{m}\left(A+B\tan\left(fx+e\right)+C\left(\tan\left(fx+e\right)\right)^{2}\right)}{c+d\tan\left(fx+e\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)),x)
```

```
[Out] int((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\tan\left(fx+e\right)^{2}+B\tan\left(fx+e\right)+A\right)\left(b\tan\left(fx+e\right)+a\right)^{m}}{d\tan\left(fx+e\right)+c}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
)),x, algorithm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m/(d
*tan(f*x + e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\tan\left(fx+e\right)^{2}+B\tan\left(fx+e\right)+A\right)\left(b\tan\left(fx+e\right)+a\right)^{m}}{d\tan\left(fx+e\right)+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
)),x, algorithm="fricas")
```

```
[Out] integral((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m/(d*
tan(f*x + e) + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**m*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x
+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\tan\left(fx+e\right)^{2}+B\tan\left(fx+e\right)+A\right)\left(b\tan\left(fx+e\right)+a\right)^{m}}{d\tan\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
)),x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m/(d
*tan(f*x + e) + c), x)
```

3.170 $\int \frac{(a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^2}\,dx$

Optimal. Leaf size=403

$$-\frac{(a+b\tan(e+fx))^{m+1}\left(ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)-b\left(Ad^2\left(c^2(2-m)-d^2m\right)-Bcd\left(c^2(1-m)-d^2(m+1)\right)+\right.\right.}{f(m+1)\left(c^2+d^2\right)^2(bc-ad)^2}$$

```
[Out] ((A - I*B - C)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a -
 I*b)]*(a + b*Tan[e + f*x])^(1 + m))/(2*(I*a + b)*(c - I*d)^2*f*(1 + m)) +
((I*A - B - I*C)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a
 + I*b)]*(a + b*Tan[e + f*x])^(1 + m))/(2*(a + I*b)*(c + I*d)^2*f*(1 + m))
- ((a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)) - b*(A*d^2*(c^2*(2 - m) - d^2*m)
- B*c*d*(c^2*(1 - m) - d^2*(1 + m)) - c^2*C*(c^2*m + d^2*(2 + m))))*Hyperge
ometric2F1[1, 1 + m, 2 + m, -((d*(a + b*Tan[e + f*x]))/(b*c - a*d))]*(a + b
*Tan[e + f*x])^(1 + m))/((b*c - a*d)^2*(c^2 + d^2)^2*f*(1 + m)) + ((c^2*C -
 B*c*d + A*d^2)*(a + b*Tan[e + f*x])^(1 + m))/((b*c - a*d)*(c^2 + d^2)*f*(c
 + d*Tan[e + f*x]))
```

Rubi [A] time = 1.21502, antiderivative size = 402, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3649, 3653, 3539, 3537, 68, 3634}

$$-\frac{(a+b\tan(e+fx))^{m+1}\left(ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)-b\left(Ac^2d^2(2-m)-Ad^4m-B\left(c^3d(1-m)-cd^3(m+1)\right)-c\right.\right.}{f(m+1)\left(c^2+d^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^m*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d
*Tan[e + f*x])^2,x]
```

```
[Out] ((A - I*B - C)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a -
 I*b)]*(a + b*Tan[e + f*x])^(1 + m))/(2*(I*a + b)*(c - I*d)^2*f*(1 + m)) +
((I*A - B - I*C)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a
 + I*b)]*(a + b*Tan[e + f*x])^(1 + m))/(2*(a + I*b)*(c + I*d)^2*f*(1 + m))
- ((a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)) - b*(A*c^2*d^2*(2 - m) - c^4*C*m
- A*d^4*m - c^2*C*d^2*(2 + m) - B*(c^3*d*(1 - m) - c*d^3*(1 + m))))*Hyperge
ometric2F1[1, 1 + m, 2 + m, -((d*(a + b*Tan[e + f*x]))/(b*c - a*d))]*(a + b
*Tan[e + f*x])^(1 + m))/((b*c - a*d)^2*(c^2 + d^2)^2*f*(1 + m)) + ((c^2*C -
 B*c*d + A*d^2)*(a + b*Tan[e + f*x])^(1 + m))/((b*c - a*d)*(c^2 + d^2)*f*(c
 + d*Tan[e + f*x]))
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
 b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
```

```
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3653

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[e + f*x])^n*(1 + Tan[e
+ f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3634

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^2}dx = \frac{\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{1+m}}{(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))} + \int \cdots$$

$$= \frac{\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{1+m}}{(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))} + \int(\cdots$$

$$= \frac{\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{1+m}}{(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))} + \frac{(A \cdots}{\cdots}$$

$$= -\frac{\left(ad^2\left(2c(A-C)d-B\left(c^2-d^2\right)\right)-b\left(Ac^2d^2(2-\cdots\right.\right.}{\cdots}$$

$$= -\frac{(iA+B-iC)\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a-ib}\right)\cdots}{2(a-ib)(c-id)^2f(1+m\cdots}$$

Mathematica [A] time = 6.18405, size = 563, normalized size = 1.4

$$-\frac{\left(Ad^2-c(Bd-cC)\right)(a+b\tan(e+fx))^{m+1}}{f\left(c^2+d^2\right)(ad-bc)(c+d\tan(e+fx))} - \frac{-\frac{(a+b\tan(e+fx))^{m+1}\left(d^2\left((cC-Bd)(ad-bc(m+1))-A\left(acd-b\left(c^2-d^2m\right)\right)\right)-cd(bc-ad)(Bc-d(A-\cdots\right.}{f(m+1)\left(c^2+d^2\right)(a\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])^m*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(c + d*Tan[e + f*x])^2,x]
```

```
[Out] -(((A*d^2 - c*(-(c*C) + B*d))*(a + b*Tan[e + f*x])^(1 + m))/((-(b*c) + a*d)
*(c^2 + d^2)*f*(c + d*Tan[e + f*x]))) - (-(((-(c*d*(b*c - a*d)*(B*c - (A -
C)*d)) - b*c^2*(c^2*C - B*c*d + A*d^2)*m + d^2*((c*C - B*d)*(a*d - b*c*(1 +
 m)) - A*(a*c*d - b*(c^2 - d^2*m))))*Hypergeometric2F1[1, 1 + m, 2 + m, (d*
(a + b*Tan[e + f*x]))/(-(b*c) + a*d)]*(a + b*Tan[e + f*x])^(1 + m))/((-(b*c
) + a*d)*(c^2 + d^2)*f*(1 + m))) + (((I/2)*(-((b*c - a*d)*(c^2*C - 2*B*c*d
- C*d^2 - A*(c^2 - d^2))) - I*(b*c - a*d)*(2*c*(A - C)*d - B*(c^2 - d^2)))*
Hypergeometric2F1[1, 1 + m, 2 + m, ((-I)*a - I*b*Tan[e + f*x])/((-I)*a + b)
]*(a + b*Tan[e + f*x])^(1 + m))/((a + I*b)*f*(1 + m)) - ((I/2)*(-((b*c - a*
d)*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2))) + I*(b*c - a*d)*(2*c*(A - C)*
d - B*(c^2 - d^2)))*Hypergeometric2F1[1, 1 + m, 2 + m, -((I*a + I*b*Tan[e +
 f*x])/((-I)*a - b))]*(a + b*Tan[e + f*x])^(1 + m))/((a - I*b)*f*(1 + m)))/
(c^2 + d^2))/((-(b*c) + a*d)*(c^2 + d^2))
```

Maple [F] time = 0.641, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tan\left(fx+e\right)\right)^m\left(A+B\tan\left(fx+e\right)+C\left(\tan\left(fx+e\right)\right)^2\right)}{\left(c+d\tan\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^2,x
)
```

```
[Out] int((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^2,x
)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\tan\left(fx+e\right)^2+B\tan\left(fx+e\right)+A\right)\left(b\tan\left(fx+e\right)+a\right)^m}{d^2\tan\left(fx+e\right)^2+2\,cd\tan\left(fx+e\right)+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^2,x, algorithm="fricas")
```

```
[Out] integral((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m/(d^
2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**m*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x
+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\tan\left(fx+e\right)^2+B\tan\left(fx+e\right)+A\right)\left(b\tan\left(fx+e\right)+a\right)^m}{\left(d\tan\left(fx+e\right)+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^2,x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m/(d
*tan(f*x + e) + c)^2, x)
```

3.171 $\int \frac{(a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^3}\,dx$

Optimal. Leaf size=702

$$\frac{(a+b\tan(e+fx))^{m+1}\left(2a^2d^3\left(d(A-C)\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)-2abd^2\left(2cd(A-C)\left(c^2(3-m)-d^2(m+1)\right)+B\left(6c\right.\right.\right.}{}$$

```
[Out] ((A - I*B - C)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a -
 I*b)]*(a + b*Tan[e + f*x])^(1 + m))/(2*(I*a + b)*(c - I*d)^3*f*(1 + m)) +
((A + I*B - C)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a +
 I*b)]*(a + b*Tan[e + f*x])^(1 + m))/(2*(a + I*b)*(I*c - d)^3*f*(1 + m)) +
((2*a^2*d^3*((A - C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)) - 2*a*b*d^2*(B*(6
*c^2*d^2 - c^4*(2 - m) - d^4*m) + 2*c*(A - C)*d*(c^2*(3 - m) - d^2*(1 + m))
) - b^2*(A*d^2*(d^4*(1 - m)*m + 2*c^2*d^2*(1 + 3*m - m^2) - c^4*(6 - 5*m +
m^2)) + B*c*d*(d^4*m*(1 + m) - 2*c^2*d^2*(3 + m - m^2) + c^4*(2 - 3*m + m^2
)) + c^2*C*(c^4*(1 - m)*m + 2*c^2*d^2*(3 - m - m^2) - d^4*(2 + 3*m + m^2)))
)*Hypergeometric2F1[1, 1 + m, 2 + m, -((d*(a + b*Tan[e + f*x]))/(b*c - a*d)
)]*(a + b*Tan[e + f*x])^(1 + m))/(2*(b*c - a*d)^3*(c^2 + d^2)^3*f*(1 + m))
+ ((c^2*C - B*c*d + A*d^2)*(a + b*Tan[e + f*x])^(1 + m))/(2*(b*c - a*d)*(c^
2 + d^2)*f*(c + d*Tan[e + f*x])^2) - ((2*a*d^2*(2*c*(A - C)*d - B*(c^2 - d^
2)) - b*(c^4*C*(1 - m) + A*d^4*(1 - m) - B*c^3*d*(3 - m) + B*c*d^3*(1 + m)
+ c^2*d^2*(A*(5 - m) - C*(3 + m))))*(a + b*Tan[e + f*x])^(1 + m))/(2*(b*c -
 a*d)^2*(c^2 + d^2)^2*f*(c + d*Tan[e + f*x]))
```

Rubi [A] time = 2.93764, antiderivative size = 702, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3649, 3653, 3539, 3537, 68, 3634}

$$\frac{(a+b\tan(e+fx))^{m+1}\left(2a^2d^3\left(d(A-C)\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)-2abd^2\left(2cd(A-C)\left(c^2(3-m)-d^2(m+1)\right)+B\left(6c\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^m*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d
*Tan[e + f*x])^3,x]
```

```
[Out] ((A - I*B - C)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a -
 I*b)]*(a + b*Tan[e + f*x])^(1 + m))/(2*(I*a + b)*(c - I*d)^3*f*(1 + m)) +
((A + I*B - C)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a +
 I*b)]*(a + b*Tan[e + f*x])^(1 + m))/(2*(a + I*b)*(I*c - d)^3*f*(1 + m)) +
((2*a^2*d^3*((A - C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)) - 2*a*b*d^2*(B*(6
*c^2*d^2 - c^4*(2 - m) - d^4*m) + 2*c*(A - C)*d*(c^2*(3 - m) - d^2*(1 + m))
) - b^2*(A*d^2*(d^4*(1 - m)*m + 2*c^2*d^2*(1 + 3*m - m^2) - c^4*(6 - 5*m +
m^2)) + B*(c*d^5*m*(1 + m) - 2*c^3*d^3*(3 + m - m^2) + c^5*d*(2 - 3*m + m^2
)) + c^2*C*(c^4*(1 - m)*m + 2*c^2*d^2*(3 - m - m^2) - d^4*(2 + 3*m + m^2)))
)*Hypergeometric2F1[1, 1 + m, 2 + m, -((d*(a + b*Tan[e + f*x]))/(b*c - a*d)
)]*(a + b*Tan[e + f*x])^(1 + m))/(2*(b*c - a*d)^3*(c^2 + d^2)^3*f*(1 + m))
+ ((c^2*C - B*c*d + A*d^2)*(a + b*Tan[e + f*x])^(1 + m))/(2*(b*c - a*d)*(c^
2 + d^2)*f*(c + d*Tan[e + f*x])^2) - ((2*a*d^2*(2*c*(A - C)*d - B*(c^2 - d^
2)) - b*(c^4*C*(1 - m) + A*d^4*(1 - m) - B*c^3*d*(3 - m) + B*c*d^3*(1 + m)
+ c^2*d^2*(A*(5 - m) - C*(3 + m))))*(a + b*Tan[e + f*x])^(1 + m))/(2*(b*c -
 a*d)^2*(c^2 + d^2)^2*f*(c + d*Tan[e + f*x]))
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
 b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3653

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[e + f*x])^n*(1 + Tan[e
+ f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3634

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^m \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^3}\,dx = \frac{\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{1+m}}{2(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))^2} + \int \frac{(a+b\cdots}{\cdots}$$

$$= \frac{\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{1+m}}{2(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))^2} - \frac{\left(2ad^2\left(\cdots\right.\right.}{\cdots}$$

$$= \frac{\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{1+m}}{2(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))^2} - \frac{\left(2ad^2\left(\cdots\right.\right.}{\cdots}$$

$$= \frac{\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{1+m}}{2(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))^2} - \frac{\left(2ad^2\left(\cdots\right.\right.}{\cdots}$$

$$= \frac{\left(2a^2d^3\left((A-C)d\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)-2abd\cdots\right.}{\cdots}$$

$$= -\frac{(A-iB-C)\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a-ib}\right)(a\cdots}{2(a-ib)(ic+d)^3f(1+m)}$$

Mathematica [B] time = 6.23456, size = 2238, normalized size = 3.19

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])^m*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(c + d*Tan[e + f*x])^3,x]
```

```
[Out] -((A*d^2 - c*(-(c*C) + B*d))*(a + b*Tan[e + f*x])^(1 + m))/(2*(-(b*c) + a*d
)*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^2) - (-(((-(c*(2*d*(b*c - a*d)*(B*c -
(A - C)*d) - b*c*(c^2*C - B*c*d + A*d^2)*(1 - m))) + d^2*(A*(2*c*(b*c - a*d
) + b*d^2*(1 - m)) + (c*C - B*d)*(2*a*d - b*c*(1 + m))))*(a + b*Tan[e + f*x
])^(1 + m))/((-(b*c) + a*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*x]))) - (-(((-(c
*d*(-(b*c) + a*d)*(-2*c*(b*c - a*d)*(B*c - (A - C)*d) - b*d*(c^2*C - B*c*d
+ A*d^2)*(1 - m) + d*(A*(2*c*(b*c - a*d) + b*d^2*(1 - m)) + (c*C - B*d)*(2*
a*d - b*c*(1 + m))))) - b*c^2*m*(-(c*(2*d*(b*c - a*d)*(B*c - (A - C)*d) - b
*c*(c^2*C - B*c*d + A*d^2)*(1 - m))) + d^2*(A*(2*c*(b*c - a*d) + b*d^2*(1 -
 m)) + (c*C - B*d)*(2*a*d - b*c*(1 + m)))) + d^2*((2*d*(b*c - a*d)*(B*c - (
A - C)*d) - b*c*(c^2*C - B*c*d + A*d^2)*(1 - m))*(-(a*d) + b*c*(1 + m)) + (
-(c*(-(b*c) + a*d)) - b*d^2*m)*(A*(2*c*(b*c - a*d) + b*d^2*(1 - m)) + (c*C
- B*d)*(2*a*d - b*c*(1 + m)))))*Hypergeometric2F1[1, 1 + m, 2 + m, (d*(a +
b*Tan[e + f*x]))/(-(b*c) + a*d)]*(a + b*Tan[e + f*x])^(1 + m))/((-(b*c) + a
*d)*(c^2 + d^2)*f*(1 + m))) + (((I/2)*(d*(-(b*c) + a*d)*(-2*c*(b*c - a*d)*(
B*c - (A - C)*d) - b*d*(c^2*C - B*c*d + A*d^2)*(1 - m) + d*(A*(2*c*(b*c - a
*d) + b*d^2*(1 - m)) + (c*C - B*d)*(2*a*d - b*c*(1 + m)))) + c*((2*d*(b*c -
 a*d)*(B*c - (A - C)*d) - b*c*(c^2*C - B*c*d + A*d^2)*(1 - m))*(-(a*d) + b*
c*(1 + m)) + (-(c*(-(b*c) + a*d)) - b*d^2*m)*(A*(2*c*(b*c - a*d) + b*d^2*(1
 - m)) + (c*C - B*d)*(2*a*d - b*c*(1 + m))) + b*m*(-(c*(2*d*(b*c - a*d)*(B*
c - (A - C)*d) - b*c*(c^2*C - B*c*d + A*d^2)*(1 - m))) + d^2*(A*(2*c*(b*c -
 a*d) + b*d^2*(1 - m)) + (c*C - B*d)*(2*a*d - b*c*(1 + m))))) + I*(c*(-(b*c
) + a*d)*(-2*c*(b*c - a*d)*(B*c - (A - C)*d) - b*d*(c^2*C - B*c*d + A*d^2)*
(1 - m) + d*(A*(2*c*(b*c - a*d) + b*d^2*(1 - m)) + (c*C - B*d)*(2*a*d - b*c
*(1 + m)))) - d*((2*d*(b*c - a*d)*(B*c - (A - C)*d) - b*c*(c^2*C - B*c*d +
A*d^2)*(1 - m))*(-(a*d) + b*c*(1 + m)) + (-(c*(-(b*c) + a*d)) - b*d^2*m)*(A
```

```
*(2*c*(b*c - a*d) + b*d^2*(1 - m)) + (c*C - B*d)*(2*a*d - b*c*(1 + m))) + b
*m*(-(c*(2*d*(b*c - a*d)*(B*c - (A - C)*d) - b*c*(c^2*C - B*c*d + A*d^2)*(1
 - m))) + d^2*(A*(2*c*(b*c - a*d) + b*d^2*(1 - m)) + (c*C - B*d)*(2*a*d - b
*c*(1 + m)))))))*Hypergeometric2F1[1, 1 + m, 2 + m, ((-I)*a - I*b*Tan[e + f
*x])/((-I)*a + b)]*(a + b*Tan[e + f*x])^(1 + m))/((a + I*b)*f*(1 + m)) - ((
I/2)*(d*(-(b*c) + a*d)*(-2*c*(b*c - a*d)*(B*c - (A - C)*d) - b*d*(c^2*C - B
*c*d + A*d^2)*(1 - m) + d*(A*(2*c*(b*c - a*d) + b*d^2*(1 - m)) + (c*C - B*d
)*(2*a*d - b*c*(1 + m)))) + c*((2*d*(b*c - a*d)*(B*c - (A - C)*d) - b*c*(c^
2*C - B*c*d + A*d^2)*(1 - m))*(-(a*d) + b*c*(1 + m)) + (-(c*(-(b*c) + a*d))
 - b*d^2*m)*(A*(2*c*(b*c - a*d) + b*d^2*(1 - m)) + (c*C - B*d)*(2*a*d - b*c
*(1 + m))) + b*m*(-(c*(2*d*(b*c - a*d)*(B*c - (A - C)*d) - b*c*(c^2*C - B*c
*d + A*d^2)*(1 - m))) + d^2*(A*(2*c*(b*c - a*d) + b*d^2*(1 - m)) + (c*C - B
*d)*(2*a*d - b*c*(1 + m))))) - I*(c*(-(b*c) + a*d)*(-2*c*(b*c - a*d)*(B*c -
 (A - C)*d) - b*d*(c^2*C - B*c*d + A*d^2)*(1 - m) + d*(A*(2*c*(b*c - a*d) +
 b*d^2*(1 - m)) + (c*C - B*d)*(2*a*d - b*c*(1 + m)))) - d*((2*d*(b*c - a*d)
*(B*c - (A - C)*d) - b*c*(c^2*C - B*c*d + A*d^2)*(1 - m))*(-(a*d) + b*c*(1
+ m)) + (-(c*(-(b*c) + a*d)) - b*d^2*m)*(A*(2*c*(b*c - a*d) + b*d^2*(1 - m)
) + (c*C - B*d)*(2*a*d - b*c*(1 + m))) + b*m*(-(c*(2*d*(b*c - a*d)*(B*c - (
A - C)*d) - b*c*(c^2*C - B*c*d + A*d^2)*(1 - m))) + d^2*(A*(2*c*(b*c - a*d)
 + b*d^2*(1 - m)) + (c*C - B*d)*(2*a*d - b*c*(1 + m)))))))*Hypergeometric2F
1[1, 1 + m, 2 + m, -((I*a + I*b*Tan[e + f*x])/((-I)*a - b))]*(a + b*Tan[e +
 f*x])^(1 + m))/((a - I*b)*f*(1 + m)))/(c^2 + d^2))/((-(b*c) + a*d)*(c^2 +
d^2)))/(2*(-(b*c) + a*d)*(c^2 + d^2))
```

Maple [F] time = 0.805, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tan\left(fx+e\right)\right)^{m}\left(A+B\tan\left(fx+e\right)+C\left(\tan\left(fx+e\right)\right)^{2}\right)}{\left(c+d\tan\left(fx+e\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^3,x
)
```

```
[Out] int((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^3,x
)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\tan\left(fx+e\right)^{2}+B\tan\left(fx+e\right)+A\right)\left(b\tan\left(fx+e\right)+a\right)^{m}}{d^{3}\tan\left(fx+e\right)^{3}+3\,cd^{2}\tan\left(fx+e\right)^{2}+3\,c^{2}d\tan\left(fx+e\right)+c^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^3,x, algorithm="fricas")
```

```
[Out] integral((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m/(d^
3*tan(f*x + e)^3 + 3*c*d^2*tan(f*x + e)^2 + 3*c^2*d*tan(f*x + e) + c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**m*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x
+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C \tan\left(fx+e\right)^{2} + B \tan\left(fx+e\right) + A\right)\left(b \tan\left(fx+e\right) + a\right)^{m}}{\left(d \tan\left(fx+e\right) + c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^3,x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m/(d
*tan(f*x + e) + c)^3, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
                print("both result and optimal complex");
             fi;
             #both result and optimal complex
             if leaf_count_result<=2*leaf_count_optimal then
                return "A";
             else
                return "B";
             end if
          else #result contains complex but optimal is not
             if debug then
                print("result contains complex but optimal is not");
             fi;
             return "C";
          end if
       else # result do not contain complex
            # this assumes optimal do not as well
            if debug then
                print("result do not contain complex, this assumes optimal do
    not as well");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
               if debug then
                  print("leaf_count_result<=2*leaf_count_optimal");
               fi;
               return "A";
            else
               if debug then
                  print("leaf_count_result>2*leaf_count_optimal");
               fi;
               return "B";
            end if
       end if
  else #ExpnType(result) > ExpnType(optimal)
       if debug then
          print("ExpnType(result) > ExpnType(optimal)");
       fi;
       return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                         #since this estimate of leaf count is bit lower than
```

```
                    #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```